AIP Journal of Applied Physics

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Citation: J. Appl. Phys. **113**, 034904 (2013); doi: 10.1063/1.4780612
View online: http://dx.doi.org/10.1063/1.4780612
View Table of Contents: http://jap.aip.org/resource/1/JAPIAU/v113/i3
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Constructal design of pedestrian evacuation from an area

C. H. Lui,[1] N. K. Fong,[1] S. Lorente,[2] A. Bejan,[3] and W. K. Chow[1]
[1]*Department of Building Services Engineering, The Hong Kong Polytechnic University, Hong Kong, China*
[2]*INSA, LMDC (Laboratoire Matériaux et Durabilité des Constructions), University of Toulouse, 135 Avenue de Rangueil, 31077 Toulouse, France*
[3]*Department of Mechanical Engineering and Materials Science, Duke University, Durham, North Carolina 27708-0300, USA*

(Received 27 November 2012; accepted 26 December 2012; published online 18 January 2013)

Pedestrian movement occurs in evolutionary patterns the effect of which is to facilitate the access of people into and out of living spaces (areas, volumes). In this paper, we rely on the philosophy of Constructal design as applied physics, in order to uncover two fundamental features of evolutionary design for pedestrian evacuation from rectangular areas (e.g., lecture halls with seated occupants). First, the paper shows analytically that the aspect ratio of the floor area can be selected such that the total evacuation time is minimal. Second, the shape of the floor area of each aisle can be tapered such that the total evacuation time is decreased further. These two architectural features are confirmed by means of extensive and systematic numerical simulations of pedestrian evacuation, by using two numerical packages (Simulex and FDS + Evac). © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4780612]

I. INTRODUCTION

The study of pedestrian movement is driving a growing research field, which is now being pursued as applied physics.[1–8] Pedestrian movement is the main "flow" phenomenon of human dynamics and life in society. It is a natural movement with *patterns*, which evolve as our level of civilization rises. We see this evolution in the designs of buildings, neighbourhoods, airports, shopping malls and all public areas. This evolutionary design is one of the flow design phenomena of nature, which are covered by the Constructal law: for a flow system to persist in time, it must evolve such that it provides greater and greater access to its currents.[9,10]

The objective of designs for pedestrian movement is to facilitate access into and out of inhabited spaces. To this end, the Constructal law is showing how to configure the paths and the areas such that the pedestrian flow occurs in less time, and with greater safety. Examples of constructal designs are the configurations of modern airports,[7] and the shapes of bifurcated walkways.[8] In this paper, we extend this deterministic approach to the discovery of the pedestrian flow configuration that facilitates the evacuation from a rectangular area (e.g., lecture hall) during an emergency event. We show analytically and through numerical simulations that the shape of the area can be optimized, and that the aisles can be shaped (tapered) to facilitate even more the evacuation flow.

II. LECTURE HALL DESIGN

Consider a rectangular area $A = HL$ on which seats are arranged in r parallel rows (Fig. 1). Each seat occupies a square floor area of size D^2. The spacing between rows is S_x. The seated persons are evacuated by walking through the spacings S_x with the speed V_x, and with the speed V_y and spacings S_y along an aisle of length H and width W.

The time required for complete evacuation is of the same order of magnitude as the time required by the farthest seated person, who must travel the longest distance,

$$t = \frac{L - W/2}{V_x} + \frac{H}{V_y}. \tag{1}$$

Note the distance $L - W/2$ traveled at speed V_x, which accounts for the fact that this movement must reach the mainstream (i.e., the middle) of the W aisle. The speeds V_x and V_y vary monotonically with the person-to-person spacings S_x and S_y, as shown in Fig. 1 of Ref. 8,

$$V_x = CV_\infty \left(1 - \frac{S_0}{S_x}\right)^{1/2}, \tag{2}$$

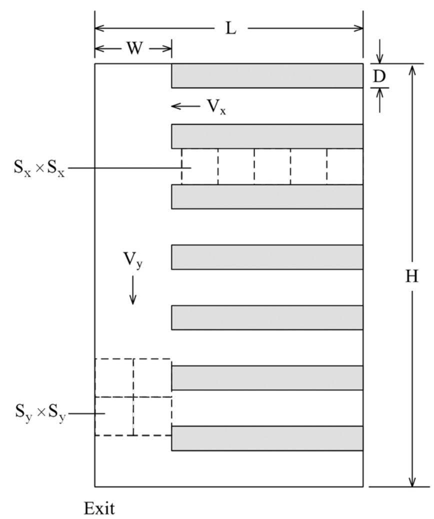

FIG. 1. Rectangular floor area with r rows of seats and one perpendicular aisle of width W.

