Mode coupling in lead zirconate titanate/epoxy 1–3 piezocomposite rings

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Lead zirconate titanate (PZT)/epoxy 1–3 piezocomposite rings were fabricated with PZT volume fractions \( \phi \) ranging from 0.82 to 0.94 and with a small epoxy width of 77 \( \mu \)m in order to investigate their resonance characteristics and to reveal the mode coupling. Four major resonance modes were observed, namely the coupled longitudinal-thickness \( f_H \) and lateral \( f_{L1} \) and \( f_{L2} \) mode resonances of individual PZT elements inside the rings as well as the radial \( f_R \) and wall-thickness \( f_W \) mode resonances of the whole rings. No stopband resonances were observed in the frequency range of 1 to 10 MHz. \( f_H \) was found to increase linearly with the decrease in element height while \( f_{L1} \) and \( f_{L2} \) remained constant. When the height and width of the elements became comparable, coupling of \( f_H \) with \( f_{L1} \) and \( f_{L2} \) occurred. The observed \( f_H \), \( f_{L1} \), and \( f_{L2} \) for all samples agreed with those calculated by the mode-coupling theory. \( f_R \) and \( f_W \) were almost independent of the ring thickness but increased as \( \phi \) increased. A guide of operating \( f_H \) in the rings without causing mode coupling was presented to optimize the composite structure for transducer design. © 2001 American Institute of Physics. [DOI: 10.1063/1.1405139]

I. INTRODUCTION

1–3 piezocomposites, consisting of one dimensionally connected piezoceramic elements embedded in a three dimensionally connected passive polymer matrix, have been an important breakthrough in overcoming the drawbacks of piezoceramics for hydrophones in underwater sonar with operating frequencies below 40 kHz as well as for pulse—echo transducers and undiced transducer arrays in medical ultrasonic imaging functioning in the megahertz frequency range.\(^1\)–\(^11\) The underwater applications require high hydrophone receiving sensitivity, which is achieved by eliminating the response from an incident acoustic pressure applied to the edges of the hydrophone due to a buckling effect and by acoustically matching the impedance between the hydrophone and water. The most important consideration in medical ultrasound is to optimally match the acoustic impedance of the transducer to that of the human tissue in order to achieve high sensitivity, wide bandwidth, and low lateral mode interference. In both cases, disk or plate-shaped composites with relatively low volume fractions of ceramic well below 0.8 are typically used at frequencies near their thickness resonances. Accordingly, the resonance characteristics of these composites have been studied.\(^5\)–\(^10\) and a good understanding of mode coupling has been established.

For solid-state driving or actuation purposes, however, little information is available on using 1–3 type piezocomposites where high volume fraction of ceramic (>0.8) should be adopted. Ring-shaped lead zirconate titanate (PZT) piezoceramics have been used extensively in various low frequency (<100 kHz) ultrasonic devices such as ultrasonic motors, welding transducers, piezoelectric transformers, etc., and their resonance characteristics have been reported elsewhere.\(^12\)–\(^15\) For some dedicated applications such as high frequency ultrasonic wire bonding of microelectronic packages, thickness-driven, axial mode transducers operating at a few hundred kilohertz are of prime importance. As PZT (typically hard PZT) rings used in state-of-the-art transducers have many nonaxial modes at these frequencies, they will couple with the desired axial mode of the transducer and cause significant degradation of its performance. By using 1–3 piezocomposite rings, the mode-coupling problem can be greatly alleviated, and pure axial mode can be attained.\(^13\)–\(^17\) Therefore, in this work, we aim at fabricating PZT/epoxy 1–3 piezocomposite rings and determining their resonance characteristics.

