Dynamic modeling of traffic incident impacts on network reliability

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Traffic incident is one of the major sources for degrading network capacity, inducing traffic congestion, and decreasing network reliability. The impacts of traffic incident on network reliability have been extensively studied with the use of static network equilibrium or dynamic simulation models. In this paper, an analytical reliability-based dynamic traffic assignment model is proposed for assessing the temporal and spatial impacts of traffic incident on network reliability. The proposed dynamic traffic assignment model can be used to estimate the stochastic link flow pattern and route travel time distribution for examining the impacts of traffic incident on the on-time arrival probability with and without dynamic speed limit control. It is shown that traffic incident on congested road during peak period will greatly decrease the on-time arrival probability, particularly when the incident has greater effects on link capacity degradation with longer duration. Under certain circumstance, speed limit control can be employed to reduce total network delay during the time intervals with traffic incident.

Keywords: reliability based dynamic traffic assignment; traffic incident; network reliability; speed limit

1. Introduction

Urban transportation systems are suffered with recurrent and non-recurrent congestions. The non-recurrent congestion caused by unexpected traffic incidents brings serious congestion and high vulnerability into transportation network. Approximately one half to two-thirds of the total travel delay in large metropolitan areas is incident relevant (Center for Urban Transportation Research, 2010). Kwon et al. (2006) analyzed the total traffic congestion and found that around 25% of the delay in their study area in the US is caused by incidents. So incidents induce serious delay and drivers dislike this uncertain delay the most because it is unexpected and, therefore, they may be late for important appointments. Furthermore, transportation network supply is natural stochastic, because link performance parameters, e.g., free flow speed, capacity, etc., have large variations due to the variability in driving behavior and the characteristics of vehicles (Wang et al., 2013). Thus travel time variations cannot be neglected. Several empirical studies paid attention to the reliability issue from travelers' perspective proved that route choice is surely determined to a large extent by the variability of travel times (Lam and Small, 2001; De Palma and Picard, 2005). It was reported by Lam and Small (2001) that travelers make route choice decision not only based on the travel time, but also on the route travel time reliability. Large travel time variation may result in unexpected late arrival and impose a high penalty on travelers. Therefore, there is a need to investigate the incident impacts on network reliability.

The network-wide travel time reliability has been intensively studied by static traffic assignment models by considering the uncertainties from demand side (Clark and Watling, 2005; Shao et al., 2006; Chen and Zhou, 2010), supply side (Chen et al., 2002; Lo 2006; Lo et al., 2006; Nie, 2011), or both (Lam et al., 2008). Traffic incident causes network capacity degradation, and it is one source of uncertainty from supply side. It is known that static traffic assignment is a widely-used technique for long-term planning. But traffic incident is short term event. The incident occurrence time, duration and the induced rerouting behaviors are all time dependent. Static traffic assignment model cannot capture time dependent features, so it is not suitable for analyzing the dynamic impacts of traffic incident on network reliability with and without dynamic traffic control measures particularly on the urban expressways.

However, dynamic traffic assignment (DTA) is commonly adopted for short-term planning or real-time traffic management. Thus it is more appropriate for assessing the impacts of traffic incident on network reliability.

Most of the existing DTA models make use of the mean travel times as the route choice criterion but ignore their variations (Friesz et al., 1989; Lo and Szeto, 2002; Long et al., 2014). Very limit works (e.g., Liu et al., 2002; Szeto et al., 2011; Gao, 2012) focus on travel time reliability in DTA. Generally, DTA consists of two components: traffic flow component and travel choice principle. In order to take account the travel time reliability for assessing the incident impacts, the stochastic traffic flow model and the reliability based travel choice principle should be considered together. Stochastic traffic flow model could be simply obtained by adding "noise" terms onto the deterministic traffic flow model (Szeto and Gazis, 1972). Sumalee et al., (2011) extended the cell transmission model to the stochastic dimension in which the parameters in the flow-density fundamental diagram are random variables with independent normal distributions. Jabari and Liu (2012) presented a stochastic traffic flow model with uncertainty in driver choice on the time gap between vehicles. As to the travel choice principle, Szeto et al., (2011) introduced the reliability based dynamic user equilibrium (DUE) principle, and investigated the time dependent travel time reliability by Monte-Carlo simulation.

The dynamic impacts of traffic incident on total delay are the main concerns of existing works. Kamga et al., (2011) investigated the impact of incident on network delay with and without information by using the simulation tool VISTA. Long et al. (2010; 2012) investigated the turning restriction and signal control methods to eliminate the impact of incident on network congestion. Corthout et al. (2010) proposed the marginal incident computation method based on the link transmission model, and studied the route travel time distribution under various incident condition. Gao (2012) proposed a DTA model to estimate the network travel time under random incident condition, in which travelers make strategic route choices in response to real time traffic information.

In the previous related works, the time-dependent impacts of traffic incident on network travel time distribution were mainly estimated by time-consuming Monte-Carlo simulations but stable solution is not guaranteed particularly under very congested condition. In order to ensure a relative stable solution, an analytical reliability-based dynamic traffic assignment model is proposed in this paper to estimate the stochastic link flow pattern and route travel times in terms of both of their mean and standard deviation (SD). Then it is employed to study the dynamic impacts of traffic incident on time dependent network reliability. Furthermore, appropriate control measures, such as change of speed limits, lane control and route guidance, etc., are always implemented to reduce the adverse impact of incident. It has been shown in literature that the change of speed limit can be efficient to alleviate congestion on both static and dynamic networks (Yang et al., 2012; Wang, 2013; Zhu and Ukkusuri, 2014). In this paper, the effects of dynamic speed limit control on network total travel time and network reliability are investigated under different incident conditions and various traffic demand scenarios.

This paper is organized as follows. Firstly, the basic notations and formulation of the reliability-based DTA model are presented together with the incident model and the dynamic speed limit control model. It follows with the solution algorithm given in section 3. In section 4, numerical tests on a small network are carried out to demonstrate the application of the proposed model and solution algorithm. The numerical results are analyzed to study the dynamic impacts of incident on network performance in terms of total network travel time and on-time arrival probability by route. Finally, conclusion is given together with recommendations for further studies.

