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平面齿轮的高阶接触啮合理论

(重庆工商大学,　重庆　400079)　邹昌平
(浙江科技学院 机电系,　浙江 杭州　310012)　刘鹄然
(香港理工大学,　中国 香港)　C. Y. Chan

摘要　根据互相啮合的一对齿廓高阶接触要求,确定媒介齿条产形轮齿廓所应满足的条件,使互相啮合的共轭齿廓达到 4 阶接触。并研究了这种媒介齿条齿廓齿形曲线的构造方法。

关键词　齿轮　高阶接触　媒介齿条

1　曲线的 4 阶近似

将曲线在一点 M_0 的邻域用 4 阶戴劳级数展开

$$\boldsymbol{r}(u)=\boldsymbol{r}(u_0)+\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}u}\Delta u+\frac{1}{2!}\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}u^2}\Delta u^2+\frac{1}{3!}\frac{\mathrm{d}^3\boldsymbol{r}}{\mathrm{d}u^3}\Delta u^3\boldsymbol{r}+\frac{1}{4!}\frac{\mathrm{d}^4\boldsymbol{r}}{\mathrm{d}u^4}\Delta u^4\cdots\cdots$$

$$\overrightarrow{MM_0}=\boldsymbol{r}(u)-\boldsymbol{r}(u_0)=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}u}\Delta u+\frac{1}{2!}\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}u^2}\Delta u^2+\frac{1}{3!}\frac{\mathrm{d}^3\boldsymbol{r}}{\mathrm{d}u^3}\Delta u^3+\frac{1}{4!}\frac{\mathrm{d}^4\boldsymbol{r}}{\mathrm{d}u^4}\Delta u^4\cdots$$

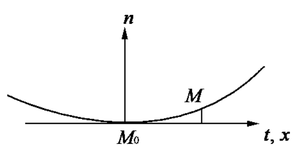


图 1　点到切线的距离

如图 1,曲线上 M 点到 M_0 点切线的距离为

$$\delta=\boldsymbol{n}\circ\overrightarrow{MM_0}$$

$$\delta=\boldsymbol{n}\circ\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}u}\Delta u+\frac{1}{2}\boldsymbol{n}\circ\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}u^2}\Delta u^2+\frac{1}{6}\boldsymbol{n}\circ\frac{\mathrm{d}^3\boldsymbol{r}}{\mathrm{d}u^3}\Delta u^3+\frac{1}{4!}\boldsymbol{n}\circ\frac{\mathrm{d}^4\boldsymbol{r}}{\mathrm{d}u^4}\Delta u^4$$

因为　$\boldsymbol{n}\circ\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}u}=0$;弧长微分 $\Delta s=\sqrt{E}\ \Delta u=\Delta x$,则

$$\Delta u=\frac{\Delta x}{\sqrt{E}}$$

$$\delta=\frac{1}{2}\frac{\boldsymbol{r}_{uu}\boldsymbol{n}}{E}\Delta x^2+\frac{1}{6}\frac{\boldsymbol{r}_{uuu}\boldsymbol{n}}{E^{\frac{3}{2}}}\Delta x^3+\frac{1}{4!}\frac{\boldsymbol{r}_u^{(4)}\circ\boldsymbol{n}}{E^2}\Delta x^4 \quad (1)$$

两曲线的诱导曲线可局部展开为

$$\delta_1-\delta_2=\frac{1}{2}\left(\frac{\boldsymbol{r}_{uu1}\boldsymbol{n}}{E}-\frac{\boldsymbol{r}_{uu2}\boldsymbol{n}}{E}\right)\Delta x^2+\frac{1}{6}\left(\frac{\boldsymbol{r}_{uuu1}\boldsymbol{n}}{E^{\frac{3}{2}}}-\frac{\boldsymbol{r}_{uuu2}\boldsymbol{n}}{E^{\frac{3}{2}}}\right)\Delta x^3+\frac{1}{4!}\left(\frac{\boldsymbol{r}_{u1}^{(4)}\circ\boldsymbol{n}}{E^2}-\frac{\boldsymbol{r}_{u2}^{(4)}\circ\boldsymbol{n}}{E^2}\right)\Delta x^4 \quad (2)$$

