Finding Energy-Efficient Paths on Uneven Terrains

Nuwan Ganganath†, Chi-Tsun Cheng, and Chi K. Tse
Department of Electronic and Information Engineering
The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
†Email: nuwan.marasinghearachchige@connect.polyu.hk

***Abstract*—Mobile robots are increasingly getting popular in outdoor applications. Long period of continuous operations are common in such applications. Therefore, robot motions need to be optimized to minimize their energy consumption. Shortest paths do not always guarantee minimum energy consumptions of mobile robots. This paper proposes a novel algorithm to generate energy-efficient paths on uneven terrains using an established energy-cost model for mobile robots. Terrains are represented using grid based elevation maps. Similar to A* algorithm, the energy-cost of traversing through a particular gird depends on a heuristic energy-cost estimation from the current location to the goal. The proposed heuristic energy-cost function makes it possible to generate zigzag-like path patterns on steep hills under the power limitations of the robot. Therefore, the proposed method can find physically feasible energy-efficient paths on any given terrain, provided that such paths exist. Simulation results presented in this paper demonstrate the performance of the proposed algorithm on uneven terrains maps.**

***Keywords*—Mobile robot, path planning, energy-efficient, uneven terrain, heuristic search.**

I. Introduction

Mobile robots are usually powered by portable energy sources, such as batteries. These sources are with limited capacities, which limits the operation duration of the mobile robots. The energy consumption is also related to the operation cost of the mobile robots. Therefore, minimizing the energy consumption is crucial in most mobile robot applications. In general, mobile robots energy consumption has two categories: electrical and mechanical [1]. The former considers the energy consumed by robot's sensors and processing units, such as microcontrollers and signal processing chips. Here, we focus on minimizing the mechanical energy consumption of mobile robots, which is mainly due to its motors and actuators. This can be achieved by using energy-efficient motors [2], [3], selecting energy-efficient velocity profiles [4]–[7], and choosing energy-efficient routes [1], [8].

In this paper, we consider wheeled mobile robots navigating on uneven terrains with the influences of friction and gravitational force. Unlike in indoor applications in which terrains are assumed to be flat [9]–[11], the energy consumption of mobile robots navigating on uneven terrains heavily depends on their routes. Therefore, our goal is to determine energy-efficient routes for these mobile robots. Despite the vast range of potential application areas, very few attentions have been devoted for energy-efficient path finding problem. This problem should not be confused with the shortest path finding problem on uneven terrains. Geodesic shortest paths often fail to capture the physical limitations imposed by the external environment and the robot itself, such as friction, gravity, stability on steep hills, and the maximum driving force of the robot.

A. Related Work

Early work on mobile robots path planning on uneven terrain maps can be found in [12]. In their work, the terrain is modeled as polygonalized isolines. The minimum-time trajectories of motion are calculated using the elevation changes between adjacent isolines. Later in [13], Rowe and Ross introduced an energy-cost model for mobile robots navigating in uneven terrains. In their model, cost of the traversal between two points is defined as the energy loss due to friction and gravity. They also considered the impermissible traversal directions due to overturn dangers and power limitations. The cost-optimal path is created by using A* search algorithm to pick appropriate path segments from path subspaces.

Lanthier *et al.* [14] introduced the terrain face weight concept, which captures the varied nature of the terrain, friction, and slope of each terrain face. They utilized Dijkstra's algorithm to path with minimum total weight in a graph. Using the terrain face weight concept introduced in [14], Sun and Reif [15] proposed an energy minimizing path planning method on uneven terrains. They used BUSHWHACK discrete search algorithm [16] to compute the optimal path in a given graph. The terrain face concept not only reduces the computational complexity, but also degrades the accuracy of the generated paths as it somehow approximates the uneven ground level with flat surfaces. Spero and Javis [17] introduced a mobile robot path planning method on elevation maps using rapidly-exploring random trees. Their proposed navigation system for controlling a nonholonomic mobile robot in unknown outdoor terrains only considers two type of occupancy on elevation maps: free or occupied.

