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A general formalism for acoustic 2-port source with multi-propagating modes in duct

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Abstract: The source characterization of fluid machines is important for calculating the acoustic field generated in duct systems and for the analysis of source-load interaction effects. Several investigators used one- or two-port model to describe fluid machines as acoustic sources and the study is only limited to the plane wave region. However, for some applications it is necessary to characterize them at frequencies beyond this region since higher order modes can propagate along the ducts coupled to the machine. In the present work, a measurement method is suggested for a linear time-invariant source with two active openings connected to a duct where multi-propagating modes exist.
Key words: source characterization; duct system; two port; modes.

1 INTRODUCTION

Acoustic source data for fluid machines such as pumps, fans etc, are of importance for calculating the acoustic field generated in duct systems and for the analysis of source-load interaction effects. Methods for analysis and measuring the relevant source data are therefore of great interest.

The fluid machine, where the external acoustic load changes only at one opening, is modeled as acoustic one-port source[1] that can be completely described by a source strength and a source reflection coefficient. When there is an acoustic coupling between the outlet and inlet, and the conditions on both sides of the fluid machine can be changed, it must be modeled as a two-port source[2-6] that is usually described by the scattering matrix and the source strength matrix. Both of the one and twoport fluid machines are defined as linear time-invariant physical systems. In the previous work of several investigators, it was usually assumed that the machines work in plane wave region, until J. Lavrentjev and M. Åbom provided the measurement method for the source with one opening connected to a duct where N modes propagate, and they named the system as an acoustic Nport source[7, 8].

However, a linear time-invariant source with two openings connected to a duct where multi-propagating modes exist, has more general applications and is defined here as the acoustic 2-port source with N modes. In this work, the relationship among all of the propagating modes in the outlet and inlet must be considered. It is the extension of the source reflection matrix for one opening described in the work of Lavrentjev and M. Åbom[8]. This source reflection matrix for one opening is substituted here by a scattering matrix to describe the acoustic coupling between every mode in two openings. The aim is therefore to propose a measurem-

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ent method for determining the source data for the acoustic 2-port source with N modes.

2 GENERAL FROMULATION

2.1 Source description

An acoustic two-port source shown in Fig.1 can be described by a system of equations that establish the relationship between its output and input openings.

The state of the flow machine can be completely described by 2N source state variables, one N for the output and another N for the input. The pressure amplitudes of acoustic modes p^S_{a+} and p^S_{b+}, are chosen as the source state variables, and can be written (in the frequency domain) as:

$$\begin{bmatrix} p_{a+} \\ p_{b+} \end{bmatrix} = S \begin{bmatrix} p_{a-} \\ p_{b-} \end{bmatrix} + \begin{bmatrix} p^S_{a+} \\ p^S_{b+} \end{bmatrix} \tag{1}$$

or $p_+ = Sp_- + p^S_{b+}$ (2)

where $p_{a+}, p_{a-}, p_{b+}, p_{b-}$ are $[N\times 1]$ state vectors of output and input openings that contain the left- and right-going acoustic pressure wave amplitudes for N modes, S is the $[2N\times 2N]$ scattering matrix to describe the acoustic coupling between every mode in two openings, p^S_{a+} and p^S_{b+}, are $[N\times 1]$ source pressure vectors.

In order to characterize the two-port acoustic source with N propagating modes in the duct, the unknown matrices S and p^S_+ must be determined.

2.2 Determination of the scattering matrix

For determination of the scattering matrix S, the state vectors p_+ and p_- should be obtained first. Two external sources (e.g., loudspeakers) which are uncorrelated to the source under test are located to the left and right of the fluid machine. They generate much higher sound pressures to suppress the primary source, so p^S_+ can be eliminated from Eq. (2).

For opening a, the acoustic pressures are measured at N points in cross-section 1a and N points in cross-section 2a. The result of such measurements can be formulated as:

$$\begin{bmatrix} p_{a1} \\ p_{a2} \end{bmatrix} = \begin{bmatrix} M_{a+} & M_{a-} \\ M_{a+}T_+ & M_{a-}T_- \end{bmatrix} \begin{bmatrix} p_{a1+} \\ p_{a1-} \end{bmatrix} \tag{3a}$$

where p_{a1} and p_{a2} are $[N\times 1]$ measured vectors, M_{a+} and M_{a-} are modal $[N\times N]$ matrices containing eigen functions for the N modes[8], and transfer matrices $T_\pm$ contain transfer functions between cross-sections 1a and 2a. For a rigid walled, the functions are given by $(T_\pm)_{mn} = \exp(\mp ik_{m\pm}z_s)\boldsymbol{\delta}_{mn}$, where $\boldsymbol{\delta}_{mn}$ is Kronecker's delta, $k_{m\pm}$ are the axial wave numbers corresponding to modes propagating in the positive/ negative z-direction and z_s is the separation between the two cross-sections.