对一般齿轮,上式系数都不为零,齿面啮合点邻域间隙为 2 阶无穷小。人们只能尽量减少该系数,例如尼曼蜗杆。如第 1 项系数为零,意味着两齿面在啮合点曲率相同,齿面啮合点邻域间隙为 3 阶无穷小。如前两项系数都为零,齿面啮合点邻域间隙将为 4 阶无穷小 。本文研究使齿面啮合点邻域间隙为 3 阶和 4 阶无穷小的条件。本文的研究思路为,先假定媒介齿条的齿形是待定的,分别求出两共轭齿廓的齿形,再根据两共轭齿廓的最佳接触条件,确定媒介齿条的齿形。

2　齿面啮合点邻域间隙为 4 阶无穷小的条件

根据啮合原理,已知媒介齿条齿廓曲线 $y=y(x)$,对应齿轮齿廓和法矢可表示为下列公式,参见图 2。

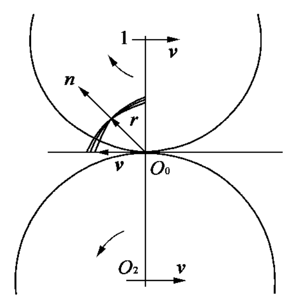


图 2　媒介齿条与共轭齿轮的产生(图中 v 表示媒介齿条不动,齿轮转动和轮心移动)

$$\begin{bmatrix}X\\Y\end{bmatrix}=\begin{bmatrix}\cos\phi & \sin\phi\\-\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}\cos\phi & \sin\phi\\-\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}-R_p\phi\\R_p\end{bmatrix} \quad (3)$$

$$\begin{bmatrix}N_x\\N_y\end{bmatrix}==\begin{bmatrix}\cos\phi & \sin\phi\\-\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}n_x\\n_y\end{bmatrix}=\begin{bmatrix}\cos\phi & \sin\phi\\-\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}-y'\\1\end{bmatrix}/\sqrt{y'^2+1} \quad (4)$$

根据齿廓法线法,齿条的移动距离 l 与齿轮转动

角度ϕ之间满足

$$\phi=\frac{l}{R_p}=\frac{x+yy'}{R_p} \tag{5}$$

将式(3)用矩阵表示

$$\{R\}=M\{r\}+M\{\Phi\} \tag{6}$$

式中, $\{R\}=\begin{bmatrix}X\\Y\end{bmatrix}$, $M=\begin{bmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{bmatrix}=\begin{bmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{bmatrix}$, $\{r\}=\begin{bmatrix}x\\y\end{bmatrix}$, $\{\Phi\}=\begin{bmatrix}-R_p\phi\\R_p\end{bmatrix}$

取$\{R\}$的1~4阶导数

$$+M\{\Phi\}+M\{\Phi\} \tag{7}$$

$$\{R\}''=M''\{r\}+2M'\{r\}'+M\{r\}''+M''\{\Phi\}+2M'\{\Phi\}'+2M\{\Phi\}'' \tag{8}$$

$$\{R\}'''=M'''\{r\}+3M''\{r\}'+3M'\{r\}''+M\{r\}'''+M'''\{\Phi\}+3M''\{\Phi\}'+3M'\{\Phi\}''+M\{\Phi\}''' \tag{9}$$

$$\{R\}^{(4)}=M^{(4)}\{r\}+4M'''\{r\}'+6M''\{r\}''+4M'\{r\}'''+M\{r\}^{(4)}+M^{(4)}\{\Phi\}+4M'''\{\Phi\}'+6M''\{\Phi\}''+4M'\{\Phi\}'''+M\{\Phi\}^{(4)} \tag{10}$$

