Exact soliton solutions for the core of dispersion-managed solitons

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(Received 9 January 2003; published 14 October 2003)

We consider the averaged dispersion-managed (DM) fiber system equation, which governs the dynamics of the core of the DM solitons. For a special case of such a system equation, we derive the exact soliton solutions using the Darboux transformation. Further, we discuss the interaction scenario between two neighboring solitons. Finally, we derive a dark soliton solution for such a system by assuming an ansatz, and the interaction between neighboring dark solitons is discussed.

DOI: 10.1103/PhysRevE.68.046605 PACS number(s): 42.81.Dp, 42.65.Tg, 05.45.Yv

The dispersion-managed (DM) fiber system has paved a new way to increase the transmitting capacity of optical fiber links [1–4]. Basically, the dispersion-management technique utilizes a fiber transmission line with a periodic dispersion map, such that each period is built up by two types of fiber, generally with different lengths and opposite group-velocity dispersion (GVD). Because of the periodic splicing of anomalous and normal dispersion fibers, there is an abrupt discontinuity in the GVD of the DM fiber system. This has left almost no way to analytically handle the DM fiber system governing equation. Hence, only numerical DM soliton solutions are being derived using the averaging method [5].

To analytically describe the evolution of the parameters of the DM solitons, the variational principle is widely used with the help of a Gaussian ansatz [1]. Based on the exact solution of the variational equations, very recently analytical methods have been reported for designing the dispersion map of the DM fiber systems [6,7]. All these techniques are fundamentally based on the feature that most of the time during the periodic evolution of the DM soliton, the core is very close to a Gaussian shape [8,9].

Hasegawa *et al.* [10] tried a different kind of approach to studying is the properties of the core of the DM solitons. In that approach they considered the lossless DM fiber system and after removing the fast varying chirp part of the DM soliton they derived the averaged DM fiber system equation which governs the dynamics of the core of the DM solitons.

In this work, we also follow a similar procedure to derive the averaged DM soliton equation for DM fiber system with loss or gain. The same system equation also governs the nonlinear pulse propagation in a uniform fiber system with loss (gain) and frequency chirp. We show that for a specific choice of the DM fiber system parameters the averaged DM soliton system equation has exact soliton solutions. Lax pair for such a soliton system is reported. Based on the Lax pair, a methodology to derive N-soliton solutions is presented by employing simple, straightforward Darboux transformation. As examples, one- and two-soliton solutions in explicit forms are generated and their properties are also analyzed. It is shown that there is an exact balance between the fiber loss (gain) and pulse chirping to achieve the compression of the soliton pulse. In addition, we discuss the interaction scenario between two neighboring solitons in detail. Finally, we derive a dark soliton solution for such a system by assuming an ansatz, and the interaction between neighboring dark solitons is discussed.

The nonlinear Schrödinger equation (NLSE) which governs the dynamics of DM fiber system is given by

$$\frac{\partial u}{\partial z}=\frac{id(z)}{2}\frac{\partial^2 u}{\partial T^2}+i|u|^2u-\Gamma u, \tag{1}$$

where u is the envelope of the axial electric field, $d(z)$ is the periodically varying GVD parameter representing anomalous and normal dispersions, and Γ is the loss (gain) coefficient. In the following we follow the same steps as Hasegawa *et al.* [10] for deriving the averaged DM soliton system equation. Here the only difference is that we explicitly retain the loss (gain) term also in the NLSE.

Because of the large variation in the GVD parameter in going from the normal dispersion fiber to the anomalous dispersion fiber and vice versa, there is a large variation in the quadratic phase chirp of the DM soliton within each dispersion map. Hence the DM soliton field can be considered in the form

$$u(z,T)=w(z,T)\exp\left[\frac{i}{2}C(z)T^2\right]. \tag{2}$$

Inserting Eq. (2) into Eq. (1), we have

$$i\left(\frac{\partial w}{\partial z}+dCT\frac{\partial w}{\partial T}\right)+\frac{d}{2}\frac{\partial^2 w}{\partial T^2}+|w|^2w-\frac{1}{2}(\dot{C}+dC^2)T^2w$$
$$=-\frac{i}{2}dCw-i\Gamma w, \tag{3}$$

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where $\dot{C}$ represents the derivative of C with respect to z. Now introduce a new coordinate t and the amplitude function $a(z)$ such as

$$t=p(z)T\equiv T\exp\left[-\int_0^z d(\zeta)C(\zeta)d\zeta\right], \quad (4)$$

$$w(T,z)=a(z)v(t,z). \quad (5)$$

Then we have

$$i\frac{\partial v}{\partial z}+\frac{dp^2}{2}\frac{\partial^2 v}{\partial t^2}+a^2|v|^2v=\frac{\kappa(z)}{2}t^2v-i\Gamma v \quad (6)$$

with the equations for $a(z)$, $p(z)$, and $C(z)$ as

$$\dot{a}=-\frac{1}{2}Cad, \quad (7)$$

$$\dot{p}=-Cpd, \quad (8)$$

$$\kappa(z)\equiv\frac{\dot{C}+C^2d}{p^2}. \quad (9)$$

Now averaging Eq. (6) for one dispersion map, we get

$$i\frac{\partial q}{\partial z}+\frac{D_0}{2}\frac{\partial^2 q}{\partial t^2}+\gamma_0|q|^2q-\kappa_0t^2q=-i\Gamma_0q, \quad (10)$$

