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A Class of Selection Criteria Achieving Full Diversity in AF Opportunistic Relaying

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***Abstract*—The way to select the "best" relay for forwarding the received signal to the destination is critical in opportunistic relaying. In this paper, we analyze the asymptotic outage probability of the amplify-and-forward opportunistic relaying (AF-OR) under a generalized selection criterion termed as the max-generalized-mean (MGM) selection criterion. We show that this generalized selection framework can be regarded as a class of selection criteria achieving full diversity in the AF-OR, encompassing the conventional selection criteria as special cases. The asymptotic outage probability can be further minimized by optimizing the parameters associated with the MGM selection criterion. It is shown that under this generalized selection framework, the conventional max-min selection criterion is optimal for the AF-OR in the sense that it achieves the minimum outage probability.**

I. Introduction

Cooperative communication is widely regarded as a promising technique to enhance signal reliability in wireless networks by exploiting the spatial diversity provided by the terminals distributed in space [1] [2] [3]. A simple and distributed opportunistic relaying approach has been proposed to select the "best" relay for forwarding the signal from the source instead of utilizing all relays to transmit at the same time [4] [5]. It is shown that the opportunistic relaying can achieve the same diversity-multiplexing tradeoff as the distributed space-time coding. Furthermore, the opportunistic relaying adapts itself to the distributed nature of cooperation communication with small cooperation overhead as opposed to other complex relaying protocols. In the opportunistic relaying, two conventional ways of selecting the best relay are (1) maximize the minimum of the source-relay channel gain and the relay-destination channel gain (referred to as the max-min selection), and (2) maximize the harmonic mean of the two channel gains (referred to as the max-harmonic-mean (MHM) selection).

The diversity-multiplexing tradeoff has been analyzed for the opportunistic relaying incorporated with the amplify-and-forward (AF-OR) and the selection decode-and-forward (SDF-OR) relaying strategies [6]. The approximate ergodic capacity and the outage probability of the AF-OR at a low to medium signal-to-noise-ratio (SNR) have also been studied recently [7]. Based on the max-min selection criterion, different relaying protocols have been designed and compared in terms of the outage performance [8]. Furthermore, the exact analytical performance of the outage probability and the symbol error probability have been derived for the AF-OR based on the max-min selection criterion [9]. Subsequently, the asymptotic analysis is generalized to that based on the MHM selection criterion [10] and it is shown that at a high SNR, the outage performance of the MHM selection and the max-min selection are identical.

While most of the previous works focus on the conventional max-min and MHM selection criteria, a weighted harmonic mean selection has been proposed to equalize the power consumption of different relays [11]. The "weighted harmonic mean" idea has inspired the proposal of a generalized selection criterion, namely the max-generalized-mean (MGM) selection criterion in [12], which provides some degrees of freedom for system behavior optimization. In [12], the performance of the MGM selection criterion in the SDF-OR has been analyzed. The results show that the MGM selection criterion outperforms the conventional max-min selection and MHM selection criteria with the optimized associated parameters. However, the performance of the MGM selection criterion for the AF-OR is still unknown and it is worth studying on its own interest. In this paper, we try to solve this problem by further investigating the asymptotic outage performance of the AF-OR based on the MGM selection criterion. Although the analysis of the MGM selection criterion for the SDF-OR has been conducted in [12], the analysis for the AF-OR is nontrivial considering the fundamental difference in these two relaying protocols [1] [4]. We show by analysis and simulation that the conventional max-min selection criterion is optimal under this generalized selection framework, while the MGM criterion can be regarded as a large class of selection criteria achieving the full diversity.

The rest of the paper is organized as follows. Section II presents the system model and the MGM criterion for selecting the best relay. Section III analyzes the

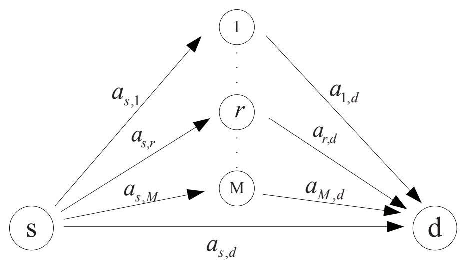

Fig. 1: The cooperative communication system model under study.

asymptotic performance of the AF-OR based on the MGM selection criterion. Section IV derives the optimal parameters to obtain the lowest outage probability at a high SNR. Section V shows the simulation and analytical results. Finally, Section VI concludes the paper.

Throughout the paper, we denote the probability density function (PDF) of a random variable X by $p_X(x)$ and the cumulative distribution function (CDF) of X by $F_X(x)$.