To eliminate the flow noise from the microphone signals and non-suppressed background noise from the source, transfer functions are taken between the pressure signals and a reference signal correlated with the sound from the external source. In this sense the electronic signale, driving the external loudspeaker, is a convenient choice. Based on the idea, Eq. (3a) is rewritten

$$\begin{bmatrix} H_{a1} \\ H_{a2} \end{bmatrix} = \begin{bmatrix} M_{a+} & M_{a-} \\ M_{a+}T_+ & M_{a-}T_- \end{bmatrix} \begin{bmatrix} H_{a1+} \\ H_{a1-} \end{bmatrix} \tag{4a}$$

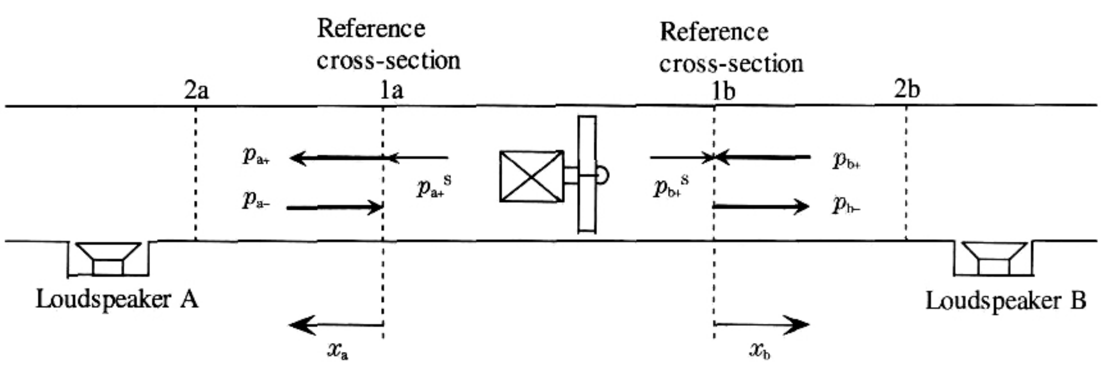

Fig.1 An acoustic two-port source with N propagating modes in the duct

or

$$H_a=M_aH_{a\pm} \quad (5a)$$

where the transfer function is equivalent to dividing the pressures in Eq. (3a) by e. To determine the matrix S with size[2N×2N], at least 2N different acoustical states must be tested, which can be created by locating the external source at different positions along the axis and at different angles over the perimeter of the duct. From Eq. (5a), for 2N different states (I, the left source "on" and the right one "off"; II, the right source "on" and the left one "off". N different states for I, and N for II)

$$[H_{1a}^{I} \cdots H_{Na}^{I} H_{1a}^{II} \cdots H_{Na}^{II}]= M_a[(H_\pm)_{1a}^{I} \cdots (H_\pm)_{Na}^{I}(H_\pm)_{1a}^{II} \cdots (H_\pm)_{Na}^{II}] \quad (6a)$$

in short

$$[H_a^I H_a^{II}]=M_a[H_{\pm a}^I H_{\pm a}^{II}] \quad (7a)$$

or

$$H_a^{\Sigma}=M_aH_{\pm a}^{\Sigma} \quad (8a)$$

where H_a^{Σ} and $H_{\pm a}^{\Sigma}$ are [2N×2N] matrices. From Eq. (8a), $H_{\pm a}^{\Sigma}$ can be solved and then H_{+a}^{Σ}, H_{-a}^{Σ} are obtained. Thus the load reflection matrix is determined by

$$R_a=H_{a-}^{II}/H_{a+}^{II} \quad (9a)$$

In the same way, acoustic pressures are measured at N measurement points in cross-section 1b and N points in cross-section 2b for opening b, and 2N different acoustical states are the same as that of opening a. The analogy equations are as follows:

$$\begin{bmatrix} p_{b1} \\ p_{b2} \end{bmatrix}=\begin{bmatrix} M_{b+} & M_{b-} \\ M_{b+}T_+ & M_{b-}T_- \end{bmatrix}\begin{bmatrix} p_{b1+} \\ p_{b1-} \end{bmatrix} \quad (3b)$$