where $D_0=\langle dp^2\rangle$, $\gamma_0=\alpha\langle p\rangle$, $\kappa_0=\langle\kappa\rangle/2$, and Γ_0 is the small residual loss or gain in one dispersion map. In regular DM systems an amplifier at the end of each dispersion map will compensate for the total power loss in the respective dispersion map. Here we consider that the periodic amplification is not exactly compensating for the loss. One can consider that either there is a small residual loss or gain in each dispersion map, which is a must for our study, which will be shown in the following. The study on Eq. (10) is restricted not only for the core of a DM soliton but also for the optical pulse propagating in a uniform fiber system with loss (gain) effect with quadratic phase chirp represented by the t^2 term. This finds application in pulse compression. It should be pointed out that without the residual loss (gain) term, Eq. (10) has been studied in different contexts in Refs. [11,12], concretely speaking, where nonlinear compression of chirped solitary waves [11] and quasisoliton propagation in DM optical fiber [12] have been discussed. With the loss (gain) term, the special case of Eq. (10) has been reported in Refs. [13–15] from the integrability point of view, where by choosing a special parameter, one soliton solution has been obtained by Bäcklund transformation. Equation (10) also describes the propagation of envelope solitons in inhomogeneous media—an example being that of electromagnetic waves in an inhomogeneous plasma [16]. In this paper, we present the general procedure to construct the N-soliton solutions, and the explicit one- and two- soliton solutions are presented.

Now, we consider the special parametric choice with $D_0=2\alpha_1$, $\gamma_0=2\mu^2\alpha_1$, $\kappa_0=-\beta^2$, and $\Gamma_0=\beta$ so that Eq. (10) becomes

$$i\frac{\partial q}{\partial z}+\alpha_1\frac{\partial^2 q}{\partial t^2}+2\mu^2\alpha_1|q|^2q+\beta^2t^2q+i\beta q=0. \quad (11)$$

In Eq. (11), one can see that the coefficients of the quadratic phase chirp term and the loss (gain) term are related to the parameter β. This relationship is a must for the complete integrability of Eq. (10) as shown through the Painlevé analysis [17]. This is why we need a residual gain or loss factor in the averaged DM soliton system, in contrast to the usual DM fiber system where the periodic amplification exactly compensates for the fiber loss in the respective span. Note that with respect to the sign of the parameter β, the averaged DM fiber system will have either a lossy or amplifying effect.

Considering the following spectral problem:

$$\psi_t=U\psi, \quad (12)$$

$$\psi_z=V\psi, \quad (13)$$

where

$$U=\lambda J+P,$$

$$V=2i\alpha_1\lambda^2J-2\beta t\lambda J+2i\alpha_1\lambda P+W,$$

with

$$J=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad P=\begin{pmatrix}0 & \mu Q\\ -\mu\overline{Q} & 0\end{pmatrix},$$

$$W=\begin{pmatrix}i\mu^2\alpha_1|Q|^2 & -2\mu\beta tQ+i\mu\alpha_1Q_t\\ 2\mu\beta t\overline{Q}+i\mu\alpha_1\overline{Q_t} & -i\mu^2\alpha_1|Q|^2\end{pmatrix}.$$

Here $\overline{Q}$ represents the complex conjugate of Q. From the compatibility condition $U_z-V_t+[U,V]=0$ one can derive Eq. (11). Where $Q=q\exp(-i\beta t^2/2)$ and λ is the variable spectral parameter given by

$$\lambda=\eta(z)+i\zeta(z), \quad \lambda_z=-2\beta\lambda, \lambda_t=0,$$

$$\lambda=\nu\exp(-2\beta z), \quad \eta(z)=\mathrm{Re}(\nu)\exp(-2\beta z),$$

$$\zeta(z)=\mathrm{Im}(\nu)\exp(-2\beta z).$$

Here $\mathrm{Re}(\nu)$ and $\mathrm{Im}(\nu)$ are the real and imaginary parts, respectively, of the hidden isospectral parameter ν. The Lax pair assures the complete integrability of a nonlinear system and is specially used to obtain integrability condition and N-soliton solutions by means of the inverse scattering transform method. In this paper, we investigate Eq. (11) by employing a simple, straightforward Darboux transformation [18–20]. In the following, we give the Darboux transformation of Eq. (11).

Introducing transformation

$$\varphi' = (\lambda I - S)\varphi, \quad S = H\Lambda H^{-1}, \quad \Lambda = \mathrm{diag}(\lambda_1, \lambda_2), \tag{14}$$

where H is a nonsingular matrix, requiring

$$\varphi'_t = U'\varphi', \quad U' = \lambda J + P', \quad P' = \begin{pmatrix} 0 & \mu Q' \\ -\mu \overline{Q'} & 0 \end{pmatrix}, \tag{15}$$

and combining Eqs. (12), (13), (14), and (15), we obtain the Darboux transformation for Eq. (11) in the form

$$P' = P + JS - SJ. \tag{16}$$

It is easy to verify that, if $(\varphi_1, \varphi_2)^T$ is a solution of Eq. (12) corresponding to eigenvalue $\lambda = \lambda_1$, $(-\overline{\varphi_2}, \overline{\varphi_1})^T$ is also a solution of Eq. (12) and the eigenvalue λ is replaced by $-\overline{\lambda_1}$, that is, if we consider

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & -\overline{\lambda_1} \end{pmatrix}, \quad H = \begin{pmatrix} \varphi_1 & -\overline{\varphi_2} \\ \varphi_2 & \overline{\varphi_1} \end{pmatrix},$$

then

$$S_{ij} = -\overline{\lambda_1}\delta_{ij} + \frac{(\lambda_1 + \overline{\lambda_1})\varphi_i \overline{\varphi_j}}{\Delta}, \quad i,j = 1,2, \tag{17}$$

where

$$\Delta = \det|H| = |\varphi_1|^2 + |\varphi_2|^2.$$

From Eq. (16) we obtain other solutions as

$$Q' = Q + \frac{2}{\mu} S_{12}; \quad \overline{Q'} = \overline{Q} + \frac{2}{\mu} S_{21}. \tag{18}$$

Thus we obtain the fundamental expression of the Darboux transformation.