II. System Model and MGM Selection Criterion

We consider a system with one source denoted by s, M relays denoted by $1, 2, \ldots, M$, and one destination denoted by d, as illustrated in Fig. 1. Due to the space constraint, all terminals are assumed to be equipped with a single antenna. We denote $a_{k,l}$ as the instantaneous channel gain of the k–l link, where k can be the source s, relay i, $i \in \{1, 2, \ldots, M\}$ or the best relay denoted by r; and l can be relay i, the best relay r or the destination d. Furthermore, the channel gains are modeled as independent but not necessarily identical complex Gaussian random variables with zero mean and variance $\sigma^2_{k,l}$. The transmission process is divided into the following two phases.

First Phase: The source broadcasts its messages to the M relays and the destination.

Second Phase:

1) The "best relay", denoted by $r \in \{1, 2, \ldots, M\}$, is selected in a distributed manner [4] according to a generalized selection criterion to be defined shortly.
2) AF-OR: The Relay r amplifies the signal according to its power constraint and forwards the amplified signal to the destination.
3) The destination combines the signal received in the two phases using maximal ratio combining (MRC) and decodes the message.

Definition 1 (Generalized Mean [13]): Given two variables x and y, we define the generalized mean of x and y as $\mu_{w,p}(x, y) = (wx^p + \bar{w}y^p)^{\frac{1}{p}}$, where $w \in [0, 1]$, $\bar{w} = 1 - w$ and p is a real number.

We assume that the channel gains remain constant during the two transmission phases. Recall that for the ith relay, the source-relay channel gain is defined as $a_{s,i}$ and the relay-destination channel gain is defined as $a_{i,d}$. The max-generalized-mean (MGM) selection criterion aims to select the relay having the largest generalized mean of $|a_{s,i}|^2$ and $|a_{i,d}|^2$, which is formally defined as follows.

Definition 2: (*Max-Generalized-Mean (MGM) Selection Criterion [12]*) For the Relay i, we define the generalized mean of $|a_{s,i}|^2$ and $|a_{i,d}|^2$ as

$$\mu_{w_i,p}(|a_{s,i}|^2, |a_{i,d}|^2) = (w_i|a_{s,i}|^{2p} + \bar{w}_i|a_{i,d}|^{2p})^{\frac{1}{p}} \quad (1)$$

where $w_i \in [0, 1]$, $\bar{w}_i = 1 - w_i$ and p is a real number. In general, w_i can be different for different values of i. In the MGM selection criterion, the Relay r is selected as the "best relay" where

$$r = \arg \max_{i \in \{1,2,\ldots,M\}} \mu_{w_i,p}(|a_{s,i}|^2, |a_{i,d}|^2).$$

Note that the max-min and the MHM selection criteria are two special cases of the MGM selection criterion. As $p \to -\infty$, $r = \arg\max_{i\in\{1,2,\ldots,M\}} \min(|a_{s,i}|^2, |a_{i,d}|^2)$ and the MGM selection criterion becomes the max-min selection criterion [4]. When $p = -1$ and $w_i = 1/2$ for all $i = 1, 2, \ldots, M$, $r = \arg\max_{i\in\{1,2,\ldots,M\}} \frac{2|a_{s,i}|^2|a_{i,d}|^2}{|a_{s,i}|^2+|a_{i,d}|^2}$ and the selection criterion is equivalent to the MHM selection criterion [4].

III. Performance Analysis

We study the asymptotic outage probability of the AF-OR based on the MGM selection criterion with the parameter $p < 0$. The outage performance for $p \geq 0$ can be derived following a similar analysis method. Define SNR as P/N_0, where P denotes the transmitting power of the source and the relays; and N_0 denotes the variance of the Gaussian noise at the receiving terminal (relay or destination). Also, we denote the transmission rate by R in bits/s/Hz.

Assuming that the channel-state-information (CSI) assisted AF protocol [1] is used, the mutual information of the AF-OR is given by

$$I_{\mathrm{AF-OR}} = \frac{1}{2}\log\left(1 + \mathrm{SNR}|a_{s,d}|^2 + \frac{\mathrm{SNR}^2|a_{s,r}|^2|a_{r,d}|^2}{\mathrm{SNR}|a_{s,r}|^2 + \mathrm{SNR}|a_{r,d}|^2 + 1}\right). \quad (2)$$

As a result, the outage probability of the AF-OR is given by

$$P^{\mathrm{out}}_{\mathrm{AF-OR}} = \Pr\{I_{\mathrm{AF-OR}} < R\} = \Pr\left\{|a_{s,d}|^2 + \frac{f(\mathrm{SNR}|a_{s,r}|^2, \mathrm{SNR}|a_{r,d}|^2)}{\mathrm{SNR}} < g(\mathrm{SNR})\right\}, \quad (3)$$

where $f(x, y) = \frac{xy}{x+y+1}$ and $g(\mathrm{SNR}) = (2^{2R}-1)/\mathrm{SNR}$.