With recent advancements in technology, processing units are getting smaller, cheaper, and with higher computing power. Such processing units are commonly used on mobile robots. Their improved computing capability has enabled the real-time processing of high resolution grid based maps. By taking the advantage of such technologies, Choi *et al.* [18] recently presented a mobile robot path planning method for grid based elevation maps using A* search algorithm. Due to the limitations of the heuristic function they used, their algorithm is unable to find paths on steep hills as robot cannot climb straightly. According to Choi *et al.* [18], A* search algorithm is inadequate for path planning in elevation maps. Nevertheless, elevations in uneven terrains have not been completely considered in the previous works and need to be further investigated.

B. Contributions and Organization of the Paper

In this paper, we represent the terrain using grid based elevation maps. It considers the problem of finding physically feasible paths on uneven terrains for mobile robots, which can minimize the energy expenditures due to both gravity and friction. The path planning algorithm proposed in this paper is inspired by A* search algorithm. A new heuristic function is proposed and used to estimate the energy cost for traveling from the current location to the destination. Notably, the heuristic function here enables the proposed method to generate *zigzag*-like paths to overcome the impermissible traversal headings due to power limitations of a mobile robot.

The paper is organized as follows. Section II elaborates the formulations of the terrain landscapes and briefly discusses the energy-cost model used in this paper. The proposed algorithm for energy-efficient path planning is proposed in Section III. Simulation settings are presented in Section IV. Simulation results are presented and performance of the proposed algorithm is analyzed in Section V. Concluding remarks are given in Section VI.

II. Problem Formulation

In this paper, path planning problem is defined as finding a path between arbitrary starting location (n_s) and arbitrary goal location (n_g) such that the energy cost of traversal is minimized. Here, we assume the robot to be holonomic and rigid with no suspension compliance. Furthermore, it is represented as a point on a given map.

A. Terrain Representation

Two different terrain maps are considered in this paper in order to represent different scenarios. The terrains under study are meshed three dimensional (3D) models mimicking hilly landscapes. The two models can be expressed using the following formulas:

$$\text{Model 1}: \quad z(x,y) = 3.79[\sin(\tfrac{y}{3\pi}+0.5) - 2\sin(\tfrac{y}{3\pi}) + 1.3\cos(\tfrac{x}{3\pi}) - 0.3\sin(3\sqrt{(\tfrac{x}{2\pi})^2+(\tfrac{y}{2\pi})^2})]^2, \tag{1}$$

$$\text{Model 2}: \quad z(x,y) = 4.81[1.5\cos(\tfrac{x}{4\pi}) + 0.5\sin(\tfrac{y}{4\pi}) - 0.5\sin(2.5\sqrt{(\tfrac{x}{4\pi})^2+(\tfrac{y}{4\pi})^2})]^2. \tag{2}$$

Here, $z(x,y)$ is the elevation of the terrain for a grid centered at (x,y). Illustrations of Model 1 and Model 2 are shown in Fig. 4 and Fig. 6, respectively. The base of the terrain is defined as a 100×100 m^2 square-shaped grid map. Similar terrain representations have been previously used in mobile robot path planning applications [19]–[21].

B. Energy-Cost Model

In this work, we use the energy-cost model developed by Rowe and Ross [13]. Let n_c be the current location (grid) of the robot in a given grid map, and n_n be a neighboring grid which the robot will move to in next time step. The length of the projection of the straight line connecting the centers of n_c and n_n on the x-y plane can be defined as

$$d(n_c,n_n) = \sqrt{(n_c.x - n_n.x)^2 + (n_c.y - n_n.y)^2}, \tag{3}$$

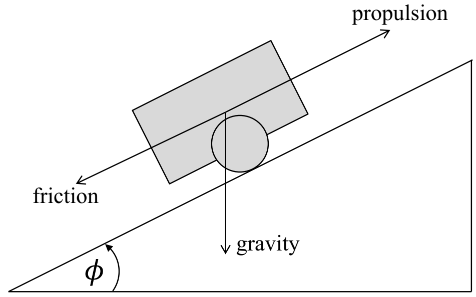

Fig. 1: Physical model of the mobile robot.