II. FABRICATION OF 1–3 PIEZOCOMPOSITE RINGS

The 1–3 piezocomposite rings were fabricated using the dice-and-fill technique.\(^1\)–\(^7\) PKI804 PZT rings (12.7 mm outer diameter, 5.0 mm inner diameter, and 2.5 mm thickness) and Araldite LY5210/HY2954 epoxy were used as the active and passive phases in the 1–3 structure, respectively. Two sets of equally spaced, parallel grooves were cut in the PZT rings in orthogonal directions using a Disco DAD 341 automatic dicing saw equipped with a diamond saw blade of 70 \( \mu \)m thickness. However, the grooves did not go right through the ring thickness. Approximately 0.19 mm of PZT was allowed to remain on the base so that the whole ring structure could be retained after cutting. Due to blade vibration, the resulting grooves in the ring were about 77 \( \mu \)m wide as measured by a traveling microscope. Thus, by applying a different number of cuts (5, 7, 9, 11, 13, and 15 cuts/direction) to the PZT rings, piezocomposite rings with different PZT element widths \( L \) of 2.03, 1.50, 1.19, 0.98, 0.83, and 0.71 mm and with different \( \phi \) values of 0.94, 0.91,
0.89, 0.87, 0.84, and 0.82 were obtained, respectively (Fig. 1). Due to the fineness of the grooves, filling of epoxy under a vacuum was carried out. The cured composite rings were lapped to grind off the PZT base and to obtain a pair of flat and parallel surfaces. It was found that a thickness of 2.30 ± 0.02 mm could generally be obtained after lapping.

III. RESONANCE MODES IN 1–3 PIEZOCOMPOSITE RINGS

Resonance modes of a 1–3 piezocomposite ring can be classified into two main categories as follows.

1. Characteristic resonances of individual PZT elements inside the ring comprise the first category.
   (a) The coupled longitudinal-thickness mode resonance \( f_H \), which is essentially determined by the height \( H \) of the PZT elements, is the mode of practical interest in ultrasonic transducers as it corresponds to the thickness mode resonance \( f_t \) of the composite ring.
   (b) The coupled lateral mode resonances \( f_{L1} \) and \( f_{L2} \) are mainly determined by the width \( L \) of the PZT elements with square cross section. When the composite ring is thick, the PZT elements are tall thin bars, and \( f_H \) is well separated from \( f_{L1} \) and \( f_{L2} \). When the elements approach the shape of a cube, coupling of these resonance modes occurs. This mode-coupling effect can be predicted using the mode-coupling theory, which will be described in the next section.

2. Cooperative resonances of the whole ring comprise the second category.
   (a) The radial \( f_R \) and wall-thickness \( f_W \) mode resonances are the lateral resonances of the composite ring as a whole and are usually the two low frequency resonance modes in a thin ring. They are essentially determined by the mean diameter and wall-thickness of the ring, respectively.
   (b) The stopband resonances \( f_{S1} \) and \( f_{S2} \) result from the periodic structure of the composite and occur at frequencies near where scattering of laterally propagating shear waves in the epoxy matrix from the planes of the regular spaced PZT elements occur. They are related to the lateral periodicity \( d \) (one epoxy width \( d_p + \) one element width \( L \)) if composites have low values of \( \phi \left(<0.25\right) \) with large \( d_p \). In contrast, they are mainly determined by \( d_p \) for composites possessing relatively high values of \( \phi \) with sufficiently small \( d_p \). The composite rings fabricated in the present work belong to the latter case.

IV. THE MODE-COUPLING THEORY

Consider a square parallelepiped with width \( L \) in the \( x \) and \( y \) directions and height \( H \) in the \( z \) direction. This parallelepiped is essentially equivalent to an individual PZT element inside a 1–3 composite with its \( z \) axis taken as the poling axis and with both square surfaces normal to the \( z \) axis being entirely electroded. The frequency equation of this parallelepiped can be expressed as

\[
[f^2 - f_a^2(1 - \gamma)]\{f^2 - [f_a^2(1 + \gamma) + f_b^2]f^2 + f_a^2f_b^2(1 + \gamma - 2\alpha^2)] \right\} = 0,
\]

(1)

where

\[
f_a = \frac{1}{2L} \sqrt{\frac{c_{11}}{\rho}},
\]

(2)

and

\[
f_b = \frac{1}{2H} \sqrt{\frac{c_{33}}{\rho}},
\]

(3)

are the uncoupled lateral and longitudinal-thickness resonance frequencies, respectively; \( \gamma = c_{12}/c_{11} \) and \( \alpha = c_{13}/\sqrt{c_{11}c_{33}} \) are the coupling constants; \( c_{11} \), \( c_{12} \), \( c_{13} \), and \( c_{33} \) are the elastic stiffness constants; and \( \rho \) is the density. Equation (1) can be factorized into a quadratic and a biquadratic equation as follows.

1. The quadratic equation is

\[
f^2 - f_a^2(1 - \gamma) = 0.
\]

(4)

It gives two solutions in which the one with negative root is discarded for physical reasons, leaving only the positive one as

\[
f_{L2} = f_a \sqrt{1 - \gamma} = \frac{1}{2L} \sqrt{\frac{c_{11} - c_{12}}{\rho}}.
\]

(5)

\( f_{L2} \) is called the coupled lower lateral resonance frequency.