2. Model formulation

2.1 Notations and assumptions

A network contains a set of nodes *N*, and a set of links *A*. Link $a = (i, j)$ is the link with the tail node *i* and the head node *j*. *A*(*i*) is the set of links leaving node *i*, and *B*(*i*) is the set of links heading to node *i*. *R* and *S* denote the sets of origin nodes and destination nodes. Each Origin-Destination (OD) pair (*r*, *s*) is connected by a set of routes P_{rs}. *P* is the set of all routes. The planning horizon is divided into a finite number of discrete time intervals of uniformly small length *Δ*, and the set of time intervals is denoted by *T*. The notations used throughout the paper are listed as follows unless otherwise specified.

$\tilde{C}_a$:	capacity of link *a*
$\tilde{T}_{a,f}$:	free flow travel time of link *a*
$v_{a,f}$:	free flow speed of link *a*
$x_a(t)$:	link flow on link *a* at the beginning of time interval *t*
$u_a(t)$:	link inflow rate to link *a* at the beginning of time interval *t*
$v_a(t)$:	link exit flow rate from link *a* at the beginning of time interval *t*
$\tilde{\tau}_a(t)$:	travel time of link *a* at the end of time interval *t*
$\tau_a^\alpha(t)$:	*α*-percentile travel time of link *a* at the end of time interval *t*
$\mu_a(t)$	the mean of travel time on link *a* at the end of time interval *t*
$\sigma_a(t)$:	the SD of travel time on link *a* at the end of time interval *t*
$d^{rs}(t)$:	travel demand from OD pair (*r, s*) at the beginning of time interval *t*
$f_p^{rs}(t)$:	traffic flow on route $p \in P_{rs}$ at the beginning of time interval *t*
$\tilde{\eta}_p^{rs}(t)$:	travel time on route $p \in P_{rs}$ at the end of time interval *t*
$\eta_p^{rs,\alpha}(t)$:	*α*-percentile travel time on route $p \in P_{rs}$ at the end of time interval *t*
$\mu_p^{rs}(t)$:	the mean of travel time on route $p \in P_{rs}$ at the end of time interval *t*

$\sigma_p^{rs}(t)$: the SD of travel time on route $p \in P_{rs}$ at the end of time interval t

$\pi^{rs,\alpha}(t)$: minimum α-percentile travel time on route $p \in P_{rs}$ at the end of time interval t

δ_{ap}: link-route incidence matrix; $\delta_{ap} = 1$, if route p uses link a and $\delta_{ap} = 0$, otherwise

To facilitate the presentation of the essential ideas without loss of generality, the following basic assumptions are made in this paper:

(A1) The time dependent travel demands between each OD pair are assumed to be given and deterministic. It is because the travel demand considered here is short term and within day, particularly during the short period with traffic incident occurred. In literature (Corthout et al., 2010; Knoop et al., 2010**;** Kamga et al., 2011**)**, it is always interested to study the impacts of traffic incident on the route choice rather than the travel demand.

(A2) The stochastic dynamic link performance function is used for the flow propagation component so as to capture the stochastic effects. Empirical results show that traffic parameters, e.g., free flow speed, capacity, etc., have large variations due to the variability in driving behavior and the characteristics of vehicles (Wang et al., 2013). Similar to Lam et al, (2008), the free flow travel time and capacity are assumed to be random parameters in the link performance function.

(A3) Travelers make route choice decision based on the percentile travel time. The percentile user equilibrium is an extension of the classic Wardrop equilibrium, and had been adopted in static models (Nie, 2011). In this paper, it is further extended to dynamic networks. The covariance between link travel times are considered in calculating the percentile route travel time.

(A4) As the proposed model is mainly used for off-line applications in order to assess the dynamic impacts of different traffic incidents in network with supply uncertainty, it is assumed that travelers will know the occurrence of incident through broadcasted information by radio, variable message signs, SMS, micro-blog, Wechat, etc.. Therefore the travelers departing before the incident occurrence do not change route, while that departing after the incident occurrence can change route to avoid incident induced congestion (Corthout et al., 2010).

(A5) In literature (Wang, 2013; Carlson et al., 2010), speed limit can be changed to alleviate the adverse impact of incident on traffic delay and network reliability. On urban expressway network with variable message signs, the dynamic speed limit control can be implemented on links upstream of the incident. The speed limit values can be selected dynamically from a finite discrete set. However, this paper mainly focuses on analyzing the dynamic impact of incident on network reliability. The dynamic optimization problem for speed control will not be pursued further in this paper.

2.2 Model formulation

2.2.1 Percentile travel time based DTA model

Empirical results show that link performance parameters, e.g., free flow speed, capacity, etc., have large variations due to the variability in driving behavior and the characteristics of vehicles (Wang et al., 2013). Thus it is assumed that the link performance function has the following form,

$$\tilde{\tau}_a(t) = \tilde{T}_{a,f} + \frac{x_a(t)}{\tilde{C}_a} \tag{1}$$

Here $\tilde{T}_{a,f}$ is free flow travel time, and $\tilde{C}_a$ represents the capacity. They are both random variables. Similar to the previous related studies (e.g. Lam et al., 2008), normal distributions are adopted to describe the stochastic characteristics of supply uncertainties, i.e.,

$$\tilde{T}_{a,f} \sim N(\mu_a^T, (\sigma_a^T)^2) \tag{2}$$

$$\frac{1}{\tilde{C}_a} \sim N(\mu_a^{\frac{1}{C}}, (\sigma_a^{\frac{1}{C}})^2) \tag{3}$$

$\mu_a^T(t)$ and $\sigma_a^T(t)$ are the mean and SD of free flow travel time. $\mu_a^{\frac{1}{C}}(t)$ and $\sigma_a^{\frac{1}{C}}(t)$ are the mean and SD of the inverse of link capacity. It should be pointed out that the actual distributions should be calibrated according to empirical traffic flow data. It is obvious that link travel time expressed by eqn.(1) also follows normal distribution.

Assume that $\tilde{T}_{a,f}$ and $\tilde{C}_a$ are independent, then one can obtain the mean and SD of random link travel time $\tilde{\tau}_a(t)$:

$$\mu_a^\tau(t) = \mu_a^T + x_a(t)\mu_a^{\frac{1}{C}}, \qquad (\sigma_a^\tau(t))^2 = (\sigma_a^T)^2 + \left(x_a(t)\sigma_a^{\frac{1}{C}} \right)^2 \tag{4}$$

The α-percentile travel time on link a is the time required to complete traversing the link with probability α (Nie, 2011). Since the link travel time follows normal distribution, the α-percentile travel time on link a is given by the following equation:

$$\tau_a^\alpha(t) = \mu_a^\tau(t) + Z_\alpha \sigma_a^\tau(t) \tag{5}$$

Z_α is the right fractile with probability α.