In the Rayleigh fading model, $|a_{s,i}|^2$ and $|a_{i,d}|^2$ are two exponentially distributed random variables with mean $\sigma_{s,i}^2$ and $\sigma_{i,d}^2$, respectively. Consequently, we can use the following lemma [12] to derive the asymptotic outage probability.

Lemma 1: Let $V = \max_{\substack{i\in\{1,2,\ldots,M\}\\ i\neq k}} \mu_{w_i,p}\left(|a_{s,i}|^2, |a_{i,d}|^2\right)$. When v is very small, the asymptotic CDF of V is given by

$$F_V(v) = \prod_{\substack{i=1\\ i\neq k}}^{M} \left(\sigma_{s,i}^{-2} w_i^{-\frac{1}{p}} + \sigma_{i,d}^{-2} \bar{w}_i^{-\frac{1}{p}}\right) v^{M-1}. \quad (4)$$

Based on Lemma 1, the asymptotic outage probability of the AF-OR under the MGM selection criterion can be derived as follows.

Theorem 1: The asymptotic outage probability of the AF-OR based on the MGM selection criterion with $p < 0$ is given by

$$P_{\mathrm{AF-OR},p<0}^{\mathrm{out}} \sim \frac{\sigma_{s,d}^{-2}}{M(M+1)} \left\{ \sum_{k=1}^{M} \left[\prod_{\substack{i=1\\ i\neq k}}^{M} \left(\sigma_{s,i}^{-2} w_i^{-\frac{1}{p}} + \sigma_{i,d}^{-2} \bar{w}_i^{-\frac{1}{p}} \right) \right] \left(\sigma_{s,k}^{-2} w_k^{-\frac{1-M}{p}} + \sigma_{k,d}^{-2} \bar{w}_k^{-\frac{1-M}{p}} \right) \right\} (g(\mathrm{SNR}))^{M+1}. \quad (5)$$

Proof: Please refer to Appendix A. ∎

By comparing Theorem 1 and the results derived in [8], it can be seen that by substituting $p = -\infty$ into (5), the asymptotic outage probabilities are reduced to those for the max-min selection criterion. By comparing Theorem 1 and the results derived in [10], it can be observed that when $p = -1$ and $w_i = 1/2\ \forall i = 1, 2, \ldots, M$, (5) gives the asymptotic outage probabilities for the MHM selection criterion. Another interesting point is that by setting $w_i = 1/2\ \forall i = 1, 2, \ldots, M$ in (5), the outage probability of the AF-OR under the MGM selection criterion becomes identical to that obtained under the max-min selection criterion regardless of the parameter p.

Definition 3 (Diversity Gain and Multiplexing Gain): The diversity gain $d(m)$ is defined as $d(m) = \lim_{\mathrm{SNR}\to\infty} -\frac{\log P^{\mathrm{out}}(\mathrm{SNR},m)}{\log \mathrm{SNR}}$ where P^{out} denotes the outage probability and m represents the multiplexing gain defined as $m = \lim_{\mathrm{SNR}\to\infty} \frac{R}{\mathrm{SNR}}$.

Corollary 1: Based on Theorem 1, the diversity-multiplexing tradeoff [14] for the the AF-OR protocol equals $d_{p<0}(m) = (M+1)(1-2m)$ under the MGM selection criterion with $p < 0$ and is plotted in Fig. 2.

Corollary 1 shows that the MGM selection criterion with $p < 0$ serves as a large class of selection criteria achieving the full diversity $M + 1$.

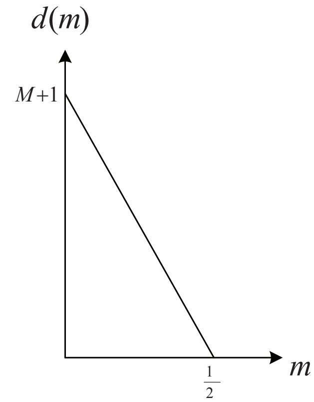

Fig. 2: Diversity-multiplexing tradeoff for the max-generalized-mean (MGM) selection criterion in the AF-OR.

IV. Optimal Parameters for MGM Selection Criterion

Although the MGM selection criterion with $p < 0$ can always provide the full diversity as shown in the previous section, the parameters $w_i\ \forall i = 1, 2, \ldots, M$ can be further optimized to minimize the asymptotic outage probability.

