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Constructal design for pedestrian movement in living spaces: Evacuation configurations

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Here we show that the configuration of an inhabited area controls the time required by all the pedestrians to vacate the space. From the minimization of the global evacuation time emerges the optimal configuration of the area. This is a fundamental principle for designing living spaces with efficient evacuation quality, and it is demonstrated here with several simple building blocks that can be used as components of more complex living structures: single walkway, corner, and T-shaped walkway. We show analytically and numerically that the ratio of the widths of the stem and branches of the T-shaped walkway has an optimal value that facilitates the evacuation of all the inhabitants. This result is fundamental, and is the crowd-dynamics equivalent of the Hess-Murray rule for the ratio of diameters in bifurcated ducts with fluid flow. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.3689771]

I. THE PHYSICS OF EVACUATION OF LIVING SPACES

Pedestrian movement is the most basic physics aspect of human life, from locomotion on the landscape to modern architecture, engineering and traffic. This is a subject that has generated a significant body of empirical work in social dynamics and animal locomotion.[1–7] In this paper, we propose a principle-based physics approach to the prediction of pedestrian movement. We also develop an applied physics method for designing the shape and structure of living spaces such that they facilitate the movement of pedestrians.

Designs that facilitate pedestrian movement are essential in all domains of human activity. They are absolutely critical in emergency situations (e.g., fire, explosions, accidents, terrorism, tornados, tsunamis), where the fast *evacuation* of the population is the chief concern.[8] Presently, the evacuation plans for living spaces are based on numerical simulations of crowd dynamics. The numerical codes range from fluid dynamics analogies[1–3] to reliance on cognitive science.[6]

The method proposed in this paper relies on the constructal law, which states that the designs of all flow systems evolve in time toward configurations that provide easier access to their currents.[9,10] The evacuation of pedestrians from an inhabited space is one such flow system, and the designing of better and better configurations for evacuation is an evolutionary design that can be aided (i.e., fast forwarded) based on constructal theory.

The method consists of discovering the configurations that tend to reduce the time needed for full or partial evacuation. The work is about discovering the relationship between the configuration of the living space and the evacuation time. We develop the method in a modular sequence that proceeds from the simplest building blocks (e.g., one straight walkway, one corner) toward more complex structures (e.g., one bifurcated walkway). The design value of this article is that more complex configurations can be designed for efficient evacuations by using fundamental building blocks of the kind treated in this paper.

II. PEDESTRIAN SPEED AND SPACING

The objective of the method is to determine the relationship between the geometry of the living space and the time needed to evacuate a finite number of inhabitants from the space. The ultimate objective is to identify the geometry that facilitates the evacuation.

Here we take the approach that the time needed for evacuation is governed not only by the geometry of the inhabited space but also by the path selected by every inhabitant in that space. The inhabitant has complete freedom to choose the path: in the following analysis we assume that every inhabitant selects the fastest escape route.

Assume that all the movement is pedestrian. The obstacles that every pedestrian must avoid are two: the rigid walls that define the living space, and the neighboring inhabitants. The monotonic relation between the walking speed of a person and the average person-to-person distance is shown in Fig. 1, where the walking speed data[2,3] can be correlated adequately with the function

$$\frac{V}{V_\infty} = C\left(1 - \frac{S_0}{S}\right)^{1/2}, \tag{1}$$

where $V_\infty \cong 1.3\,\mathrm{m/s}$ is the walking speed in the limit of sparse populations ($S \gg S_0$) and $S_0 \cong 0.5\,\mathrm{m}$ is the cutoff spacing below which the population is so dense that it stops moving. The constant C is dimensionless and of order 1: for example, noting that in Fig. 1 the speed V is roughly 1 m/s

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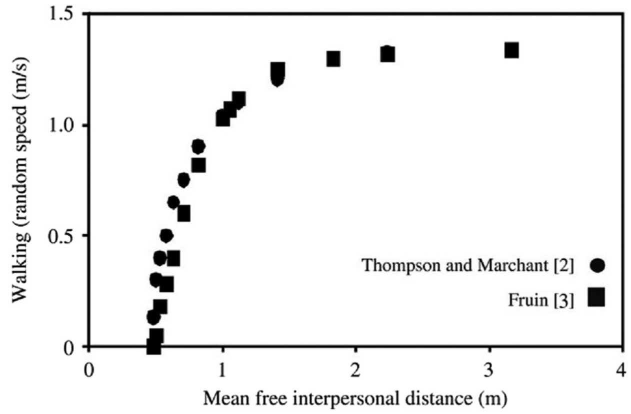

FIG. 1. The average pedestrian speed V vs the average person-to-person spacing S.[1]

when $S = 1$ m, we find that Eq. (1) fits the data near the knee of the curve.