where $(n_c.x, n_c.y)$ and $(n_n.x, n_n.y)$ are coordinates of the grids n_c and n_n, respectively. Let,

$$\Delta(n_c,n_n) = z(n_n.x, n_n.y) - z(n_c.x, n_c.y) \tag{4}$$

be the elevation difference between these two grids. Therefore, the Euclidean distance s between n_c and n_n in a 3D space can be defined as

$$s(n_c,n_n) = \sqrt{d(n_c,n_n)^2 + \Delta(n_c,n_n)^2}, \tag{5}$$

and the angle of inclination ϕ (positive for uphilling, negative for downhilling) as

$$\phi(n_c,n_n) = \arctan\left[\frac{\Delta(n_c,n_n)}{d(n_c,n_n)}\right]. \tag{6}$$

Assuming a constant velocity v (no acceleration) for the entire traversal, two major external forces applying on the robot are gravity and friction (see Fig. 1) [13]. Sum of the two forces can be given as $F = mg(\mu\ \cos\phi + \sin\phi)$, where m is the mass of the robot, μ is the friction coefficient, and g is the gravitational field strength. This formula is well adopted in [13]–[15] and has been confirmed experimentally within 1% for wheeled vehicles on slopes of less than 20% in [22]. Therefore, the energy cost for traversing distance s can be defined as

$$E = mgs(\mu\ \cos\phi + \sin\phi). \tag{7}$$

This model assumes no energy cost for making turns.

Rowe and Ross [13] add anisotropism to their model by considering the impermissible travel directions due to power limitations or overturn dangers. In the uphill traversal, the robot may fail to climb steep inclination due to its constrained force expenditure. For the physical model considered in this paper, the inclined angle of the robot cannot exceed $\phi_f = \arcsin(F_{\max}/mg\sqrt{\mu^2+1}) - \arctan(\mu)$. The maximum force available to overcome gravity and friction ($F_{\max}$) can be obtained by $P_{\max}/v$, where $P_{\max}$ is the maximum available motion power of the robot. This has been experimentally confirmed within 2% for wheeled vehicles on shallow slopes in [22]. Again in an uphill traversal, the robot may face a danger of losing traction which is governed by the static friction coefficient μ_s of the surface. It can be proved that the anisotropic traction-loss phenomena will arise if the inclined angle is greater than $\phi_s = \arctan(\mu_s - \mu)$ [13]. Considering aforementioned scenarios, the critical impermissible angle for the uphill traversal can be defined as $\phi_m = \min(\phi_f, \phi_s)$, which is the maximum inclined angle that the robot is capable of overcoming.

For a downhill traversal, when $F = 0$, $\phi = \phi_b$, which is defined as critical breaking angle. Using simple mathematics, it can be shown that $\phi_b = -\arctan(\mu)$. A special scenario can be observed when $\phi < \phi_b$, as $mg(\mu \ \cos\phi + \sin\phi) < 0$. That means, the robot starts to gain energy and accelerate. However, since Rowe and Ross model assumes the robot moves at constant velocity v, it has to apply breaking force to avoid acceleration. Generally, breaking requires negligible energy [13]. Therefore, it is assumed that the robot consumes no energy when it is traveling in the breaking region ($\phi \leq \phi_b$). Another special situation, the catastrophic overturn of the mobile robot, may occur while moving perpendicular to an inclined plane. However, we neglect such possibility by assuming that the robot avoids traversing sideslopes with more than certain steepness. This also depends on the design of the mobile robot, especially on the location of the center of gravity of the robot. For further details, please refer [13].

