

## **Fuzzy Set Theory Approach for Measuring the Performance of Relationship-Based Construction Projects in Australia**

\*John F.Y. Yeung<sup>1</sup>, Albert P.C. Chan<sup>2</sup>, Daniel W.M. Chan, M.ASCE<sup>3</sup>

<sup>1</sup> Lecturer I, College of International Education, School of Continuing Education, Hong Kong Baptist University, Hong Kong, China (formerly Postdoctoral Fellow, Department of Building and Real Estate, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China)

<sup>2</sup> Professor and Associate Dean, Faculty of Construction and Environment, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China

<sup>3</sup> Associate Professor, Department of Building and Real Estate, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China

\* Corresponding author.

E-mail address: jfyeyung@hkbu.edu.hk (John F.Y. Yeung)

Telephone number: 852-3411-3140

Fax number: 852-3411-3253

**Abstract:** Research into performance measures for relationship-based construction projects becomes crucial because there exhibits an increasing trend of client organizations to adopt relationship-based (or relational contracting) approach to their construction projects worldwide over the last decade. However, few, if any, comprehensive and systematic research studies focus on developing a comprehensive, objective, reliable and practical performance evaluation model for relationship-based construction projects. A Performance Index (PI), which comprises eight weighted Key Performance Indicators (KPIs) and a set of corresponding Quantitative Indicators (QIs) for measuring the performance of relationship-based construction projects have been developed in Australia. The PI and QIs can assist in developing a benchmark for measuring the performance of relationship-based construction projects. However, the establishment of a set of QIs cannot fully solve the subjectivity of performance evaluation. In order to remedy this deficiency, the aim of this paper is to adopt a Fuzzy Set Theory (FST) approach to establish a well-defined range of Quantitative Requirements (QRs) for each QI within each of the five performance levels. By using the modified horizontal approach, Fuzzy Membership Functions (FMFs) have been constructed through three various methods, namely Constrained Regression Line with the Vertical Error Method (VEM), the Horizontal Error Method (HEM), and the Bisector Error Method (BEM). It was shown that the results derived from the three methods were similar and it seems that the Bisector Error Method is the best technique to construct the FMFs as it considers both the errors created by the residual sum of squares by both vertical and horizontal distances. The newly developed performance evaluation model is not only innovative in nature but it can also improve the objectiveness, reliability and practicality in evaluating the performance of relationship-based construction projects.

**DOI:** [http://dx.doi.org/10.1061/\(ASCE\)ME.1943-5479.0000083](http://dx.doi.org/10.1061/(ASCE)ME.1943-5479.0000083)

**CE Database subject headings:** Fuzzy sets; Construction management; Evaluation; Australia.

**Author keywords:** Fuzzy sets; Relationship-based approach; Relational contracting; Evaluation; Australia.

## **Introduction**

A rising trend has been observed from client organizations in adopting relationship-based approach (Different researchers use the terms “Relationship-based”, “Relational Contracting”, “Relational Contracts”, “Relationship Contracting”, and “Relationship Contracts” with the same meaning and therefore they are considered interchangeable in this paper. ) to their construction projects worldwide during the last decade (Rahman and Kumaraswamy 2002, 2004a, 2004b & 2008; Rowlinson and Cheung 2004). Owing to a number of perceived benefits of adopting relationship-based method to procure construction projects (Alsagoff and McDermott 1994; Jones 2000; Palaneeswaran et al. 2003; Kumaraswamy et al. 2005), research into performance measures for relationship-based construction projects becomes vital because it could assist in establishing a benchmark for measuring the performance of relationship-based construction projects. However, few, if any, comprehensive and systematic research studies focus on developing a comprehensive, objective, reliable and practical performance evaluation model for relationship-based construction projects.

The research team has developed a Performance Index (PI) for relationship-based construction projects in Australia, which is shown by the following formula (Yeung et al. 2009):

**Performance Index (PI) for Relationship-based Construction Projects in Australia**

$$\begin{aligned} \text{PI} = & 0.151 * \text{Client's Performance} + 0.131 * \text{Cost Performance} \\ & + 0.130 * \text{Quality Performance} + 0.125 * \text{Time Performance} \\ & + 0.124 * \text{Effective Communications} + 0.124 * \text{Safety Performance} \\ & + 0.110 * \text{Trust and Respect} + 0.105 * \text{Innovation and Improvement} \end{aligned}$$

The PI consists of eight weighted Key Performance Indicators (KPIs) identified in the Round 4 of the Delphi questionnaire survey and the coefficients are their individual weightings, which are calculated by their mean ratings divided by the total mean ratings. The PI is derived based on the assumption that this is a linear model. It is rational and valid to derive this linear model because the correlation matrix as shown in Table 1 reveals that the eight weighted KPIs are not highly correlated with each other at 5% significance level (more than half of them are even insignificantly correlated with each other). In addition, the measurement units for the eight weighted KPIs are different so it is unlikely to have any multiplier effect between them. Though it appears more sophisticated to use a nonlinear model to fit the data obtained, over-fitting is a common problem with nonlinear models particularly when the sample size is small (Neter et al. 2005; Weisberg 2005; Yeung et al. 2009). This is a major reason behind a linear, but not a nonlinear model, is recommended if the relationship amongst variables is not proved to be nonlinear. A linear model is assumed to be a linearized model of an unknown nonlinear model if it really exists (Morrison 1991; Griffiths et al. 1993). From practical point of view, it is simpler and easier to use this model to measure the performance of relationship-based construction projects within the Australian construction industry.

**Please insert Table 1 here.**

At a next research stage, corresponding Quantitative Indicators (QIs) for each of the eight weighted KPIs have been established for measuring the performance of relationship-based construction projects in Australia, including: (1) "Perceived Cost Satisfaction Scores of Clients by Using a 10-point Likert Scale"; (2) "Variation of Actual Project Cost Expressed as a Percentage of Finally Agreed Project Cost"; (3) "Average Number of Non-conformance Reports Generated Per Month"; (4) "Variation of Actual Completion Time Expressed as a Percentage of Finally Agreed Completion Time"; (5) "Perceived Key Stakeholders' Satisfaction Scores on Effective Communication Performance by Using a 10-point Likert Scale"; (6) "Accident Rate (in terms of Lost Time Injury Frequency Rate)"; (7) "Average Duration for Settling Variation Orders"; and (8) "Perceived Key Stakeholders' Satisfaction Scores on Innovation and Improvement Performance by Using a 10-point Likert Scale". The PI and QIs can be used to formulate a benchmark measure for the performance of relationship-based construction projects. However, the establishment of a series of QIs cannot fully solve the subjectivity problem of performance evaluation. For example, in terms of cost performance, 2% reduction in project cost may be perceived as "good performance" in one case while 5% reduction in project cost may be regarded as "very good performance" in other case. Should a relationship-based construction project be classified as "good" or "very good" in case of 3.5% reduction in project cost? The research question for this study is

to find out whether there exist some well-defined quantitative ranges which can be applied to objectively, reliably and practically measure the performance of relationship-based construction projects.

The aim of this study is to establish well-defined quantitative ranges/requirements (QRs) for each QI within each of the five performance levels that are used to classify various levels of achievement and these five levels are: (1) “poor”; (2) “average”; (3) “good”; (4) “very good”; and (5) “excellent” by the application of Fuzzy Set Theory (FST). By doing so, assessors could evaluate the performance of relationship-based construction projects with greater flexibility while maintaining objectivity. With the establishment of well-defined Fuzzy QRs (FQRs) by using the FST, different relationship-based construction projects can be assessed and compared more objectively, reliably and practically. Thus, it assists in setting a benchmark for measuring the performance of relationship-based projects, and developing a best practice model to achieve excellence in relational contracting performance. In fact, the FST could help model vagueness intrinsic in human cognitive process and may solve ill-posed and complicated problems due to incomplete and vague information that characterize many real-world systems. Certainly, it looks suitable for uncertain or approximate reasoning that involves human intuitive thinking (Zimmermann 2001) since much of our natural language is fuzzy in nature. The research findings of an empirical questionnaire survey will be discussed in this paper, followed by highlighting the significance and limitations of the research study.