$$V_y = CV_\infty \left(1 - \frac{S_0}{S_y}\right)^{1/2}. \tag{3}$$

Here, S_0 is the critical spacing below which the walking speed is zero, $C \cong 1$ and $V_\infty \cong 1.3$ m/s is the speed in the limit of wide spacings, $(S_x, S_y) \gg S_0$.[8] The evacuation from a crowded theatre or airplane corresponds to the opposite limit where spacings are close to S_0.

The continuity of individuals from the S_x spacings to the S_y aisle dictates the following relationship. Let Δt be the time needed by one occupant to leave one S_x space and enter the aisle area,

$$\Delta t = \frac{S_x}{V_x}. \tag{4}$$

Because there are r rows, the number of individuals entering the aisle during Δt is equal to r. In the aisle, this group fills an area of width W and longitudinal length $Y = V_y \Delta t$. The size of the area occupied by this group is $WY = r S_y^2$, where S_y^2 is the area allocated to one person in the aisle, and

$$r = \frac{H}{D + S_x}. \tag{5}$$

From this argument follows:

$$\frac{WS_x}{S_y^2} = r\frac{V_x}{V_y}. \tag{6}$$

There are two area constraints. One is the overall floor area

$$A = HL, \text{ constant} \tag{7}$$

and the other is the area available for walking:

$$A_w = HW + rS_xL, \text{ constant} \tag{8}$$

First, we minimize t of Eq. (1) by varying H and L subject to constraint (7), and the results are

$$\left(\frac{H}{L}\right)_{opt} = \frac{V_y}{V_x}, \tag{9}$$

$$H_{opt} = \left(A\frac{V_y}{V_x}\right)^{1/2}, \tag{10}$$

$$L_{opt} = \left(A\frac{V_x}{V_y}\right)^{1/2}, \tag{11}$$

$$t_{min} = 2\left(\frac{A}{V_xV_y}\right)^{1/2} - \frac{W/2}{V_x}. \tag{12}$$

Equation (12) can also be written as

$$t_{min} = \frac{3}{2}\left(\frac{A}{V_xV_y}\right)^{1/2} - \frac{A - A_w}{2V_xrD}, \tag{13}$$

where $(A - A_w)$ is the floor area occupied by seats. Before proceeding further, we ask under what conditions the first term dominates the second in Eq. (12) [or in Eq. (13)], i.e., when

$$2\left(\frac{A}{V_xV_y}\right)^{1/2} \gg \frac{W/2}{V_x}. \tag{14}$$

Because W is comparable with L but smaller than L, we substitute $W = \varepsilon L_{opt}$ in the above inequality, with the understanding that $\varepsilon < 1$. After these substitutions, the inequality (14) becomes

$$4 \gg \varepsilon \tag{15}$$

which is amply satisfied by the assumption $\varepsilon < 1$.

In conclusion, we retain only the first term in the t_{min} expression (12) and note that to decrease the evacuation time t_{min} further, we must maximize the product V_xV_y subject to the walking area constraint (8), which can be written as

$$\frac{2A_w}{A} = \left(\frac{S_y}{S_x}\right)^2 + 1, \text{ constant}. \tag{16}$$

In this way, we discover that the further minimization of t_{min} subject to constraint (8) reduces to maximizing the product V_xV_y subject to Eq. (13), which is the same as

$$\frac{S_y}{S_x} = \text{constant}. \tag{17}$$

As a consequence, the product V_xV_y increases monotonically as either V_x or V_y increases.

III. NARROWING AISLE

In the pedestrian flow of Fig. 1, the individuals who enter the W aisle from the top-left of the figure encounter fewer pedestrians than those who enter the aisle near the lower-left. The density of people walking down the $W \times H$ aisle is not uniform. Does this nonuniformity have a negative effect on the total travel time along H? Should the aisle be wider near its exit, to accommodate the higher density of pedestrians?