III. Proposed Path Planner

In this section, we introduce the proposed path planner for uneven terrains. The objective of the proposed search algorithm is to find an energy-efficient path from the start grid n_s to the goal grid n_g. The main routine of the proposed algorithm is explained in Algorithm 1. There are three other sub-routines in this algorithm: Algorithm 3 calculates the energy-cost for traversing from the current grid n_c to a given neighboring grid n_n using the energy-cost model described in Section II-B. Algorithm 4 estimates the expected energy-cost for traversing from n_n to n_g. Algorithm 2 recursively construct the path from n_g to n_s at the end of the exploration.

A. Process

Similar to A* algorithm, the proposed algorithm also uses best-first search to traverse the graph and follows the lowest expected energy-cost. The expected energy-cost of traversing to n_g through n_c is defined as

$$f(n_c) = g(n_c) + h(n_c), \tag{8}$$

where $g(n_c)$ is a calculated energy-cost of traveling from n_s to n_c, and $h(n_c)$ is a heuristic estimate of the energy-cost of traveling from n_c to n_g.

It starts with pushing n_s to an OPEN set (Algo. 1, Line 2) and obviously $f(n_s) = h(n_s)$ since $g(n_s) = 0$ (Algo. 1, Line 3). Once a grid is explored, it will be removed from the OPEN set (Algo. 1, Line 11) and all of its neighbors which are not already in the OPEN set will be pushed to the OPEN set unless the current grid is the goal grid (Algo. 1, Line 8), or the expected energy-cost of traveling through the current grid is an infinitely large value (Algo. 1, Line 6) (this is equivalent to the isotropic obstacle phenomena described in [13]). Furthermore, it updates already calculated f cost values if later routes returns a smaller value than the existing one. In the next iteration, the node in the OPEN set with the lowest f cost value will be explored (Algo. 1, Line 5). This idea can be easily implemented using a priority queue. The proposed algorithm does not borrow the idea of CLOSED set from A* [18]. A*-like heuristic search algorithms are guaranteed to find an optimal solution with a CLOSED set only if heuristics are consistent. Therefore, the absence of the CLOSED set in proposed algorithm enables it to revisit already explored nodes and ultimately helps to find an energy-efficient path. There is no possibility of an infinite loop due to node revisits as it happens only if the previous cost can be improved.

Algorithm 1: Pseudocode of proposed search algorithm

```
1: function PATH_FINDER(n_s, n_g)
2:     OPEN ← {n_s}
3:     f_cost[n_s] ← CALCULATE_H_COST(n_s, n_g)
4:     while OPEN ≠ ∅ do
5:         n_c ← argmin_{n∈OPEN} f_cost[n]
6:         if f_cost[n_c] == ∞ then
7:             return failure
8:         else if n_c == n_g then
9:             return CONSTRUCT_PATH(previous, n_g)
10:        end if
11:        remove n_c from OPEN set
12:        for each neighbor n_n of n_c do
13:            g_cost_temp ← g_cost[n_c]+
               CALCULATE_COST(n_c, n_n)
14:            f_cost_temp ← g_cost_temp+
               CALCULATE_H_COST(n_n, n_g)
15:            if f_cost[n_n] is undefined  or
               f_cost_temp < f_cost[n_n] then
16:                previous[n_n] ← n_c
17:                g_cost[n_n] ← g_cost_temp
18:                f_cost[n_n] ← f_cost_temp
19:                if n_n ∉ OPEN then
20:                    add n_n to OPEN set
21:                end if
22:            end if
23:        end for
24:    end while
25:    return failure
26: end function
```

Algorithm 2: Pseudocode of CONSTRUCT_PATH function

```
function CONSTRUCT_PATH(previous, n_i)
    if previous[n_i] is defined then
        p ← CONSTRUCT_PATH(previous, previous[n_i])
        return (p + n_i)
    else
        return n_i
    end if
end function
```