Theorem 2: Under the MGM selection criterion, the optimal asymptotic outage probability of the AF-OR is attained by setting $w_i = 1/2\ \forall i = 1, 2, \ldots, M$ and is given by

$$P_{\mathrm{AF-OR},p<0}^{\mathrm{out,opt}} = \frac{1}{M+1} \sigma_{s,d}^{-2} (g(\mathrm{SNR}))^{M+1} \sum_{(j_1,\ldots,j_M)\in\mathcal{J}} \prod_{i=1}^{M} \sigma_{(i,j_i)}^{-2}, \quad (6)$$

where $j_i \in \{0, 1\}\ \forall i = 1, 2, \ldots, M$; $\sigma_{(i,j_i)}$ equals $\sigma_{s,i}$ and $\sigma_{i,d}$ when $j_i = 0$ and 1, respectively; and $\mathcal{J}$ denotes the set of all combinations of $(j_1, \ldots, j_M)$. Consequently, $|\mathcal{J}| = 2^M$.

Proof: By expanding the asymptotic outage probability of the AF-OR in (5), we obtain

$$\begin{aligned} P_{\mathrm{AF-OR},p<0}^{\mathrm{out}} &\sim \frac{\sigma_{s,d}^{-2}}{M(M+1)} (g(\mathrm{SNR}))^{M+1} \times \\ &\sum_{(j_1,\cdots,j_M)\in\mathcal{J}} \left(\prod_{i=1}^{M} \sigma_{(i,j_i)}^{-2} w_{(i,j_i)}^{-\frac{1}{p}} \right) \left(\sum_{k=1}^{M} w_{(k,j_k)}^{\frac{M}{p}} \right) \\ &\overset{(a)}{\geq} \frac{1}{M+1} \sigma_{s,d}^{-2} (g(\mathrm{SNR}))^{M+1} \sum_{(j_1,\cdots,j_M)\in\mathcal{J}} \prod_{i=1}^{M} \sigma_{(i,j_i)}^{-2}, \end{aligned} \quad (7)$$

where $w_{(i,j_i)}$ equals w_i and $\bar{w}_i$ when $j_i = 0$ and 1, respectively; and (a) follows from the arithmetic-geometric-mean inequality, i.e., $x_1 + x_2 + \cdots + x_M \geq M(x_1 x_2 \cdots x_M)^{\frac{1}{M}}$. It is readily shown that the equality is attained when $w_{(i,j_i)} = w_{(k,j_k)}\ \forall i, k = 1, 2, \ldots, M$ and $j_i, j_k \in \{0, 1\}$, yielding $w_i = 1/2\ \forall i = 1, 2, \ldots, M$. ∎

It can be seen that the optimal selection parameters w_i are independent of the channel parameters $\sigma_{s,i}^2$ and $\sigma_{i,d}^2$. Setting $w_i = 1/2\ \forall i = 1, 2, \ldots, M$ yields the minimum outage probability for the AF-OR in the high SNR region. The minimum outage probability is also

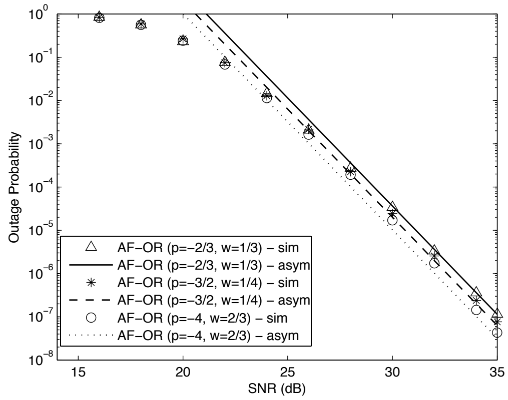

Fig. 3: Comparison of the simulated (sim) and the analytical asymptotic (asym) outage probabilities of the AF-OR under the max-generalized-mean (MGM) selection criterion when $p < 0$. Number of relays $M = 4$.

independent of the parameter p, as shown in (6). Moreover, the optimized asymptotic outage probability based on the MGM selection criterion is identical to that based on the max-min selection criterion [8] [9] and that based on the MHM selection criterion [10].

V. Simulation Results

In this section, we show the simulated and the analytical outage performance of the AF-OR under the MGM selection criterion for $p < 0$. In our simulations, we assume that the number of relays is $M = 4$ and the transmission rate is $R = 1$ (bit/s/Hz). The channel parameters being used are as follows: $\sigma^2_{s,d} = 0.025$; $\sigma^2_{s,1} = 1.25\sigma^2_{s,d}$ $\sigma^2_{s,2} = 1.5\sigma^2_{s,d}$ $\sigma^2_{s,3} = 2.5\sigma^2_{s,d}$ and $\sigma^2_{s,4} = 2\sigma^2_{s,d}$; $\sigma^2_{1,d} = 2.5\sigma^2_{s,d}$ $\sigma^2_{2,d} = 2\sigma^2_{s,d}$ $\sigma^2_{3,d} = 1.25\sigma^2_{s,d}$ and $\sigma^2_{4,d} = 1.5\sigma^2_{s,d}$. Also, we use $w_i = w\ \forall i = 1, \ldots, M$.