To answer these questions, consider Fig. 2, in which the aisle width is an unknown function of longitudinal position, W(y). Both S_y and V_y may vary along the aisle length H. The objective is to minimize the total travel time

$$t = \int_0^H \frac{dy}{V_y} \tag{18}$$

subject to the total aisle area constraint

$$A_a = \int_0^H W\,dy. \tag{19}$$

The objective is to find the optimal width function W(y) such that t is minimum.

Another approach is based on the observation that (because of the equally spaced rows of seats) people enter

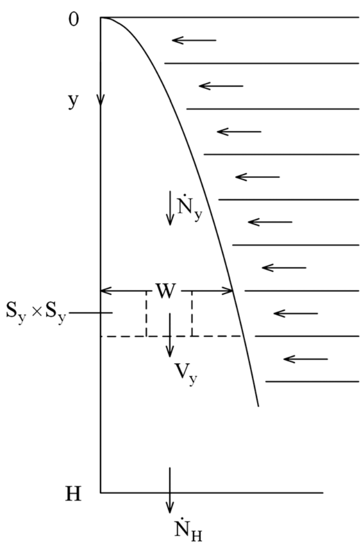

FIG. 2. Tapered aisle W(y) with uniform flow rate of pedestrians coming from many rows of seats. This model corresponds to the limit $W \ll L$ in Fig. 1.

the aisle from the right at a uniform rate per unit of length in the y direction, and this means that the numerical flow rate of people ($\dot{N}_y$) through the W(y) cross section is proportional to y,

$$\frac{\dot{N}_y}{\dot{N}_H} = \frac{y}{H}. \tag{20}$$

The numerical flow rate at the location y through the aisle of width W(y) is

$$\dot{N}_y = \frac{\text{number of individuals passing}}{\text{unit time}} \tag{21}$$

and $\dot{N}_H$ is the value of $\dot{N}_y$ at $y = H$, which is a constant dictated by the flow rate of people entering the aisle from the right in Fig. 2.

Assume that the spacing around one individual in the aisle is S_y, longitudinally and transversally. This means that the number of individuals in a transversal cut W(y) is W/S_y, and this number passes over the W line during the time interval S_y/V_y. From this follows, the numerical flow rate defined in Eq. (21),

$$\dot{N}_y = \frac{W/S_y}{S_y/V_y}. \tag{22}$$

Combining Eqs. (20) and (22), we conclude that

$$\frac{y}{H}\,\dot{N}_H = \frac{W\,V_y}{S_y^2}. \tag{23}$$

A first consequence of Eq. (23) is that if the spacing between individuals in the aisle is y-independent ($S_y = \text{constant}$), the walking speed V_y is also y-independent [cf. Eq. (3)], and therefore the aisle width W must increase in proportion with y in the downward direction,

$$W(y) = \frac{y}{H}\,\dot{N}_H\,\frac{S_y^2}{V_y} = y \times (\text{constant}). \tag{24}$$

This is the first theoretical instance where the tapering of the aisle promises to be an attractive feature of the floor evacuation design.

IV. NUMERICAL SIMULATIONS

To validate the conclusions of Secs. II and III, we conducted numerical simulations of pedestrian evacuation scenarios on two-dimensional floor areas with the features shown in Figs. 3(a)–3(g). The total hall area, number of seats, number of occupants and number of exits are the same as in Ref. 11. The arrangement of the seating layout was varied in order to study the impact on the evacuation time of the occupants. These floor configurations have two exits, and are equivalent to using the right hand side of Fig. 1 as a symmetry axis. The width of the exits is maintained constant at $W = 1.80$ m. The objectives of the simulations were to illustrate the existence of an optimal floor area shape H/(2 L) for minimum evacuation time (Sec. II), and the attractiveness of tapering the aisle shape toward decreasing the evacuation time (Sec. III).