B. Estimating the Heuristic Energy-Cost

In the A* implementation described in [18], the $h(n_c)$ is calculated using a similar method as described in Algorithm 3 by connecting n_n and n_g with a virtual straight line. Therefore, $h(n_c)$ can sometime be infinitely large depending on the gradient of this straight line with respect to the x-y plane. Since the value of $h(n_c)$ will eventually affect the value of $f(n_c)$, such situations can result in false impermissible traversal headings. Even though the robot is unable to travel on a straight line from one point to another if $\phi > \phi_m$, it may still reach the target by following a series of zigzag movements as illustrated in Fig. 2. The headings in the zigzag pattern is permissible if $\phi \leq \phi_m$. The proposed method of estimating the

Algorithm 3: Pseudocode of CALCULATE_COST function

function CALCULATE_COST(n_i, n_j)
  calculate $\phi(n_i, n_j)$ using Equation (6)
  if $\phi(n_i, n_j) > \phi_m$ **then**
    return ∞
  else if $\phi(n_i, n_j) <= \phi_m$ **and** $\phi(n_i, n_j) > \phi_b$ **then**
    calculate E using Equation (7)
    return E
  else
    return 0
  end if
end function

Algorithm 4: Pseudocode of CALCULATE_H_COST function

function CALCULATE_H_COST(n_n, n_g)
  $E_h(n_n, n_g) \leftarrow$ CALCULATE_COST(n_n, n_g)
  if $E_h(n_n, n_g) \neq \infty$ **then**
    return $E_h(n_n, n_g)$
  else
    calculate $E_h(n_n, n_g)$ using Equation (9)
    return $E_h(n_n, n_g)$
  end if
end function

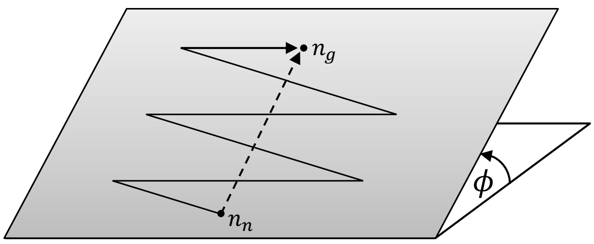

Fig. 2: The robot is unable to climb straightly from n_n to n_g on a steep surface (dashed line). However, it may be able to reach n_g by following a series of zigzag movements. Here, $\phi = \phi(n_n, n_g) > \phi_m$.

heuristic energy-cost (E_h) is developed based on such zigzag movements and presented in Algorithm 4. The example in Fig. 3 illustrates how to obtain the mathematical formulation of E_h for a false impermissible heading described earlier. Since $\phi(n_n, n_g) > \phi_m$, it is impossible to estimate a finite value for $E_h(n_n, n_g)$ using the regular method. Hence, we propose a new path to n_g via n_i for obtaining a finite value of $E_h(n_n, n_g)$ which results in a zigzag-like path pattern.

Let n_j be a point on the x-y plane which goes through n_n, such that $\angle n_g n_n n_j = \pi/2$ and $\angle \acute{n}_g n_j n_g = \phi_m$. Here, $\acute{n}_g$ is the projection of n_g on the same x-y plane. Therefore, $\angle \acute{n}_g n_n n_j = \pi/2$ as well. Let n_i be the middle point of the straight line connecting n_g and n_j. Using simple geometry, it is possible to show that $s(n_i, n_g) = s(n_j, n_i) = s(n_n, n_i)$ and $\phi(n_n, n_i) = \phi_m$. In Fig. 3, $\acute{n}_i$ is the projection of n_i on the x-y plane which goes through n_n, n_j, and $\acute{n}_g$. The total heuristic energy-cost is equal to the summation of energy expenditure of traversing $n_n n_i$ and $n_i n_g$. Note that $s(n_j, n_i) = s(n_n, n_i)$ and $\phi(n_n, n_i) = \phi(n_j, n_i) = \phi_m$. Hence, we can define $E_h(n_n, n_g)$ be the energy-cost of traversing $n_j n_i n_g$. By definition, $s(n_g, \acute{n}_g) = \Delta(n_n, n_g)$ and