Based on the α-percentile link travel time, one can obtain the α-percentile route travel time. Assume that route p consists of nodes (r, 1, 2, …,n, s), and link a=(i,j) is on route p. There are spatial and temporal correlations among the links on the same route. As to a traveler who just arrives at node i, the α-percentile travel time on the forthcoming link (i, j) is correlated to the α-percentile travel time on the sub-route of p from r to i. The α-percentile travel time of the first link on route p is

$$\eta_p^{r1,\alpha}(t) = \mu_p^{r1}(t) + Z_\alpha \sigma_p^{r1}(t) \tag{6}$$

The α-percentile travel time from r to 2 via the sub-route of route p is

$$\eta_p^{r2,\alpha}(t) = \mu_p^{r2}(t) + Z_\alpha \sigma_p^{r2}(t) \tag{7}$$

Where

$$\mu_p^{r2}(t) = \mu_p^{r1}(t) + \mu_{(1,2)}(t + \eta_p^{r1,\alpha}(t)) \tag{8}$$

$$\left[\sigma_p^{r2}(t)\right]^2 = \left[\sigma_p^{r1}(t)\right]^2 + \left[\sigma_{(1,2)}(t + \eta_p^{r1,\alpha}(t))\right]^2 + 2\operatorname{cov}(\tilde{\eta}_p^{r1}(t), \tilde{\tau}_{(1,2)}^{p}(t + \eta_p^{r1,\alpha}(t))) \tag{9}$$

Given the mean and SD of route travel time form r to i, the α-percentile travel time from r to j via the sub-route of route p is

$$\eta_p^{rj,\alpha}(t) = \mu_p^{rj}(t) + Z_\alpha \sigma_p^{rj}(t) \tag{10}$$

where

$$\mu_p^{rj}(t) = \mu_p^{ri}(t) + \mu_{(i,j)}(t + \eta_p^{ri,\alpha}(t)) \tag{11}$$

$$\left[\sigma_p^{rj}(t)\right]^2 = \left[\sigma_p^{ri}(t)\right]^2 + \left[\sigma_{(i,j)}(t + \eta_p^{ri,\alpha}(t))\right]^2 + 2\operatorname{cov}(\tilde{\eta}_p^{ri}(t), \tilde{\tau}_{(i,j)}^p(t + \mu_p^{ri,\alpha}(t))) \tag{12}$$

$$\operatorname{cov}(\tilde{\eta}_p^{ri}(t), \tilde{\tau}_{(i,j)}^p(t + \eta_p^{ri,\alpha}(t))) = \sum_{l<i} \operatorname{cov}(\tilde{\tau}_{(l-1,l)}^p(t + \eta_p^{rl,\alpha}(t)), \tilde{\tau}_{(i,j)}^p(t + \eta_p^{ri,\alpha}(t))) \tag{13}$$

Through a step by step method, one can obtain the mean of route travel time, i.e., $\mu_p^{rj+1}(t)$, $\mu_p^{rj+2}(t)$,.., $\mu_p^{rs}(t)$, the SD of route travel time, i.e., $\sigma_p^{rj+1}(t)$, $\sigma_p^{rj+2}(t)$,.., $\sigma_p^{rs}(t)$, and the α-percentile route travel time $\eta_p^{rj+1,\alpha}(t)$, $\eta_p^{rj+2,\alpha}(t)$,.., $\eta_p^{rn,\alpha}(t)$, $\eta_p^{rs,\alpha}(t)$.

Chen et al., (2012) investigated the variance–covariance matrix of link travel times from the RTIS data collected at a morning peak hour in Hong Kong. It was found that the spatial correlation between the subject link and the 1st neighbor link is 0.29, while the spatial correlation between the subject link and the 4th neighbor link decreases to 0.04. Cheng et al. (2012) studied the spatio-temporal autocorrelation of journey time data collected on London's road network. It was also shown that the temporal cross-covariance coefficients between two neighbor links decreases as the temporal lags increases within a small extent. According to those facts, in this paper, it is assumed that the correlation just exists between the subject link and its 3 nearest neighboring links. The correlation decreases as the spatial distance and the temporal lags increase. For simplicity, the link travel time correlation between link a and link b is expressed as:

$$\operatorname{corr}(\tau_a(k_1), \tau_b(k_2)) = \lambda \exp(-\left|\operatorname{order}(a) - \operatorname{order}(b)\right| \bullet \left|\frac{k_1 - k_2}{T_{ab}}\right|) \tag{14}$$

Links a and b are both on the route p. The function order(a) is the ordinal number of link a on the route p. T_{ab} is the free flow travel time from the tail node of link a to the tail node of link b. There is no correlation among links on different route. The parameter λ determines the sign and magnitude of correlation. When $\lambda > 0$, the link travel times are positively correlated; when $\lambda < 0$, the link travel times are negatively correlated; and when $\lambda = 0$, there is no correlation between link travel times.

If all travelers choose route based on the α-percentile travel time, the dynamic user equilibrium condition implies that at each time interval any used route has the identical and minimum percentile route travel time, i.e.,

$$\eta_p^{rs,\alpha}(t)\begin{cases} = \pi^{rs,\alpha}(t), & \text{if} \quad f_p^{rs}(t) > 0 \\ \geq \pi^{rs,\alpha}(t), & \text{if} \quad f_p^{rs}(t) = 0 \end{cases} \tag{15}$$

Route flow assignment constraints:

$$d^{rs}(t) = \sum_{p \in P_{rs}} f_p^{rs}(t), \quad \forall r, s \tag{16}$$

$$u_a(t) = \sum_{p \in P_{rs}} \delta_{ap} f_{\ p}^{rs}(t), \ \forall r, a \in A(r) \tag{17}$$

Flow conservation and propagation

$$\sum_{a \in A(j)} u_{a,p}^{rs}(t) = \sum_{a \in B(j)} v_{a,p}^{rs}(t), \quad \forall r, s; \ \forall j \neq r, s; \ \forall p \tag{18}$$

$$\frac{dx_{a,p}^{rs}(t)}{dt} = u_{a,p}^{rs}(t) - v_{a,p}^{rs}(t), \ v_{a,p}^{rs}(t + \tau_{a,p}^{rs,\alpha}(t)) = \frac{u_{a,p}^{rs}(t)}{1 + d\tau_{a,p}^{rs,\alpha}(t)/dt} \quad \forall r, s; \ \forall a, p \tag{19}$$