## **Research Methodology**

The research method adopted in this research study included questionnaire survey and FST. As the QIs chosen are fuzzy in nature which requires evaluators’ subjective value judgment, the aim of this empirical questionnaire survey is to define Fuzzy Quantitative Requirements (FQRs) for each QI by seeking the perception of the same group of 17 construction experts, who participated in the first (including 4 rounds) and second (encompassing 2 rounds) Delphi surveys, on the performance evaluation criteria for relationship-based construction projects based on the selected KPIs and QIs previously identified by the 17 construction experts. At this research stage, a total of 12 valid responses were collected with a response rate of 70.59% and 5 Delphi experts withdrew from the study because of the heavy commitment of their current workload.

After collecting all the research data, FST was adopted for the data analysis. In fact, FST is a branch of modern mathematics that was originated by Zadeh (1965) to model vagueness intrinsic in human cognitive process. Afterwards, it has been used to solve ill-defined and complicated problems because of incomplete and imprecise information that characterize many real-world systems (Baloi and Price 2003). Contrary to binary or non-binary logic, the center of fuzziness is that the transition from a full-membership to non-membership state of an element of a set is gradual rather than sudden (Baloi and Price 2003). Thus, FST allows a generalization of the classical set concept to model complex and ill-defined systems. The main concepts associated with FST, as applied to decision systems, are: (1) membership functions; (2) linguistic variable; (3) natural language computation; (4) linguistic approximation; (5) fuzzy set arithmetic operations; (6) set operations; and (7) fuzzy weighted average (Bandemer and Gottwald 1995; Jamshidi 1997; Grima 2000; Piegat 2001; Zimmermann 2001; Ng et al. 2002; Baloi and Price 2003; Seo et al. 2004; Zheng and Ng 2005). Linguistic variable and membership functions are increasingly applied in construction management.

## **Mean Value of the Quantitative Assessment against the Five Different Performance Levels**

A view on the mean value (Table 2) could reveal the general perception of experts on the “poor”, “average”, “good”, “very good” and “excellent” performance level for each QI. A closer inspection to the standard deviation (SD) shows that there are slight to moderate deviations from the mean value in most of the performance levels depicting the QIs. However, the deviations are high for “variation of actual completion time expressed as a percentage of finally agreed completion time” (SD for the poor performance = 0.09); “variation of actual project cost expressed as a percentage of finally agreed project cost” (SD for the poor performance = 0.12); “average number of non-conformance reports generated per month for civil works” (SD for the poor performance = 4.38); “average number of non-conformance reports generated per month for building works” (SD for the poor performance = 11.10); and “accident rate” (in terms of Lost Time Injury Frequency Rate) (SD for the poor performance = 3.70).

The results indicate that differences in expectation exist between the Delphi experts in the perceived performance level of each QI. Therefore, although the mean value can serve as a quick rule-of-thumb for assessors to differentiate an “average” from “good” performance of a relationship-based construction project, it is more suitable to identify a range of reasonable expectations for each performance level. For instance, a relationship-based construction project with “good” time performance may range from ahead of schedule by 1% to 4%. If such a range can be defined, quantitative requirements for each QI against the five different performance levels could be established which would certainly provide greater flexibility for evaluators to objectively, reliably and practically evaluate the performance of a relationship-based construction project.

**Please insert Table 2 here.**

## **Establishment of Fuzzy Membership Functions**

A fuzzy set is a set whose elements having varying degrees of membership (Bharathi and Sarma 1985; Civanlar and Trussell 1986; Ng et al. 2002; Cross and Sudkamp 2002; Niskanen 2004). A fuzzy membership function enables one to perform quantitative calculations in fuzzy decision making. The degrees of membership of an element are expressed by a membership function. A membership function is a function that maps a universal set of objects,  $X$ , into the unit interval  $[0, 1]$  (Godal and Goodman 1980; Dubois and Prade 1983; Bharathi and Sarma 1985; Civanlar and Trussell 1986; Zimmermann, 2001). The universal set of objectives represents all the elements of the set and the interval corresponds to the set of grades. The grades of membership in fuzzy sets may fall anywhere within the interval  $[0, 1]$ . A degree of 0 (zero) means that an element is not a member of the set at all while a degree of 1 (one) represents full membership. Unlike “crisp” sets that have only one membership function, fuzzy sets have a vast number of membership functions. Membership functions comprising straight segments are always used in practice for their simplicity (Piegat 2001). Piegat (2001) stated that there are four major advantages of polygonal membership functions. Firstly, a small amount of data is needed to define the membership function. Secondly, it is easy to modify parameters (modal values) of membership functions on the basis of measured values of the input and output of a system. Thirdly, it is possible to obtain input and output mapping of a model which is a hyper-surface consisting of linear segments.

Finally, polygonal membership functions mean that the condition of a partition of unity (it means that the sum of membership grades for each value  $x$  amounts to one) is easily satisfied. However, it should be noted that polygonal membership functions are not continuously differentiable. Therefore, it is suitable to adopt Fuzzy Membership Function to define QRs with Fuzzy ranges because they are vague in nature and it is not good and unrealistic to use a single figure to measure each of the performance levels. These non-uniform ranges define a better classification properly. For more detailed discussions on FST, interested readers are encouraged to refer to the well-known textbooks by Zimmermann (2001) and Piegat (2001). Four major methods have been used for establishing the fuzzy membership function, including: (1) the horizontal approach (Godal and Goodman 1980; Bharathi-Devi and Sarma 1985); (2) the vertical approach (Civanlar and Trussel 1986); (3) the pairwise comparison approach (Saaty 1980); and (4) the membership function estimation approach with the aid of probabilistic characteristics (Dubois and Prade 1983). In addition, Ng et al. (2002) proposed a “modified horizontal approach” to develop the fuzzy membership function to evaluate the performance of engineering consultants, which is based on an amalgamation of the horizontal and graphical approaches (Bandemer and Gottwald 1995). In this research study, the modified horizontal approach was adopted for developing the fuzzy membership functions due to its simplicity and accuracy (Figure 1). Unlike the other methods for establishing the fuzzy membership functions, this approach allows the final outcome to be derived from simple probability functions (Ng et al. 2002; Chow 2005; Chow and Ng 2007). While the horizontal approach allows the computation of an optimal value of  $k$  (i.e. the number of bands) which is vital to the accuracy of estimation (Bharathi-Devi and Sarma 1985), the graphical approach further solves the problem of discontinuity in the transition from full membership to absolute exclusion in pure horizontal methods (Othnes and Enochson 1972). The fuzzy membership functions developed in this research study is firstly presented in a tabular form as shown in Table 3. Based on the value in the universe of discourse that defines the fuzzy set ( $X$ ) and the degree of membership of that fuzzy set ( $A$ ), a scatter diagram for the membership function is plotted (Figure 2) and the best-fit lines are produced to join all the separate points (i.e. lines AB and AC in Figure 2) using the MATLAB 7.0 to plot the fuzzy membership functions. Both Chow (2005) and Chow and Ng (2007) stated that it is rational to construct the best-fit lines passing through the highest point with full membership (point A in Figure 2) because there must be a peak in a Fuzzy membership function. When the line of best-fit for each of the five performance levels corresponding to a quantitative indicator is generated (Figure 3), the intersections of the best-fit lines between two consecutive performance levels represent a same degree of membership for both performance levels (e.g. points A, B, C and D). As a result, it is logical to choose these intersecting points to identify the QRs of each QI for the five different performance levels (i.e. “poor”, “average”, “good”, “very good” and “excellent”).