$$\begin{bmatrix} H_{b1} \\ H_{b2} \end{bmatrix}=\begin{bmatrix} M_{b+} & M_{b-} \\ M_{b+}T_+ & M_{b-}T_- \end{bmatrix}\begin{bmatrix} H_{b1+} \\ H_{b1-} \end{bmatrix} \quad (4b)$$

$$H_b=M_bH_{b\pm} \quad (5b)$$

$$[H_{1b}^{I} \cdots H_{Nb}^{I} H_{1b}^{II} \cdots H_{Nb}^{II}]= M_b[(H_\pm)_{1b}^{I} \cdots (H_\pm)_{Nb}^{I}(H_\pm)_{1b}^{II} \cdots (H_\pm)_{Nb}^{II}] \quad (6b)$$

$$[H_b^I H_b^{II}]=M_b[H_{\pm b}^I H_{\pm b}^{II}] \quad (7b)$$

$$H_b^{\Sigma}=M_bH_{\pm b}^{\Sigma} \quad (8b)$$

$$R_b=H_{b-}^{II}/H_{b+}^{II} \quad (9b)$$

Matrices $H_+^{\Sigma}=\begin{bmatrix} H_{a+}^{\Sigma} \\ H_{b+}^{\Sigma} \end{bmatrix}$ and $H_-^{\Sigma}=\begin{bmatrix} H_{a-}^{\Sigma} \\ H_{b-}^{\Sigma} \end{bmatrix}$ are defined, so the scattering matrix is formally written as

$$S=H_-^{\Sigma}(H_+^{\Sigma})^{-1} \quad (10)$$

In order to reduce the influence of measurement errors in the results, it is useful to use more than 2N test states and obtain an over determined problem.

2.3 Determination of the source strength vector

After knowing S, the source pressure p_+^S is found from Eq. (2) when external sources are turned off. Defining the reflection matrix $R=\begin{bmatrix} R_a & 0 \\ 0 & R_b \end{bmatrix}$, source pressure vectors can be expressed in measurable quantities obtained at measurement points in reference cross-sections 1a and 1b

$$\begin{bmatrix} p_{a+}^S \\ p_{b+}^S \end{bmatrix}=C\begin{bmatrix} p_{a1} \\ p_{b1} \end{bmatrix} \quad (11)$$

where $C=(E-SR)\begin{bmatrix} M_{a+}M_{a-}R_a & 0 \\ 0 & M_{b+}M_{b-}R_b \end{bmatrix}^{-1}$ and E is the unit matrix.

To get a formulation which is valid for both random and periodic types of signals, the source-spectrum matrix describing the source strength is introduced

$$G^s=(Cp)(Cp)^C=C(pp^C)C^C= C\begin{bmatrix} G_{a1a1} & \cdots & G_{ana1} & G_{b1a1} & \cdots & G_{bna1} \\ \vdots & \ddots & \vdots & \vdots & & \\ G_{a1an} & \cdots & G_{anal} & G_{b1an} & \cdots & \vdots \\ G_{a1b1} & \cdots & G_{anb1} & G_{b1b1} & \cdots & \vdots \\ \vdots & & \vdots & \vdots & \ddots & \\ G_{a1bn} & & \cdots & \cdots & & G_{anbn} \end{bmatrix} C^C \quad (12)$$

where the superscript "c" denotes a transposed and complex conjugated quantity. The pressure cross-spectrum matrix $G=(pp^C)$ is a [2N×2N] matrix and the values of its elements can be measured directly from N microphones located in crosssection 1a and N microphones in cross-section 1b.

3 EXAMPLE

In order to demonstrate the use of the measurement method for the two-port source with multi-propagating modes, one case is simulated by BEM software to obtain the measured data for the following calculations.

3.1　Description of the model

The length of the duct model is 10m, and the dimension of the square cross-section is 0.2m. It can been seen in Fig.1 that the measurement points located at four positions (i.e. dotted cross-section lines 2a, 1a, 1b and 2b shown in the figure) are 2.95m, 3m (for opening a) and 7m, 7.05m (for opening b) distance away from the left exit. Fig.2 shows the volume of the structure, which are meshed by unit $0.04\times0.04\times0.04m^3$. It means that the dimensions and the length of the duct are divided into 5 segments and 250 segments respectively. Vibra-tions of the units at corresponding positions in the duct are taken as the primary source and the external sources A and B. Different test states are created by imposing displacements of the vibration on different units.