Definitional constraints:

$$\sum_{rsp} u_{a,p}^{rs}(t) = u_a(t), \quad \sum_{rsp} v_{a,p}^{rs}(t) = v_a(t), \quad \sum_{rsp} x_{a,p}^{rs}(t) = x_a(t) \qquad \forall a \qquad (20)$$

Nonnegative conditions:

$$u_{a,p}^{rs}(t) \geq 0, v_{a,p}^{rs}(t) \geq 0, x_{a,p}^{rs}(t) \geq 0 \quad \forall r,s,a,p; \qquad f_p^{rs}(t) \geq 0, \quad \forall\, p \qquad (21)$$

Note that constraint (16) means that the sum of route flows of OD pair *rs* departing at time *t* equals the demand of OD pair *rs* at time *t*, and constraint (17) denotes that the sum of route flows passing link *a* at time *t* equals to the inflow of link *a* at time *t* as shown in eqn.(17). Constraint (18) makes sure that the sum of inflow to node *j* at time *t* equals the sum of outflow from node *j* at time *t*. Constraint (19) bounds the flow changing rate of link *a* at time *t* equals that the inflow of link *a* at time *t* minus the outflow of link *a* at time *t*, and the outflow. These constraints are used to generate path and link flows when route departure flows are determined. That is to say the route departure flow $f_p^{rs}(t)$ is the basic decision variables.

VI formulation of the percentile travel time based DTA model

The discrete time version of the percentile travel time based DTA model can be formulated as an equivalent variational inequality (VI) problem. And it is to find a vector $\mathbf{f}^* \in \mathbf{F}$, such that for all $\mathbf{f} \in \mathbf{F}$

$$\sum_t \sum_p \pi^{rs,\alpha}(t)[f_p^{rs}(t) - f_p^{rs*}(t)] \geq 0 \qquad (22)$$

Where $\mathbf{F}$ is a closed convex set $\mathbf{F} = \{\mathbf{f} \geq 0 : \sum_{p \in P_{rs}} f_p^{rs}(t) = d^{rs}(t), \forall r,s\}$

2.2.2 Traffic incident model

Traffic incident will block certain number of lanes before it is cleared. Assumed that traffic incident occurrence time, duration and link capacity drop ratio are denoted as T_S, T_D, and r_c, respectively. The capacity drop ratio is defined as the ratio of the dropped capacity to the original capacity. Under traffic incident conditions, it is assumed that travelers had perfect knowledge of the incident conditions and could select routes to avoid the incident location.

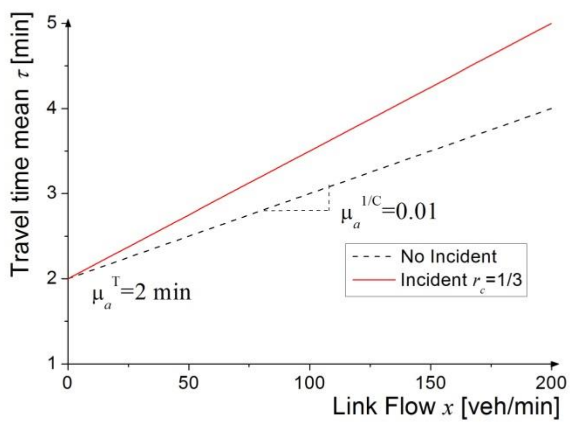

Fig.1 Link travel time mean as a function of link flow with and without incident. The parameters are μ_a^T =2.0 and $\mu_a^{\frac{1}{C}}$ =0.01.

During the incident, the link performance function becomes:

$$\tilde{\tau}_a(t) = \tilde{T}_{a,f} + \frac{x_a(t)}{(1-r_c)\tilde{C}_a} \tag{23}$$

Fig.1 shows the link travel time mean as a function of link flow with and without incident. Note that the linear type link performance function (1) satisfies first-in-first-out (FIFO) constraint (Carey et al., 2003). But when the incident link recovers to normal state, the link capacity increases to a high value and the travel time may decrease sharply. This will cause

$d\tau_a(t)/dt \le -1$, and the FIFO constraint is violated. Here, a special rule is adopted to guarantee that the flow propagation satisfies FIFO constraint. Denote the incident recover time interval as *t'*. The link travel time at time interval *t'* is restricted by

$$\tilde{\tau}_a(t') = \max(\tilde{\tau}_a(t'-1)-1, \tilde{T}_{a,f} + \frac{x_a(t)}{\tilde{C}_a}) \tag{24}$$

Then the link travel time $\tilde{\tau}_a(t')$ is not lower than $\tilde{\tau}_a(t'-1)-1$, which means that the flow entering link *a* at time interval *t'* will not leaving link *a* earlier than $t'+\tilde{\tau}_a(t'-1)-1$, which equals the leaving time of the flow entering link *a* at time interval *t'*-1.

2.2.3 Speed limit model

The speed limit control measures only affect the free flow travel time. With a speed limit (SL) $\bar{v}_a$, the free flow travel time is $\bar{T}_{a,f}$. The link performance function becomes

$$\tilde{\tau}_a(t) = \max(\bar{T}_{a,f}, \tilde{T}_{a,f} + \frac{x_a(t)}{\tilde{C}_a}) \tag{25}$$

where $\bar{T}_{a,f} = L_a / \bar{v}_a$. Fig.2 shows the link travel time as a function of link flow with and without speed limit. Note that in dynamic network, the speed limit is time dependent and chosen from a limited discrete set. Here for simplicity, only the case with constant speed limit value is considered. That is to say, the speed limit value kept unchanged during the period of speed limit control measures.

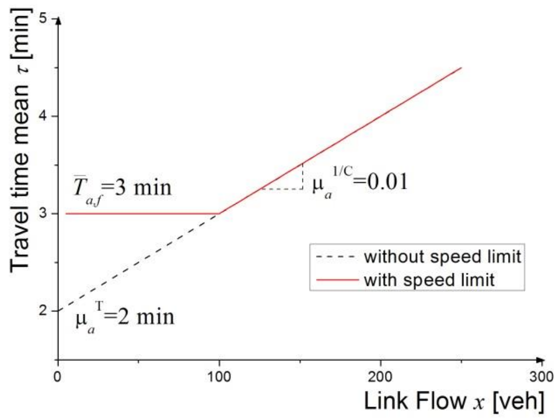

Fig.2 Link travel time mean as a function of link flow with and without speed limit. The parameters are μ_a^T =2.0, $\bar{T}_{a,f}$ =3.0 and $\mu_a^{\frac{1}{C}}$ =0.01.

