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Node importance for dynamical process on networks: A multiscale characterization

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Defining the importance of nodes in a complex network has been a fundamental problem in analyzing the structural organization of a network, as well as the dynamical processes on it. Traditionally, the measures of node importance usually depend either on the local neighborhood or global properties of a network. Many real-world networks, however, demonstrate finely detailed structure at various organization levels, such as hierarchy and modularity. In this paper, we propose a multiscale node-importance measure that can characterize the importance of the nodes at varying topological scale. This is achieved by introducing a kernel function whose bandwidth dictates the ranges of interaction, and meanwhile, by taking into account the interactions from all the paths a node is involved. We demonstrate that the scale here is closely related to the physical parameters of the dynamical processes on networks, and that our node-importance measure can characterize more precisely the node influence under different physical parameters of the dynamical process. We use epidemic spreading on networks as an example to show that our multiscale node-importance measure is more effective than other measures. © *2011 American Institute of Physics*. [doi:10.1063/1.3553644]

Node importance is a basic measure in characterizing the structure and dynamics of general complex networks. Real-world networks, however, usually demonstrate finer and finer structures at smaller spatial scales. To better understand node importance at different topological scales of a network, we propose a multiscale measure that can evaluate the importance of a node at varying scale. We use epidemic spreading on networks to show that the scale of this node-importance measure corresponds well to the physical parameter of epidemic spreading, and our measure provides a more systematic characterization of node importance for dynamical processes on networks.

Complex networks, which are composed of a number of nodes that are interconnected by a set of edges, are widely observed in a vast range of natural and artificial systems in recent years, ranging from the brain through the Internet to human society,[1–4] which have been shown to demonstrate universal features such as small-world[5] and scale-free[6] effects. Meanwhile, the past decade have also witnessed significant advances in which the general concept of complex networks has spurred many other research areas.[7–12]

A fundamental problem in analyzing the complex networks is to identify the most important nodes or to define the importance of the nodes,[13–15] which is immediately related to network resilience to attacks and immunization of epidemics. The important nodes usually play a crucial role in the global organization of the network which, in turn, have significant consequences to the dynamical processes taking place on it, such as synchronization,[16–21] epidemic spreading,[22,23] navigation,[24] random walks,[25–27] and so on.[28]

A variety of centrality measures have been proposed to determine the relative importance of a vertex within a graph,[15,29–31] particularly for social networks. Examples are degree centrality (DC, defined as the degree of a vertex),[32] betweenness centrality (BC, measures the number of times that a shortest path travels through the node),[33,34] subgraph centrality[35] (SC, characterizes the participation of each node in all subgraphs in a network), eigenvector centrality[36,37] (EVC, defined as the dominant eigenvector of the adjacency matrix), andcloseness centrality (CC, reciprocal of the sum of the lengths of the geodesic distance to every other vertex). Another way to define node importance is the famous PageRank[38,39] (PR). The PageRank of a page (or a node) depends not only on how many pages point to it but on the PageRank of these pages as well. PageRank is defined as:

$$PR_i = (1-d) + d\sum_j \frac{PR_j}{k_j}, \tag{1}$$

where PR_i is the centrality of node i, j runs for all neighbors of i, and k_j is the out-degree of node j. The parameter d is a damping factor between 0 and 1, which is usually set to 0.85.

These measures characterize the node importance from different angles. DC measures the immediate influence among the nodes, rather than long-term effect by counting the number of immediate neighbors of a node. Comparatively, BC and SC take into account more global information like the number of shortest path and the subgraphs containing that node. Generally, a good node-importance measure should include information from different scales, both local and global. That is, both immediate neighbors and higher order neighbors of a given node should be considered. Moreover, most real-world networks, either biological or technological, demonstrate the so-called hierarchical structure,[40] that is, more and more detailed structure of the network is present at finer topological scales. Considering both the "higher order" argument and the hierarchical nature of many real-world networks, we are naturally led to a multiscale characterization of node importance, which will be introduced next.

In our previous work, we have introduced the kernel function into networks for community detection.[42] A new framework based on a kernel function has also been developed to evaluate the correlation pattern emergent in dynamical processes on networks.[41] The kernel function K is a monotonically decreasing function[42,43] used to describe the fact that the strength of interaction between node i and j diminishes with their shortest path distance x_{ij} (see Fig. 1 in Ref. 41). Directly coupled nodes usually exert a stronger influence on each other, while this influence tends to weaken for indirectly linked nodes due to intermediaries. For the Gaussian kernel, the interaction between node i and j will be $R(i,j) = \exp(-x_{ij}^2/2h^2)$, where x_{ij} is the shortest path distance, h is the bandwidth of the kernel which controls its width, and R is the interaction matrix.