3.2　Modal decomposition for rectangular cross-section duct

J. Lavrentjev and M. Åbom[8] provided the modal decomposition for circular cross-section duct. However, it is necessary to have modal decomposition of rectangular duct for this example, as well as for more applications. The acoustic pressure field in a uniform straight rectangular duct can be generally written as (in the frequency domain)

$$p(x,y,x)=\sum_m\sum_n[p_{mn+}(z)\cos k_{m+}x\cos k_{n+}y+p_{mn-}(z)\cos k_{m-}x\cos k_{n-}y] \tag{13}$$

where z is a co-ordinate along the duct axis.

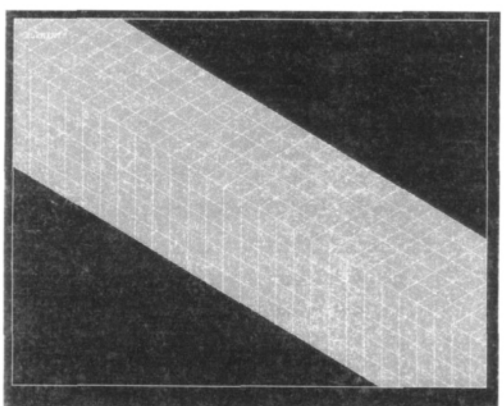

Fig.2　Surfaces are meshed by unit $0.04\times0.04m^2$

When N points in the sound field are sampled, the equation for one of the points will be

$$p_n=\sum_m\sum_n[p_{mn+}(z)\cos k_{m+}x_n\cos k_{n+}y_n+p_{mn-}(z)\cos k_{m-}x_n\cos k_{n-}y_n] \tag{14}$$

In matrix notation, for all of the sampled points,

$$p=M_+p_++M_-p_- \tag{15}$$

where the pressures are written as $[N\times1]$ vectors and M_+, M_- are $[N\times N]$ modal matrices given by $(M_\pm)_{mn}=\cos k_{m+}x_n\cos k_{n+}y_n$.

3.3　Measurement and result

The frequency chosen for the simulation is 1360Hz, so the modes propagate in the duct are (0,0), (0,1), (1,0) and (1,1). In that way, 4 points should be measured in every cross-section, and their positions in X-Y plane are shown in Fig.3.

In the first step, a vibrational displacement is imposed to the external source A. After running the BEM software, the sound pressures on measured points are obtained directly from the result table. Change the position of the external source for different acoustical states (marked by I) until four groups of the result data are read. The same operation is done for external source B (marked by II).

When eight groups of sound pressures for eight acoustical states are obtained, calculations are ready to be carried. In this simulation, vibrations of external sources are driven by imposing vibrational displacements to planes regarded as sources, rather than electric signals driving loudspeakers, so in equations below, the variable will be the sound pressure, instead of H=p/e. In fact, the receiver is not microphone in this simulation, so no flow noise will affect the result, and then it is not necessary to apply H as the variable.

$$[p_a^{I}p_a^{II}]=M_a[p_{\pm a}^{I}p_{\pm a}^{II}] \tag{16a}$$

$$[p_b^{I}p_b^{II}]=M_b[p_{\pm b}^{I}p_{\pm b}^{II}] \tag{16b}$$

$p_{\pm a}^{I}$, $p_{\pm a}^{II}$ and $p_{\pm b}^{I}$, $p_{\pm b}^{II}$ can be obtained by solving Eqs. (16a), (16b), and the load reflection matrices for two openings are $R_a=p_{a-}^{II}/p_{a+}^{II}$ and

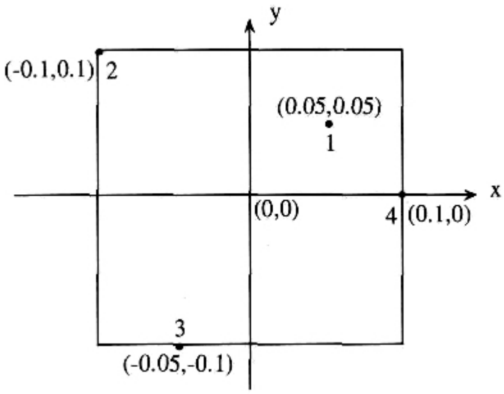

Fig.3 Positions of the measured points in X-Y plane

$R_b = p_{b-}^I / p_{b+}^I$ respectively, thus the scattering matrix

$$S=\begin{bmatrix} p_{a+}^{I} & p_{a+}^{II} \\ p_{b+}^{I} & p_{b+}^{II} \end{bmatrix}\begin{bmatrix} p_{a-}^{I} & p_{a-}^{II} \\ p_{b-}^{I} & p_{b-}^{II} \end{bmatrix}^{-1} \tag{17}$$