In sum, the moving crowd moves faster when the interpersonal distances are of order 1 m or greater. This means that the moving population must spread itself over the available space, and must cover the space with sparse traces of minimal travel time so that the entire space is evacuated fast. In this study we pursue this human dynamics design as a sequence of fundamental constructal design problems concerning simple (building block) configurations.

III. LONG WALKWAY

Consider first a long walkway with unspecified variable width $W(x)$ and length L (Fig. 2). The width is considerably smaller than the length. The total floor area represents the size of the space, and it is fixed,

$$A = \int_0^L W(x)dx. \tag{2}$$

A stream of pedestrians moves from $x = 0$ to $x = L$, as shown in Fig. 2. Because the number of pedestrians who pass through every constant-x plane is the same for any x, then the interpersonal spacing varies as $W(x)$,

$$S(x) = \frac{1}{n}W(x), \tag{3}$$

where n is the number of pedestrians who fit transversally, within $W(x)$. Equations (1) and (3) establish $V(x)$ as a function that depends on x via $S(x)$ and $W(x)$,

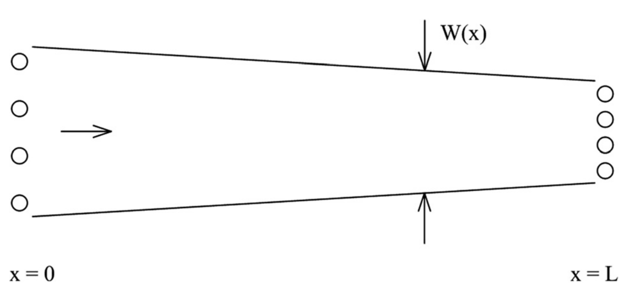

FIG. 2. Slender walkway with arbitrary width $W(x)$ and fixed length L.

$$V(S) = V\left[\frac{1}{n}W(x)\right]. \tag{4}$$

The time of travel between $x = 0$ and $x = L$ is

$$t = \int_0^L \frac{dx}{V(x)}. \tag{5}$$

According to the method of variational calculus,[4] to minimize this integral by selecting $W(x)$ subject to the constraint (2) is equivalent to minimizing the aggregate integral formed as a linear combination of the integrands of integrals (2) and (5),

$$\Phi = \int_0^L \left(\frac{1}{V} + \lambda W\right)dx. \tag{6}$$

The solution for the optimal walkway width is obtained by solving

$$\frac{\partial F}{\partial W} = 0, \tag{7}$$

where F is the integrand of (Φ). We obtain

$$-\frac{1}{V^2}\frac{\partial V}{\partial S}\frac{\partial S}{\partial W} + \lambda = 0, \tag{8}$$

where $\partial S/\partial W$ is a constant equal to $1/n$ and, according to Fig. 1, $\partial V/\partial S$ is a monotonic function of S or V, for example $f(V)$. The conclusion from Eq. (8) is that $V^{-2}f(V) = \text{constant}$, and this means that V must have one value (independent of x). This also means that the spacing S and walkway width W must be uniform, independent of x. The best walkway is the walkway with constant width.

IV. TURNING A CORNER

Next, consider the design of a walkway that must make a 90 deg turn. To save time, every pedestrian is tempted to cut the corner, but this tends to increase the pedestrian density in the turn region, and to decrease the speed of the entire flow. How should the pedestrians space themselves in the turn region so that the crowd moves most easily?

In Fig. 3, the pedestrian with the shortest path is the one who walks along the wall, P_0. This pedestrian and those behind him pass through the sharp corner. It is relative to the sharp corner that the spacing S_1 between P_0 and the closest pedestrian (P_1) is measured.