2. The biquadratic equation is

\[
f^4 - [f_a^2(1 + \gamma) + f_b^2]f^2 + f_a^2f_b^2(1 + \gamma - 2\alpha^2) = 0
\]

(6)

or

\[
f^4 - f_a^2f_b^2(1 + \gamma - 2\alpha^2) = 0.
\]

FIG. 1. Fabricated PK804/Araldite LY5210/HY2954 piezocomposite rings are shown. The black epoxy dot is a polarity marking to indicate the anode (connected to positive voltage during poling).
where \( G = L/H \) is the configuration ratio and \( R = \sqrt{c_{11}/c_{33}} \) is the coupling constant. Equation (7) has two solutions \( f_+ \) and \( f_- \), which give two frequency branches: upper \( f_+ \) and lower \( f_- \). The coupled longitudinal-thickness resonance \( f_H \) and upper lateral \( f_{L1} \) resonance frequencies are governed by these two frequency branches in terms of \( G \).

Table I shows the material parameters of PKI804 PZT.\(^5\) Hence, the theoretical curves of the uncoupled [Eqs. (2) and (3)] and coupled [Eqs. (5) and (7)] resonance frequencies at different \( G \) can be derived using parameters with superscripts \( E \) and \( D \) for the short- and open-circuit conditions, respectively. The calculated curves will be compared with experimental results in a later section.

### V. THINNING TEST

Thinning test was carried out to study the resonance characteristics of the composite rings. It involves lapping each composite sample to different thicknesses in steps and at each thickness applying a thin silver layer of 5 to 10 \( \mu m \) (G3691 quick drying silver paint from Agar Scientific Ltd. in U.K.) on the two major surfaces to form electrodes and then measuring the electrical impedance spectrum using a HP 4194A impedance analyzer. It should be noted that the measured series resonance frequency was compared with the theoretical predictions calculated using the short-circuit (constant \( E \)) parameters. The measured parallel resonance frequency was compared with the theoretical predictions deduced using the open-circuit (constant \( D \)) parameters. \( f_H \) of the PZT elements is a piezoelectric stiffened mode while the other resonance modes are unstiffened modes.

### VI. RESULTS AND DISCUSSION

#### A. Characteristic resonances of individual PZT elements

Figure 2 shows the thinning test results of a composite ring with \( \phi = 0.82 \) and \( L = 0.71 \) mm. The electrical impedance spectrum of the ring with \( H = 2.30 \) mm \((G = 0.31)\) is plotted in Fig. 2(a). Guided by the mode-coupling theory, the strongest resonance mode at 0.93 MHz is identified as \( f_H \). The mode at about 2.70 MHz is its third harmonic and is labeled as \( 3f_H \) since it appears at about three times \( f_H \). Even harmonics are not expected to be observed because the applied electric field has opposite polarity on the two faces of the sample. The two lateral modes \( f_{L1} \) and \( f_{L2} \) are weak resonances found at 3.73 MHz and 2.48 MHz, respectively. They are often mixed with other high frequency resonances. For example, \( f_{L2} \) has merged with \( 3f_H \) in Fig. 2(a). Besides, there are several weak resonances above \( f_{L1} \) and \( f_{L2} \). These may be other lateral modes, which arise from the PZT elements with smaller lateral sizes \( L \) such as those elements located near the inner and outer circumferences of the ring as shown in Fig. 1. Moreover, the two small resonances appeared in the lower frequency band of 75 kHz and 306 kHz are \( f_g \) and \( f_w \) of the whole composite ring, respectively. Detailed behavior of these modes will be discussed later.

When the sample is subsequently thinned down [Figs. 2(b)–2(d)], \( f_H \) increases accordingly while \( f_{L1} \) and \( f_{L2} \) stay quite constant. Also, \( 3f_H \) increases at approximately three times the rate of \( f_H \). If there is no other resonance close to \( f_{L1} \) and \( f_{L2} \), the resonance peaks associated with them will be very weak and sometimes even disappear [Fig. 2(b)]. For \( H < 0.95 \) mm \((G > 0.75)\), \( f_H \) starts to couple with \( f_{L2} \) [Fig. 2(c)] and later continues to merge with the subsequent lateral modes [Fig. 2(d)], thereby forming a strong but complex resonance mode covering a broad range of frequency. Consequently, \( f_{L2} \) serves to impose an upper frequency bound where mode coupling begins.

