# Equalization-enhanced phase noise induced timing jitter

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In electronic digital signal processing based optical communication systems, digital equalization for chromatic dispersion interacts with local oscillator phase noise to produce equalization-enhanced phase noise (EEPN). In addition to both phase and intensity noises, EEPN also induces timing jitter to the equalized signal. For a 100 Gbit/s quadrature-phase-shift keying signal with laser linewidth of 300 kHz, the timing jitter is up to 20% of the symbol interval for a transmission distance of 1500 km. 

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Single-carrier quadrature-phase-shift keying (QPSK) signals are used for long-haul lightwave systems [1,2]. Performing close to the matched-filter limit [3,4], coherently detected QPSK signals may use digital signal processing (DSP) to compensate for chromatic and polarization-mode dispersions [5–7]. However, for lasers with finite linewidth, the electronic equalizer produces equalization-enhanced phase noise (EEPN), which gives additional phase and intensity noises to the equalized signal [8–10], limiting system performance. Recently, EEPN distortion was first measured in [11]. EEPN also induces timing jitter to the signals that is also measured in [11].

Following the mathematical model of both [8,10], a single symbol of the system is considered. The overall pulse response of the system with a receiver phase noise $\phi(t)$ is

$$p(t) = h_1(t)e^{j\phi(t)} \otimes h_2(t), \tag{1}$$

where $h_1(t)$ is the combined impulse response due to the transmitter pulse shaping filter and the fiber chromatic dispersion, $h_2(t)$ is ideally the matched filter to $h_1(t)$ and includes the equalizer and the receiver filter as shown in Fig. 1, and $\otimes$ denotes convolution. As shown in Fig. 1, $h_1(t)$ and $h_2(t)$ are the responses before and after the mixing of the received signal with the LO laser, respectively.

Without phase noise, the pulse is $p_0(t) = h_1(t) \otimes h_2(t)$, and the difference between $p_0(t)$ and $p(t)$ is studied in [8–10] as the EEPN impairment. The timing jitter of the pulse $p(t)$ is studied here.

Without phase noise and for a typically real signal of $p_0(t)$, the timing shift may be defined as $\int tp_0(t)\mathrm{d}t$ with the normalization of $\int p_0(t)\mathrm{d}t = 1$, where the integration interval throughout this letter is from $-\infty$ to $+\infty$. When $h_2(t)$ is the filter matched to $h_1(t)$, $p_0(t)$ is typically a raised-cosine pulse. The normalization $\int p_0(t)\mathrm{d}t = 1$ is equivalently $H_1(0) = H_2(0) = 1$, where $H_1(f)$ and $H_2(f)$ are the Fourier transforms of $h_1(t)$ and $h_2(t)$, respectively. The definition of timing shift can be verified by the relationship $\int tp_0(t - t_0)\mathrm{d}t = t_0 + \int tp_0(t)\mathrm{d}t$ regardless of the shape of $p_0(t)$.

With phase noise $\phi(t)$, the pulse $p(t)$ generally has a small imaginary part even after the rotation of $p(t)e^{-j\phi(0)}$. The timing shift may be defined as $\mathrm{sign}(\mathrm{Re}\{\tau e^{-j\phi(0)}\})|\tau|$ with $\tau = \int tp(t)\mathrm{d}t$, where $\mathrm{Re}\{\}$ denotes the real part of a complex number. The random part of the timing shift is timing jitter that may be defined as $\mathrm{sign}(\mathrm{Re}\{\tau e^{-j\phi(0)}\})|\tau - E\{\tau\}|$, where $E\{\}$ denotes expectation. If both impulse responses of $h_1(t)$ and $h_2(t)$ are symmetrical with respect to $t = 0$, even with phase noise, the zero-mean timing shift is the same as timing jitter.

The timing jitter variance is

$$\sigma_j^2 = \iint t_1 t_2 E\{p(t_1)p^*(t_2)\}\mathrm{d}t_1\mathrm{d}t_2. \tag{2}$$

Substituting in (1), the expectation $E\{p(t_1)p^*(t_2)\}$ is

$$\iint h_2(t_1 - \tau_1)h_1(\tau_1)h_2^*(t_2 - \tau_2)h_1^*(\tau_2) \times E\{e^{j\phi(\tau_1) - j\phi(\tau_2)}\}\mathrm{d}\tau_1\mathrm{d}\tau_2. \tag{3}$$