3. Solution algorithm

In the percentile route travel time calculation process, one can note that there is not additive properties, thus the link based algorithm cannot be used to solve the percentile travel time based DTA model. We used the route-based algorithm based on the Method of Successive Average (MSA). The column generation method is used to update the route set. The detailed algorithms are listed as follows.

Step 0 Initialization: Initialize all link flows $x_a(t), u_a(t), v_a(t)$ to zero and calculate initial link travel time $\tau_a^0(t)$. Set the iteration counter *n*=1, the maximum iteration *N*, and the convergence criterion *ε*.

Step 1 Shortest route: Find *α*-percentile shortest route $p_{rs}^{\alpha,n}(t)$ for each OD at each time interval. If the shortest route is a new one, then update the route set *P*.

Step 2 Flow assignment: Assign all OD demand on the *α*-percentile shortest route, then get $\hat{f}_p^{rs,n}(t)$. Calculate the new route flow

$$f_p^{rs,n}(t) = f_p^{rs,n-1}(t) + \frac{1}{n+1}\left[\hat{f}_p^{rs,n}(t) - f_p^{rs,n-1}(t)\right]$$

Step 3 Network flow loading: update all link flow $x_a^n(t), u_a^n(t), v_a^n(t)$, and the link travel time $\tilde{\tau}_a^n(t)$. Then calculate *α*-percentile route travel time $\eta_p^{rs,\alpha}(t)$ considering link travel time covariance.

Step 4 convergence check: if $\frac{\sum_{rstp}\left|\eta_p^{rs,\alpha}(t) - \pi^{rs,\alpha}(t)\right| f_p^{rs}(t)}{\sum_{rstp}\pi_p^{rs,\alpha}(t) f_p^{rs}(t)} \le \varepsilon$ or *n*=*N*, stop; otherwise, *n*=*n*+1, go to step 1.

Note that *ε* is a given positive value, *N* is the maximum iterations. In the numerical text, *ε*=0.001 and *N*=1000 are used.

4. Numerical example

The numerical test network is shown in Fig.3. There are two OD pairs, one from node 1 to node 3, the other one from node 2 to node 4. A parabolic-shaped curve is employed to represent the OD demand for each pair. And the demand rate is assumed to be calculated through eq.(26).

$$d^{rs}(t) = 8 + T_D * \left(1 - \left(\frac{t - T_D/2}{T_D/2}\right)^2\right) \qquad \forall 1 \le t \le T_D, \forall r \in R, \forall s \in S \tag{26}$$

Where T_D denotes the total number of intervals during which OD trips will be generated. In the numerical test, each time interval is set as 1 min, and the total number of time intervals with OD demand T_D=60. To reflect the level of OD demand, the parameter demand ratio r_D is introduced. Then the actual OD demand equals to the base demand defined in Eqn.(26) multiplied by r_D. In most of the case, r_D is set as 1.0.

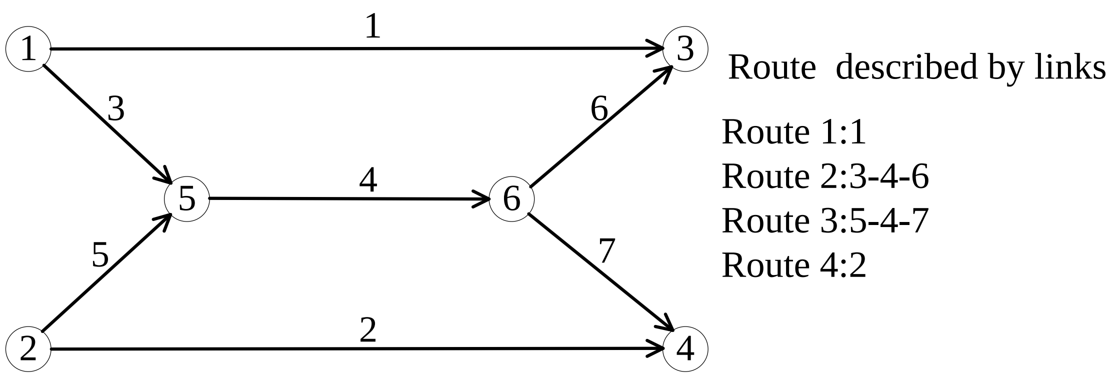

Fig.3 Test network

Table.1 Link performance function parameters

Link #	μ_a^T (min)	σ_a^T	$\mu_a^{\frac{1}{C}}(E(\frac{1}{\tilde{C}_a}))$ (min/veh)	$\sigma_a^{\frac{1}{C}}$
1	12.0	1.0	0.01	0.001
2	12.0	1.0	0.01	0.001
3	3.0	0.5	0.02	0.004
4	4.0	0.5	0.0125	0.003
5	3.0	0.5	0.02	0.004
6	2	0.4	0.02	0.004
7	2	0.4	0.02	0.004

The link properties are shown in Table.1. All the links have the free flow speed of 60km/h. And according to the given free flow travel time mean given in Table.1, one can obtain the length of each link. In calculating the route travel time, the correlation between link travel times in eqn.(14) is set as $\lambda = 1.0$, if not specially mentioned. From Fig.3, one can see that the network is symmetric. Route 1 is symmetric to route 4 and route 2 is symmetric to route 3. The two OD pairs also have the same travel demand, so the route flow patterns on routes 1 and 4 are the same, so do those on routes 2 and 3. Considering those facts, only the two routes 1 and 2 on OD pair (1, 3) are analyzed in numerical test. One can also note that route 1 has longer free flow travel time and smaller SD, while route 2 has shorter free flow travel time and larger SD. All the links are urban expressway, and variable message signs (VMS) are installed on the links. The dynamic speed limits can be displayed on the VMS and the drivers should comply with the speed limit. In the whole paper, the 90%-percentile travel time is used to obtain reliable dynamic traffic flow pattern.