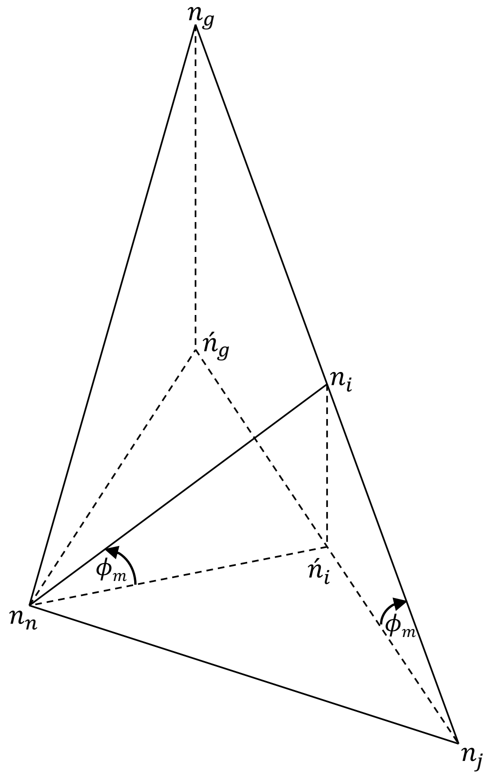

Fig. 3: It is unable to estimate the heuristic energy-cost of the $n_n n_g$ traversal since $\phi(n_n, n_g) > \phi_m$. Therefore, the proposed method selects a heading for the robot such that $\phi = \phi_m$, which results in a finite heuristic energy-cost.

$s(n_j, n_g) = \Delta(n_n, n_g)/\sin\phi_m$. Using (7), $E_h(n_n, n_g)$ can be derived as

$$E_h(n_n, n_g) = \frac{mg(\mu\ \cos\phi_m + \sin\phi_m)\Delta(n_n, n_g)}{\sin\phi_m}. \quad (9)$$

It should be noted that (9) is used for estimating the heuristic energy-cost only when the regular method fails to do so. Details are given in Algorithm 4.

IV. Simulations

In this section, the proposed algorithm is tested and evaluated against the energy-efficient path planning algorithm proposed by Choi *et al.* [18] (we denote it as A*-E_{opt} and it employs the energy-cost model proposed in Section II-B) and the Dijkstra's algorithm. The original distance-cost [23] in Dijkstra's algorithm is replaced with the 3D Euclidean distance in order to obtain the shortest path on the terrains under study. Paths generated from the Dijkstra's algorithm are aimed to illustrate the differences between shortest paths and energy-efficient paths. Simulations were conducted in MATLAB using the terrain models introduced in Section II-A.

In the simulations, we assume that the mass of the robot m = 22 kg, velocity v = 0.35 ms^{-1}, and maximum motion power $P_{\max}$ = 72 W. The friction coefficients between the terrain and the robot wheels are taken as $\mu = 0.01$ and μ_s = 1.0 [18]. The gravitational field strength (g) is assumed to be 9.81 ms^{-2}. Four different situations (I-IV) were setup using the two terrain models and simulation parameters are shown in Table I.

TABLE I: SIMULATION PARAMETERS AND RESULTS.

Setup	Terrain model	n_s (m)	n_g (m)	Energy-cost (J)			Path length (m)		
				Dijkstra	A*-E$_{\text{opt}}$	Proposed algorithm	Dijkstra	A*-E$_{\text{opt}}$	Proposed algorithm
I	Model 1	(4,75)	(53,12)	617.49	202.39	202.39	83.53	99.56	99.56
II	Model 1	(5,43)	(92,51)	N/A	221.63	221.63	107.33	138.25	137.59
III	Model 2	(20,10)	(78,88)	N/A	N/A	6674.33	123.71	N/A	152.84
IV	Model 2	(82,25)	(4,85)	N/A	N/A	5893.9	130.88	N/A	238.93