A great advantage of using such kernel function is its adjustable bandwidth h, which renders spontaneously a multiscale framework. A narrow kernel indicates that a node will have impact only on its immediate neighbors; while under a wide kernel, a node can affect its higher order neighbors and this order can be conveniently specified by parameter h. Generally, the importance of a node depends on not only its immediate neighbors but also the higher order neighbors. Combining this with the kernel framework, the importance of node i [which we denote as $u(i)$] can then be conveniently defined as the sum of all the influences it sends out (or receives): $u(i) = \sum_{j=1}^{N} R(i,j)$.

Here $u(i)$ only involves the influences going through the shortest path between node i and other nodes. To take into account other alternative paths between node i and other nodes (i.e., non-shortest-path route, also called the generalized connectivity[44]), we add another term into $u(i)$

$$u(i) = \sum_{j=1}^{N} R(i,j) + \sum_{p} \exp\left(\frac{-L(p)^2}{2h^2}\right), \tag{2}$$

where p indicates the non-shortest-path routes between node i and all other nodes, and $L(p)$ is the length of these non-shortest-path routes. This way our measure $u(i)$ takes into account the influences from all the paths associated with node i. The range of interaction is conveniently controlled by the width h of the kernel function. Under a small h, only short-range interactions will be counted, while a large h favors long-range interactions.

Traditional node-importance measures are purely graph-theoretic, which are seldom associated directly with dynamical processes on networks. However, the dynamical processes on networks is an important topic in the study of complex systems.[29] Understanding the relationship between the structure of complex networked systems and their dynamics (or function) is expected to shed more lights onto relevant issues in neuroscience, ecology, sociology, and engineering.[45] In this paper, we will show that $u(i)$ serves as an effective, multiscale node-importance measure that can evaluate the influence of a node in dynamical processes on networks under different physical parameters. Specifically, we use epidemic spreading on networks as an example.

We consider the susceptible–infected–susceptible (SIS) model, in which node have two states: susceptible (S) and infected (I). A susceptible node is infected with rate v at each time step, and can recover and become again susceptible with rate δ. Epidemic spreading on networks here is a good example to illustrate the multiscale concept: an infected node can spread the disease not only to its immediate neighbors but also to its higher order neighbors through the intermediaries. This large scale interaction will be more relevant when the effective spreading rate λ (defined as v/δ) is high. Numerically, the probability that a node transmits the disease to its nth order neighbor is v^n, if the recovery rate $\delta = 0$. Under the circumstances that λ is relatively large, a hub node will be persistently infective and can affect even its higher order neighbors.

To validate the effectiveness of our node-importance measure $u(i)$ for epidemic spreading on networks, we first need to find a quantity that originates purely from epidemic

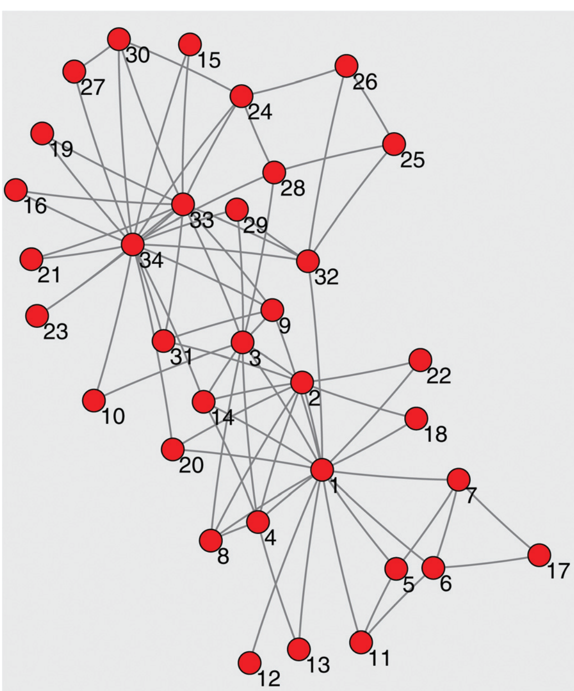

FIG. 1. (Color online) The Zachary karate club network. The top five nodes (according to degree) are 34, 1, 33, 3, and 2, with their degrees being 17, 16, 12, 10, and 9, respectively. The number of their second-order neighbors are 6, 9, 12, 20, and 13, respectively.