4.1 No incident

Firstly, the base case without incident is analyzed. Fig.4 shows the inflow rates of routes 1 and 2 at each time as a result of the 90%-percentile travel time based DTA model. One can see that at the beginning, all the departure flows choose route 2, because it has shorter free flow travel time. Several minutes later, some flows choose route 1 for route 2 has been congested. As the traffic demand grows, more flows will choose route 1 because link 1 has larger capacity and smaller SD.

Fig.5 shows the cumulative route travel time probability for the flow departure at the beginning of 1 min and 30 min. It can be seen that for the flow departing from the origin at 1 min, the 90%-percentile travel time on routes 2 is lower than that on route 1. Thus no flow chooses routes 1. As the OD demand increases, all the routes should be used to improve the on-time arrival probability. For the flow departing the origin at 30 min, at which the OD demand is the largest, the 90%-percentile travel times on the routes 1 and 2 are equal. However, the 50%-percentile (mean) travel time on routes 2 is lower than that on route 1. That is to say, if the route choice principle is based on the mean travel time, route 1 will not be used.

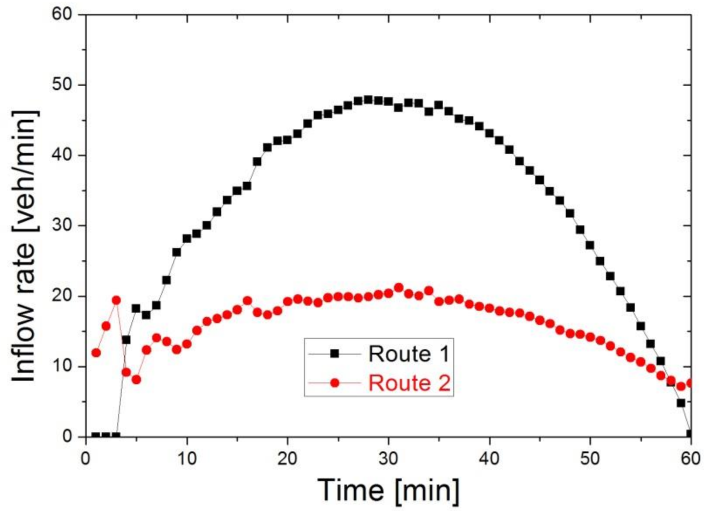

Fig.4 90%-percentile travel time based DTA solutions for inflow rates on routes 1 and 2 without incident.

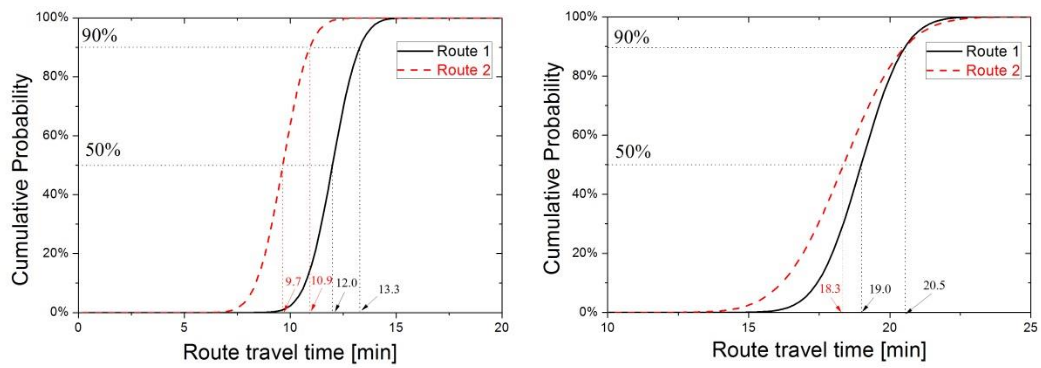

Fig.5 Route travel time cumulative probability for flow departing from origin at (a) 1 min and (b) 30 min.

Next the flow distributions on the two routes influenced by percentile value are analyzed. Fig.6 shows the proportion of flow on route 1 to the total flow from node 1 to node 3 with different percentile value α. Note that the case with α=50% is equivalent to deterministic DTA model. One can see that as the percentile value α becomes larger, which means that more reliability issue is being concerned with, more flow choose route 1. Such results indicate that the proposed percentile travel time based DTA model can reflect the realistic route choice behavior when reliability issue is concerned.

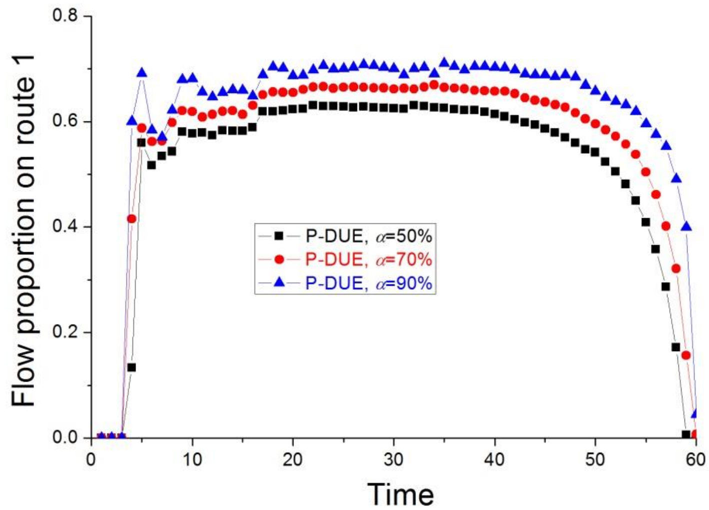

Fig.6 The flow proportion on route 1 at each departure time with different percentile value α.

In all, the percentile travel time based DTA model can predict the stochastic link flow pattern and route travel time distribution, so it can be used to analyze the reliability issues on dynamic time dependent network.

4.2 Impact of incident

It is true that traffic incident also has stochastic features. Here for simplicity and as a preliminary work, the traffic incident is deemed as deterministic event with given occurrence time, duration, and capacity drop ratio. The on time arrival probability is the main concern to study the dynamic impact of traffic incident on network reliability. The 90%-percentile route travel time without incident are deemed as the preferred arrival time. That is to say, if no incident happens, the on-time arrival probability is 90%. When incident happens, there will be traffic delay, and the on-time arrival probability is recalculated according to the preferred arrival time.

It is assumed that the incident happens on link 4. In this case, both the two OD pairs could be influenced by the incident. Note that there are three parameters, i.e., T_D, r_c, and T_S, which characterize traffic incident. So the impact of the three parameters on the on time arrival probability is analyzed in detail.