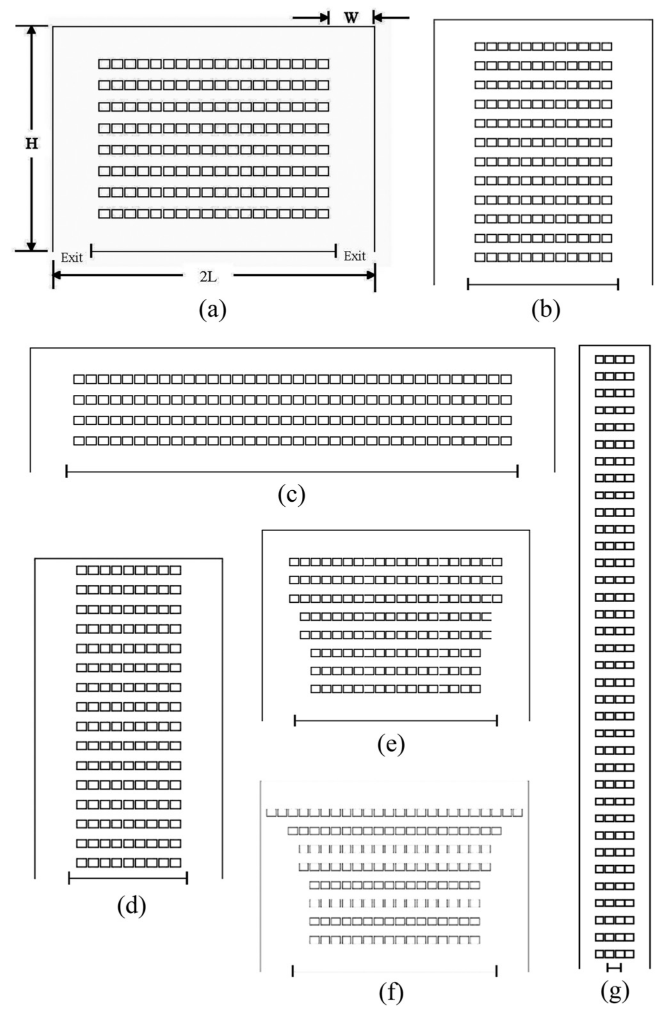

FIG. 3. Rectangular floor plans of lecture halls with fixed area and number of seats.

In each scenario, the total floor area was fixed, $H \times 2L = 157.5\ m^2$. The number of seats was fixed at 144. The number of individuals filling these seats was also fixed at 144, i.e., there were no empty seats. For greater confidence in the numerical results, the simulations were performed using two different numerical codes: SIMULEX[2] and FDS+Evac.[12,13]

SIMULEX is a computer package simulating the evacuation movement of occupants from large, geometrically complex building structures. By using the computer-aided-designed floor plans of buildings, final external exits can be defined outside the buildings. A distance map, with $0.2\ m \times 0.2\ m^2$ square mesh sizes, is spread over the entire plan. The value of distance-to-exit from the center of each square is calculated. Occupants are present in this space individually or in groups. When the building population has been defined, the potential routes of the occupants can be calculated and then a simulation can be carried out to find the total evacuation time. The occupants move toward the exits with individual walking speeds dependent upon individual characteristics and the proximity of other people. The algorithms for the individual movement are based on real-life data, collected by using computer-based techniques for the analysis of human movement, observed in real-life footage.

The FDS + Evac model is used to simulate the evacuation of the occupants. The software considered each occupant as an individual agent. Each agent is composed of three elastic circles to represent the shape of human body. These agents are moving in a two dimensional plane representing the floors of the building. The shape of the agent is similar to the one used in SIMULEX. The agents are treated as autonomous bodies with their own physical properties and escape decisions. The contact forces, torques, psychological and motive forces of the agents are also included in the equations of motion of individual agents to simulate their movement. To ensure the agents move toward the exits, a walking direction vector field is generated first. This vector field is obtained by solving a two dimensional potential flow problem for incompressible fluid with all boundary conditions (walls, openings) specified. The movement algorithm includes many factors concerning the agents such as body sizes, the mass, the walking speed, and the moment of inertia. In addition to these physical factors, the social forces and the contact forces are also considered.

The model for studying the interaction of the agents is developed based on Ref. 14. It considers the "social force" between agent and agent and between agent and walls. The social force is represented by a vectorial quantity in the equation of motion. It is used to simulate the acceleration force or deceleration force of the agent movement as the agent reacted to the perceived information obtained from its environment. In addition to the model of Ref. 14, a short range counterflow model is also adopted to avoid agent collision. After setting the translational and rotational equation of motion, the modified velocity-Verlet algorithm[15] is used to solve the equations. The total evacuation time can then be obtained from the excel files. The software SMOKEVIEW[13] is used to obtain the animation and graphical output.