Figure 3 shows the simulated outage probability of the AF-OR together with the analytical results derived in (5). We can observe that the simulated results are close to the analytical ones in the high SNR region. The results indicate that the asymptotic expressions derived in (5) can accurately predict the outage performance of the AF-OR in the high SNR region. In Fig. 4, we present the simulated outage performance of the AF-OR under the MGM selection criterion for different values of p and w. It is observed that the MGM selection criterion can always achieve the full diversity.

In Fig. 5, we examine the outage performance of the AF-OR for different w with the SNR fixed at 30 dB. The results in Fig. 5 have illustrated that the lowest outage probability for the AF-OR is achieved by setting $w = 1/2$, which is in accord with Theorem 2. Moreover, the minimum outage probabilities for different p are almost identical, which again matches the conclusion drawn in Theorem 2.

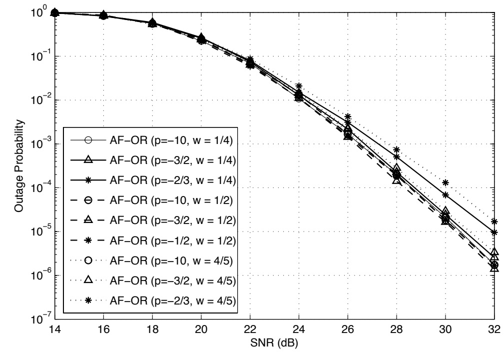

Fig. 4: Simulated outage probability of the AF-OR under the max-generalized-mean (MGM) selection criterion when $p < 0$. Number of relays $M = 4$.

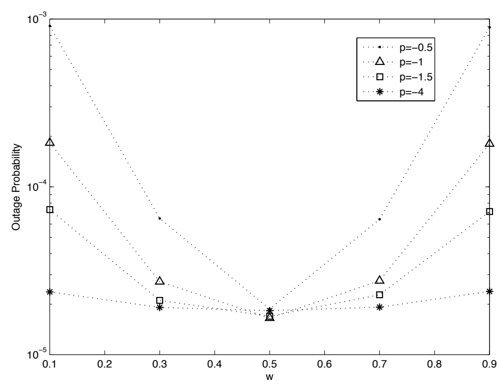

Fig. 5: Simulated outage probability of the AF-OR under the max-generalized-mean (MGM) selection criterion when SNR = 30 dB. Number of relays $M = 4$.

VI. Conclusion

The max-generalized-mean (MGM) selection criterion characterized by two parameters p and w_i is applied to the opportunistic relaying (OR) incorporated with the amplify-and-forward (AF) relaying mode. For $p < 0$, the asymptotic outage probability of the AF-OR is derived analytically and it is shown that the full diversity can be achieved. By optimizing w_i for each relay under this generalized selection framework, the outage performance of the AF-OR converges to that based on the max-min selection at a high SNR. An interesting question is: How will the MGM selection criterion together with the parameters p and w_i affect the outage probability in the low and medium SNR regions? This issue will be investigated and the results will be reported in future publications.

Appendix
Proof of Theorem 1

Claim 1: Suppose that we are given $p < 0$, a function $g(t)$ and $0 < \epsilon < 1$. When $0 \leq x \leq g(t)$ and $y \geq$

$\left(\frac{w}{\bar{w}}\right)^{\frac{1}{p}}[(1-\epsilon)^p-1]^{\frac{1}{p}}g(t)$, we have $\mu_{w,p}(x,y)\geq(1-\epsilon)w^{\frac{1}{p}}x$. When $0\leq y\leq g(t)$ and $x\geq\left(\frac{\bar{w}}{w}\right)^{\frac{1}{p}}[(1-\epsilon)^p-1]^{\frac{1}{p}}g(t)$, we will have $\mu_{w,p}(x,y)\geq(1-\epsilon)\bar{w}^{\frac{1}{p}}y$.

Proof: We show the proof of the first part of the Claim and the second part can be proved similarly. To prove $\mu_{w,p}(x,y)\geq(1-\epsilon)w^{\frac{1}{p}}x$ is the same as proving

$$y\geq\left(\frac{w}{\bar{w}}\right)^{\frac{1}{p}}[(1-\epsilon)^p-1]^{\frac{1}{p}}x. \tag{8}$$

Obviously, when $y\geq\left(\frac{w}{\bar{w}}\right)^{\frac{1}{p}}[(1-\epsilon)^p-1]^{\frac{1}{p}}g(t)$ and $0\leq x\leq g(t)$, the inequality (8) must hold. ■