The general behaviors of \( f_H \), as well as \( f_{L1} \) and \( f_{L2} \), at different sample thicknesses for the remaining five batches of composite rings with \( \phi \) values of 0.84, 0.87, 0.89, 0.91, and 0.94 are quite similar to each other and also to the one with \( \phi = 0.82 \). The corresponding spectra for the electrical impedance are shown in Figs. 3–7. The observed results are summarized as follows.

(1) For the sample with \( \phi = 0.84 \) and \( L = 0.83 \) mm (Fig. 3), the initial thickness of the composite is \( H = 2.31 \) mm \((G = 0.36)\) [Fig. 3(a)]. A clean \( f_H \) is observed at 0.93 MHz, then no other resonance is detected until \( f_{L2} \) appears at 2.22 MHz. Resonance modes following \( f_{L2} \) include other high frequency lateral modes, \( 3f_H \) (that has mixed with these lateral modes at \(~2.80 \) MHz), and \( f_{L1} \) \((3.35 \) MHz). All these lateral resonances are found at frequencies lower than those of the sample with \( \phi = 0.82 \) due to an increment of \( L \) from 0.71 to 0.83 mm. When \( H \) is reduced, \( f_H \) and \( 3f_H \) rise almost linearly while \( f_{L1} \) and \( f_{L2} \) remain nearly unchanged [Figs. 3(b)–3(c)]. Coupling of \( f_w \) with \( f_{L2} \), as well as the subsequent lateral modes, occurs at \( H < 1.05 \) mm \((G > 0.79)\) [Fig. 3(c)].

(2) For the sample with \( \phi = 0.87 \) and \( L = 0.98 \) mm (Fig. 4), \( f_H \), \( 3f_H \), and \( f_{L1} \) are found at 0.94 MHz, 2.78 MHz, and 2.96 MHz, respectively for \( H = 2.31 \) mm \((G = 0.42)\) [Fig. 4(a)]. \( f_{L2} \) is not observed because of no nearby resonances but reappears at 1.92 MHz when the
sample is thinned down to 1.70 mm ($G = 0.58$) [Fig. 4(b)]. Clearly, the lateral mode resonances, including $f_{L1}$ and $f_{L2}$, have lower frequencies as compared to the previous batches, but $f_H$ is similar. It increases continuously in response to the thinning process and tends to couple with $f_{L2}$ at $H = 1.26$ mm ($G = 0.78$) [Fig. 4(c)].

(3) For the sample with $\phi = 0.89$ and $L = 1.19$ mm (Fig. 5), $f_H$ and $3f_H$ are clearly observed at 0.94 and 2.89 MHz when $H = 2.31$ mm ($G = 0.52$), and they increase with decreasing sample thickness. Both $f_{L1}$ and $f_{L2}$ remain quite stationary at lower frequencies of 2.42 MHz and
1.61 MHz, respectively. Also, merging of \( f_H \) with \( f_{L2} \) results at a comparatively larger thickness of \( H = 1.56 \text{ mm} \) (\( G = 0.76 \)) [Fig. 5(c)].

(4) For the sample with \( \phi = 0.91 \) and \( L = 1.50 \text{ mm} \) (Fig. 6), there are no major changes in \( f_H \) (0.94 MHz) and \( 3f_H \) (2.83 MHz) at \( H = 2.31 \text{ mm} \) (\( G = 0.65 \)). However, the operation bandwidth of \( f_H \) is even smaller since \( f_{L2} \) is detected at a frequency as low as 1.30 MHz and \( f_{L1} \) at 1.87 MHz. The corresponding sample thickness of \( f_H \) that starts to couple with \( f_{L2} \) is \( H < 2.01 \text{ mm} \) (\( G > 0.75 \)) [Fig. 6(b)].

(5) For the sample with \( \phi = 0.94 \) and \( L = 2.03 \text{ mm} \) (Fig. 7), mode coupling is apparent, and complex peaks are found even for sample as thick as 2.31 mm (\( G = 0.88 \)) [Fig. 7(a)]. Nevertheless, the mode-coupling theory still can provide a good guide for locating the position of \( f_H \), \( f_{L1} \), and \( f_{L2} \).