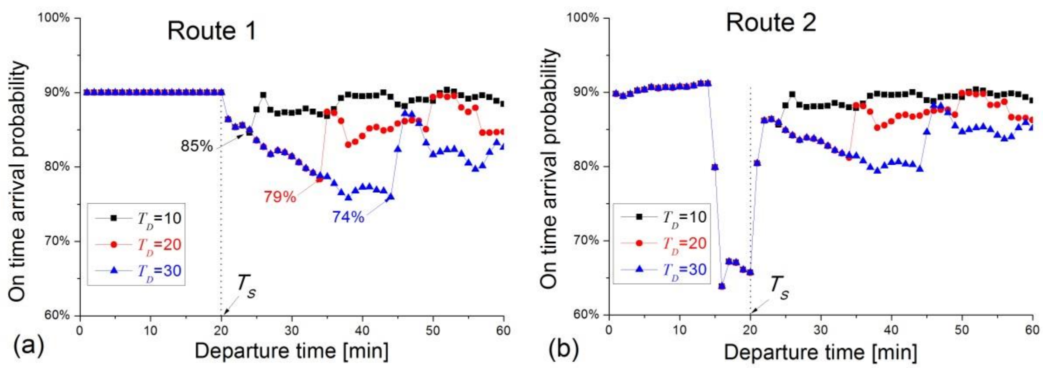

Fig. 7 The on time arrival probability with different duration T_D for flow on (a) route 1 and (b) route 2. The other parameters are T_S=20 and r_c =1/3.

Fig.7 shows the on time arrival probability with different duration T_D. One can see that as duration increases, the on time arrival probability will decreases. That is to say longer incident duration will have more serious impact on network reliability. Note that the incident happens on route 2, thus some flow will divert from route 2 to route 1 to avoid incident induced traffic congestion. But the travelers departing before the incident occurrence do not change route because they cannot predict the occurrence of incident. Therefore, around the incident occurrence time, the on time arrival probabilities for the two routes are different. The on time arrival probability for flow on route 2 will be reduced to very low values, while that for flow on route 1 keeps unchanged when departure time is earlier than T_S. A few minutes later than T_S, the on time arrival probabilities for the two routes are equal, which indicates that equilibrium state has been reached as a result of flow diversion.

The above results show that the incident has more serious impact on flows on route 2. In the following section, only the on time arrival probabilities on route 2 are shown. In Fig.8, the on time arrival probability with different duration r_c, and T_S are plotted. One can see that as the incident induced capacity drop becomes large, the on time arrival probability decreases (Fig.8 (a)). However as to the incident occurrence time, the extent of impact is hard to judge, because the lines cross each other several times as time going (see Fig.8 (b)). The impact of capacity drop ratio is easily to understand. Larger capacity drop induces more serious traffic congestion and results in lower on time arrival probability. However, the impact of the occurrence time is highly related to the given OD demand pattern. Next a network level index will be introduced the impact of incident on network reliability.

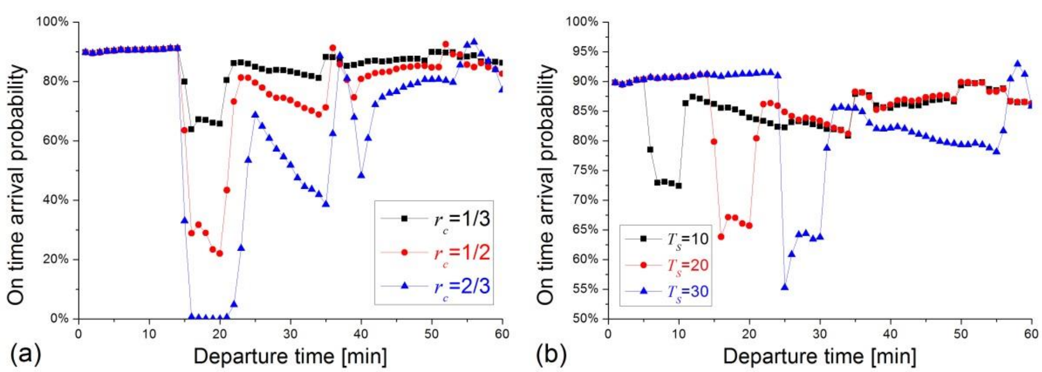

Fig.8 The on time arrival probability with different incident parameters (a) capacity drop ratio r_c, given T_D=20, T_S=20; (b) occurrence time T_S, given T_D=20, r_c =1/3.

In order to clearly show the impact of incident on network reliability, a reliability index reflecting the whole time horizon and network scale is introduced in Eqn. (27).

$$\mathrm{Re} = \sum_{rstp} \alpha_{rs}^{p}(t) f_{rs}^{p}(t) / \sum_{rstp} f_{rs}^{p}(t) \tag{27}$$

Re denotes the average on-time arrival probability over all traffic demand and time horizon. It is a direct indicator for network reliability and has a similar form with that in Yin et al. (2004), and is a direct indicator for network reliability. Besides the network reliability, the total travel delay, which is defined as the extra total travel time under traffic incident over the total travel time without traffic incident, is also investigated to show the impact of traffic incident on network performance.

Fig.9 shows the impacts of incident occurrence time and duration on network performance. From Fig.9 (a), one can see that the traffic incident happens at 25 min and lasts 20 minutes has the most serious impact on network performance, because the total delay is the largest and the Re is the lowest. As mentioned above, the impact of incident occurrence time is highly related to the given OD demand pattern. The OD demand reaches the peak value at 30 min. Thus the incident happens at 25 min and lasts 20 minutes just covers the period with the highest traffic demand. And this is the main cause for the most serious impact on network performance. From Fig.9 (b), one can see that as the incident duration grows, the total delay also increases while the network reliability decreases. That is to say, the longer the incident duration is, the more serious impact on network performance is. Such results are consistent with that in Fig.7.