The Fourier transform of $\iint E\{p(t_1)p^*(t_2)\}e^{j2\pi t_1 f_1 - j2\pi t_2 f_2} \times \mathrm{d}t_1\mathrm{d}t_2$ is

$$P_2(f_1, f_2) = H_2(f_1)H_2^*(f_2)\int \Phi_\phi(f)H_1(f_1 - f)H_1^*(f_2 - f)\mathrm{d}f, \tag{4}$$

where $\Phi_\phi(f)$ is the spectral density of the laser field $e^{j\phi(t)}$. The timing jitter variance (2) becomes

$$\sigma_j^2 = \frac{1}{(2\pi)^2}\frac{\partial^2}{\partial f_1 \partial f_2}P_2(f_1, f_2)\bigg|_{f_1=0, f_2=0}, \tag{5}$$

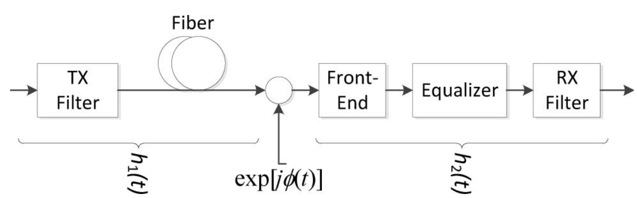


Fig. 1. The system diagram here, showing that $h_1(t)$ and $h_2(t)$ include all linear responses before and after LO laser, respectively.

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which can be further simplified to

$$\sigma_j^2 = \frac{1}{(2\pi)^2}\int \Phi_\phi(f)\left|\frac{dH_1^*(f_1)}{df_1}\right|_{f_1=-f}\right|^2 df, \qquad (6)$$

if $H_2(0) = 1$ and $dH_2(f_1)/df_1|_{f_1=0} = 0$ when $h_2(t)$ is symmetrical with respect to $t = 0$.

As a numerical example, let the transmitted filter be an ideal low-pass filter such that

$$H_1(f) = H_2^*(f) = \begin{cases} e^{j2\pi^2\beta_2 L f^2} & |f| < \frac{1}{2T} \\ 0 & \text{otherwise} \end{cases}, \qquad (7)$$

where $\beta_2$ is group velocity dispersion parameter of the fiber, $L$ is length of the system, and $T$ is the symbol period of the QPSK signals. When the phase noise is modeled as a Wiener process, the spectral density of laser with linewidth $\Delta f$ is [4,8]

$$\Phi_\phi(f) = \frac{\Delta f}{2\pi[(\Delta f/2)^2 + f^2]}. \qquad (8)$$

Substituting (7) and (8) in (6), the timing jitter variance is

$$\sigma_j^2 = 2\pi\Delta f L^2\beta_2^2\left[\frac{1}{T} - \Delta f \tan^{-1}\left(\frac{1}{T\Delta f}\right)\right]. \qquad (9)$$

The second term of (9) is generally negligible compared with the first term and also the timing jitter is only meaningful when normalized to the symbol period. Therefore, we focus on

$$\frac{\sigma_j}{T} = \sqrt{2\pi\Delta f T}\,\frac{|\beta_2|L}{T^2}. \qquad (10)$$

Figure 2 shows the timing jitter $\sigma_j/T$ as a function of distance for laser linewidth of 100 kHz, 300 kHz, and 1 MHz. The system assumes a dispersion coefficient of 17 ps/km/nm and no optical dispersion compensation. The QPSK signal has a symbol rate of 27 Gbaud/s, sufficient for dual-polarization 100 Gbit/s system with overhead from forward error correction. For a distance of up to 1500 km, the laser linewidth must be less than 300 kHz to ensure that the timing jitter is less than about $\sigma_j/T = 0.2$.

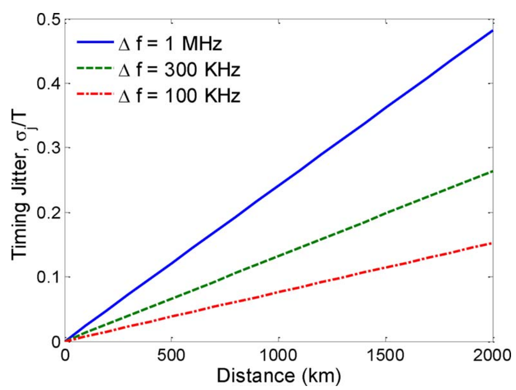


Fig. 2. (Color online) Timing jitter as a function of transmission distance for a 100 Gb/s QPSK signal. The laser linewidth is 100 kHz, 300 kHz, and 1 MHz.

Figure 3 shows the simulated timing jitter as a function of laser linewidth. The fiber distance is 1200 km. As shown in Fig. 3, the timing jitter obtained from simulation is in agreement with theoretical prediction.