The observed \( f_H \), \( f_{L1} \), and \( f_{L2} \) for all composite samples at different thicknesses are plotted in Fig. 8 as the short- and open-circuit cases together with the mode-coupling theory predictions, and good agreements are obtained. The small offsets from the theoretical curves may be owed to the clamping effect imposed on the PZT elements by the epoxy matrix. At \( G > 0.8 \), significant mode coupling of \( f_H \) with \( f_{L1} \) and \( f_{L2} \) occurs, so the data exhibit some scattering. According to the predictions, \( f_H \) will decouple with \( f_{L1} \) and \( f_{L2} \) for \( G > 1.5 \). However, it is not the case in practice.
because the PZT used in the present study has strong lateral coupling. For ultrasonic wire bonding transducer applications, piezoelectric rings with thicknesses of less than 1 mm are seldom used as they are susceptible to breakage during fabrication. Therefore, \( f_L \) acts as an upper frequency bound for the operation of \( f_H \), in practice.

B. Cooperative resonances of composite rings

1. Radial and wall-thickness mode resonances

\( f_R \) and \( f_W \) are the two low frequency resonance modes in a thin ring resonator as shown in Figs. 2–7. The several weak resonances following \( f_R \) and \( f_W \) are their harmonics. They have frequencies approximately equal to an integral multiple of them. Due to the mechanical absorption of the epoxy matrix, both \( f_R \) and \( f_W \) become weaker in comparison with the pure PZT ring (Fig. 9). So, it is an advantage to use 1–3 composites instead of PZT in order to fabricate transducers with low lateral mode-coupling characteristics.

As shown in Fig. 10, the observed \( f_R \) and \( f_W \) taken at the series resonance frequency decrease slightly as the ring thickness increases but increase as \( \phi \) increases. The dependence of \( f_R \) and \( f_W \) on \( \phi \) elucidates the existence of different degrees of lateral clamping of the PZT elements by the epoxy matrix at different \( \phi \). If the composite ring thickness becomes comparable to its wall thickness (3.85 mm), \( f_H \) will couple with \( f_W \). Hence, \( f_W \) acts as a lower frequency bound, which limits the selection of the thickness of the ring.
Stopband resonances

The empirical resonance frequency equations proposed by Geng et al.\textsuperscript{18} were employed to estimate the two lateral stopband resonances $f_{S1}$ and $f_{S2}$. They are given as follows:

\[
f_{S1} = \frac{f_{S2}}{\sqrt{2}},
\]
\[
f_{S2} = \frac{v_s}{2d_p},
\]

where $f_{S2}$ can be viewed as a half-wave resonance in the epoxy gap width $d_p$ while $f_{S1}$ can be considered as another half-wave resonance across the diagonal direction $\sqrt{2}d_p$, and $v_s$ denotes the shear wave velocity in the epoxy.

Using $v_s = 1207\text{ m/s}$ for Araldite LY5210/HY2954 epoxy,\textsuperscript{13} $f_{S1}$ and $f_{S2}$ were calculated to be 5.54 MHz and 7.84 MHz, respectively. However, an investigation into the electrical impedance spectra for frequencies up to 10 MHz found no observable resonance peaks for all the samples studied. The reasons are probably that the PZT elements are much larger than the epoxy gap width and are spaced so closely that the possible reflection of shear waves is weakened and the resulting standing waves cannot be greatly enhanced in this limited epoxy gap width.

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VII. CONCLUSION

PKI804/Araldite LY5210/HY2954 1–3 piezocomposite rings with \( \phi \) varying from 0.82 to 0.94 have been fabricated, and their resonance characteristics have been studied theoretically and experimentally to elucidate the mode-coupling behavior. These composite rings have sufficiently small epoxy width (~77 \( \mu \)m) and can be treated as an effective homogeneous medium in which \( f_{S1} \) and \( f_{S2} \) are not observed. \( f_H \) has been found to increase linearly with the decrease in element height while \( f_{L1} \) and \( f_{L2} \) remained quite stable. \( f_R \) and \( f_W \) have been observed to be independent of ring thickness but increased as \( \phi \) increases. In designing a transducer for ultrasonic wire bonding, for example, in addition to optimizing the homogeneity, material properties, and manufacturing cost of the composite rings, one should also avoid other resonance modes being coupled to \( f_H \). This in turn requires a ring thickness not too thin in order to keep \( f_H \) smaller than all lateral modes of the PZT elements (\( G <0.8 \)) but not too thick to avoid coupling of \( f_H \) with \( f_W \) and \( f_R \) of the ring. Based on the current findings, an optimal set of composite rings has been obtained and used in the development of various ultrasonic wire bonding transducers with low mode coupling and wide bandwidth. Details are reported in separated articles.\(^{13–16}\)

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