式中,$\{r\}$、$\{\Phi\}$的各阶导数分别是

$$\{r\}'=\begin{bmatrix}1\\y'\end{bmatrix},\{r\}''=\begin{bmatrix}0\\y''\end{bmatrix},\{r\}'''=\begin{bmatrix}0\\y'''\end{bmatrix},$$

$$\{r\}^{(4)}=\begin{bmatrix}0\\y^{(4)}\end{bmatrix} \tag{11}$$

$$\{\Phi\}=R_p\begin{bmatrix}-\phi\\1\end{bmatrix},\{\Phi\}'=R_p\begin{bmatrix}-\phi'\\1\end{bmatrix},$$

$$\{\Phi\}''=R_p\begin{bmatrix}-\phi''\\1\end{bmatrix},\{\Phi\}'''=R_p\begin{bmatrix}-\phi'''\\1\end{bmatrix},$$

$$\{\Phi\}^{(4)}=R_p\begin{bmatrix}-\phi^{(4)}\\1\end{bmatrix} \tag{12}$$

由于媒介齿条的坐标原点取在齿廓法线与齿条节线的交点,在此处 $\phi=\frac{l}{R_p}=\frac{x+yy'}{R_p}=0$,故有

$$M|_{\phi=0}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \tag{13}$$

$$M'|_{\phi=0}=\begin{bmatrix}-\phi'\sin\phi & \phi'\cos\phi\\ -\phi'\cos\phi & -\phi'\sin\phi\end{bmatrix}_{\phi=0}=\begin{bmatrix}0 & \phi'\\ -\phi' & 0\end{bmatrix} \tag{14}$$

$$M''|_{\phi=0}=\begin{bmatrix}-\phi''\sin\phi-\phi'^2\cos\phi & \phi''\cos\phi-\phi'^2\sin\phi\\ -\phi''\cos\phi+\phi'^2\sin\phi & -\phi''\sin\phi-\phi'^2\cos\phi\end{bmatrix}_{\phi=0}=\begin{bmatrix}0 & \phi''\\ -\phi'' & 0\end{bmatrix} \tag{15}$$

$$\{R\}''=\phi''\begin{bmatrix}y\\-x\end{bmatrix}+2\phi'\begin{bmatrix}y'\\-1\end{bmatrix}+\begin{bmatrix}0\\y''\end{bmatrix}+R_p\begin{bmatrix}0\\2\phi'^2\end{bmatrix} \tag{16}$$

齿廓法线矢量取决于媒介齿条的齿廓法线矢量,即

$$\begin{bmatrix}N_x\\N_y\end{bmatrix}_{\phi=0}=\frac{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}-y'\\1\end{bmatrix}}{\sqrt{y'^2+1}}=\frac{\begin{bmatrix}-y'\\1\end{bmatrix}}{\sqrt{y'^2+1}} \tag{17}$$

齿廓曲线局部展开式的2阶参数 $n\circ\frac{d^2\boldsymbol{r}}{du^2}$为

$$\{N\}^{\mathrm{T}}\{R\}''=\phi''[-y'\ \ 1]\begin{bmatrix}y\\-x\end{bmatrix}+2\phi''[-y'\ \ 1]\begin{bmatrix}y'\\-1\end{bmatrix}+[-y'\ \ 1]\begin{bmatrix}0\\y''\end{bmatrix}+R_p[-y'\ \ 1]\begin{bmatrix}0\\2\phi'^2\end{bmatrix}$$

$$=\phi''(-y'y-x)+2\phi'(-y'^2-1)+y''+2R_p\phi'^2$$

$$=-2\phi'(y'^2+1+2R_p\phi')+y'' \tag{18}$$

式中,$\phi=0$, $l=x+yy'=0$。

对轮1,2分别有

$\{N_1\}^{\mathrm{T}}\{R_1\}''=-2\phi'_1(y'2+1+2R_{p1}\phi'_1)+y''$,

$\phi'_1=\frac{l}{R_{p1}}=\frac{x+yy'}{R_{p1}}$,

$\{N_2\}^{\mathrm{T}}\{R_2\}''=-2\phi'_2(y'2+1+2R_{p2}\phi'_2)+y''$,

$\phi'_2=\frac{x+yy'}{R_{p2}}$

令2阶系数为0:$\{N_1\}^{\mathrm{T}}\{R_1\}''-\{N_2\}^{\mathrm{T}}\{R_2\}''=0$ 则必有

$$\phi'_1=\frac{l'}{R_{p1}}=0,\ \phi'_2=\frac{2'}{R_{p2}}=0,\ l'=0$$

$$x+yy'=0,\ 1+y'^2+y''=0 \tag{19}$$

曲率中心坐标 $y_c=y+\frac{1+y'^2}{y''}=0$,根据曲率中心坐标和渐曲线性质,知曲率中心必在$x$轴上。故欲使2阶系数为0,媒介齿条曲率中心必处处落在齿条节线上。为求3阶参数