Analogous to this procedure and taking the Darboux transformation n times, we find the following formula:

$$Q[n] = Q + \frac{2}{\mu} \sum \frac{(\lambda_m + \overline{\lambda_m})\psi_1[m,\lambda_m]\overline{\psi_2}[m,\lambda_m]}{\psi[m,\lambda_m]^T \overline{\psi}[m,\lambda_m]}, \tag{19}$$

where $m = 1, \ldots, n$ and

$$\psi[m,\lambda] = (\lambda - S[m-1]) \cdots (\lambda - S[1])\psi[1,\lambda],$$

$$S_{ij}[k] = -\overline{\lambda_k}\delta_{ij} + \frac{(\lambda_k + \overline{\lambda_k})\psi_i[k,\lambda_k]\overline{\psi_j}[k,\lambda_k]}{(\psi[k,\lambda_k], \psi[k,\lambda_k])},$$

$i,j = 1,2$, $k = 1,2, \ldots, m-1$, $m = 2,3, \ldots, n$, where $\psi[1,\lambda]$ is the eigenfunction corresponding to λ for φ_1 and φ_2. Substituting the zero solution of Eq. (11) as $q = 0$ into Eq. (19), one can derive the one-soliton solution for Eq. (11). Using that one soliton solution as the seed solution in Eq. (19), we can derive the two-soliton solution. Thus in recursion, one can generate up to N-soliton solution.

By setting $n = 1$ in Eq. (19), the one-soliton solution can be derived as

$$q = \frac{2\eta(z)}{\mu}\mathrm{sech}[2\xi(z,t)]\exp\left[i2\theta(z,t) + \frac{i\beta t^2}{2}\right], \tag{20}$$

where

$$\xi(z,t) = \eta(z)t - 4\alpha_1 \int \eta(z)\zeta(z)dz + T_1, \tag{21}$$

$$\theta(z,t) = \zeta(z)t + 2\alpha_1 \int [\eta^2(z) - \zeta^2(z)]d; \tag{22}$$

T_1 is an integration constant. The explicit form of $\xi(z,t)$ and $\theta(z,t)$ can be derived from Eqs. (21) and (22), respectively, using the expression for the spectral parameter $\lambda(z)$. Thus we have derived the exact soliton solution for the core of the DM solitons propagating in a DM fiber system with residual loss (gain), using the Darboux transformation. In Ref. [14], such a simple soliton solution with $\mu = 1$ has been derived.

Similarly, setting $n = 2$, the two-soliton solution can be written in an explicit form as follows:

$$q[2] = \frac{G}{F}\exp(i\beta t^2/2), \tag{23}$$

where

$$\begin{aligned} G = {} & [a_1(z) + a_3(z)]\cosh[2\xi_2(z,t)]\exp[i2\theta_1(z,t)] + [a_2(z) \\ & + a_4(z)]\cosh[2\xi_1(z,t)]\exp[i2\theta_2(z,t)] + a_5(z) \\ & \times\{\sinh[2\xi_1(z,t)]\exp[2i\theta_2(z,t)] \\ & - \sinh[2\xi_2(z,t)]\exp[2i\theta_1(z,t)]\}, \end{aligned}$$

$$F = b_1(z)\cosh(2\xi^+) + b_2(z)\cosh(2\xi^-) + b_3(z)\cos(2\theta^-),$$

with

$$\xi^+ = \xi_1(z,t) + \xi_2(z,t), \quad \xi^- = \xi_2(z,t) - \xi_1(z,t);$$

$$\theta^+ = \theta_2(z,t) + \theta_1(z,t), \quad \theta^- = \theta_2(z,t) - \theta_1(z,t).$$

The explicit form of $\xi_k(z,t)$ and $\theta_k(z,t)$ can be, respectively, derived from the following equations using the expression for the spectral parameter $\lambda(z)$:

$$\lambda_k(z) = \eta_k(z) + i\zeta_k(z),$$

$$\xi_k(z,t) = \eta_k(z)t - 4\alpha_1 \int \eta_k(z)\zeta_k(z)dz + T_k,$$

$$\theta_k(z,t) = \zeta_k(z)t + 2\alpha_1 \int [\eta_k^2(z) - \zeta_k^2(z)]dz, \quad k = 1,2,$$

where the T_k's are integration constants, and introducing notations with the spectral parameter $\lambda(z)$ as follows:

$$a_1(z) = -\frac{\eta_1(z)\eta^+\eta^-}{\mu},$$

$$a_2(z)=\frac{\eta_2(z)\,\eta^+\eta^-}{\mu},$$

$$a_3(z)=\frac{\eta_1(z)(\zeta^-)^2}{\mu},$$

$$a_4(z)=\frac{\eta_2(z)(\zeta^-)^2}{\mu},$$

$$a_5(z)=\frac{2i\,\eta_1(z)\,\eta_2(z)\zeta^-}{\mu};$$

$$b_1(z)=\frac{(\eta^-)^2+(\zeta^-)^2}{4},$$

$$b_2(z)=\frac{(\eta^+)^2+(\zeta^-)^2}{4},$$

$$b_3(z)=-\eta_1(z)\,\eta_2(z);$$

$$\eta^+=\eta_2(z)+\eta_1(z),$$

$$\eta^-=\eta_2(z)-\eta_1(z),$$

$$\zeta^-=\zeta_2(z)-\zeta_1(z).$$

Thus we have derived the exact two-soliton solution for the wave propagation in the uniform optical fiber system equation with the fiber loss (gain) and pulse chirping or the core of the DM solitons propagating in a DM fiber system with residual loss (gain) using the Darboux transformation. These solutions will also be useful for the study of soliton interactions under the influence of perturbations.