spreading (i.e., not graph-theoretic measures) and can reflect the importance or influence of each node in the spreading of epidemics. Then we should check the extent to which this quantity coincides with our measure $u(i)$, which is defined on a graph-theoretic sense. One quantity that can properly characterize the node importance for epidemic spreading is the total number of times a node gets infected [denoted by $T(i)$] during a fixed time interval. If we use 0 and 1 to represent the S and I state of the nodes, the dynamics of node i is encoded in a stochastic process, in which $T(i)$ can be conveniently computed by counting the transitions from 0 to 1.

The reason why we use the number of infection a node receives as an importance indicator for epidemic spreading can be explained from the viewpoint of network immunization: the node that are most easily infected [i.e., those with the largest $T(i)$] should be segregated or vaccinated in the first place for effective immunization of the network. Therefore, the nodes with larger $T(i)$ can readily be taken as more "influential" or "important." It should be noted that this node-importance quantity for epidemic spreading $T(i)$ naturally suggests the need to consider multiscale interactions on a network, as the infection of node i may come not only from its immediate neighbors but also from its higher order neighbors.

In the following text, we will use epidemic spreading on a simple network, that is, the Zachary karate club network[46] (see Fig. 1) to show the effectiveness of our measure $u(i)$. For a given parameter ($\lambda = v/\delta$), we first run the SIS model on the network and compute the number of times a node gets infected ($T(i)$). Then we derive the node importance $u(i)$ using an exponential kernel function according to Eq. (2). Since the kernel function has a tunable bandwidth h, we vary h in a continuous manner until we get a maximum correlation coefficient ρ between $u(i)$ and $T(i)$. We denote this h as $h_{\max}$. Figure 2 shows the correlation coefficient ρ between $u(i)$ and$T(i)$ using two different λ. As can be seen, ρ takes a high value near 0.94, indicating the good correspondence between the node influence $T(i)$ and our estimation $u(i)$. The correlation coefficient of $T(i)$ with other centrality measures are $\rho_{(T,\mathrm{DC})} = 0.83$, $\rho_{(T,\mathrm{CC})} = 0.81$, $\rho_{(T,\mathrm{BC})} = 0.76$, and $\rho_{(T,\mathrm{EVC})} = 0.90$. The most remarkable feature of our measure

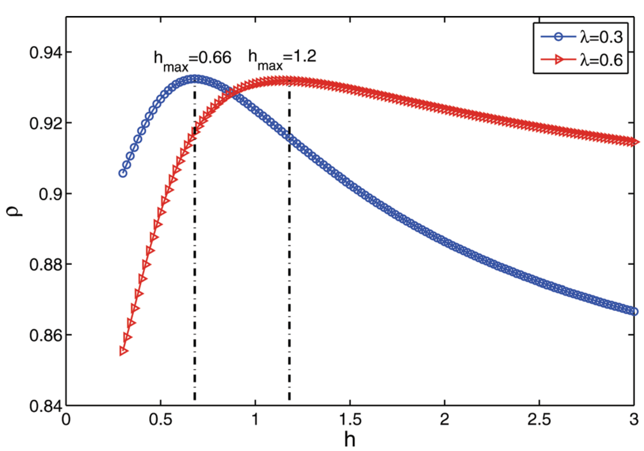

FIG. 2. (Color online) The correlation coefficient between $u(i)$ and $T(i)$ under $\lambda = 0.3$ and $\lambda = 0.6$, respectively, for Karate network.

is its multiscale nature. For epidemic spreading on networks, λ determines the range or scale over which a node can exert influence. When λ is low, each node tends to infect only its immediate neighbors, which is described well by a narrow kernel (having a small h); when λ is relatively large, the infection can span to higher order neighbors of a node, therefore, the epidemic spreading can be better characterized by a wider kernel function. Using kernel function with different width h, we can estimate the node importance for epidemic spreading on networks under various effective spreading rate λ.