Each numerical simulation began with randomly selected initial speeds and directions of the individuals leaving the seats. As a consequence, the total evacuation time (t) depended on the initial set of conditions. For each of the configurations of Fig. 3 we conducted additional simulations while averaging the evacuation time over the number of simulations. For example, if the number of simulations was n, then the evacuation time $\bar{t}$ reported in Table I is the average of the evacuation times produced by the n simulations, namely $t_1, t_2, \ldots, t_n$. As shown in Fig. 4, the average evacuation time $\bar{t}$ becomes insensitive to adding more simulations to the calculation when n is approximately 30. All the $\bar{t}$ values reported in Table I were calculated by performing 30 simulations of the same evacuation scenario. Figure 4 also shows how the standard deviation of t_i relative to $\bar{t}$ stabilizes as n approaches 30.

The results of Table I confirm the two effects anticipated theoretically. First, the $\bar{t}$ values of Figs. 3(a)–3(d) and 3(g) show the effect of varying the floor area shape H/(2L), while holding the aisle width W fixed. Varying in this sequence of five scenarios are the length of the aisle and the length of the rows. Figure 5 shows that $\bar{t}$ exhibits a minimum with respect to the shape of the floor area. The optimal shape H/(2L) is in the range 1.2–1.7. This agrees qualitatively with Eq. (9), which can be written as $H/(2L) = V_y/(2V_x)$, and where the walking speed in the aisle (V_y) is expectedly of the same order or greater than twice the speed in the space between rows ($2V_x$).

The second objective of the numerical simulations was to document the effect of tapering the aisle. This effect was studied in the sequence of configurations shown as Figs. 3(a), 3(e), and 3(f). The degree of tapering increases from

TABLE I. Numerical results for the average evacuation times in the configurations of Figs. 3(a)–3(j).

Figures 3	Evacuation time SIMULEX, $\bar{t}_S$(s)	Evacuation time FDS+Evac, $\bar{t}_F$(s)	Standard deviation SIMULEX σ_S	Standard deviation FDS+Evac σ_F	H (m)	2L (m)	Aspect ratio of floor H/(2L)	Rows	Seats in a row	Aspect ratio of seats area
(a)	30.5	38.8	0.79	1.57	10.5	15	0.7	8	18	0.44
(b)	29.4	38.71	0.62	0.77	13.8	11.4	1.21	12	12	1
(c)	35.8	40.6	1.14	1.71	6.1	25.8	0.24	4	36	0.11
(d)	29.6	37.7	0.87	0.85	16.4	9.6	1.71	16	9	1.78
(e)	29.6	37.8	0.80	1.02	10.5	15	0.7	8	Varied	N/A
(f)	29.8	38.67	0.56	2.19	10.5	15	0.7	8	Varied	N/A
(g)	64.5	70.1	0.65	3.04	36.6	4.3	8.52	36	4	9
(h)	29.2	36.2	0.87	1.84	10.5	15	0.7	8	18	0.44
(i)	23.9	34.6	0.68	2.03	10.5	15	0.7	8	Varied	N/A
(j)	24.3	34.4	0.54	2.72	10.5	15	0.7	8	Varied	N/A

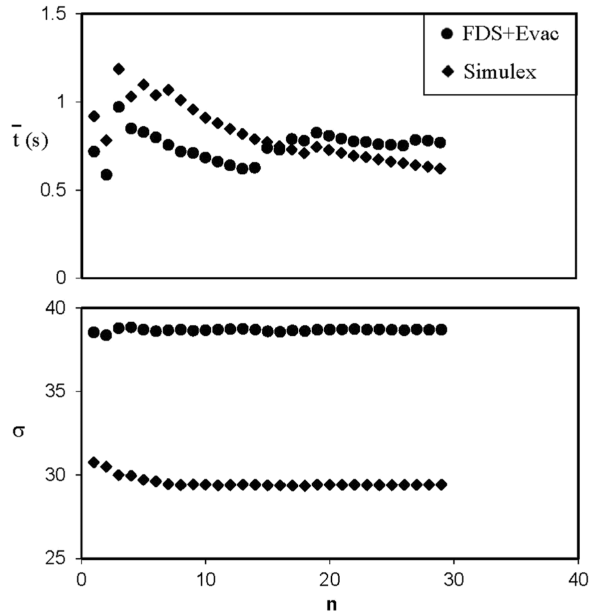

FIG. 4. The effect of the number of simulations of one evacuation scenario on the average evacuation time $\bar{t}$ and the standard deviation (from the case with hall aspect ratio being 1.21) in the configuration of Fig. 3(b).