**Please insert Figure 1 here.**

**Please insert Table 3 here.**

**Please insert Figure 2 here.**

**Please insert Figure 3 here.**

Though the modified horizontal approach as adopted by Ng et al. (2002), Chow (2005) and Chow and Ng (2007) is theoretically sound, there is a major limitation in this method because the establishment of best-fit lines (the QIs against the five different performance levels) constrained to pass through the point with full membership has only considered the minimization of the residual sum of squares by vertical distance (taking the effect of dependent variable into consideration) (namely Vertical Error Method (VEM) in this research

study) (please refer to Appendix A). It is obvious that this method does not take the effect of independent variable into account. Therefore, although the modified horizontal approach was used in this research study, fuzzy membership functions have been constructed through constrained best-fit lines not only with the VEM, but also with the Horizontal Error Method (HEM) (please refer to Appendix B) (Minimizing the Residual Sum of Squares by Horizontal Distance – taking the effect of independent variable into consideration), and the Bisector Error Method (BEM) (please refer to Appendix C) (Minimizing the Residual Sum of Squares by Bisector Error – taking both the effects of dependent and independent variables into consideration) (Figure 2). Since the BEM considers the errors created by both the VEM and the HEM, it is taken as superior to the other two methods so it was selected to establish the fuzzy membership functions and calculate the FQRs in this research study.

## **Procedures for Defining the Fuzzy Quantitative Requirements (FQRs)**

Figure 1 elicits the six major steps of the “adapted” modified horizontal approach to define the FQRs, including: (1) establishing the most suitable quantitative interpretation for each KPI; (2) quantifying fuzzy QI; (3) identifying the “X” values of the fuzzy membership functions; (4) identifying the “A” values of the fuzzy membership functions; (5) formulating fuzzy membership functions; (5) deriving fuzzy membership functions graphs; and (6) identifying the QRs for each QI with respect to the five performance levels (through constrained best-fit lines with the VEM, HEM, and BEM).

### ***Establishment of the Most Suitable QI for each Weighted KPI***

As described earlier, QIs for each of the eight weighted KPIs, based on the previous study of the research team, have been established for measuring the relationship-based performance of construction projects in Hong Kong.

### ***Quantification of the Fuzzy Quantitative Indicators***

Through the questionnaire survey, a total of 17 previously identified Delphi experts were requested to provide a numerical figure ( $f_0$ ) for each QI with respect to the five different performance levels, namely “poor”, “average”, “good”, “very good” and “excellent”. As mentioned earlier, at this research stage, a total of 12 valid responses were collected with a response rate of 70.59% and 5 Delphi experts subsequently withdrew from the study because of the heavy commitment of their current workload.

### ***Identification of the “X” Values of the Fuzzy Membership Functions***

A fuzzy membership function is principally formulated by two values: X and A. X represents the value in the universe of discourse that defines the fuzzy set while A stands for the degree of membership of that fuzzy set.  $X_i$  values are defined as the means of bands  $B_i$  ( $i = 1, 2, \dots, k$ ), where  $B_i$  ( $i = 1, 2, \dots, k$ ) are the bands of values  $f_0$  given by the respondents of the empirical questionnaire survey to the QI pertinent to the five performance levels. The  $X_i$  values are defined according to the lowest and highest values of  $f_0$  for each QI and the number of bands  $k$ . To find the number of bands  $k$  for estimation, a widely used approach was proposed by Bharathi-Devi and Sarma (1985) with the following equation:

$$k = 1.87 (N - 1)^{\frac{2}{5}} \quad (\text{Equation 1})$$

where N is the total number of valid replies to the corresponding QI.

For example, Table 4 shows the perceived “excellent” time performance of a relationship-based construction project as suggested by each of the 12 respondents.

**Please insert Table 4 here.**

By using Equation 1, the number of bands  $k$  is calculated as follows:

$$\begin{aligned} k &= 1.87(N - 1)^{\frac{2}{5}} \\ k &= 1.87(12 - 1)^{\frac{2}{5}} \\ &= 4.880 \end{aligned}$$

By rounding off the value, there should be 5 numbers of bands for “excellent” time performance.

Sine the lowest value of this data set is -3% (ahead of schedule by 3%) and the greatest value is -20% (ahead of schedule by 20%). The range of each band should be:

$$(-20\% - (-3\%))/5 = -3.4\%$$

Therefore, after calculations, the Band 1 ranges from -3% to -6.4%; the Band 2 ranges from -6.4% to -9.8%; the Band 3 ranges from -9.8% to -13.2%; the Band 4 ranges from -13.2% to -16.6%; and the Band 5 ranges from -16.6% to -20.0%.

After defining the ranges of each band, the total number of values that falls into each band is counted and then their average value is taken so that the “X” values of the fuzzy membership function for the perceived “excellent” time performance are derived (Table 5).

**Please insert Table 5 here.**

#### ***Identification of the “A” Values of the Fuzzy Membership Functions***

The degree of membership  $A_i$  was computed according to the equation (Ng et al. 2002; Chow 2005):

$$A_i = n(B_i) / n_{\max} \quad \text{for } i=1, 2, 3, \dots, k \quad (\text{Equation 2})$$

In Equation 2,  $n(B_i)$  corresponds to the number of valid replies that have the values of  $f_0$  belonging to a certain band  $B_i$  and  $n_{\max}$  represents the maximum value of all the  $n(B_i)$  with  $i=1, 2, 3, \dots, k$ . Referring to Table 5, the maximum value of all  $n(B_i)$  is 4 and the “A” value for each band is calculated by the total number of values that falls into a particular band divided by 4. Table 5 also shows the calculations and results of “A” values. In order to examine whether the estimation of membership is valid, the standard deviation  $std(A)$  was calculated. The  $std(A)$  is found out by Equation 3. If the  $std(A)$  has a lower value than  $A_i$  computed in Equation 2, the estimation of membership is considered

acceptable (Ng et al. 2002; Chow 2005). If the  $std(A)$  does not have a lower value than  $A_i$ , the result is considered to be unacceptable, and a possible solution is to delete the outliers. In this research study, only 4 out of 225 number of the values of  $std(A)$  do not have a lower value than their  $A_i$ . However, by deleting those possible error points (outliers), all the estimations of membership become valid for further calculations.

$$std(A_i) = A_i \times \left(1 - A_i^{\frac{1}{2}}\right) / N \quad (\text{Equation 3})$$

### ***Formulation of the Fuzzy Membership Functions***

The fuzzy membership functions established in this research study are firstly presented in a tabular form as indicated in Table 3. Based on the X and A values, scatter diagrams for the membership function are plotted (Figure 2), with the horizontal axis representing the X values and the vertical axis representing the A values. After determining the degree of membership for all bands  $k$ , best-fit lines are plotted to join all the discrete points (i.e. lines AB and AC in Figure 2) using the MATLAB 7.0 to plot the fuzzy membership functions. As mentioned earlier, both Chow (2005) and Chow and Ng (2007) stated that it is logical to construct the best-fit lines passing through the point with full membership (point A in Figure 2) because there must be a vertex in a Fuzzy membership function. When the best-fit line for each of the five performance levels with respect to a quantitative indicator is drawn up (Figure 3), the intersections of the best-fit lines between two consecutive performance levels manifest a same degree of membership for both performance levels (e.g. points A, B, C and D). Consequently, it is logical to choose these intersecting points to identify the QRs of each QI for the five different performance levels (i.e. “poor”, “average”, “good”, “very good” and “excellent”). The QRs for each performance level are defined in Table 6.