In order to understand the influence of frequency chirp parameter β on the interaction between neighboring solitons, here we investigate their transmission properties. Figure 1 shows the interaction scenario between neighboring solitons with larger initial pulse separation $=15$ ps. As shown in Fig. 1(a), since β is smaller (here we take $\beta=0.000\,01$), the transmission property of the two-soliton solution, Eq. (23), is similar to the one without frequency chirp effect as shown in Fig. 2(a) of Ref. [18]. However, with increasing β, it can be seen in Figs. 1(b) and 1(c) that the effect of frequency chirp leads to the splitting of the two-soliton solution. This property has been confirmed by direct numerical simulation for Eq. (11). In fact, from the exact two-soliton solution (23) it is shown that the group velocity varies by the exponential law $\exp(-4\beta z)$. In addition, we also note that the pulses undergo broadening or compression depending on the sign of the frequency chirp parameter β as they propagate along the fiber.

However, as the initial separation of two solitons decreases further, the interaction between neighboring solitons becomes much stronger. In the following, two different cases are discussed. Figure 2(a) depicts the in-phase injection of the two solitons with equal amplitudes. From Fig. 2(a), we can note that the transmission properties of the two-soliton solution are the same as the ones without the frequency chirp as shown in Fig. 3 of Ref. [18] except that their amplitudes

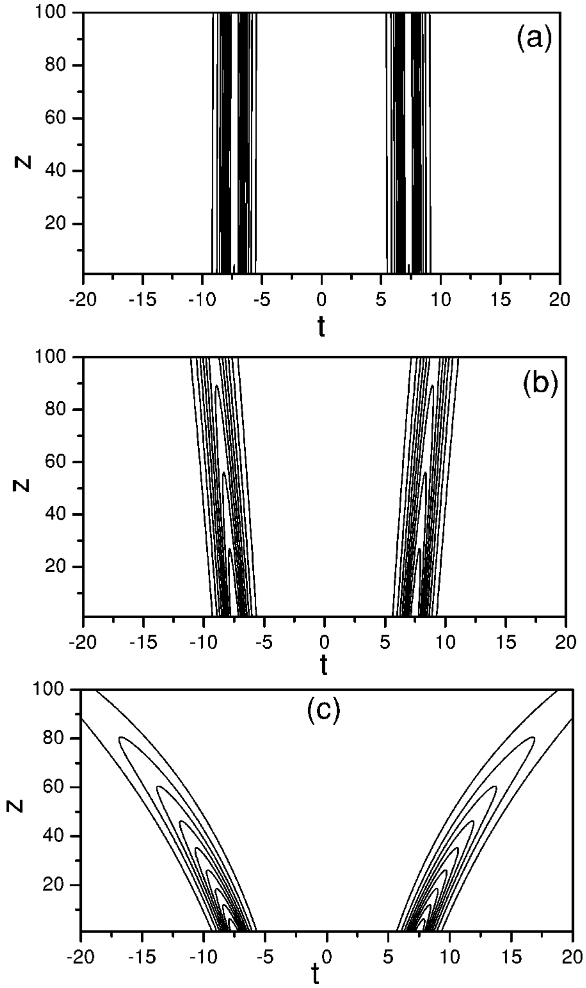

FIG. 1. Interaction of two equal amplitude pulses with initial pulse separation equal to 15. The parameters are as follows: $\mathrm{Re}(\nu_1)=-0.500\,56$, $\mathrm{Re}(\nu_2)=0.499\,451\,5$, $\mathrm{Im}(\nu_1)=\mathrm{Im}(\nu_2)=0$, $\alpha_1=0.5$, $\mu=1$, $T_1=T_2=0$. Frequency chirp parameter: (a) $\beta=0.000\,01$, (b) $\beta=0.001$, (c) $\beta=0.005$.

decrease in an exponential way. Consequently, there is a pulse broadening during the propagation. However, by changing the sign of parameter β, one can achieve the compression of soliton pulses. Figure 2 (b) shows the contour of the interaction between neighboring solitons. In another case, when the amplitudes of two-soliton become unequal as shown in Fig. 3(a), the interaction between neighboring solitons is suppressed as a result of the unequal amplitudes. In addition, we clearly note that the two solitons experience a periodic evolution due to the effect of frequency chirp. Figure 3(b) represents the contour of the interaction between neighboring solitons in this unequal amplitude case. Here, it should be pointed that there is a special relation (integrable condition) between the frequency chirp and fiber loss (gain) in the model we considered. However, by direct numerical simulation, we found that when this constraint condition is not valid, the soliton solution still exists in the model, as shown in Fig. 4. Hence, a more detailed study on this issue is under way and will be published elsewhere.