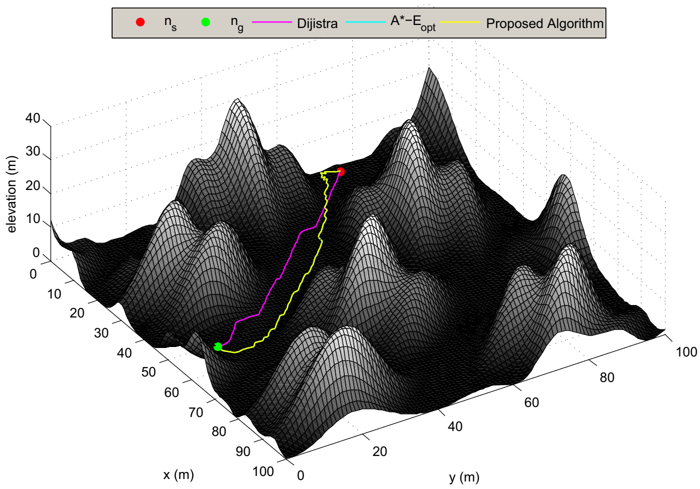

Fig. 4: Paths generated in Setup I.

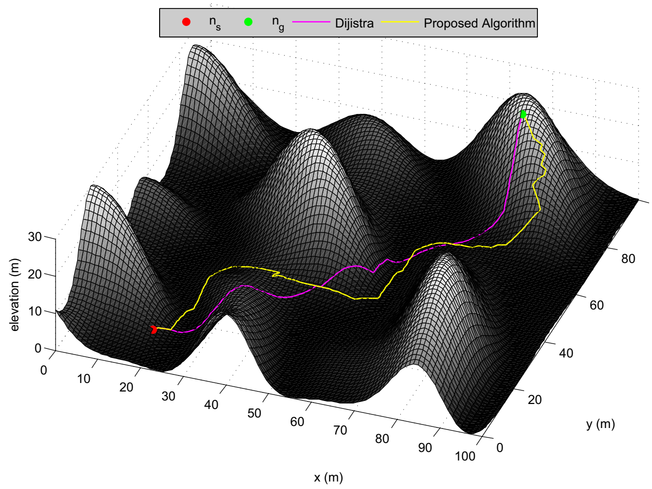

Fig. 6: Paths generated in Setup III.

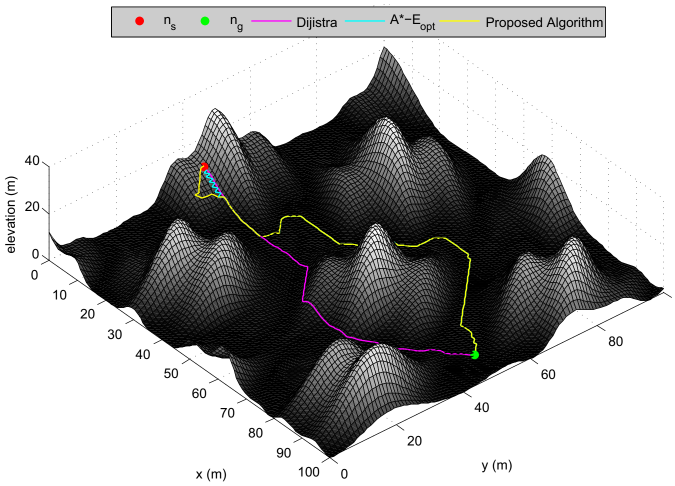

Fig. 5: Paths generated in Setup II.