**Please insert Table 6 here.**

### ***Identification of the FQRs for Each QI***

#### **Client’s Satisfaction**

The fuzzy membership functions of “poor”, “average”, “good”, “very good” and “excellent” performance for measuring the client’s satisfaction of relationship-based construction projects are all triangular shaped. The results indicate that the full memberships of the five different performance levels occur at 3, 5, 7, 8.9 and 10 respectively as perceived by client’s cost satisfaction scores based on a 10-point Likert scale. In order to cater for the vagueness of various performance levels, a range of allowable values for each performance level as shown in Table 4 should be referred to. For example, the Client’s Satisfaction of a relationship-based project would be classified as “poor” if the perceived client’s cost satisfaction scores based on a 10-point Likert scale is smaller than 4.2 scores; “average” if between 4.2 scores and less than 6 scores; “good” if between 6 scores and less than 7.6 scores; “very good” if between 7.6 scores and less than 8.9 scores; and “excellent” if equal to or greater than 8.9 scores.

## **Cost Performance**

The fuzzy membership functions of “poor”, “average”, “good”, and “excellent” for the cost performance of relationship-based projects are triangular shaped, whereas the membership function of “very good” performance is trapezoidal. The full memberships for the five different performance levels emerge at -10%, 0%, 4.6%, 5% and 15%. The results also indicate that the Cost Performance of a relationship-based project would be categorized as “poor” if the project is overrun budget by more than 5.8% (in terms of variation of actual project cost expressed as a percentage of finally agreed project cost); “average” if between overrun budget by 5.8% and less than 0% (on budget); “good” if between on budget and underrun budget by less than 3%; “very good” if between underrun budget by 3% and by less than 10.7%; and “excellent” if underrun budget by equal to or greater than 10.7%.

## **Quality Performance (Civil Works)**

The highest memberships of “poor”, “average”, “good”, “very good” and “excellent” performance are at 10.6, 8, 6, 6 and 0 average number of non-conformance reports generated per month. The least memberships of the five different performance levels take place at (1.19 and 29.66), (3.33 and 14.03), (-0.41 and 13.99), (-0.10 and 6), and (0 and 4.42). All membership functions of quality performance for civil works are triangular shaped. It was reflected from the results that the Quality Performance for Civil Works of a relationship-based project would be classified as “poor” if the average number of non-conformance reports generated per month is more than 8; “average” if between more than 7 and 8; “good” if between more than 6 and 7; “very good” if between more than 3 and 6; and “excellent” if less than or equal to 3.

## **Quality Performance (Building Works)**

The full memberships of “poor”, “average”, “good”, “very good” and “excellent” Quality Performance for building works of a relationship-based project are located at 18.33, 15, 10, 4.4 and 0 average number of non-conformance reports generated per month. The least memberships of the five different performance levels are set at (18.33 and 65.24), (3.33 and 27.16), (4.44 and 17.32), (7.50 and 34.63) and (0 and 14.71). All membership functions of quality performance for building works are triangular except that the membership function of “very good” performance is trapezoidal. The Quality Performance for Building Works of a relationship-based project would be categorized as “poor” if the average number of non-conformance reports generated per month is more than 18; “average” if between more than 13 and 18; “good” if between more than 8 and 13; “very good” if between more than 4 and 8; and “excellent” if less than or equal to 4.

## **Time Performance**

The fuzzy membership function of “poor”, “average”, “good”, “very good” and “excellent” performance for measuring the time performance of relationship-based construction projects are all triangular shaped. The results show that the full memberships of the five various performance levels occur when -10%, -0.2%, 4.8%, 10% and 15% respectively of time performance achieved. The Time Performance of a relationship-based project would be classified as “poor” if the project is behind schedule by more than 3.8% (in terms of variation of actual completion time expressed as a percentage of finally agreed completion time); “average” if between behind schedule by 3.8% and less than 0% (on time); “good” if between

on time and ahead of schedule by no less than 4.8%; “very good” if between ahead of schedule by 4.8% and less than 12%; and “excellent” if ahead of schedule by equal to or greater than 12%.

### **Safety Performance**

The highest memberships of “poor”, “average”, “good”, “very good” and “excellent” performance are at 4.80, 3.63, 0.50, 0.10 and 0 working hours per 1,000,000 working hours (accident rate measured in terms of Lost Time Injury Frequency Rate (LTIFR) per 1,000,000 working hours). All membership functions of safety performance are triangular shaped. Similar to other QIs, a range of allowable values for each performance level is calculated by using the modified horizontal approach with the BEM in order to cater for the vagueness of the five different performance levels. The research findings manifest that the safety performance of a relationship-based construction project would be regarded as “poor” if the accident rate is more than 4.5 working hours per 1,000,000 working hours; “average” if between greater than 2.1 working hours and 4.5 working hours; “good” if between greater than 0 working hour and 2.1 working hours; “very good” and “excellent” if it is equal to 0 working hour.

### **Effective Communications Performance**

The full memberships of “poor”, “average”, “good”, “very good” and “excellent” effective communications performance of a relationship-based construction project are located at 3, 5, 6, 8.1 and 10. The lowest memberships of the five different performance levels occur at (0.70 and 4.90), (2.31 and 9.38), (3.92 and 8.92), (6.55 and 10) and (7.56 and 10), respectively. The results reflect that all the membership functions are triangular and the effective communications performance of a relationship-based construction project is discerned as “poor” if the perceived key stakeholders’ satisfaction scores on effective communications performance according to a 10-point Likert scale is less than 3.8 scores; “average” if between 3.8 scores and less than 5.7 scores; “good” if between 5.7 scores and less than 7.4 scores; “very good” if between 7.4 scores and less than 9 scores; and “excellent” if equal to or more than 9 scores.

### **Trust and Respect Performance**

The membership functions of “poor”, “averaged”, “good” and “excellent” trust and respect performance are triangular, whereas the membership function of “very good” performance is trapezoidal. The full memberships of the five different performance levels occur at 6.33, 3.5, 2.4, 2 and 1 week on average to settle variation orders. The lowest memberships of the five different performance levels are set at (2.84 and 18.70), (3.5 and 10.12), (2.4 and 7.44), (1.17 and 7.6) and (1 and 3.79). The results reveal that the Trust and Respect Performance of a relationship-based construction project would be perceived as “poor” if the average duration for settling variation orders is longer than 5.4 weeks; “average” if between longer than 3.5 weeks and 5.4 weeks; “good” if longer than 2.4 weeks and 3.5 weeks; “very good” if between longer than 1.2 weeks and 2.4 weeks; and “excellent” if equal to or shorter than 1.2 weeks.

### **Innovation and Improvement Performance**

The fuzzy membership functions for “poor”, “average”, “very good” and “excellent” Innovation and Improvement Performance are triangular, whereas the membership function

of “good” is trapezoidal. The highest memberships for each level are at 0.02%, 1.14%, 5%, 9.3% and 9.89%. The full memberships of the five various performance levels occur at 2, 5, 7, 8 and 9 scores as perceived by key project stakeholders’ satisfaction scores on innovation and improvement performance based on a 10-point Likert scale. The results reveal that the Innovation and Improvement Performance of a relationship-based construction project would be classified as “poor” if the perceived key stakeholders’ satisfaction scores on innovation and improvement performance according to a 10-point Likert scale is less than 3.3 scores; “average” if between 3.3 scores and less than 5.5 scores; “good” if between 5.5 scores and less than 7.4 scores; “very good” if between 7.4 scores and less than 8.5 scores; and “excellent” if equal to or more than 8.5 scores.

## **Case Studies – Application of KPIs, QIs, QRs, and PI**

The QRs have been defined using fuzzy set theory in the previous section. For the sake of demonstrating the applicability of QRs together with KPIs, QIs, and PI to assess the performance of relationship-based construction projects in Australia, three case studies were investigated and all the data were provided by the Delphi experts from another questionnaire survey conducted by the same research team. The scope of analysis under each case study includes client’s satisfaction, cost performance, quality performance, time performance, communications performance, safety performance, trust and respect, and innovation and improvement. Table 7 shows the summary of the background information and the results of different KPIs, QIs, QRs, and PI of these three case studies. Details of each case study are discussed in the following subsections (Yeung et al. 2009).