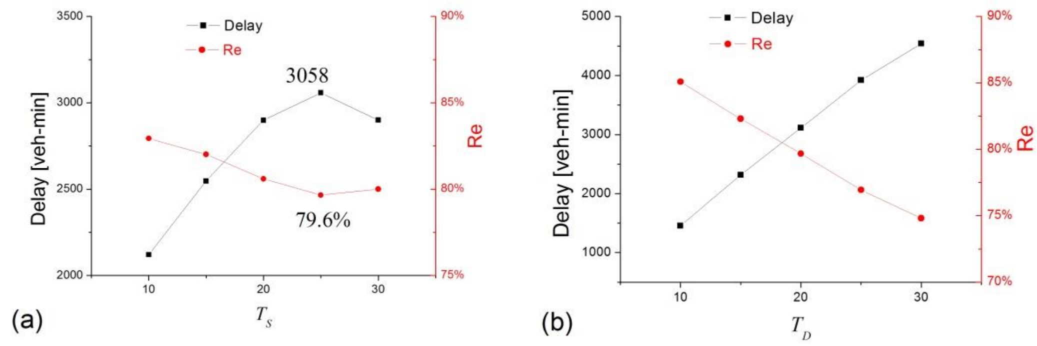

Fig.9 Network performance influenced by (a) incident occurrence time and (b) incident duration. The capacity drop ratio r_c =1/2.

4.3 Impact of speed limit

Note that all the links have the same free flow speed of 60km/h. Assumed that the speed limit can be selected from the following discrete values: 50km/h, 40km/h, 30km/h, and 20km/h. Here we just study the impacts of speed limit on improving the on time arrival probability. The optimal speed limit control is not the main concern of this paper, and they will be investigated in the future works, so several measures with different speed limits are analyzed and compared here. The speed limit control is applied on link 3, which is upstream the incident link, from the time 5 minutes after the incident happens to the time when the incident is cleared. The speed limit value is kept unchanged during the control period.

In static network, Yang et al. (2012) proposed that speed limit can play the same role as a toll charge and reduce the total travel time. The effect of speed limit on dynamic network has not been investigated. Here the impacts of speed limit on total delay and network reliability

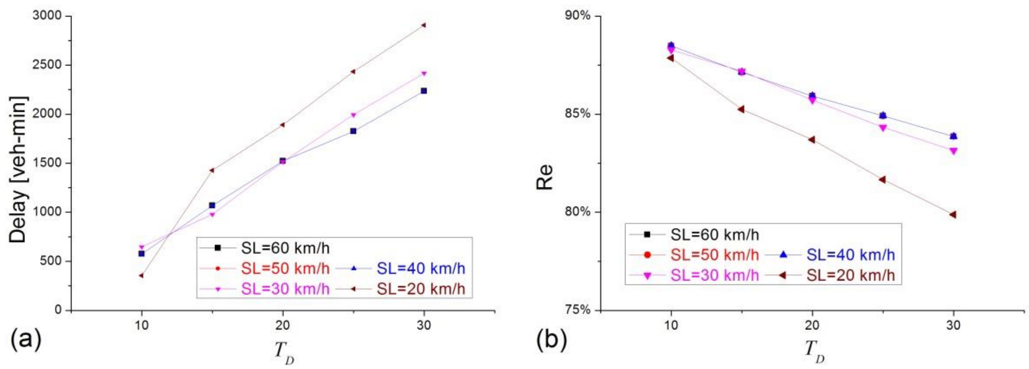

Fig.10 The effect of speed limit control under different incident duration on (a) total delay and (b) network reliability. The parameters are T_S=20, r_c =1/2.

Fig.10 shows the effect of speed limit on network performance with different incident duration. One can see that, in general, lower speed limit will increase the delay and decrease the network reliability. There are some exceptions. When T_D=10, speed limit SL=20 km/h will decrease the delay and the reliability at the same time. When T_D=15, speed limit SL=30 km/h will decrease the delay but do not reduce the reliability. That is to say, appropriate speed limit could improve network performance for the resulting lower delay and not lower network reliability. Furthermore, the traffic demand also impacts the implementation of speed limit measures. Table.2 shows the total travel time with different speed limit and demand ratios. One can see that when travel demand ratio is 0.5 and 1.5, speed limit SL=30 km/h does not reduce traffic delay.

Table.2 Network performance with different speed limit control and demand ratios.

	r_D=0.5		r_D=1.0		r_D=1.5	
	Delay	**Re**	**Delay**	**RE**	**Delay**	**Re**
No SL 60 km/h	296	88.2%	**1068**	**87.1%**	2070	86.8%
SL 30 km/h	348	87.6%	**979**	**87.1%**	2091	86.8%

Note that here a very simple speed limit rule is adopted, thus the effect is not obvious. Mathematical programming model should be established to get optimal solutions for variable speed limits. This is will be done in future works.

5. Conclusions

In this paper, a reliability-based DTA model was proposed to estimate the stochastic link flow pattern and route travel time distribution in road network with uncertainty and incident. The reliability-based DTA model was formulated as an equivalent VI problem in terms of route flow, and a solution algorithm with MSA method was adapted to solve the equivalent problem. The proposed model was then employed to investigate the impacts of traffic incident on the on-time arrival probability with and without dynamic speed limit control.

As the proposed model was developed for off-time applications so as to assess the dynamic impacts of traffic incident in network with uncertainty, it was assumed in the proposed model that travelers have perfect information on the dynamic or time-dependent network with and without traffic management control. During the period with incident, the link capacity is dropped by certain percentage or ratio. The incident occurrence time and duration are the two major concerns for assessing their impacts on on-time arrival probabilities on critical routes for travel with different OD pairs. In practice, change of speed limit on urban expressway is the commonly used control measures to alleviate the adverse impacts of traffic incident. However, it is interested to know that under what circumstances, whether the speed limit control could or could not be used to reduce the total network travel time and improve the on-time arrival probabilities.

In this paper, numerical tests are carried out on a small symmetric network. It was shown that: 1) the proposed reliability-based DTA model could estimate the route travel time distribution on time-dependent network; 2) traffic incident would greatly decrease the on-time arrival probability, especially when the incident has greater reduction on network capacity and longer duration; 3) dynamic speed limit control could reduce the total traffic delay and simultaneously do not deteriorate network reliability when the OD demand is falling within a certain range. Hence, there is a need to use the proposed reliability-based DTA model to assess the dynamic impacts of traffic incident on the network reliability with and without various dynamic traffic management and control measures.

Further studies should be carried out to apply the proposed model to a realistic network with empirical traffic incident data so as to evaluate the effects of various dynamic traffic management and control measures. Apart from dynamic speed limit control, some other dynamic traffic control strategies such as dynamic lane allocation and ramp metering should also be investigated with the proposed model under different traffic incident conditions. On the other hand, the proposed model can also be extended to estimate the dynamic multi-class OD demand matrices based on classified vehicular traffic counts over the time of the day.

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