Figs. 3(a)–3(f); while the floor area, its shape and number of seats, remain fixed. The results for the averaged evacuation times are listed in Table I and show that tapered aisles are an attractive feature for facilitating the movement of pedestrians out of the lecture hall. Table I also shows that the numerical results generated with the SIMULEX code are nearly the same as the results obtained with the FDS+Evac code.

To highlight the effect of tapering the aisles, we decided to enlarge the width of the two symmetric exits so that the strangulation effect caused by the two exits is avoided. To achieve this, the exit width was doubled. The average evacuation time is also plotted in Fig. 6. Note that the shape of the aisle still corresponds to the configuration shown in Figs. 3(e) and 3(f). The evacuation time in the case of a rectangular configuration (case 3a) is slightly smaller than in Fig. 5: this is due to the wider exit. As expected, increasing the width of the exit accentuates the impact of tapering the aisles: the tapered aisles lead to a decrease of 20% in the average evacuation time.

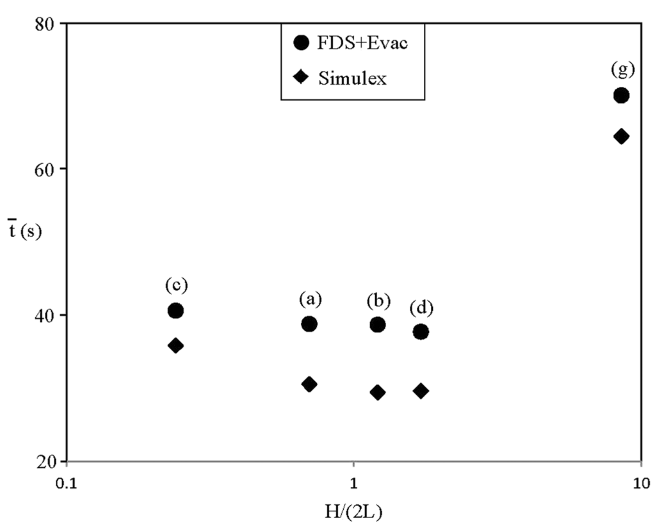

FIG. 5. The effect of the hall aspect ratio on the average evacuation time in the configurations of Fig. 3.

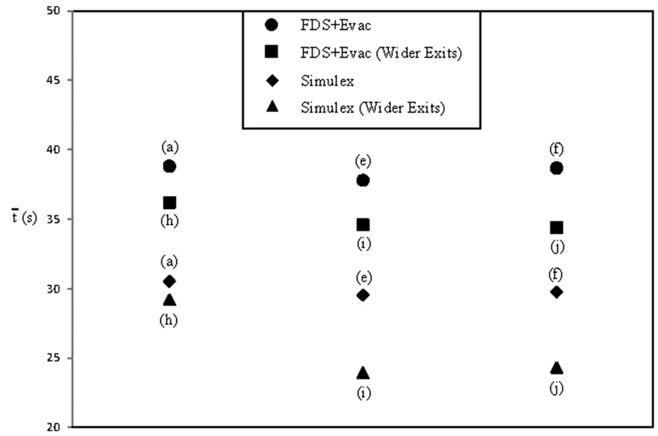

FIG. 6. The effect of the tapered aisle and the exit width (1.8 m and 3.6 m) on the average evacuation time in the configurations of Figs. 3(a), 3(e), and 3(f).

V. CONCLUSIONS

In this paper, we showed how the constructal law guides the search for the configuration of a lecture hall so that the evacuation time during an emergency is minimum. For a given number of people and seats, and on a fixed lecture hall floor area, we demonstrated that an optimal shape of the room can be predicted.

The evacuation time reached a minimum value when the hall aspect ratio, calculated as the ratio of the width and length, is about 1. We also showed that more progress can be made toward a more efficient evacuation pattern by tapering the aisles of the seated area. The shaping of the lecture hall leads to a 20% decrease in the evacuation time. These theoretical predictions based on applying the Constructal Law to pedestrian evolution were confirmed with full numerical simulations by using two different numerical models, Simulex and FDS+Evac.

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