**Please insert Table 7 here.**

### ***Case 1 – A Building Project***

It is a building work. The total contract duration was 28 months and the total contract sum was AUD\$100 million. The project was procured with management contracting and the form of contract is guaranteed maximum price. It is a collaborative project, in which it received 9 out of 10 client’s satisfaction scores (Excellent Performance) provided by a survey respondent and it was estimated to be under-run budget by 7.5% (Very Good Performance). The average number of non-conformance reports generated per month was 3 (Excellent Performance) and it was constructed on time (Good Performance). The accident rate was 1 out of 1,000,000 working hours (measured in terms of Lost Time Injury Frequency Rate) (Good Performance). The effective communications score was 8 (Very Good Performance) and the average duration for settling variation orders was 4 weeks (Average Performance). In addition, it received 8 innovation and improvement scores based on a 10-point Likert scale (Very Good Performance). As a whole, the PI was scored at 3.812 out of a total of 5, which can be concluded to be a good project.

### ***Case 2 – A Civil Project***

It is a civil work. The total contract duration was 15 months and the total contract sum was AUD\$250 million. The project was tendered with negotiated tendering method and the form of contract is target cost. It is an alliancing project, in which it received 9 out of 10 client’s satisfaction scores (Excellent Performance) provided by a respondent of a questionnaire and it was estimated to be on budget (Good Performance). The average number of non-conformance reports generated per month was 10 (Poor Performance) and it was constructed

on time (Good Performance). The accident rate was 1 out of 1,000,000 working hours (measured in terms of LTIFR) (Good Performance). The effective communications score was 8 (Good Performance) and the average duration for settling variation orders was 3 weeks (Good Performance). In addition, it received 8 innovation and improvement scores based on a 10-point Likert scale (Very Good Performance). As a whole, the PI was found to be 3.271 out of a total of 5, which can be perceived to be a project with average performance.

### ***Case 3 – A Civil Project***

It is a civil work. The total contract duration was 30 months and the total contract sum was AUD\$80 million. It is an alliancing project, in which it received 8 out of 10 client's satisfaction scores (Very Good Performance) provided by a respondent of a questionnaire and it was estimated to be under-run budget by 8% (Very Good Performance). The average number of non-conformance reports generated per month was 2 (Excellent Performance) and it was constructed ahead of schedule by 12% (Excellent Performance). The accident rate was 0 out of 1,000,000 working hours (measured in terms of LTIFR) (Excellent Performance). The effective communications score was 9.5 (Excellent Performance) and the average duration for settling variation orders was less than 1 week (Excellent Performance). In addition, it received 9.5 innovation and improvement scores based on a 10-point Likert scale (Excellent Performance). As a whole, the PI stood at 4.849 out of a total of 5, which can be regarded as an excellent project.

## **Validation of the Research Findings**

Due to the time and cost constraints, the proposed performance evaluation model for relationship-based construction projects in Australia was unable to be presented to other new field expert interviewees in Australia for proper direct validation. However, a similar performance evaluation model developed for partnering projects in Hong Kong using the same research methodology (Literature review, content analysis, Delphi questionnaire survey, another empirical questionnaire survey and Fuzzy Set Theory) was validated by seven independent expert interviewees who had not been involved in the Delphi questionnaire surveys. It should be noted that the surveyed group and validation group comprised of separate and independent groups of experts so that no biases existed for the validation results. In addition, since the seven independent expert interviewees had gained abundant hands-on experiences in procuring partnering projects in Hong Kong, they are highly qualified and representative enough to validate the research findings derived. The validation results derived from Hong Kong can serve as a useful reference for deducing that the performance evaluation model developed for relationship-based construction projects in Australia is comprehensive, objective, reliable and practical because the research methodology adopted between these two regions are the same and their research findings are quite similar.

## **Significance and Limitations of the Study**

By using a Fuzzy Set Theory approach (modified horizontal approach with the BEM), this research study has established well-defined FQRs for each of the QIs with each of the five performance levels (i.e. "poor", "average", "good", "very good" and "excellent") to measure the performance of relationship-based construction projects in Australia. The FQRs defined provides a more practical and objective way for assessors to evaluate relationship-based construction projects, thus preventing various assessors from applying subjective

interpretation to each QI during performance evaluation. As a result, assessors could determine which performance levels should be assigned in accordance with the actual performance of a relationship-based project. The proposed performance evaluation model is not only innovative in nature but it could also improve the objectiveness, reliability and practicality in evaluating relationship-based projects.

The performance evaluation model not only provides better understanding to clients, contractors and consultants on how to run a successful relationship-based project, but it also helps set a benchmark for measuring the performance of their relationship-based projects. Both the clients and contractors could apply the model for monitoring purposes when the relationship-based project is implemented at the very early beginning of the construction phase. And the results could be applied to compare the performance of other relationship-based construction projects for benchmarking purposes.

Despite the novel approach, there are still some limitations in using the newly developed model as the variability of project nature could affect its applicability. For instance, a particular range of cost savings may be suitable to evaluate one project type but less suitable for another. Therefore, it is important to note that project and environmental specifics at the time may have great effect on the adopted fuzzy ranges of QIs. For example, the effect of different project sizes, as well as various project natures may have an impact on the success of the project. Therefore, the performance evaluation model developed from this study should be taken as a prototype and the same research methodologies could be replicated to develop similar performance evaluation models to suit the unique application. The research approach can be scaled up to formulate another performance evaluation model for larger, more complex construction projects. By doing so, different performance evaluation models, such as private, public and infrastructure sector relationship-based construction projects can be generated for international comparisons.

## **Conclusion**

This research study has applied a Fuzzy Set Theory approach to establish well-defined quantitative ranges/requirements (QRs) for each Quantitative Indicator (QI) with each of the five performance levels that are used to classify various levels of achievement and these five levels are “poor”, “average”, “good”, “very good” and “excellent”. The “adapted” Modified Horizontal Approach combined with the Bisector Error Method was used to develop the fuzzy membership functions of each QI from all the data collected. The Quantitative Requirements (QRs) of each performance level were defined by the intersecting points of the best-fit lines between two consecutive performance levels of the fuzzy membership functions. With the development of a reliable set of QRs, evaluators could perform their evaluation based on the established fuzzy ranges rather than applying their subjective value judgment. As a result, assessors could determine which performance levels should be assigned in accordance with the actual performance of a relationship-based construction project. It should be pointed out that the results derived from this study are similar for the three methods (the Vertical Error Method, the Horizontal Error Method and the Bisector Error Method). However, it is theoretically better to use the Bisector Error Method to establish the Fuzzy membership functions because it considers both the errors created by the residual sum of squares by vertical and horizontal distances. It is concluded that the performance evaluation model for relationship-based construction projects is not only novel in nature but it could also significantly enhance the objectiveness, reliability and practicality in measuring the performance of relationship-based construction projects.

## Acknowledgement

The work described in this paper was fully supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (RGC Project No. PolyU 5158/04E). Special gratitude is devoted to those construction academics and industrial practitioners who have responded to and contributed their valuable input in completing the questionnaire surveys in Hong Kong.