$$M'''|_{\phi=0}=\begin{bmatrix}-\phi'''\sin\phi-3\phi''\phi'\cos\phi+\phi'^3\sin\phi & \phi'''\cos\phi-3\phi''\phi'\sin\phi-\phi'^3\cos\phi\\ -\phi'''\cos\phi+3\phi''\phi'\sin\phi+\phi'^3\cos\phi & -\phi'''\sin\phi-3\phi''\phi'\cos\phi-\phi'^3\sin\phi\end{bmatrix}_{\phi=0}$$

$$=\begin{bmatrix}0 & -\phi'''\\ -\phi''' & 0\end{bmatrix} \tag{20}$$

$$\{R\}'''=\phi'''\begin{bmatrix}y\\-x\end{bmatrix}+3\phi''\begin{bmatrix}y'\\-1\end{bmatrix}+\begin{bmatrix}0\\y'''\end{bmatrix}+R_p\left\{\phi'''\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}-\phi'''\\0\end{bmatrix}\right\} \tag{21}$$

齿廓曲线局部展开式的3阶参数 $n\circ\frac{d^3\boldsymbol{r}}{du^3}$为

$$\{N\}^{\mathrm{T}}\{R\}'''=\phi'''[-y\ \ 1]\begin{bmatrix}y\\-x\end{bmatrix}+3\phi''[-y'\ \ 1]\begin{bmatrix}y'\\-1\end{bmatrix}+[-y'\ \ 1]\begin{bmatrix}0\\y'''\end{bmatrix}$$

$$=\phi'''(-y'y-x)+3\phi''(-y'^2-1)+y'''$$
$$=3\phi''(-y'^2-1)+y''' \tag{22}$$

对轮 1, 2 分别有

$$\{N_1\}^{\mathrm T}\{R_1\}'''=3\phi_1''(-y'^2-1)+y''',$$
$$\{N_2\}^{\mathrm T}\{R_2\}'''=3\phi_2''(-y'^2-1)+y'''$$

同样, 欲使 3 阶系数为 0, 必有

$\{N_1\}^{\mathrm T}\{R_1\}'''-\{N_2\}^{\mathrm T}\{R_2\}'''=0$, 又由于$(-y'^2-1)$为法矢的模, 不等于 0, 故有

$$\phi_1''=\frac{l''}{R_{p1}}=0,\ \phi_2''=\frac{l''}{R_{p2}}=0,$$
$$l''=0,\ (1+y'^2+y'')'=0,\ 3y'y''+yy'''=0 \tag{23}$$

欲让 3 阶参数恒为 0, 必同时满足式(19)和式(23)两条件。为求 4 阶参数的系数, 有

$$M^{(4)}\big|_{\phi=0}=$$
$$\begin{bmatrix} -\phi^{(4)}\sin\phi+4\phi'''\phi'\sin\phi-3\phi''^2\cos\phi+6\phi'^2\phi''\sin\phi-\phi'^4\cos\phi & -m_{12}^{(4)} \\ -\phi^{(4)}\cos\phi+4\phi'''\phi'\sin\phi+3\phi''^2\phi\sin\phi+6\phi'^2\phi''\cos\beta\phi-\phi'^{(4)}\sin\phi & m_{11}^{(4)} \end{bmatrix}_{\phi=0}$$
$$=\begin{bmatrix} 0 & -\phi^{(4)} \\ -\phi^{(4)} & 0 \end{bmatrix} \tag{24}$$
$$\{R\}^{(4)}=\phi^{(4)}\begin{bmatrix} y \\ -x \end{bmatrix}+4\phi'''\begin{bmatrix} y' \\ -1 \end{bmatrix}+\begin{bmatrix} 0 \\ y^{(4)} \end{bmatrix}$$