In the following, we construct a dark solitary wave solutions for Eq. (11). For that we rewrite Eq. (11) as follows:

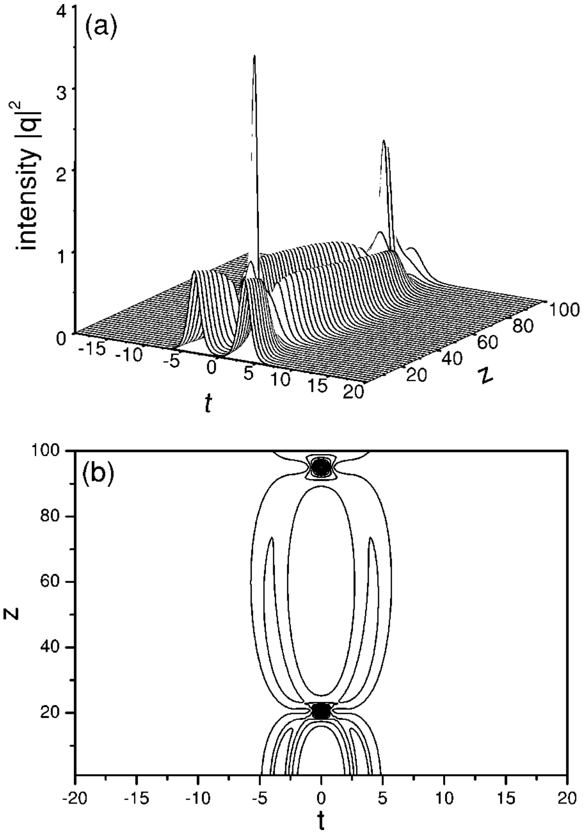

FIG. 2. Interaction of two equal amplitude pulses with initial pulse separation equal to 7. The parameters are as follows: $\text{Re}(\nu_1)=-0.536\,555$, $\text{Re}(\nu_2)=0.476\,214$, $\text{Im}(\nu_1)=\text{Im}(\nu_2)=0$, $\alpha_1=0.5$, $\mu=1$, $T_1=T_2=0$. Frequency chirp parameter: $\beta=0.0017$.

$$i\frac{\partial q}{\partial z}+\alpha_1\frac{\partial^2 q}{\partial t^2}+\alpha_2|q|^2q=\beta_1(z)t^2q-i\beta_2(z)q. \quad (24)$$

In order to proceed, we first analyze Eq. (24) by separating $q(z,t)$ into the complex amplitude function $A(z,t)$ and the phase function $\phi(z,t)$ as

$$q(z,t)=A(z,t)\exp[i\phi(z,t)], \quad (25)$$

and we consider that the phase is given by

$$\phi(z,t)=\delta t^2+\kappa(z)t+\Omega(z).$$

Thus we have the following equation:

$$i\frac{\partial A}{\partial z}+\alpha_1\frac{\partial^2 A}{\partial t^2}+2i\alpha_1\frac{\partial \phi}{\partial t}\frac{\partial A}{\partial t}+\alpha_2|A|^2A-\left[\frac{\partial \phi}{\partial z}+\alpha_1\frac{\partial \phi}{\partial t}\frac{\partial \phi}{\partial t}+\beta_1(z)t^2\right]A+i\left[\alpha_1\frac{\partial^2\phi}{\partial t^2}+\beta_2(z)\right]A=0. \quad (26)$$

In the following, we look for the solitary wave solutions for Eq. (26) by introducing an ansatz similar to Refs. [21,22]:

$$A(z,t)=i\beta(z)+\lambda(z)\tanh\theta+i\rho(z)\text{sech}\,\theta, \quad (27)$$

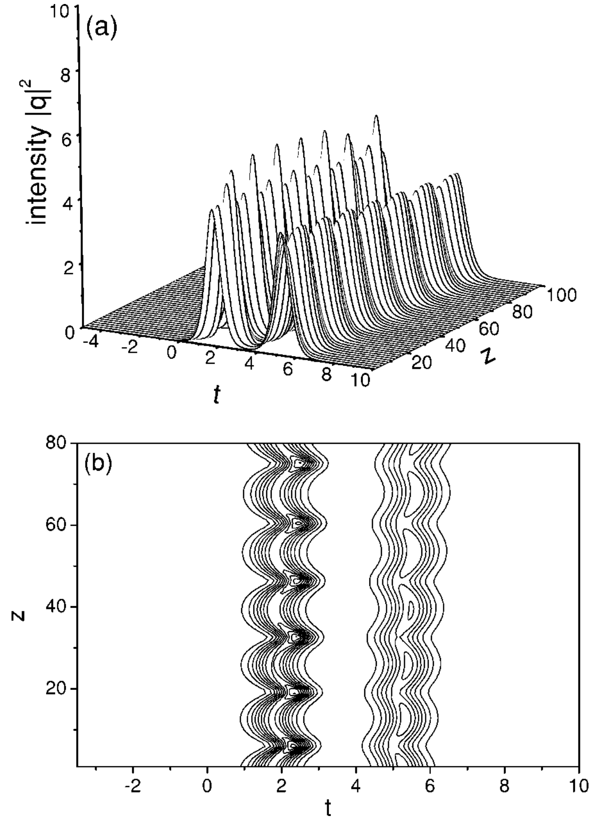

FIG. 3. Interaction of two unequal amplitude pulses, the parameters are as follows: $\text{Re}(\nu_1)=-1.073\,11$, $\text{Re}(\nu_2)=0.952\,427$, $\text{Im}(\nu_1)=\text{Im}(\nu_2)=0$, $\alpha_1=0.5$, $\mu=1$, $T_1=3.5$, $T_2=-3.5$. Frequency chirp parameter: $\beta=0.0005$.

where

$$\theta=\eta(z)[t-\chi(z)],$$

and $\eta(z)$ and $\chi(z)$ are the pulse width and shift of inverse group velocity, respectively. The solitary wave amplitude is given by $|A|^2=(\beta^2+\lambda^2)+2\beta\rho(\text{sech}\,\theta)+(\rho^2-\lambda^2)\text{sech}^2\theta$.