## References

- Alsagoff, S.A., and McDermott, P. (1994). In Rowlinson, S. (ed.) "Relational contracting: A prognosis for the UK construction industry?" *Proceedings of CIB W92, Procurement Systems – East Meets West*, The University of Hong Kong, 11-19.
- Baloi, D., and Price, A.D.F. (2003). "Modeling global risk factors affecting construction cost performance." *International Journal of Project Management*, 21(4), 261-269.
- Bandemer, H., and Gottwald, S. (1995). "Fuzzy Sets, Fuzzy Logic, Fuzzy Methods with Application." John Wiley and Sons, New York.
- Bharathi-Devi, B., and Sarma, V.V.S. (1985). "Estimation of fuzzy memberships from histograms." *Information Sciences*, 35(1), 43-59.
- Chow, L.K. (2005). "Incorporating fuzzy membership functions and gap analysis concept into performance evaluation of engineering consultants – Hong Kong study." Unpublished PhD thesis, The University of Hong Kong.
- Chow, L.K., and Ng, S.T. (2007) "A fuzzy gap analysis model for evaluating the performance of engineering consultants", *Automation in Construction*, 16(4), 425-435.
- Civanlar, M.R., and Trussel, H.J. (1986). "Constructing membership functions using statistical data." *Fuzzy Sets and Systems*, 18(1), 1-13.
- Cross, V.V., and Sudkamp, T.A. (2002). "Similarity and compatibility in fuzzy set theory: assessment and application." Heidelberg, New York.
- Dubois, D., and Prade, H. (1983). "Unfair coins and necessity measures: towards a probabilistic interpretation of histograms." *Fuzzy Sets and Systems*, 10(1), 15-20.
- Godal, R.C., and Goodman, T.J. (1980). "Fuzzy sets and borel." *Transactions on Systems, Man, and Cybernetics*, 10(10), 637-640.
- Griffiths, H.B. (1993). "Mathematics of models: Continuous and discrete dynamics systems." New York: Ellis Horwood Ltd.
- Grima, M.A. (2000). "Neuro-fuzzy modeling in engineering geology." Netherlands: A.A. Balkema Publishers.
- Jamshidi, M. (1997). "Large-scale Systems: Modeling, Control and Fuzzy Logic." Prentice Hall.
- Jones, D. (2000). "Project alliances." *Proceedings of Conference on "Whose risk? Managing risk in construction – who pays?"*, Hong Kong, November.
- Kumaraswamy, M.M., Rahman, M.M., Ling, F.Y.Y., and Pheng, S.T. (2005). "Reconstructing cultures for relational contracting." *Journal of Construction Engineering and Management, ASCE*, 131(10), 1065-1075.
- Morrison, F. (1991). "The art of modeling dynamics systems: Forecasting for chaos, randomness and determinism" New York: John Wiley & Sons.
- Neter, J., Kutner, M., Nachtsheim, C., and Wasserman, W. (2005). "Applied linear statistical models." (5<sup>th</sup> Ed), McGraw-Hill, New York.
- Niskanen, V.S. (2004). "Soft computing methods in human sciences." New York: Springer.

- Ng, S.T., Luu, D.T., Chen, S.E., and Lam, K.C. (2002). "Fuzzy membership functions of procurement selection criteria." *Construction Management and Economics*, 20(2), 285-296.
- Otnes, R., and Enochson, L. (1972). "Digital Times Series Analysis." Wiley, New York.
- Palaneeswaran, E., Kumaraswamy, M., Rahman, M., and Ng, T. (2003). "Curing congenital construction industry disorders through relationally integrated supply chains." *Building and Environment*, 38(4), 571-582.
- Piegat, A. (2001). "Fuzzy modeling and control." Heidelberg, New York.
- Rahman, M.M., and Kumaraswamy, M.M. (2002). "Joint risk management through transactionally efficient relational contracting." *Construction Management and Economics*, 20(1), 45-54.
- Rahman, M.M., and Kumaraswamy, M.M. (2004a). "Potential for implementing relational contracting and joint risk management." *Journal of Management in Engineering, ASCE*, 20(4), 178-189.
- Rahman, M.M., and Kumaraswamy, M.M. (2004b). "Contracting Relationship Trends and Transitions." *Journal of Management in Engineering, ASCE*, 20(4), 147-161.
- Rahman, M.M., and Kumaraswamy, M.M. (2008). "Relational Contracting and Teambuilding: Assessing Potential Contractual and Noncontractual Incentives." *Journal of Management in Engineering, ASCE*, 24(1), 48-63.
- Rowlinson, S., and Cheung, F.Y.K. (2004). "A review of the concepts and definitions of the various forms of relational contracting." *Proceedings of the International Symposium of the CIB W92 on Procurement Systems 'Project Procurement for Infrastructure Construction'*, 7-10 January 2004, Chennai, India.
- Saaty, T.L. (1980). "The Analytical Hierarchy Processes." McGraw-Hill, New York.
- Seo, S., Aramaki, T., Hwang, Y., and Hanaki, K. (2004). "Fuzzy decision-making tool for environmental sustainable buildings." *Journal of Construction Engineering and Management, ASCE*, 130(3), 415-423.
- Weisberg, S. (2005). "Applied linear regression (3<sup>rd</sup> Ed)." John Wiley & Sons, New York.
- Yeung, J.F.Y., Chan, A.P.C., Chan, D.W.M., and Li, Leong-kwan. (2007). "Development of a Partnering Performance Index (PPI) for construction projects in Hong Kong: A Delphi Study." *Construction Management and Economics*, 25(12), 1219-1237.
- Yeung, J.F.Y., Chan, A.P.C., and Chan, D.W.M. (2009). "Developing a Performance Index of Relationship-based Construction Projects in Australia: Delphi Study." *Journal of Management in Engineering*, 25(2), 59-68.
- Zadeh, L.A. (1965). "Fuzzy Sets." *Information and Control*, 8(3), 338-353.
- Zheng, D.X.M., and Ng, T. (2005). "Stochastic time-cost optimization model incorporating Fuzzy Sets Theory and Non-replacement Front", *Journal of Construction Engineering and Management, ASCE*, 131(2), 176-186.
- Zimmermann, H.J. (2001). "Fuzzy Set Theory and Its Application." London: Kluwer Academic Publishers.

**Table 1. Correlation Matrix Amongst the 8 Weighted KPIs (for Round 4 of the Delphi Survey)**

Correlation Matrix	Client's Satisfaction	Cost Performance	Quality Performance	Safety Performance	Effective Communications	Time Performance	Trust and Respect	Innovation and Improvement
Client's Satisfaction	1	0.124	0.086	0.174	-0.211	0.203	-0.257	-0.669**
Cost Performance		1	0.201	0.367	-0.269	0.653**	-0.322	-0.383
Quality Performance			1	0.255	0.212	0.341	-0.288	-0.073
Safety Performance				1	-0.099	0.423	0.134	0.124
Effective Communications					1	0.352	0.557*	0.403
Time Performance						1	-0.085	-0.256
Trust and Respect							1	0.721**
Innovation and Improvement								1

\*\* Correlation is significant at the 0.01 level (2-tailed)

\* Correlation is significant at the 0.05 level (2-tailed)

**Table 2. Mean Values of the Quantitative Assessment Figures for the Five Various Performance Levels of each KPI**

KPIs	The Most Important Quantitative Indicator (QI) for each of the Eight Selected KPIs	Performance Level									
		Poor		Average		Good		Very Good		Excellent	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Client's Satisfaction	Perceived cost satisfaction scores of clients by using a 10-point Likert scale	2.88	0.92	4.75	1.05	6.46	0.90	8.00	0.88	9.08	1.00
Cost Performance	Variation of actual project cost expressed as a percentage of finally agreed project cost	-15.83%	0.12	-4.58%	0.05	2.75%	0.05	8.75%	0.05	13.79%	0.05
Quality Performance (Civil Works)	Average number of non-conformance reports generated per month	13.27	4.38	8.73	3.23	6.09	2.86	3.73	2.01	1.82	1.89
Quality Performance (Building Works)	Average number of non-conformance reports generated per month	30.00	11.10	20.00	6.60	13.50	5.81	8.20	5.58	4.30	4.99
Time Performance	Variation of actual completion time expressed as a percentage of finally agreed completion time	-10.75%	0.09	-1.83%	0.05	3.67%	0.06	8.71%	0.05	12.38%	0.05
Effective Communications	Perceived key stakeholders' satisfaction scores on effective communication by using a 10-point Likert scale	2.79	0.95	4.79	1.25	6.33	1.14	8.08	1.13	9.25	1.02
Safety Performance	Accident rate (in terms of Lost Time Injury Frequency Rate)	7.25	3.70	4.63	2.69	3.00	2.37	1.88	1.96	0.75	1.22
Trust and Respect	Average duration for settling variation orders	8.64	2.76	5.73	2.41	4.18	2.05	2.68	1.53	1.82	0.98
Innovation and Improvement	Perceived key stakeholders' satisfaction scores on innovation and improvement by using a 10-point Likert scale	2.33	1.07	4.46	1.43	6.17	1.11	7.71	0.85	8.83	0.79