Lemma 2: Given a function $g(t)$ that is continuous at $t=t_0$ and satisfies $g(t)\to 0$ as $t\to t_0$, we have $\Pr\left\{|a_{s,r}|^2<g(t)\right\}\sim\frac{1}{M}\sum_{k=1}^{M}\prod_{\substack{i=1\\i\neq k}}^{M}(\sigma_{s,i}^{-2}w_i^{-\frac{1}{p}}+\sigma_{i,d}^{-2}\bar{w}_i^{-\frac{1}{p}})\sigma_{s,k}^{-2}w_k^{-\frac{1-M}{p}}\left(g(t)\right)^M$ and $\Pr\left\{|a_{r,d}|^2<g(t)\right\}\sim\frac{1}{M}\sum_{k=1}^{M}\prod_{\substack{i=1\\i\neq k}}^{M}(\sigma_{s,i}^{-2}w_i^{-\frac{1}{p}}+\sigma_{i,d}^{-2}\bar{w}_i^{-\frac{1}{p}})\sigma_{k,d}^{-2}\bar{w}_k^{-\frac{1-M}{p}}\left(g(t)\right)^M$ as $t\to t_0$.

Proof: Define P_k as the probability of the event $\left\{(r=k)\cap\left(|a_{s,k}|^2<g(t)\right)\right\}$ and let $V=\max_{\substack{i\in\{1,2,\ldots,M\}\\i\neq k}}\mu_{w_i,p}\left(|a_{s,i}|^2,|a_{i,d}|^2\right)$. Hence P_k is given by $P_k=\Pr\left\{|a_{s,k}|^2<g(t),V\leq\mu_{w_k,p}(|a_{s,k}|^2,|a_{k,d}|^2)\right\}$ and $\Pr\left\{|a_{s,r}|^2<g(t)\right\}=\sum_{k=1}^{M}P_k$. For a positive and very small $g(t)$, the upper-bound of P_k can be calculated as

$$P_k\leq\Pr\left\{|a_{s,k}|^2<g(t),V\leq w_k^{\frac{1}{p}}|a_{s,k}|^2\right\} \tag{9}$$

$$=\int_0^{g(t)}\prod_{\substack{i=1\\i\neq k}}^{M}\left(\sigma_{s,i}^{-2}w_i^{-\frac{1}{p}}+\sigma_{i,d}^{-2}\bar{w}_i^{-\frac{1}{p}}\right)w_k^{\frac{M-1}{p}} x^{M-1}\sigma_{s,k}^{-2}e^{-\sigma_{s,k}^{-2}x}\mathrm{d}x \tag{10}$$

$$\sim\frac{1}{M}\prod_{\substack{i=1\\i\neq k}}^{M}\left(\sigma_{s,i}^{-2}w_i^{-\frac{1}{p}}+\sigma_{i,d}^{-2}\bar{w}_i^{-\frac{1}{p}}\right)\sigma_{s,k}^{-2}w_k^{\frac{M-1}{p}}\left(g(t)\right)^M \tag{11}$$

where the inequality (9) is due to $\mu_{w_k,p}(|a_{s,k}|^2,|a_{k,d}|^2)\leq w_k^{\frac{1}{p}}|a_{s,k}|^2$ and (11) follows from Corollary 1 for a very small $g(t)$.

The lower-bound of P_k is derived as

$$P_k\geq\Pr\Big\{|a_{s,k}|^2<g(t),V\leq\mu_{w_k,p}(|a_{s,k}|^2,|a_{k,d}|^2), |a_{k,d}|^2\geq\left(\frac{w}{\bar{w}}\right)^{\frac{1}{p}}[(1-\epsilon)^p-1]^{\frac{1}{p}}g(t)\Big\} \tag{12}$$

$$\geq\Pr\left\{|a_{s,k}|^2<g(t),V\leq(1-\epsilon)w^{\frac{1}{p}}|a_{s,k}|^2, |a_{k,d}|^2\geq\left(\frac{w}{\bar{w}}\right)^{\frac{1}{p}}[(1-\epsilon)^p-1]^{\frac{1}{p}}g(t)\right\} \tag{13}$$

where (13) is due to Claim 1. Following the same reasoning in deriving (11), we can obtain

$$\lim_{g(t)\to 0}\frac{P_k}{(g(t))^M}\geq \frac{1}{M}\prod_{\substack{i=1\\i\neq k}}^{M}\left(\sigma_{s,i}^{-2}w_i^{-\frac{1}{p}}+\sigma_{i,d}^{-2}\bar{w}_i^{-\frac{1}{p}}\right)\sigma_{s,k}^{-2}(1-\epsilon)^{M-1}w_k^{\frac{M-1}{p}}. \tag{14}$$