Now we explicate how and why this multiscale measure outperforms other node-importance measures. Table I gives the ranking of the top five nodes in Karate network according to $T(i)$ (obtained by running SIS model on Karate network), $u(i)$ (obtained by our kernel method), and other frequently used measures. As can be seen, when λ is small ($\lambda = 0.3$), each node has a small range of influence, and the most important one [according to $T(i)$, see the first line in the table] is node 34, followed by node 1. In this case, $T(i)$ can be well fitted by a narrow kernel function ($h_{\max} = 0.66$, see the third line). When λ increases to 0.6, however, the most important node turns out to be node 1 (see the second line). This is because the nodes now have a wider range of influence and node 1, which has far more second-order neighbors than node 34, becomes the most important node. Degree centrality, which is a local measure, provides a good prediction of node importance when λ is relatively small (see the fifth line). Betweenness and closeness centralities, which take into account global structures, rank node 1 and 34 as the top two, which is consistent with a larger λ. Because these traditional centrality measures offer no choice of scale, they are not able to capture the change in node importance at varying scales.

The validity of our measure is further verified on more general networks, including a scale-free network and an email network.[47] First we generate a scale-free network with 500 nodes and mean degree 3. Then we assortatively mix this network so that it has a positive assortativity coefficient[48,49] ($r = 0.60$). The reason why we induce assortative mixing is that the node importance will be dependent on the degree–degree correlation, and it is, therefore, more complex and can test different measures effectively. We run the SIS model on this network (under two different λ) and get $T(i)$. As can be seen in Fig. 3, the infection times $T(i)$ at different λ can be well fitted by $u(i)$ with corresponding $h_{\max}$, indicated by a large correlation coefficient (ρ near 0.95) between

TABLE I. Rankings of the nodes (top five) according to SIS simulation (the first and second lines), our multiscale node-importance measure $u(i)$, and traditional centrality measures.

Measure	Ranking				
T ($\lambda = 0.3$)	34	1	33	3	2
T ($\lambda = 0.6$)	1	34	33	3	2
u ($h = 0.66$)	34	1	33	3	2
u ($h = 1.2$)	1	34	3	33	2
DC	34	1	33	3	2
BC	1	34	33	3	32
CC	1	34	33	3	32
EVC	34	1	3	33	2

$T(i)$ and $u(i)$. We also calculated the correlation coefficient of $T(i)$ ($\lambda = 0.6$) with other node-importance measures, and we found $\rho_{(T,\mathrm{DC})} = 0.62$, $\rho_{(T,\mathrm{CC})} = 0.72$, $\rho_{(T,\mathrm{EVC})} = 0.52$, and $\rho_{(T,\mathrm{BC})} = 0.06$, which are much lower than $\rho_{(T,u)}$ and indicate the effectiveness our multiscale measure. Next we use a larger email (with 1133 nodes) network as an example.[47] The numerical results are shown in Fig. 4, where we note that the correlation coefficient between $T(i)$ and u is about 0.94. In comparison, $\rho_{(T,\mathrm{DC})} = 0.79$, $\rho_{(T,\mathrm{CC})} = 0.86$, $\rho_{(T,\mathrm{EVC})} = 0.69$, $\rho_{(T,\mathrm{BC})} = 0.56$. These results suggest our node-importance measure $u(i)$ can capture the influence of nodes in epidemic spreading under different λ more reliably than other frequently used importance measures.

In the following text, we will show that our multiscale node-importance measure $u(i)$ incorporates various sources of information that are usually captured by traditional centrality measures separately. Take the email network as an example, when the kernel bandwidth h is small, each node is influencing only its immediate neighbors. In this case, $u(i)$ is very much like degree, see Fig. 5, and the correlation coefficient between the degree sequence and $u(i)$ is 0.99. Under a large kernel bandwidth h (which means that each node is exerting influence to its higher order neighbors), our measure takes into account more global information like betweenness centrality (BC). The correlation coefficient between the BC and $u(i)$ is 0.80. By choosing an appropriate bandwidth (e.g., at mesoscale), our measure is expected to reflect both the local and global organization of the network.

Finally, we check PageRank in detail, as it also has a tuning parameter d defined between 0 and 1, which changes the global behavior of this metric. First, we examine the correlation between PR and node degree as d changes (see Fig. 6), which we find to reach a maximum at $d = 1$. This indicates that PR captures more local structure of the network with a larger d, as degree measures only local influence.