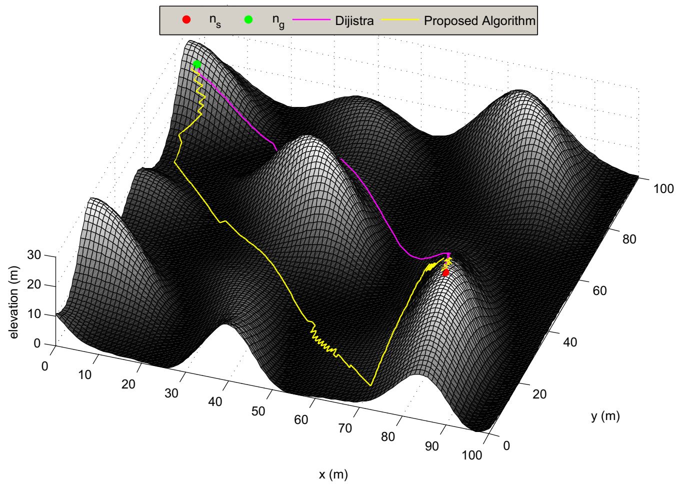

Fig. 7: Paths generated in Setup IV.

V. RESULTS AND PERFORMANCE ANALYSIS

In the first setup, both n_s and n_g are located on comparatively lower elevations in terrain model 1. Paths generated from the algorithms under test are shown in Fig. 4. As expected, the minimum energy-cost paths generated using both A*-E$_{\text{opt}}$ and the proposed algorithm are coincide with each other. The results given in the Table I further clarify it. The shortest path generated using Dijkstra's algorithm is clearly distinguishable from other two paths. According to Table I, it is shorter than the other two. However, as our objective is to obtain the minimum energy-cost path, the results clearly show that the energy-cost of the shortest path is much higher than the minimum energy-cost paths.

In the second setup, n_g is placed on a lower elevation and n_s is on a comparatively higher elevation in terrain model 1. Paths generated from the algorithms under test are shown in Fig. 5. The minimum energy-cost paths generated using A*-E$_{\text{opt}}$ and proposed algorithm are slightly different from each other, especially at the downhill close to n_s. This is due to the fact that the robot consumes no energy when $\phi(n_c, n_n) < \phi_b$. If there is more than one such n_n, the algorithm arbitrarily select one of them, which may results in different routes, but associated with the same energy-cost. This can be verified using the results given in the Table I. Similar to the previous case, the shortest path generated by the Dijkstra's algorithm is significantly different from the other two paths. However, such path is practically impermissible on the given terrain model due to the motion power limitations of the robot. Therefore, the energy-cost of the shortest path is incalculable.

Setups III and IV were carried out using terrain model 2. In the third setup, n_s is placed on a lower elevation and n_g is on a comparatively higher elevation. In the fourth setup, both n_s and n_g are placed on comparatively higher elevations among local peaks. In both simulations, A*-$\mathrm{E}_{\mathrm{opt}}$ is unable to find a path from n_s to n_g due to false impermissible headings resulted in the heuristic energy estimation. The minimum energy-cost paths generated by proposed algorithm and the Dijkstra's shortest paths are shown in Figs. 6 and 7. According to the illustrated results of these two simulations, the proposed method has utilized zigzag-like path patterns to climb steep hills in the terrain. Such paths are facilitated by the novel heuristic energy-cost function proposed in this paper, which helps to overcome the power limitations of the robot in uphilling. Therefore, the proposed method can find a minimum-energy path virtually in any situation as long as a practical path exists. Performances of all grid based path planning methods are discussed and evaluated in given discrete domains. All these algorithms, including the proposed algorithm, can achieve comparatively better results which are closer to the optimum in real-world continuous domain, with higher resolution grid maps.

VI. Conclusion

In this paper, we present an algorithm for energy-efficient path planning on uneven terrains. Traditional path planning methods cannot find physically feasible paths on due to instability and motion power limitations of robots on steep terrains. We represent the uneven terrains using grid based elevation maps. The proposed algorithm is inspired by A* heuristic search algorithm. The cost of traversing from a start point to a goal point through a chain of connected nodes is calculated as the summation of the energy-cost of traveling to the current node and the heuristic energy-cost of traveling from the current node to the goal. Using the novel heuristic energy-cost function used in this paper, the proposed approach can find energy-efficient paths on such terrains using zigzag-like path patterns which is feasible under mobile robot's motion power constraints. Its energy-efficient nature prolongs the operation duration of mobile robots, which is very useful in many practical applications.

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