齿廓曲线局部展开式的 4 阶参数 $n\circ\dfrac{\mathrm d^4\boldsymbol r}{\mathrm du^4}$为

$$\{N\}^{\mathrm T}\{R\}^{(4)}=\phi^{(4)}[-y\quad 1]\begin{bmatrix} y \\ -x \end{bmatrix}+4\phi'''[-y'\quad 1]\begin{bmatrix} y' \\ -1 \end{bmatrix}+[-y'\quad 1]\begin{bmatrix} 0 \\ y^{(4)} \end{bmatrix}$$
$$=\phi^{(4)}(-y'y-x)+4\phi'''(-y'^2-1)+y^{(4)}$$
$$=4\phi'''(-y'^2\quad 1)+y^{(4)} \tag{25}$$

对轮 1, 2 分别有

$$\{N_1\}^{\mathrm T}\{R_1\}^{(4)}=4\phi_1''(-y'^2-1)+y^{(4)},$$
$$\{N_2\}^{\mathrm T}\{R_2\}^{(4)}=4\phi_2''(-y'^2-1)+y^{(4)}$$
$$\{N_1\}^{\mathrm T}\{R_1\}^{(4)}-\{N_2\}^{\mathrm T}\{R_2\}^{(4)}=$$
$$4l'''(-y'^2-1)\left(\frac{1}{R_{p1}}+\frac{1}{R_{p2}}\right)$$

齿面啮合点邻域间隙为 4 阶无穷小

$$\delta=k_4\Delta x^4 \tag{26}$$

3　媒介齿条齿廓的构造

一条曲线的曲率中心所在的曲线称为渐缩线, 对应的曲线则称为渐伸线。直线既没有渐伸线也没有渐缩线。但我们可以人为地构造一小段一小段曲线, 使这些曲线段的首末端曲率中心过直线, 满足式(19)和式(23)两条件。

3.1　满足 3 阶接触的媒介齿条

设在平面上给定一点(x_0, y_0), 以及过该点的微段曲线的切线方向 y_0', 由于微段曲线段曲率过 x 轴

$$y_c=y_0+\frac{1+y'^2_0}{y_0''}=0,\ y_0''=-\frac{y_0}{1+y'^2_0},$$
$$y_0'''=\frac{-3y'y''}{y}=\frac{3y'(1+y'^2)}{y} \tag{27}$$
$$x_c=x-\frac{y_0'(1+y_0'^2)}{y_0''}=x_0+y_0y'_0$$

给定曲率中心横坐标的 1 个增量 Δx_c 和对应的曲线坐标增量 Δx, Δy。得到(x_1, y_1)点。则

$$x_{c1}=x_{c0}+\Delta x_c,\ x_1=x_0+\Delta x,\ y_1=y_0+\Delta y,$$
$$\tan\gamma_1=\frac{\mathrm dy_1}{\mathrm dx_1}=\frac{x_{c1}-x_1}{y_1},\ y_1''=\frac{-y_1}{1+y'^2},$$
$$y_1'''=\frac{3y'(1+y'^2)}{y_1} \tag{28}$$

二阶导数 y'', y'''应由式(28)确定。故该微段可展开成泰勒级数

$$y=y_0+y_0'(x-x_0)+\frac{y_0''}{2!}(x-x_0)^2+\frac{y_0'''}{6}(x-x_0)^3\cdots$$

过 x_0y_0 点再构造一曲线

$$y=y_0+y_0'(x-x_0)+\frac{y_0''}{2!}(x-x_0)^2+$$
$$\frac{y_0'''}{3!}(x-x_0)^3+\frac{a_4}{4!}(x-x_0)^4+\frac{a_5}{5!}(x-x_0)^4 \tag{29}$$

显然, 在 $x=x_0$点处, 该曲线与原曲线有相同的坐标、斜率、曲率和 3 阶导数。给定曲线一个增量 $x-x_0=\Delta x$, 这时, $x=x_1$, $y=y_1$, 要求在该微曲线段终点处, 仍然满足式(28)。

$$y_1'=y_0'+y_0''(x-x_0)+\frac{y_0'''}{2!}(x-x_0)^2+$$
$$\frac{a_4}{3!}(x-x_0)^3+\frac{a_5}{4!}(x-x_0)^4 \tag{30}$$
$$y_1''=y_0''+\frac{y_0'''}{1!}(x-x_0)+\frac{a_4}{2!}(x-x_0)^2$$
$$+\frac{a_5}{3!}(x-x_0)^3 \tag{31}$$
$$y_1'''=y_0'''+a_4(x-x_0)+\frac{a_5}{2!}(x-x_0)^2 \tag{32}$$