In the next step, a vibrational displacement is imposed to the plane at the middle cross-section of the duct which is regarded as the flow machine. The sound pressures at measurement points in cross-sections 1a and 1b are read from the result of the BEM software, and the final characterization of the given source is $\begin{bmatrix} p_{a+}^{S} \\ p_{b+}^{S} \end{bmatrix}=C\begin{bmatrix} p_{a1} \\ p_{b1} \end{bmatrix}$, where C has been defined in section 2.3. All of the concerned data are listed in appendix.

It is noted that the BEM software is incapable to obtain the pressure cross-spectrum matrix, although the source-spectrum matrix is more general for both random and periodic signals. However, the pressure cross-spectrum can be measured in practical work, and then the formulation of the source-spectrum matrix will be obtained.

4 CONCLUSION

In this work, the measurement method of acoustic two-port source is proposed and the working region of fluid machines is extended to frequencies above the first cut-off frequency of the duct. Based on the method of J. Lavrentjev, M. Åbom and H. Bodén[3], There are two steps. In the first step, the external sound source dominates the duct sound field, which is uncorrelated with the field of the source under test. To obtain the scattering matrix S, at least 2N different incident fields are created by using movable external sources. In the second step, external sources are turned off, and the source strength matrix G^s is calculated from measuring the pressure cross-spectrum matrix in the sound field created by flow machine. An important part of the measurement method is a modal decomposition procedure that is based on the early the work of J. Lavrentjev and M. Åbom[8, 9].

After the determination of the acoustic source data of fluid machines, further works can be done. Calculation of the sound pressure in every branch of the duct networks, where the combination of the sound fields from branches must be considered when they meet at a joint, and obviously more complex matrices should be deduced. Part of the work has been done by R. Glav and M. Åbom[10] for plane wave region, and more work is needed when higher order modes propagate in duct. Moreover, it is predictable that the measurement method for fluid machines with more than two openings can be settled in the same way as that with two openings, either in plane wave region or with higher order modes.

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References

[1] Bodén H, Åbom M and Lavrentjev J. Experimental method for determining the source characteristics of fan modeled as acoustic one-port sources[J]. SENLIS Fan Conference, 1994, 351-358.

[2] Åbom M, Bodén H and Lavrentjev J. Source characterization of fans using acoustic 2-port modes[J]. SENLIS Fan Conference, 1994, 359-363.

[3] Lavrentjev J, Åbom M and Bodén H. A measurement method for determining the source data of acoustic two-port sources[J]. Sound Vib, 1995, 183: 517-531.

[4] Allam S, Bodén H. Methods for accurate determination of acoustic two-port data in flow ducts[J]. Twelfth International Congress on Sound and Vibration, 2005, 612.

[5] Åbom M, Bodén H. A note on the aeroacoustic source character of in-duct axial fans[J]. Sound Vib, 1995, 186: 589-598.

[6] Bodén H, Åbom M Maximum sound power from in-duct sources with applications to fans[J]. Sound Vib, 1995, 187: 543-550.

[7] Lavrentjev J, Åbom M. On the modeling of fluid machines as acoustic N-port sources[J]. Proceedings of INTERNOISE 94, Yokahama, Japan, 1994, 379-382.

[8] Lavrentjev J, Åbom M. Characterization of fluid machines as acoustic multi-port sources[J]. Sound Vib, 1996, 197: 1-16.

[9] Åbom M. Modal decomposition in ducts based on transfer function measurement between microphone pairs[J]. Sound Vib, 1989, 135: 95-114.

[10] Glav R,, Åbom M. A general formalism for analyzing acoustic 2-port networks[J]. Sound Vib, 1997, 202: 739-747.

管道高阶模态下一类二端口声源的通用描述形式

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摘要: 就管道通风系统而言,在研究其中的声场分布和同管道末端负载的相互作用影响时,对作为噪声源的流体风机的特征描述是一个关键环节。已往的一些研究往往采用基于管道中平面声波传播的一端或二端模型。而在很多实用场合, 由于声源耦合入管道中的声波频率较高,需要考虑所激发出的高阶声场模态。文中提出了一种新的方法,用来在二端耦合的管道中存在高阶声波模态传播的情况下,测量描述这类稳态声源特征。

关键词: 声源特征;管道系统;二端;模态

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