SD = Standard Deviation

**Table 3. X and A Values of the ‘Excellent’ Time Performance of a Relationship-based Construction Project in Australia (Q4 in the Questionnaire Survey)**

Percentage (X)	-4%	-7.5%	-11%	-15%	-20%
Degree of membership (A)	0.5	0.25	0.75	1	0.5

Note: ‘-’ represents ‘ahead of schedule’

**Table 4. The Perceived “Excellent” Time Performance of a Relationship-Based Construction Project As Suggested by Each of the 12 Construction Delphi Experts**

Expert “Excellent” performance	1	2	3	4	5	6	7	8	9	10	11	12
time	-10%	-3%	-15%	-13%	-7.5%	-10%	-20%	-20%	-5%	-15%	-15%	-15%

Note: “-” represents “ahead of schedule”

**Table 5. “X” and “A” Values of the “Excellent” Time Performance of a Relationship-Based Construction Project as Perceived by the 12 Construction Delphi Experts**

Band	Range	Total no. of Values Falling in Each Band	Value of $X_i$	Value of $A_i$	Std(Ai)
1	-3% to -6.4%	2	$\frac{-3\% + (-5\%)}{2} = -4.0\%$	$\frac{2}{4} = 0.5$	0.0122
2	-6.4% to -9.8%	1	$\frac{-7.5\%}{1} = -7.5\%$	$\frac{1}{4} = 0.25$	0.0104
3	-9.8% to -13.2%	3	$\frac{-10\% \times 2 + (-13\%)}{3} = -11\%$	$\frac{3}{4} = 0.75$	0.0084
4	-13.2% to -16.6%	4	$\frac{-15\% \times 4}{4} = -15\%$	$\frac{4}{4} = 1$	0.0000
5	-16.6% to -20.0%	2	$\frac{-20\% \times 2}{2} = -20\%$	$\frac{2}{4} = 0.5$	0.0122

**Table 6. The Fuzzy QRs of each QI Against the Five Different Performance Levels for each KPI**

The Top-8 KPIs (with corresponding weightings) (total weighting equal to 1)	The Most Important Quantitative Indicator (QI) for each of the top-8 KPIs	Performance Level				
		Poor	Average	Good	Very Good	Excellent
Client's Satisfaction	Perceived Client's Cost Satisfaction Scores by Using a 10-point Likert Scale	<4.1	4.1-5.9*	5.9-7.6*	7.6-8.9*	≥8.9
		<4.3	4.3-6.1*	6.1-7.6*	7.6-8.9*	≥8.9
		<4.2	4.2-6.0*	6.0-7.6*	7.6-8.9*	≥8.9
Cost Performance (0.160)	Variation of Actual Project Cost Expressed as a Percentage of Finally Agreed Project Cost	<-5.8%	-5.8%-0*%	0%-3*%	3%-10.7*%	≥10.7%
		<-5.8%	-5.8%-0*%	0%-3*%	3%-10.8*%	≥10.8%
		<-5.8%	-5.8%-0*%	0%-3*%	3%-10.7*%	≥10.7%
Quality Performance (0.143)	Average Number of Non-conformance Reports Generated Per Month (for Civil Works)	>9	7 <sup>+</sup> -9	6 <sup>+</sup> -7	3 <sup>+</sup> -7	≤3
		>8	7 <sup>+</sup> -8	6 <sup>+</sup> -7	2 <sup>+</sup> -6	≤2
		>8	7 <sup>+</sup> -8	6 <sup>+</sup> -7	3 <sup>+</sup> -6	≤3
Quality Performance (0.143)	Average Number of Non-conformance Reports Generated Per Month (for Building Works)	>18	14 <sup>+</sup> -18	8 <sup>+</sup> -14	4 <sup>+</sup> -8	≤4
		>18	13 <sup>+</sup> -18	8 <sup>+</sup> -13	4 <sup>+</sup> -8	≤4
		>18	13 <sup>+</sup> -18	8 <sup>+</sup> -13	4 <sup>+</sup> -8	≤4
Time Performance (0.167)	Variation of Actual Completion Time Expressed as a Percentage of Finally Agreed Completion Time	<-3.9%	-3.9%-0*%	0%-4.8*%	4.8%-12.0*%	≥12.0%
		<-3.8%	-3.8%-0*%	0%-4.8*%	4.8%-12.0*%	≥12.0%
		<-3.8%	-3.8%-0*%	0%-4.8*%	4.8%-12.0*%	≥12.0%
Safety Performance	Accident rate (in Terms of Lost Time Injury Frequency Rate (LTIFR) per 1,000,000 Working Hours)	>4.5	2.2 <sup>+</sup> -4.5	0 <sup>+</sup> -2.2	0	0
		>4.5	2.0 <sup>+</sup> -4.5	0 <sup>+</sup> -2.0	0	0
		>4.5	2.1 <sup>+</sup> -4.5	0 <sup>+</sup> -2.1	0	0
Effective Communications (0.131)	Perceived Key Stakeholders' Satisfaction Scores on Effective Communication by Using a 10-point Likert Scale.	<3.8	3.8-5.7*	5.7-7.3*	7.3-9*	≥9
		<3.8	3.8-5.7*	5.7-7.5*	7.5-9*	≥9
		<3.8	3.8-5.7*	5.7-7.4*	7.4-9*	≥9

Trust and Respect (0.143)	Average Duration for Settling Variation Orders.	>5.5	3.5 <sup>+</sup> -5.5	2.4 <sup>+</sup> -3.5	1.2 <sup>+</sup> -2.4	≤1.2
		>5.3	3.5 <sup>+</sup> -5.3	2.4 <sup>+</sup> -3.5	1.2 <sup>+</sup> -2.4	≤1.2
		>5.4	3.5 <sup>+</sup> -5.4	2.4 <sup>+</sup> -3.5	1.2 <sup>+</sup> -2.4	≤1.2
Innovation and Improvement (0.106)	Perceived Key Stakeholders' Satisfaction Scores on Innovation and Improvement by Using a 10-point Likert Scale.	<3.4	3.4-5.5 <sup>*</sup>	5.5-7.4 <sup>*</sup>	7.4-8.3 <sup>*</sup>	≥8.3
		<3.2	3.2-5.5 <sup>*</sup>	5.5-7.4 <sup>*</sup>	7.4-8.7 <sup>*</sup>	≥8.7
		<3.3	3.3-5.5 <sup>*</sup>	5.5-7.4 <sup>*</sup>	7.4-8.5 <sup>*</sup>	≥8.5

Note: M<sup>\*</sup>% represents “less than M%” while M<sup>+</sup> represents “more than M”

The first figure of each cell is calculated by the Vertical Error Method, the second figure by the Horizontal Error Method, and the third figure by the Bisector Error Method