Then we examine the relation between PageRank and our measure u. We first set d at a high value ($d = 0.85$, which is commonly adopted in PR). We find that the correlation coefficient between PR and u reaches maximum at a small kernel bandwidth h ($h = 0.9$, corresponding roughly to

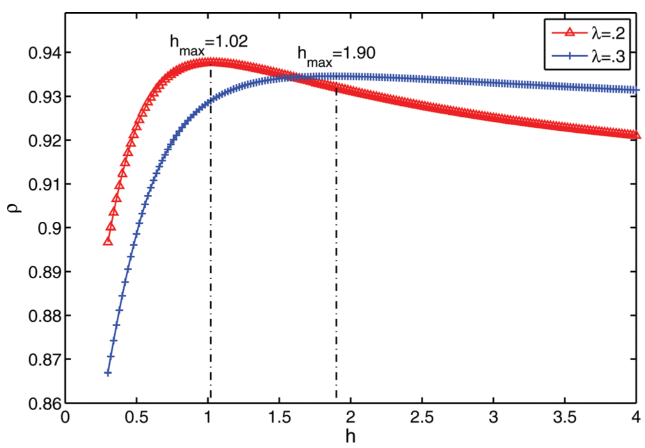

FIG. 4. (Color online) The correlation coefficient between $u(i)$ and $T(i)$ under $\lambda = 0.3$ and $\lambda = 0.6$, respectively, for an email network (Ref. 47).

first-order neighbors), see Fig. 7. This result is consistent with that in Fig. 6, that is, a high d renders PR more of a local metric. When d takes a lower value, however, we find the relation between PR and u to be complicated, which will be discussed elsewhere. In fact PageRank can be interpreted in terms of a random walk of a web surfer wandering on the web. At each step, it either jumps to any other page on the web randomly, or jumps to a page that is linked to previous page randomly. The former occurs with probability $1-d$, the latter with probability d. When d is close to 0, all nodes have the same centrality, $PR_i = 1$. Therefore, no information about the topology is included. On the contrary, when d is close to 1, the centrality of the node will read $PR_i = \sum_j PR_j / k_j$. The surfer can just walk from a node to one of its neighbors, that is, local information of the network is taken into account. In this case, PR is the dominant eigenvector of the normalized Laplacian of the network.

In summary, we have proposed a multiscale node-importance measure for a network, which can characterize the influence of a node at various organization levels or scales. This is achieved by introducing the kernel function into networks, which quantifies the interaction among the nodes in a multiscale manner. Since the kernel function is tunable in its

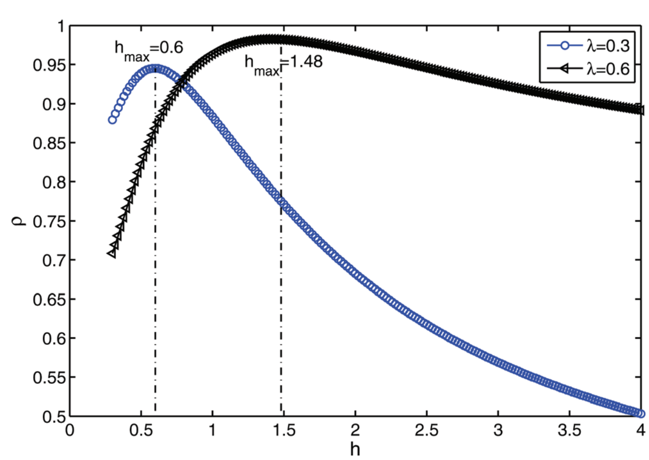

FIG. 3. (Color online) The correlation coefficient between $u(i)$ and $T(i)$ under $\lambda = 0.3$ and $\lambda = 0.6$, respectively, for assortative Barabási-Albert (BA) network.

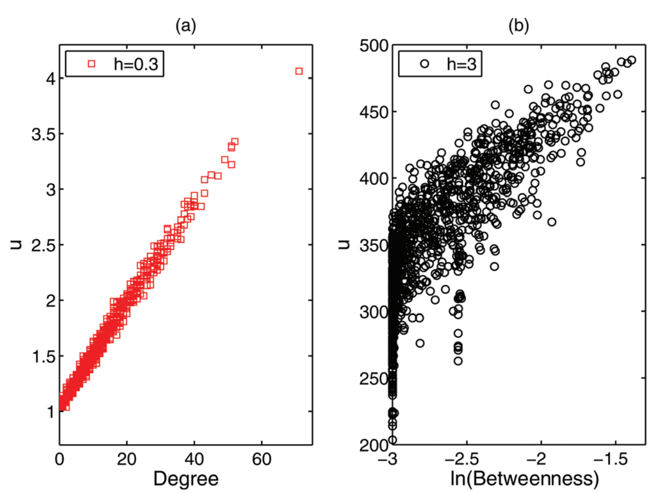

FIG. 5. (Color online) The relationship between $u(i)$ and traditional centrality measures. (a) $u(i)$ and degree centrality (b) $u(i)$ and betweenness centrality.