Since (14) holds for all $0<\epsilon<1$, by taking $\epsilon\to 0$, we can prove that the asymptotic lower-bound of P_k is the same as the upper-bound. Hence,

$$\Pr\left\{|a_{s,r}|^2<g(t)\right\}=\sum_{k=1}^{M}P_k \sim\frac{1}{M}\sum_{k=1}^{M}\prod_{\substack{i=1\\i\neq k}}^{M}(\sigma_{s,i}^{-2}w_i^{-\frac{1}{p}}+\sigma_{i,d}^{-2}\bar{w}_i^{-\frac{1}{p}})\sigma_{s,k}^{-2}w_k^{-\frac{1-M}{p}}\left(g(t)\right)^M. \tag{15}$$

The asymptotic expression of $\Pr\left\{|a_{r,d}|^2<g(t)\right\}$ can also be derived in an analogous way. ■

Lemma 3: Given a function $g(t)$ that is continuous at $t=t_0$ and satisfies $g(t)\to 0$ as $t\to t_0$, if we have $\Pr\left\{|a_{s,r}|^2<g(t)\right\}\sim\alpha\left(g(t)\right)^M$ and $\Pr\left\{|a_{r,d}|^2<g(t)\right\}\sim\beta\left(g(t)\right)^M$ as $t\to t_0$, then $\Pr\left\{r_t<g(t)\right\}\sim(\alpha+\beta)\left(g(t)\right)^M$ as $t\to t_0$, where $r_t=\frac{1}{t}f(t|a_{s,r}|^2,t|a_{r,d}|^2)$.

Proof: Since $\left\{|a_{s,r}|^2<g(t)\cup|a_{r,d}|^2<g(t)\right\}$ implies that $\{r_t<g(t)\}$, we have $\Pr\left\{r_t<g(t)\right\}\geq\Pr\left\{|a_{s,r}|^2<g(t)\cup|a_{r,d}|^2<g(t)\right\}=\Pr\left\{|a_{s,r}|^2<g(t)\right\}+\Pr\left\{|a_{r,d}|^2<g(t)\right\}-2\Pr\left\{|a_{s,r}|^2<g(t)\cap|a_{r,d}|^2<g(t)\right\}$. Note that $\Pr\left\{|a_{s,r}|^2<g(t)\right\}+\Pr\left\{|a_{r,d}|^2<g(t)\right\}\sim(\alpha+\beta)\left(g(t)\right)^M$ and that $|a_{s,r}|^2$ and $|a_{s,r}|^2$ are not independent. We can use the method in deriving Lemma 2 to calculate $\Pr\left\{|a_{s,r}|^2<g(t)\cap|a_{r,d}|^2<g(t)\right\}$.

Let $V=\max_{\substack{i\in\{1,2,\ldots,M\}\\i\neq k}}\mu_{w_i,p}\left(|a_{s,i}|^2,|a_{i,d}|^2\right)$. Therefore, we have $\Pr\left\{|a_{s,r}|^2<g(t)\cap|a_{r,d}|^2<g(t)\right\}=\sum_k\Pr\left\{|a_{s,k}|^2<g(t),|a_{r,k}|^2<g(t),V\leq\mu_{w_k,p}(|a_{s,k}|^2,|a_{k,d}|^2)\right\}$. It can be obtained that $\Pr\left\{|a_{s,r}|^2<g(t)\cap|a_{r,d}|^2<g(t)\right\}$ scales in the order of $\left(g(t)\right)^{M+1}$. Hence we have

$$\lim_{t\to t_0}\frac{\Pr\left\{r_t<g(t)\right\}}{\left(g(t)\right)^M}\geq\alpha+\beta. \tag{16}$$

Next we derive an upper bound of $\Pr\left\{r_t<g(t)\right\}$. Given any $\gamma_2>\gamma_1>1$, $\Pr\left\{r_t<g(t)\right\}$ can be written as $\Pr\left\{r_t<g(t)\right\}=I_1+I_2+I_3$, where $I_1=\Pr\left\{r_t<g(t),|a_{r,d}|^2<\gamma_1 g(t)\right\}$, $I_2=\Pr\left\{r_t<g(t),\gamma_1 g(t)\leq|a_{r,d}|^2<\gamma_2 g(t)\right\}$ and $I_3=\Pr\left\{r_t<g(t),|a_{r,d}|^2\geq\gamma_2 g(t)\right\}$.

The first term I_1 can be upper-bounded as $I_1 \leq \Pr\left\{|a_{r,d}|^2 < \gamma_1 g(t)\right\} \sim \beta\left(\gamma_1 g(t)\right)^M$. The second term I_2 can be upper-bounded as

$$\begin{aligned} I_2 &\leq \Pr\left\{|a_{s,r}|^2 < \frac{g(t)+1/t\gamma_1}{1-1/\gamma_1}, \gamma_1 g(t) \leq |a_{r,d}|^2 < \gamma_2 g(t)\right\} \\ &= \sum_{k=1}^{M} \Pr\left\{|a_{s,k}|^2 < \frac{g(t)+1/t\gamma_1}{1-1/\gamma_1},\right. \\ &\left.\gamma_1 g(t) \leq |a_{k,d}|^2 < \gamma_2 g(t), V \leq \mu_{w_k,p}(|a_{s,k}|^2, |a_{k,d}|^2)\right\} \qquad (17) \\ &\sim (g(t))^{M+1}, \qquad (18) \end{aligned}$$