**Table 7. Case Studies - Application of KPIs and PI (Yeung et al. 2009)**

	<b>Case 1</b>	<b>Case 2</b>	<b>Case 3</b>
<b>Background</b>			
Nature of project	Building work	Civil work	Civil work
Type of project	Other types of collaborative project	Alliancing project	Alliancing project
Procurement method	Management contracting	Unknown	Unknown
Tendering method	Selective tendering	Negotiated tendering	Unknown
Form of contract	Guaranteed maximum price	Target cost	Unknown
Total contract duration	28 months	15 months	2.5 years
Total contract sum	AUD\$100 million	AUD\$250 million	AUD\$80 million
<b>KPIs survey result</b>			
Client's satisfaction Score	9 out of 10 client's satisfaction scores	9 out of 10 client's satisfaction scores	8 out of 10 client's satisfaction
Cost Performance	Underrun budget by 7.5%	On budget	Underrun budget by 8%
Quality Performance	3 average number of non-conformance reports generated per month	10 average number of non-conformance reports generated per month	2 average number of non-conformance reports generated per month
Time Performance	On schedule	On schedule	Ahead schedule by 12%
Safety Performance	1 (measured in terms of LTIFR 1,000,000 working hours)	1 (measured in terms of LTIFR 1,000,000 working hours)	0 (measured in terms of LTIFR 1,000,000 working hours)
Effective Communications	8 out of 10 effective communications scores	8 out of 10 effective communications scores	9.5 out of 10 effective communication scores
Trust and Respect	4 weeks (average duration for settling variation orders)	3 weeks (average duration for settling variation orders)	Less than 1 week (average duration for settling variation order)
Innovation and Improvement	8 out of 10 innovation and improvement scores	8 out of 10 innovation and improvement scores	9.5 out of 10 innovation and improvement scores
<b>Performance Index</b>			
	3.812 out of 5 scores	3.271 out of 5 scores	4.849 out of 5 scores

## Appendix A. Constructing Fuzzy Membership Function by Using Modified Horizontal Approach through Calculating Constrained Regression Line by Minimizing Residual Sum of Square by Vertical Error

Suppose given a constraint that a best-fit line must pass through  $(x_0, 1)$  (full membership function when  $y = 1$ ) and  $(x_i, y_i)$  for  $1 \leq i \leq N$ , we need to minimize  $\sum_{i=1}^N (Y_i - y_i)^2$

Let the error function be  $E = \sum_{i=1}^N (Y_i - y_i)^2$  and theoretically,  $Y_i = mX_i + b$  (slope-intercept form) and so  $y_0 = mX_0 + b$

Hence, we need to find  $m, b$  such that  $E(m, b)$  is minimized and  $y_0 = mX_0 + b$

Based on the above constraint,  $b = y_0 - mX_0 = 1 - mX_0$

$$E = \sum_{i=1}^N (mX_i + b - y_i)^2 = \sum_{i=1}^N (mX_i + (1 - mX_0) - y_i)^2$$

Now, the error function is a single variable of  $m$ .

To compute error derivative with respect to  $m$ ,

$$\begin{aligned} \frac{dE}{dm} &= \frac{d}{dm} \left( \sum_{i=1}^N (m(x_i - x_0) + 1 - y_i) \right)^2 \text{ by using composite function rule in calculus.} \\ &= \sum_{i=1}^N 2(m(x_i - x_0) + 1 - y_i) \times (x_i - x_0) \end{aligned}$$

Since it is a necessary condition to set  $\frac{dE}{dm} = 0$  for finding minimum/maximum value, the following equation is set.

$$\sum_{i=1}^N 2(m(x_i - x_0) + 1 - y_i)(x_i - x_0) = 0$$

$$m \times \sum_{i=1}^N (x_i - x_0)^2 + \sum_{i=1}^N (1 - y_i)(x_i - x_0) = 0$$

$$m \times \sum_{i=1}^N (x_i - x_0)^2 = \sum_{i=1}^N (y_i - 1)(x_i - x_0)$$

$$m = \frac{\sum_{i=1}^N (y_i - 1)(x_i - x_0)}{\sum_{i=1}^N (x_i - x_0)^2}$$

By substitution,  $b = y_0 - mX_0 = 1 - \frac{\sum_{i=1}^N (y_i - 1)(x_i - x_0)}{\sum_{i=1}^N (x_i - x_0)^2} \times x_0$

## Appendix B. Constructing Fuzzy Membership Function by Using Modified Horizontal Approach through Calculating Constrained Regression Line by Minimizing Residual Sum of Square by Horizontal Error

Suppose given a constraint that a best-fit line must pass through  $(x_0, 1)$  (full membership function when  $y=1$ ) and  $(x_i, y_i)$  for  $1 \leq i \leq N$ , we need to minimize

$$\sum_{i=1}^N (X_i - x_i)^2$$

Let the error function be  $E = \sum_{i=1}^N (X_i - x_i)^2$  and theoretically,  $X_i = pY_i + q$  (slope-intercept form) and so  $x_0 = pY_0 + q$  and  $y_i = \frac{1}{p}X_i - \frac{q}{p}$

Therefore,  $m = \frac{1}{p}$  and  $b = \frac{q}{p}$

Now, we need to find  $p, q$  such that  $E(p, q)$  is minimized and  $x_0 = py_0 + q$

Based on the above constraint,  $q = x_0 - p$

$$\begin{aligned} E(p, q) &= \sum_{i=1}^N (py_i + q - x_i)^2 = \sum_{i=1}^N (py_i + x_0 - p - x_i)^2 \\ &= \sum_{i=1}^N (p(y_i - 1) + (x_0 - x_i))^2 \end{aligned}$$

Now, the error function is a single variable of  $p$ .

To compute error derivative with respect to  $p$ ,

$\frac{dE}{dp} = \frac{d}{dp} \left( \sum_{i=1}^N (p(y_i - 1) + (x_0 - x_i)) \right)^2$  by using composite function rule in calculus.

$$= \sum_{i=1}^N 2(p(y_i - 1) + (x_0 - x_i))(y_i - 1)$$

Since it is a necessary condition to set  $\frac{dE}{dp} = 0$  for finding minimum/maximum value, the following equation is set.

$$\sum_{i=1}^N 2(p(y_i - 1) + (x_0 - x_i))(y_i - 1) = 0$$

$$\sum_{i=1}^N (p(y_i - 1) + (x_0 - x_i))(y_i - 1) = 0$$

$$p \times \sum_{i=1}^N (y_i - 1)^2 + \sum_{i=1}^N (x_0 - x_i)(y_i - 1) = 0$$

$$p \times \sum_{i=1}^N (y_i - 1)^2 = \sum_{i=1}^N (x_i - x_0)(y_i - 1)$$

$$p = \frac{\sum_{i=1}^N (x_i - x_0)(y_i - 1)}{\sum_{i=1}^N (y_i - 1)^2}$$

By substitution,  $q = x_0 - p = x_0 - \frac{\sum_{i=1}^N (x_i - x_0)(y_i - 1)}{\sum_{i=1}^N (y_i - 1)^2}$

Therefore,  $m = \frac{1}{\frac{\sum_{i=1}^N (x_i - x_0)(y_i - 1)}{\sum_{i=1}^N (y_i - 1)^2}}$

$$b = \left( x_0 - \frac{\sum_{i=1}^N (x_i - x_0)(y_i - 1)}{\sum_{i=1}^N (y_i - 1)^2} \right) \times \frac{\sum_{i=1}^N (y_i - 1)^2}{\sum_{i=1}^N (x_i - x_0)(y_i - 1)}$$

### **Appendix C. Constructing Fuzzy Membership Function by Using Modified Horizontal Approach through Calculating Constrained Regression Line by Minimizing Residual Sum of Square by Bisector Error**

$$m = \tan\left(\frac{\vartheta_1 + \vartheta_2}{2}\right) = \frac{\tan \frac{\vartheta_1}{2} + \tan \frac{\vartheta_2}{2}}{1 - \tan \frac{\vartheta_1}{2} \times \tan \frac{\vartheta_2}{2}}$$

$$(x_0 \times m) + c = 1$$

$$x = 1 - (x_0 \times m)$$