令 $x_1-x_0=\Delta x$, $y_1-y_0=\Delta y$, 4 个未知数 Δx, Δy, a_4, a_5 应满足式(29～32)4 个方程。

3.2　满足 2 阶接触的媒介齿条

设在平面上给定一点(x_0, y_0), 以及过该点的微段曲线的切线方向 y_0', 由于微段曲线段曲率过 x 轴

$$y_c=y_0+\frac{1+y'^2_0}{y_0''}=0,\ y_0''=\frac{y_0}{1+y'^2_0} \tag{33}$$
$$x_c=x-\frac{y'_0(1+y'^2)}{y_0''}=x_0+y_0y'_0$$

给定曲线坐标 1 个增量 Δx、Δy, 得到(x_1, y_1)点。

对应的曲率中心横坐标的 1 个增量 $\Delta x_c = \dfrac{\Delta x}{\cos^2\beta} = (1 + y'^2_0)\Delta x$，可保持原压力角不变。考虑到压力角的增量，$\Delta x_c = k\dfrac{\Delta x}{\cos^2\beta} = k(1 + y'^2_0)\Delta x$。

式中，k 为压力角变化系数，$0 < k < 1$。则

$$x_{c1} = x_{c0} + \Delta x_c,\ x_1 = x_0 + \Delta x,\ y_1 = y_0 + \Delta y,$$

$$\tan\gamma_1 = \frac{dy_1}{dx_1} = \frac{x_{c1} - x_1}{y_1};\ y_1'' = \frac{y_1}{1 + y_1'^2} \tag{34}$$

二阶导数应由式(28)确定。故该微段可展开成泰勒级数

$$y = y_0 + y_0'(x - x_0) + \frac{y_0''}{2!}(x - x_0)^2 + \frac{y_0'''}{6}(x - x_0)^3 \cdots$$

过 $x_0 y_0$ 点再构造一曲线

$$y = y_0 + y_0'(x - x_0) + \frac{y_0''}{2!}(x - x_0)^2 + \frac{a_3}{3!}(x - x_0)^3 + \frac{a_4}{4!}(x - x_0)^4 \tag{35}$$

显然，在 $x = x_0$ 点处，该曲线与原曲线有相同的坐标、斜率、曲率。给定曲线一个增量 $x - x_0 = \Delta x$，这时，$x = x_1$，$y = y_1$，要求在该微曲线段终点处仍然满足式(34)。

$$y_1' = y_0' + y_0''(x - x_0) + \frac{a_3}{2!}(x - x_0)^2 + \frac{a_4}{3!}(x - x_0)^3 \tag{36}$$

$$y_1'' = y_0'' + \frac{a_3}{1!}(x - x_0) + \frac{a_4}{2!}(x - x_0)^2 \tag{37}$$

令 $x_1 - x_0 = \Delta x$，2 个未知数 a_3，a_4 应满足式(36)、式(37)2 个方程。解得

$$a_4 = \frac{12}{\Delta x^3}\left[(y_0' - y_1') + \frac{\Delta x}{2}(y_0'' - y_1'')\right]$$

$$a_3 = \frac{1}{\Delta x}\left(y_1'' - y_0'' - \frac{a_4}{2}\Delta x^3\right)$$

最后由式(34)求 y_1，就是下一个点。但必须与 $\tan\gamma_1 = \dfrac{dy_1}{dx_1} = \dfrac{x_{c1} - x_1}{y}$ 的 y_1 一致，这要反试算才可实现。计算结果为

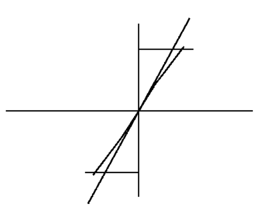


图 3　产生高阶接触齿廓的媒介齿条(产形轮)

$x_0 = 0$，$y_1 = 0$，$dy_1 = 2.3439$，$ddy_1 = 0$，$a_3 = -1.514777$，$a_4 = 2.157533$