For coherent systems, the electric waveform determines the system performance. There are alternative definitions of timing jitter. For systems using intensity detection, the timing jitter may be defined as $\int t|p(t)|^2 dt$ with normalization of $\int |p_0(t)|^2 dt = 1$, applicable to that for soliton [12]. The timing jitter based on intensity should be smaller than that based on electric waveform. Although phase noise may convert to intensity noise, mostly due to chromatic dispersion in the fiber, phase distortion of the signal before the receiver does not degrade nor give timing jitter to the intensity-detected system. The timing jitter may also be given by the peak of either the electric field $p(t)$ or the intensity $|p(t)|^2$ [11]. In some studies, the timing jitter may also be defined by first-order perturbation of some prototype pulses [13] that ignores all distortion orthogonal to the prototype pulses. For instance, the continuum is excluded in jitter analysis [14]. In all definitions, the timing jitter alone may not be sufficient to determine the system performance. A pulse may have very big timing jitter with very small system degradation. Conversely, a small timing jitter pulse may have a big system penalty. The usage of the timing jitter solely here for system performance evaluation may double count the EEPN for signal distortion and also for timing jitter. Nevertheless, asymmetric properties of the random waveform are alone an important effect.

The timing jitter variance (6) also may have a simple physical meaning. In the receiver, the LO laser adds phase noise and randomly shifts the spectrum of the received signal. The DSP equalizer compensates for the chromatic dispersion and gives rise to timing jitter due to the random spectral shift. As a simple but not necessary rigorous example including only the phase response, if $H_1(f) = H_2^*(f) = e^{j2\pi\theta(f)}$, the variance of timing jitter at each frequency may be $\Phi_\phi(f)|d\theta(f)/df|^2$, or the combination of the group delay $d\theta(f)/df$ with the spectral shift of $\Phi_\phi(f)$. If $\theta(0) = 0$ and $d\theta(f)/df|_{f=0} = 0$, the variance of timing jitter (6) is given by

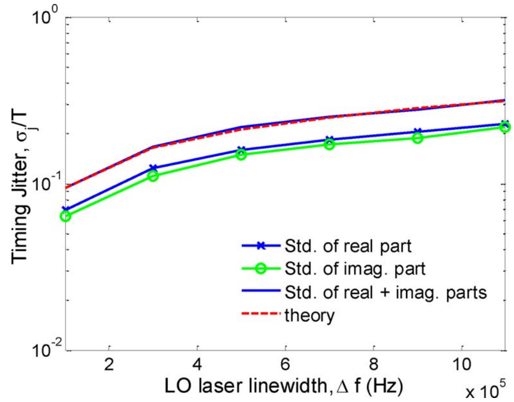


Fig. 3. (Color online) Comparison of simulated and theoretical timing jitter of 1200 km of optical fiber. The simulation result also includes the timing jitter of the real and imaginary parts of the signal.

$$\sigma_j^2 = \frac{1}{(2\pi)^2} \int \Phi_\phi(f) \left| \frac{\mathrm{d}\theta(f_1)}{\mathrm{d}f_1} \Big|_{f_1=-f} \right|^2 \mathrm{d}f. \quad (11)$$

The timing jitter variance of (11) is more or less the same as our intuitive understanding of timing jitter. However, our intuitive understanding of timing jitter is induced by the DSP equalizer $H_2(f)$. The timing jitter of both (6) and (11) is induced by the phase response of $H_1^*(f)$ with phase conjugation for timing reversal and negative frequency shift, equivalently, the time reversal pulse at the transmitter input instead of the equalized pulse at the receiver output. If a matched filter is used as an equalizer, there is no difference between the two interpretations.

Timing recovery is essential for all digital communication systems. The timing jitter gives difficulty to the timing recovery. The timing recovery of the receiver is normally designed with a very low bandwidth that may not able to optimally reduce correlated timing jitter of consecutive pulses. Outside the scope of this Letter, optimally designed timing recovery may potentially reduce the timing jitter significantly.

The timing jitter variance is the same as the variance of $|\int tp(t)\mathrm{d}t|$. The real and imaginary parts of $\int tp(t)e^{-j\phi(0)}\mathrm{d}t$ quantify the asymmetric property and the timing spreading of the real and imaginary parts of the distortion. Combined to (2), the variance due to the real and imaginary part of $\int tp(t)\mathrm{d}t$ is approximately the same, as confirmed in Fig. 3. This may be verified by $E\{t_1 t_2 \mathrm{Re}\{p(t_1)\}\mathrm{Re}\{p(t_2)\}\} = \frac{1}{4}E\{t_1 t_2 p(t_1)p^*(t_2) + t_1 t_2 p(t_2)p^*(t_1)\} + \frac{1}{2}E\{t_1 t_2 \mathrm{Re}\{p(t_1)\times p(t_2)\}\}$ with a negligible second term.

The EEPN induced timing jitter is derived analytically for a system using DSP techniques to compensate for chromatic dispersion. Simulation is conducted to confirm the theoretical analysis. The timing jitter may be up to 20% for a QPSK system with 27 Gbaud/s symbol rate, a transmission distance of 1500 km, and an LO laser linewidth of 300 kHz.

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