The distance traveled by P_1 around the turn is $(\pi/2)S_1$. The speed V_1 (S_1) is given by Eq. (1). The travel time $t_1 = (\pi/2)S_1/V_1$ is minimal when

$$\sigma_1 = \frac{S_1}{S_0} = \frac{3}{2}, \tag{9}$$

$$t_1 = \frac{\pi S_0}{2CV_\infty}\frac{\sigma_1}{(1-\sigma_1^{-1})^{1/2}}. \tag{10}$$

Similarly, the distance traveled by the next (outer) pedestrian P_2 is $(\pi/2)(S_1 + S_2)$, and the speed is V_2 (S_2) follows from Eq. (1). The travel time of P_2 is minimal when

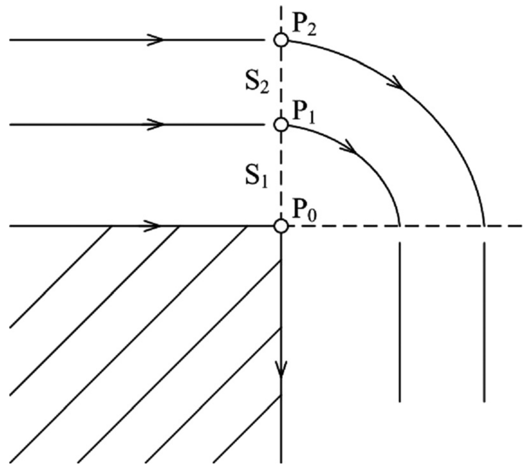

FIG. 3. Round paths followed by strings of pedestrians around a 90-deg turn.

$$\sigma_2 = \frac{S_2}{S_0} = \frac{3 + 21^{1/2}}{4} = 1.896, \tag{11}$$

$$t_2 = \frac{\pi S_0}{2CV_\infty} \frac{\sigma_1 + \sigma_2}{(1 - \sigma_2^{-1})}. \tag{12}$$

The next pedestrians $P_i (i \geq 3)$ walk the fastest when their spacings $\sigma_i = S_i/S_0$ are given by the recurrence formula

$$\sigma_1 = \frac{1}{4}\left[3 + (9 + 8\sum_{k=1}^{i-1} \sigma_k)^{1/2}\right], \tag{13}$$

which validates Eqs. (9) and (11) for $i = 1$ and 2. The corresponding minimal travel times are

$$t_1 = \frac{\pi S_0}{2CV_\infty} \frac{\sum_{k=1}^{i} \sigma_k}{(1 - \sigma_i^{-1})^{1/2}}. \tag{14}$$

The spacings and times are summarized in Table I. Proceeding away from the sharp corner, the person-to-person spacings, walking times, and speeds increase.

V. BIFURCATED WALKWAY

Another basic configuration is the bifurcated walkway shown in Fig. 4. This configuration is a combination of the straight walkways (Sec. III) and the movement around corners (Sec. IV). The pedestrian flow from the walkway of length L_1 and width W_1 is divided equally into two streams, each stream proceeding along a walkway of length L_2 and width W_2. The direction of pedestrian flow can be either from stem (L_1) to branches (L_2), or from branches to stem. The objective of the design is to provide the shortest time for travel through the T-shaped construct,

TABLE I. The optimal interpersonal spacings and minimal times for walking around the 90-deg turn shown in Fig. 3.

Pedestrians	$\sigma_i = \frac{S_i}{S_0}$	$t_i = \frac{2CV_\infty}{\pi S_0}$
P_1	1.5	2.60
P_2	1.90	4.94
P_3	1.97	8.14
P_4	2.55	10.76