where the first inequality is due to the fact that $r_t > \frac{t|a_{s,r}|^2\gamma_1 g(t)}{t|a_{s,r}|^2+t\gamma_1 g(t)+1}$ if $|a_{r,d}|^2 \geq \gamma_1 g(t)$. The third term I_3 is upper-bounded as

$$\begin{aligned} I_3 &\leq \Pr\left\{|a_{s,r}|^2 < \frac{g(t)+1/t\gamma_2}{1-1/\gamma_2}, |a_{r,d}|^2 \geq \gamma_2 g(t)\right\} \\ &= \sum_{k=1}^{M} \Pr\left\{|a_{s,k}|^2 < \frac{g(t)+1/t\gamma_2}{1-1/\gamma_2}, |a_{k,d}|^2 \geq \gamma_2 g(t),\right. \\ &\qquad \left. V \leq \mu_{w_k,p}(|a_{s,k}|^2, |a_{k,d}|^2)\right\} \\ &\leq \sum_{k=1}^{M} \Pr\left\{|a_{s,k}|^2 < \frac{g(t)+1/t\gamma_2}{1-1/\gamma_2}, |a_{k,d}|^2 \geq \gamma_2 g(t),\right. \\ &\qquad \left. V \leq w_k^{\frac{1}{p}} |a_{s,k}|^2\right\} \\ &\sim \alpha\left(\frac{g(t)+1/t\gamma_2}{1-1/\gamma_2}\right)^M. \qquad (19) \end{aligned}$$

Combining the upper bounds derived for I_1, I_2 and I_3, we obtain

$$\lim_{t\to t_0} \frac{\Pr\{r_t < g(t)\}}{(g(t))^M} \leq \beta\left(\gamma_1\right)^M + \alpha\left(\frac{1}{1-1/\gamma_2}\right)^M. \qquad (20)$$

Since the inequality holds for any $1 < \gamma_1 < \gamma_2$, the upper bound converges to the lower bound derived in (16) as $\gamma_1 \to 1$ and $\gamma_2 \to \infty$, i.e., $\Pr\{r_t < g(t)\} \sim (\alpha+\beta)(g(t))^M$. ■

Making use of Lemma 3, it can be shown that

$$\begin{aligned} &\lim_{\mathrm{SNR}\to\infty} \frac{\Pr\{r_{\mathrm{SNR}} < g(\mathrm{SNR})\}}{(g(\mathrm{SNR}))^M} = \\ &\frac{1}{M}\left\{\sum_{k=1}^{M}\prod_{\substack{i=1\\i\neq k}}^{M}\left(\sigma_{s,i}^{-2} w_i^{-\frac{1}{p}} + \sigma_{i,d}^{-2}\bar{w}_i^{-\frac{1}{p}}\right)\left(\sigma_{s,k}^{-2} w_k^{-\frac{1-M}{p}} + \right.\right. \\ &\left.\left.\sigma_{k,d}^{-2}\bar{w}_k^{-\frac{1-M}{p}}\right)\right\}, \qquad (21) \end{aligned}$$

where $r_{\mathrm{SNR}} = \frac{1}{\mathrm{SNR}} f(\mathrm{SNR}|a_{s,r}|^2, \mathrm{SNR}|a_{r,d}|^2)$. As a result, when $p < 0$, the outage probability of the AF-OR in (3) at a high SNR is given by

$$\begin{aligned} &P_{\mathrm{AF-OR},p<0}^{\mathrm{out}} = \\ &\int_0^{g(\mathrm{SNR})} \Pr\{r_{\mathrm{SNR}} < g(\mathrm{SNR}) - u\}\, \sigma_{s,d}^{-2} e^{-\sigma_{s,d}^{-2} u} \mathrm{d}u \\ &\sim \frac{\sigma_{s,d}^{-2}}{M(M+1)}\left\{\sum_{k=1}^{M}\left[\prod_{\substack{i=1\\i\neq k}}^{M}\left(\sigma_{s,i}^{-2} w_i^{-\frac{1}{p}} + \sigma_{i,d}^{-2}\bar{w}_i^{-\frac{1}{p}}\right)\right] \times\right. \\ &\left.\left(\sigma_{s,k}^{-2} w_k^{-\frac{1-M}{p}} + \sigma_{k,d}^{-2}\bar{w}_k^{-\frac{1-M}{p}}\right)\right\}(g(\mathrm{SNR}))^{M+1}. \qquad (22) \end{aligned}$$

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