Substituting ansatz (27) into Eq. (26) and equating the coefficients of independent terms, one obtains

$$\beta_1=-4\alpha_1\delta^2,\quad \beta_2=2\alpha_1\delta, \quad (28)$$

$$\beta=C_1\exp(-4\alpha_1\delta z),$$
$$\lambda=C_2\exp(-4\alpha_1\delta z),$$
$$\rho=C_3\exp(-4\alpha_1\delta z), \quad (29)$$
$$\eta=C_4\exp(-4\alpha_1\delta z),$$
$$\kappa=C_5\exp(-4\alpha_1\delta z),$$

where the C_k's ($k=1,\ldots,5$) are arbitrary constants, and independent equations as follows:

$$\rho[-2\alpha_1\eta^2+\alpha_2(\rho^2-\lambda^2)]=0,$$
$$\lambda[-2\alpha_1\eta^2+\alpha_2(\rho^2-\lambda^2)]=0, \quad (30)$$

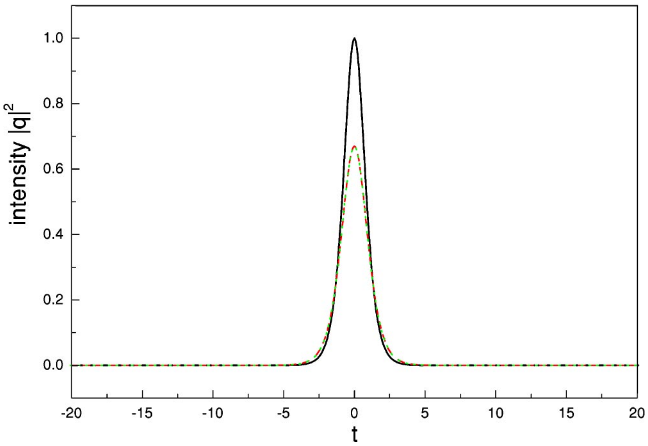

FIG. 4. Pulse shape of a bright soliton for Eq. (10) by direct nemerical simulation. The parameters are as follows: $\alpha_1=0.5$, $D_0=2\alpha_1$, $\gamma_0=2\mu^2\alpha_1$, $\Gamma_0=0.001$, and (a) for integrable condition $\kappa_0=-\Gamma_0^2$ and (b) for nonintegrable condition $\kappa_0=-0.01^2$. The solid line corresponds to the input pulse shape, the dotted curves, red and green, correspond to cases (a) and (b), respectively.

$$[\Omega_z+\alpha_1\kappa^2-\alpha_2(\beta^2+\lambda^2)]\lambda=0,$$

$$[\Omega_z+\alpha_1\kappa^2-\alpha_2(\beta^2+\lambda^2)]\beta=0,$$

$$[\Omega_z+\alpha_1\kappa^2-\alpha_2(\beta^2+\lambda^2)]\rho-\rho(\alpha_1\eta^2+2\beta^2\alpha_2)=0, \tag{31}$$

$$-\lambda\eta\chi_z+4\alpha_1\delta\lambda\eta\chi+2\alpha_1\lambda\eta\kappa+2\alpha_2\beta\rho^2+\alpha_2\beta(\rho^2-\lambda^2)=0,$$

$$\rho(-\eta\chi_z+4\alpha_1\delta\eta\chi+2\eta\alpha_1\kappa+2\lambda\beta\alpha_2)=0. \tag{32}$$

For the sake of simplicity, here we consider only the following two cases.

(1) Taking $\beta=\lambda=0$, namely, $C_1=C_2=0$, we obtain the bright soliton solution for Eq. (26) as follows:

$$A(z,t)=i\rho(z)\operatorname{sech}\theta, \tag{33}$$

where

$$\rho=C_3\exp(-4\alpha_1\delta z),$$

$$\theta=C_4\left[t\exp(-4\alpha_1\delta z)+\frac{1}{4\delta}C_5\exp(-8\alpha_1\delta z)-C_{11}\right].$$

This solution is the same as the one, Eq. (20), obtained by Darboux transformation.

(2) Taking $\rho=0$, namely, $C_3=0$, we obtain the dark (black) or gray soliton solution for Eq. (26) as follows:

$$A(z,t)=i\beta(z)+\lambda(z)\tanh\theta, \tag{34}$$

where

$$\beta=C_1\exp(-4\alpha_1\delta z),\quad \lambda=C_2\exp(-4\alpha_1\delta z),$$

$$\eta=C_4\exp(-4\alpha_1\delta z),$$

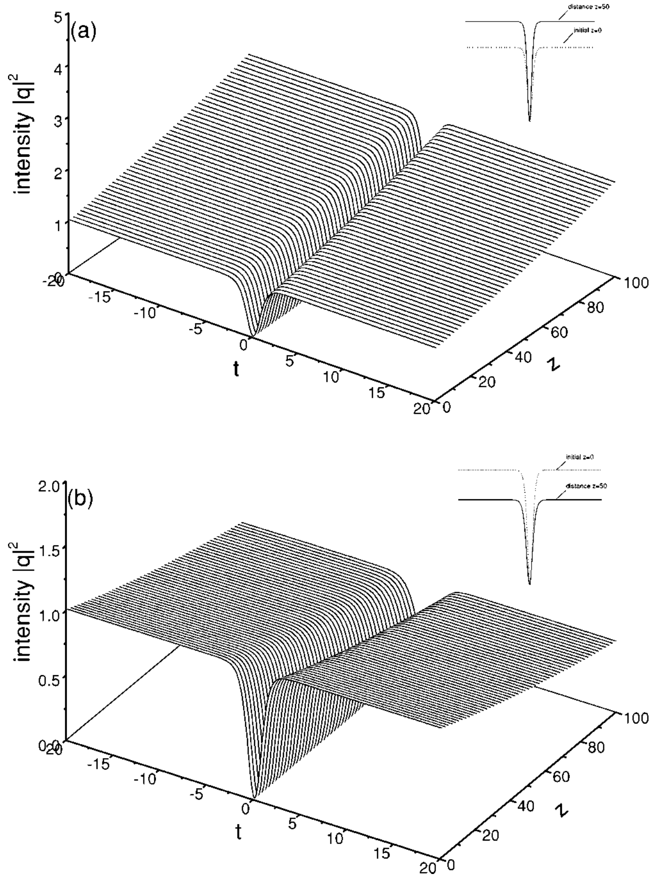

FIG. 5. Pulse evolution of a black dip with chirping frequency and gain (loss). The parameters are as follows: $\alpha_1=-0.5$, $C_1=0$, $C_2=1$, $C_4=1$, $C_5=0.003$, $C_{10}=0$, and (a) $C_{11}=0.5$, $\delta=0.0015$ and (b) $C_{11}=-0.5$, $\delta=-0.0015$.

$$\alpha_2=-2\alpha_1\frac{C_4^2}{C_2^2}, \tag{35}$$

$$\theta=C_4\exp(-4\alpha_1\delta z)[t-\chi(z)],$$

$$\chi=C_{11}\exp(4\alpha_1\delta z)-\frac{1}{4C_2\delta}(C_4C_1+C_5C_2)\exp(-4\alpha_1\delta z),$$

$$\phi(z,t)=\delta t^2+\kappa(z)t+\Omega(z),$$

$$\Omega=-\frac{1}{8\alpha_1\delta}[\alpha_2(C_1^2+C_2^2)-\alpha_1C_5^2]\exp(-8\alpha_1\delta z)+C_{10}.$$

To the best of our knowledge, this kind of solution has not been reported earlier. Figure 5(a) depicts the surface of the amplitude of the dark soliton solution for the different signs of parameter δ. As shown in Fig. 5(a), the depth of the dark soliton increases exponentially due to the exponentially increasing nature of $\lambda(z)=C_2\exp(-4\alpha_1\delta z)$ for $\delta<0$, while for $\delta>0$ as shown in Fig. 5(b), the depth of the dark soliton decreases exponentially and also the width of the dark soliton gets compressed during its propagation. Furthermore, we investigate the interaction between neighboring dark solitons by direct numerical simulation for Eq. (24). Figure 6(a)

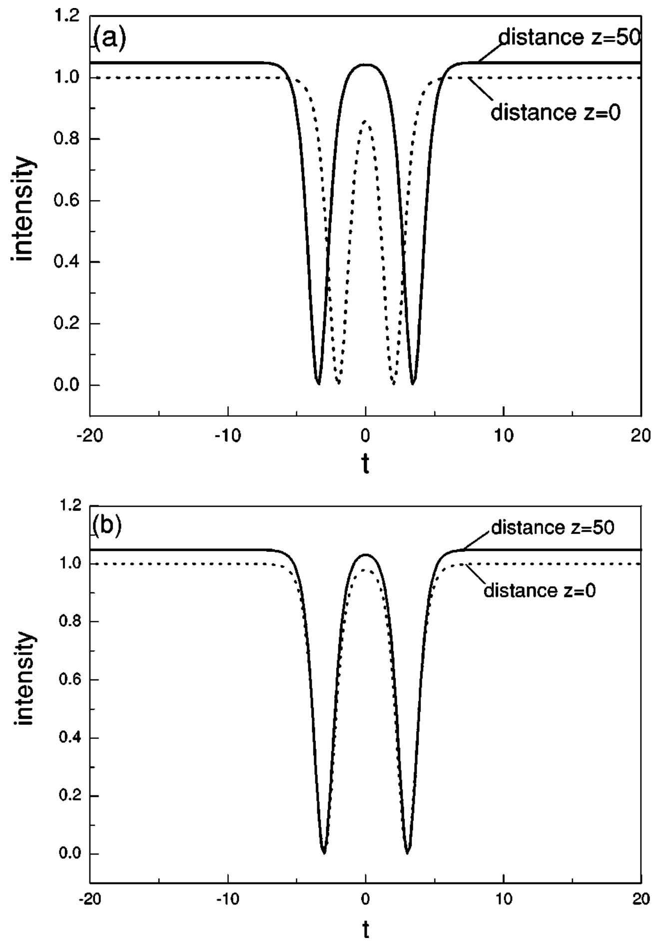

FIG. 6. Pulse shapes of a pair of dark solitons. The parameters are as follows: $\alpha_1 = -0.5$, $\alpha_2 = 1$, $\delta = 0.0005$. (a) with the initial separation equal to 4 and (b) with the initial separation equal to 6. The dotted curves correspond to the input pulse shapes.

shows the pulse shape of the output pulse when the initial soliton separation is equal to 4 after it propagates a distance of $z = 50$ in a fiber. As shown in Fig. 6(a), as the pulse travels further down the fiber, the separation between two solitons keeps increasing. However, when we increase the separation of dark solitons further up to 6, the repulsive force between two soltions is decreased, which is shown in Fig. 6(b). This property for dark solitons is similar to that in Ref. [23], where the NLS equation has been considered. Here as we add the loss (gain) in the model, as shown in Fig. 6, the intensity of the pulses will decrease (increase) depending on the sign of δ after a distance. In addition, we also find that unlike the case of bright solitons, the frequency chirp effect does not completely influence the dark soliton, which shows that dark pulses in optical fibers are more stable than bright pulses with respect to frequency chirp effect.