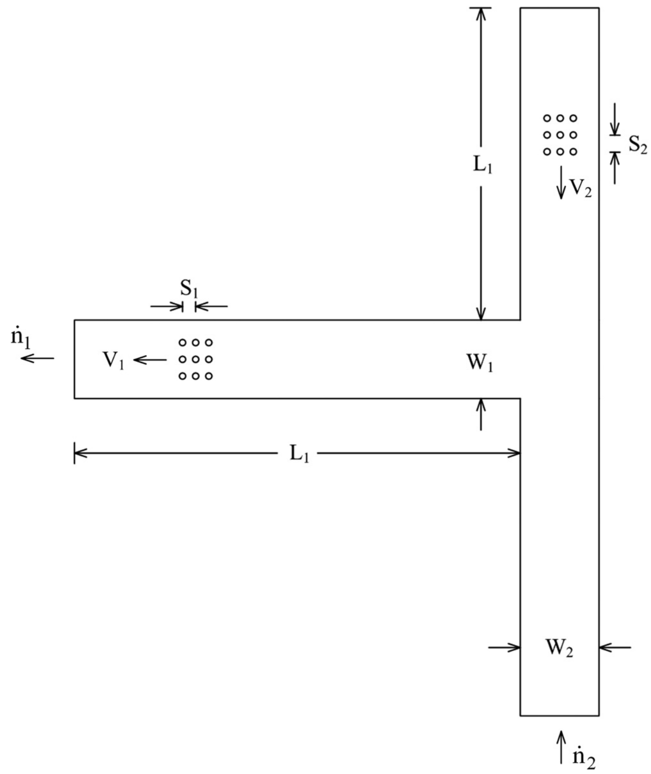

FIG. 4. T-shaped walkway with uniformly spaced pedestrian flow.

$$t = \frac{L_1}{V_1} + \frac{L_2}{V_2} \tag{15}$$

subject two constraints, total walkway surface (assuming $W_1 \times W_2$ is negligible),

$$A_w = L_1 W_1 + 2L_2 W_2 \tag{16}$$

and total rectangular territory inhabited by the T-shaped construct,

$$A_T = 2L_2 L_1. \tag{17}$$

The speeds along the stem (V_1) and branches (V_2) depend on the spacings between pedestrians, in accord with Eq. (1),

$$V_1 = CV_\infty \left(1 - \frac{S_0}{S_1}\right)^{1/2}, \tag{18}$$

$$V_2 = CV_\infty \left(1 - \frac{S_0}{S_2}\right)^{1/2}. \tag{19}$$

The spacings depend on the respective widths of the walkways,

$$S_1 = \frac{W_1}{n_1}, \quad S_2 = \frac{W_2}{n_2}, \tag{20}$$

where n_1 and n_2 are the numbers of pedestrians across each walkway. As shown in Fig. 4, we assume that the distance between pedestrians during walking is on an average S_1. The flow rate of pedestrians along the stem is

$$\dot{n_1} = \frac{n_1}{\Delta t_1} = n_1 \frac{V_1}{S_1}, \tag{21}$$

where $\Delta t_1 = S_1/V_1$ is the time step between successive rows on n_1 pedestrians. Similarly, the flow rate of pedestrians along one of the branches is

$$\dot{n_2} = \frac{n_2}{\Delta t_2} = n_2 \frac{V_2}{S_2}. \tag{22}$$

The conservation of pedestrian flow rate at the bifurcation requires

$$\dot{n_1} = 2\dot{n_2} \tag{23}$$

which, in view of Eqs. (20)–(22) yields

$$2 = \frac{W_1}{W_2}\left(\frac{S_2}{S_1}\right)^2 \frac{V_1}{V_2}. \tag{24}$$

To summarize, the objective is to minimize Eq. (15) subject to constraints (16) and (17), and the fact that V_1/V_2 and W_1/W_2 are related via Eq. (24). To make analytical progress, we use Eqs. (18) and (19) to write

$$\frac{V_1}{V_2} = \left(\frac{1-\alpha}{1-\alpha\beta}\right)^{1/2}, \tag{25}$$

where α and β are two ratios that account for the spacings S_1 and S_2, which vary,

$$\alpha = \frac{S_0}{S_1} < 1, \quad \beta = \frac{S_1}{S_2} \sim 1\,. \tag{26}$$

With this notation, Eq. (24) becomes

$$2\beta^2 = \frac{W_1}{W_2}\left(\frac{1-\alpha}{1-\alpha\beta}\right)^{1/2} \cong \frac{W_1}{W_2}. \tag{27}$$

We start by assuming that α is fixed, which means that V_1 is fixed. Then, to minimize t means to maximize V_2, or to minimize β and W_1/W_2, cf. Eq. (27). Next, to minimize W_1/W_2 subject to the walkway surface constraint (16) is the same as seeking the extremum of the function

$$\phi = \frac{W_1}{W_2} + \mu(L_1W_1 + 2L_2W_2), \tag{28}$$

where μ is a Lagrange multiplier. The solution obtained with the method of undetermined coefficients (Ref. 4, pp. 493 – 495) is

$$\frac{W_1}{W_2} = 2\frac{L_2}{L_1}. \tag{29}$$

This result means that the surface of the stem (L_1W_1) must equal the combined surface of the two branches ($2L_2W_2$), or that A_w must be divided equally into one stem and two branches.