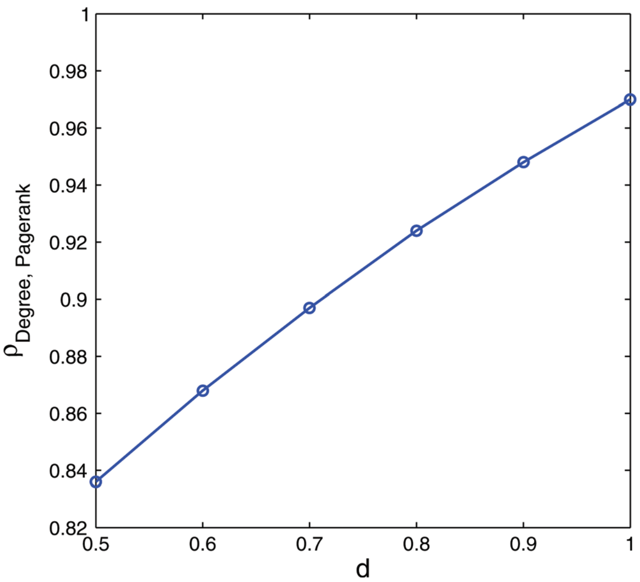

FIG. 6. (Color online) The correlation coefficient between PR and degree as d varies [for an email network (Ref. 47)].

bandwidth, we can define a node-importance measure that is multiscale in nature. We find that our measure can provide a good estimation of the node influence in terms of epidemic spreading on networks. In particular, the scale that is described by kernel bandwidth h corresponds well to the effective spreading rate λ. Usually, in order to evaluate precisely the node importance associated dynamical processes (like epidemic spreading on a network) at a given physical parameter (such as λ), we will need a priori knowledge of the functional relation between this physical parameter (i.e., λ) and $h_{\max}$. For general networks that are not related to dynamical processes, we can use Eq. (2) directly to obtain the multiscale node importance by varying kernel bandwidth h.

The complexity of our algorithm is about $O(n^2)$. For each node (node i), the complexity of finding its first-order neighbor N_i is $O(n)$. The second-order neighbor of node i can be easily identified from the first-order neighbor of N_i, and the third-order neighbor of node i can be identified from the first-order neighbor of the second-order neighbor of node i, and so on. Therefore, for each node, the complexity of finding neighbors (up to higher order) is generally bounded by $O(n)$. The overall complexity is $O(n^2)$. Furthermore, our multiscale node-importance measure can also be extended to characterize other important topological properties of a network, such as rich-club organization and degree–degree mixing in a multiscale manner,[49,50] where the degree of a node can be readily substituted with our node importance $u(i)$.

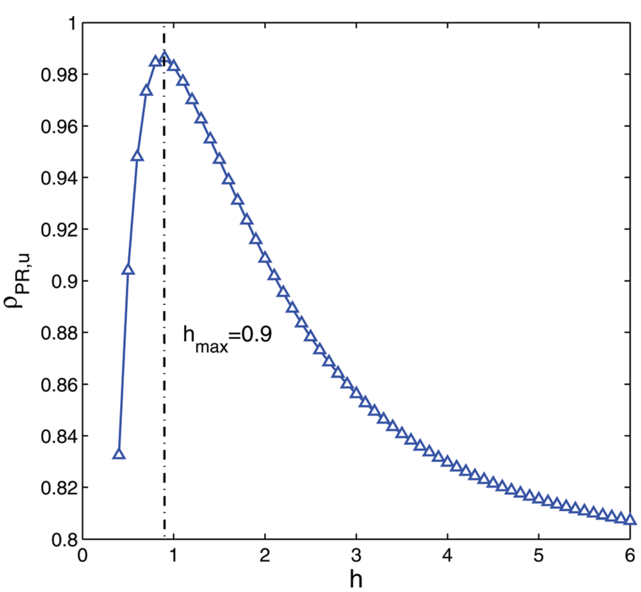

FIG. 7. (Color online) The correlation coefficient between $u(i)$ (under different h) and PageRank (with d = 0.85) for an email network (Ref. 47).

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