In conclusion, we have considered a special case of an averaged DM soliton system equation with residual loss (gain). The same system equation also governs the nonlinear pulse propagation in a uniform fiber system with fiber loss (gain) where the effects due to fiber loss (gain) and chirping of the pulse exactly balance each other. We have presented the explicit Lax pair for such a system equation using a variable spectral parameter. We have constructed the Darboux transformation on the basis of this Lax pair, and a simple procedure to derive the N-soliton solutions has been presented. For instance, the explicit one-soliton and two-soliton solutions have been generated. The interaction scenario between neighboring solitons has been discussed in detail and the influence of the frequency chirp on the soliton interaction has also been presented. We also showed that the amplitude of the pulse tends to decrease or increase in an exponential way with the same amount of broadening in the pulse width during its propagation such that the area of the pulse envelope remains constant. Furthermore, we have derived the dark soliton solution for such a system with the help of an ansatz. Finally, we have discussed the compression or broadening of the dark solitons. On the other hand, we have discussed the interaction between neighboring dark solitons. Hence, we believe that the bright soliton solutions reported here can be used for propagating the ‘‘sech’’ form of pulses in the special DM fiber system having residual loss (gain).

One of the authors (Z.Y.X.) is grateful to Professor Yuri Kivshar for suggestions in simulation of dark solitons. This research was supported by the National Natural Science Foundation of China through Grant No. 10074041 and the Provincial Natural Science Foundation of Shanxi through Grant No. 20001003 as well as the Provincial Youth Science Foundation of Shanxi through Grant No. 20011015. K.N. acknowledges support from the Research Grants Council (RGC) of the Hong Kong Special Administrative Region, China (Project No. PolyU5132/99E). K.N. is also grateful to P.K.A. Wai and S. Wabnitz for fruitful discussions.

[1] V. E. Zakharov and S. Wabnitz, *Optical Solitons: Theoretical Challenges and Industrial Perspectives* (Springer-Verlag, Berlin, 1998).
[2] M. Nakazawa, H. Kubota, K. Suzuki, and E. Yamada, Chaos **10**, 486 (2000).
[3] A. Maruta, Y. Yamamoto, S. Okamoto, A. Suziki, T. Morita, A. Agata, and A. Hasegawa, Electron. Lett. **36**, 1947 (2000).
[4] L.J. Richardson, W. Forysiak, and N.J. Doran, IEEE Photonics Technol. Lett. **13**, 209 (2001).
[5] J.H.B. Nijhof, W. Forysiak, and N.J. Doran, IEEE J. Sel. Top. Quantum Electron. **6**, 330 (2000).
[6] K. Nakkeeran, A.B. Moubissi, P. Tchofo Dinda, and S. Wabnitz, Opt. Lett. **20**, 1544 (2001).
[7] A.B. Moubissi, K. Nakkeeran, P. Tchofo Dinda, and S. Wabnitz, IEEE Photonics Technol. Lett. **14**, 1041 (2002).
[8] P. Tchofo Dinda, A.B. Moubissi, and K. Nakkeeran, J. Phys. A **34**, L103 (2001).
[9] P. Tchofo Dinda, A.B. Moubissi, and K. Nakkeeran, Phys. Rev. E **64**, 016608 (2001).
[10] A. Hasegawa, Y. Kodama, and A. Maruta, Opt. Fiber Technol.

3, 197 (1997).
[11] J.D. Moores, Opt. Lett. **21**, 555 (1996).
[12] S. Kumar and A. Hasegawa, Opt. Lett. **22**, 372 (1997).
[13] M.N. Vinoj, V.C. Kuriakose, and K. Porsezian, Chaos, Solitons Fractals **21**, 2569 (2001).
[14] K. Nakkeeran, J. Phys. A **34**, 5111 (2001).
[15] R. Ganapathy, V.C. Kuriakose, and K. Porsezian, Opt. Commun. **194**, 299 (2001).
[16] H.H. Chen and C.S. Liu, Phys. Fluids **21**, 377 (1978).
[17] P.A. Clarkson, Proc. R. Soc. Edinburgh, Sect. A: Math. **109**, 109 (1988).
[18] Z.Y. Xu, L. Li, Z.H. Li, and G.S. Zhou, Opt. Commun. **210**, 375 (2002).
[19] Z.Y. Xu, L. Li, Z.H. Li, and G.S. Zhou, Phys. Rev. E **67**, 026603 (2003).
[20] V. B. Matveev and M. A. Salli, *Darboux Transformations and Solitons*, Springer Series in Nonlinear Dynamics (Springer-Verlag, Berlin, 1991).
[21] L. Li, Z.H. Li, Z.Y. Xu, G.S. Zhou, and Karl H. Spatschek, Phys. Rev. E **66**, 046616 (2002).
[22] Z.H. Li, L. Li, H.P. Tian, and G.S. Zhou, Phys. Rev. Lett. **84**, 4096 (2000).
[23] W. Zhao, and E. Bourkoff, Opt. Lett. **14**, 1371 (1989).