The next challenge is to determine L_1/L_2. We continue with the assumption that V_1 is fixed because α is fixed. Consequently, the total walking time Eq. (15) can be written as

$$tV_1 = L_1 + L_2\frac{V_1}{V_2}, \tag{30}$$

where V_1/V_2 is given by Eq. (25),

$$tV_1 = L_1 + L_2\left(\frac{1-\alpha}{1-\alpha\beta}\right)^{1/2}. \tag{31}$$

From Eq. (24) we obtain

$$2\beta^2 = \frac{W_1}{W_2}\left(\frac{1-\alpha}{1-\alpha\beta}\right)^{1/2}, \tag{32}$$

which means that approximately [i.e., based on the same approximation as in Eq. (27)]

$$2\beta^2 \cong \frac{W_1}{W_2} = 2\frac{L_2}{L_1} \tag{33}$$

namely,

$$\beta \cong \left(\frac{L_2}{L_1}\right)^{1/2}. \tag{34}$$

In sum, in place of Eq. (30) we minimize the function

$$tV_1 = L_1 + L_2\left(\frac{1-\beta}{1-\alpha(L_2/L_1)^{1/2}}\right)^{1/2} \tag{35}$$

by selecting L_2/L_1 subject to the territory constraint Eq. (17). The solution is based on the method of undetermined coefficients, and is found by solving the implicit equation

$$\frac{1}{x^2} = \frac{(1-\alpha)^{1/2}}{(1-\alpha x)^{3/2}}\left(1 - \frac{\alpha x}{2}\right), \tag{36}$$

where $x = (L_2/L_1)^{1/2}$ and, as noted earlier, $\alpha = S_0/S_1$. The solution of Eq. (36) expresses the optimal L_2/L_1 as a function of S_0/S_1, and is reported in Fig. 5. For example, when $\alpha \sim 0.5$ [i.e., when S_1 is of order $2S_0$, or V_1 is high enough to be comparable with V_∞] the optimal L_2/L_1 is of order 0.7. In view of Eq. (29), in this S_1/S_0 range W_1/W_2 is of order 1.4, and $\beta \sim 0.84$. The fact that β is comparable with 1 confirms the initial assumption (26), which led to the approximations made in Eqs. (27) and (33).

Figure 5 also shows that at both ends of the abscissa range, $\alpha = 0$ and $\alpha = 1$, the optimal L_2/L_1 is 1. This corresponds to $W_1/W_2 = 2$ and $\beta = 1$, which again validates the simplifying assumption made in Eqs. (26), (27), and (33). One can show that in the vicinity of $S_0/S_1 \cong 1$, the L_2/L_1 curve (36) behaves as

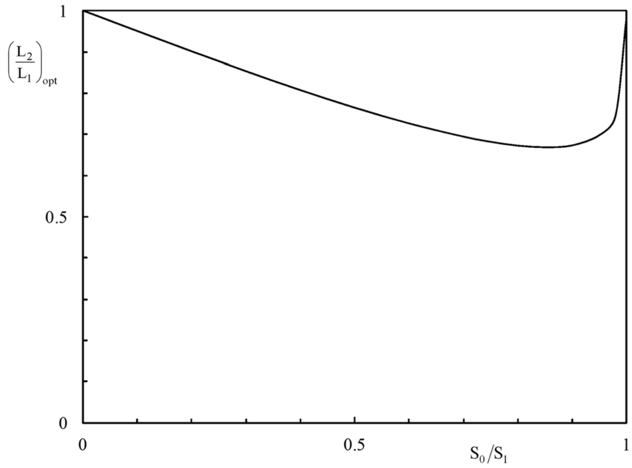

FIG. 5. The optimal ratio L_2/L_1 for the bifurcated walkway of Fig. 4.

$$1 - \left(\frac{L_2}{L_1}\right)^{1/2} = \left[\frac{1}{4}\left(1 - \frac{S_0}{S_1}\right)\right]^{1/3}. \tag{37}$$

Finally, we examine the effect of the last parameter that can still vary, namely $\alpha = S_0/S_1$. Substituting in Eq. (35) the two optimization results obtained so far [namely Eq. (34) and Eq. (36), or Fig. 5], we can write the total time Eq. (35) as

$$tCV_\infty\left(\frac{2}{A_T}\right)^{1/2} = \frac{1}{x(1-\alpha)^{1/2}} + \frac{x}{(1-\alpha x)^{1/2}}. \tag{38}$$

The function $x(\alpha)$ is provided by Eq. (36), or Fig. 5. In sum, the dimensionless time $tCV_\infty(2/A_T)^{1/2}$ increases monotonically as the ratio S_0/S_1 increases, i.e., as α decreases, Fig. 6. When S_0/S_1 is lower than 0.5 the dimensionless time is of order 2 and is insensitive to S_0/S_1.

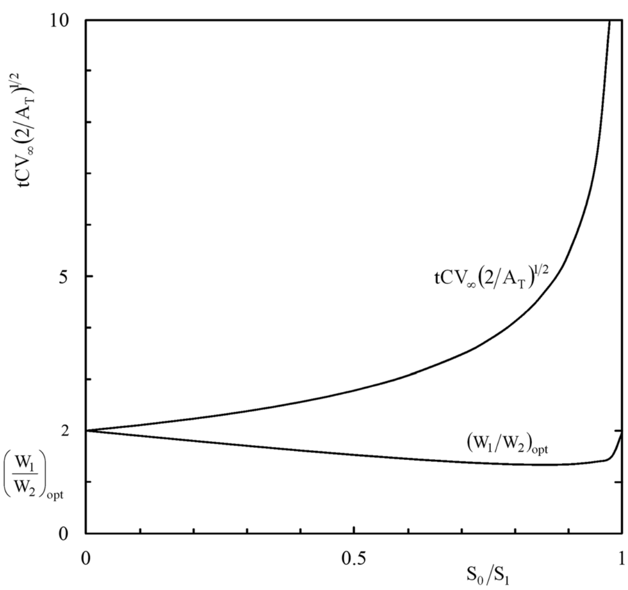

FIG. 6. The effect on the spacing on the stem (S_1/S_0) on the minimized travel time through the bifurcated walkway of Fig. 4.

VI. NUMERICAL SIMULATIONS OF T-SHAPED WALKWAY MOVEMENT AND OPTIMIZATION

To demonstrate the existence of the opportunity to optimize the configuration of bifurcated walkways, we simulated numerically the movement of pedestrians on the T-shaped walkway area defined in Fig. 4. The shape of the overall area was square, $L_1 + W_2 = 2L_2 + W_1 = 50$ m. The total area of the walkway floor was fixed at 250 m^2, which represents one tenth of the overall area. Variable is the ratio of the two widths, W_1/W_2.

We simulated the pedestrian movement in several T-shaped configurations with different W_1/W_2 ratios and estimated the time (t) of evacuating the population from the T-shaped walkway area (the branches) to the end of the trunk (the stem). The objective was to identify the configuration with the W_1/W_2 ratio that offers the shortest evacuation time. We performed this program of simulations for a scenario of 20 persons distributed over areas of 10 m^2 at the two extremities of the $W_2 \times L_2$ branches of the T, which corresponds to a density of 2 persons/m^2. This total population of 40 persons is constituted of males aged 40. At $t = 0$, the people start vacating the area toward the left side of the $W_1 \times L_1$ walkway.

We performed simulations with Simulex,[2] which is a computer package that allows simulations of escape movement of occupants from large, geometrically complex building structures. By using the computer-aided-designed floor plans of buildings with staircases, final external exits can be defined outside the buildings. A distance map representing a mesh of squares 0.2 m × 0.2 m in size is spread over the entire building space. The value of distance-to-exit from the center of each square is calculated with Simulex. Occupants are assigned inside the premises individually or as groups. When the building population has been defined, the potential routes of the occupants can be calculated and then a simulation can be carried out to find the total evacuation time. The occupants move toward the pre-defined exits with individual walking speeds dependent upon individual characteristics and the proximity of other people. The algorithms for the occupant individual movement are based on real-life data, collected by using computer-based techniques for the analysis of human movement, observed in real-life footage.

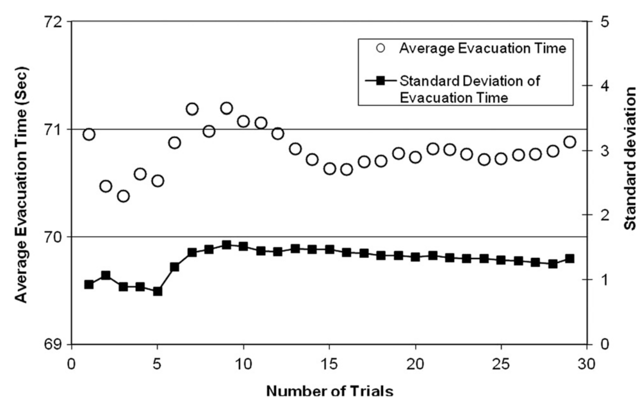

FIG. 7. The evolution of the standard deviation and evacuation time vs the number of simulations $W_1/W_2 = 1.83$).

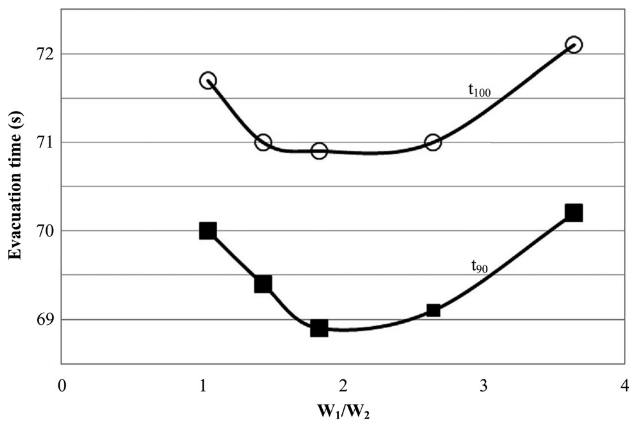

FIG. 8. Evacuation time as a function of the W_1/W_2 ratio.

Furthermore, because each simulation is based on a set of initial parameters that have random character (for example, initial speed and direction), the simulations that are conducted for a given configuration (W_1/W_2) produce evacuation times that differ from one simulation to the next. We calculated the evacuation time by averaging it over a number (n) of simulations that are large enough such that increasing n does not cause significant changes in the averaged travel time. Plotting the standard deviation of the average evacuation time as a function of the number of simulations, we found that $n = 30$ is large enough, as demonstrated in Fig. 7. To strengthen the numerical results further, we calculated two evacuation time values: t_{100}, the time when 100% of the population has exited through the end of the trunk, and t_{90}, when only 90% of the pedestrians have reached the exit.

The evacuation times calculated in this manner are reported in Fig. 8. They show that there is one ratio of walkway widths W_1/W_2 for which the evacuation time is minimum. This value is approximately $W_1/W_2 = 2$, and it is robust because the minimum is shallow. Along with its analytical derivation in Sec. V, the optimal ratio of walkway widths emerges as the crowd dynamics counterpart of the Hess-Murray rule for selecting the ratio of diameters in a bifurcated duct with fluid flow.[4,10–12]

VII. CONCLUSIONS

In this paper we constructed a physics-based method for the fundamental design of evacuation from living spaces. The method focuses on the relation between the configuration (shape, structure) of the living space and the time needed for pedestrian evacuation. The application of the method was organized into modules that proceed from the simple to the more complex (walkway, corner, T-shaped walkway). These modules can be assembled and used in evacuation designs for more complex structures.

The fundamental contribution made with this approach is that pedestrian movement, evacuation, and the design of the living space are recognized as flow systems with changing (evolutionary) configurations, in accord with other animate and inanimate flow designs unified by the constructal law.[9]

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