$x_1 = 1$，$y_1 = 2.485$，$dy_1 = 2.3497$，$ddy_1 = -0.436011$，$a_3 = 2.319431$，$a_4 = -5.528967$

$x_1 = 2$，$y_1 = 4.673$，$dy_1 = 2.15191$，$ddy_1 = -0.881063$，$a_3 = 5.412808$，$a_4 = -11.716919$

$x_1 = 3$，$y_1 = 6.6984$，$dy_1 = 2.02443$，$ddy_1 = -1.32671$，$a_3 = 8.292781$，$a_4 = -17.474321$

$x_1 = 4$，$y_1 = 8.6136$，$dy_1 = 1.93172$，$ddy_1 = -1.77109$，$a_3 = 11.078202$，$a_4 = -23.041517$

每段之间平分 10 份，求得各 x 值对应的 y 值，可得媒介齿条齿形。

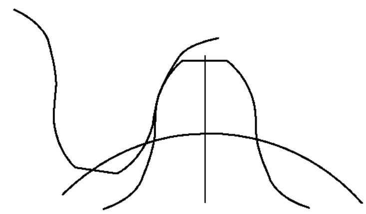
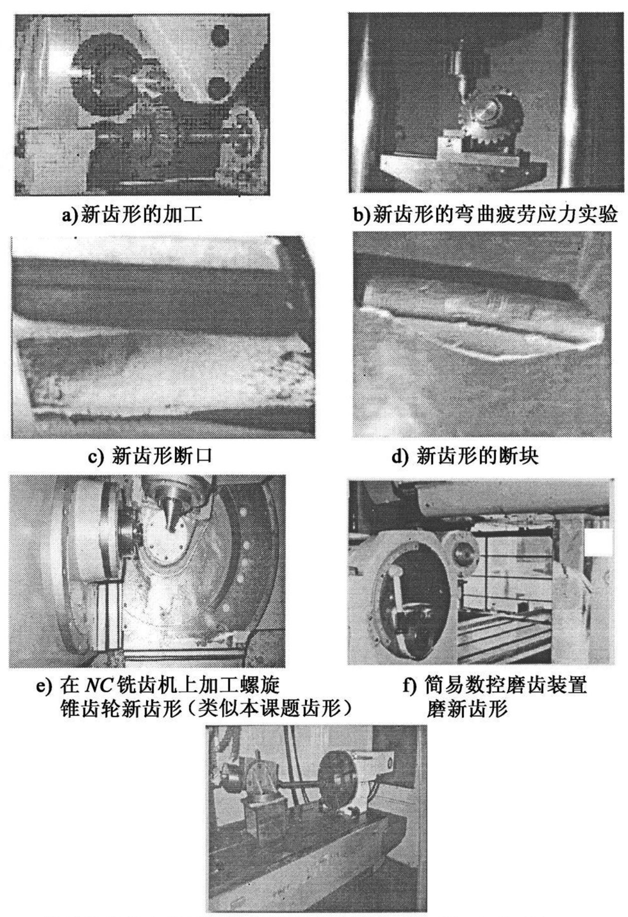


图 4　高阶接触齿轮的啮合



a) 新齿形的加工　b) 新齿形的弯曲疲劳应力实验

c) 新齿形断口　d) 新齿形的断块

e) 在 *NC* 铣齿机上加工螺旋锥齿轮新齿形（类似本课题齿形）　f) 简易数控磨齿装置磨新齿形

g) 简易数控铣齿装置加工螺旋锥齿轮新齿形（类似本课题齿形）

图 5

4　结论

根据齿廓法线法，啮合条件为 $x + yy' = 0$，对媒介齿条形状没有要求，齿面啮合点邻域间隙为 2 阶无穷

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行星齿轮系统的运动分析及动力学仿真

（重庆大学 机械传动国家重点实验室，　重庆　400044）　袁　敏　李润方
（立德管理学院 资讯传播学系 应用资讯研究所，　台湾 台南）　林建德

摘要　应用行星轮系的图画表示法与基本回路方法，对行星齿轮变速器进行了运动分析与效率计算；采用键合图理论，建立了系统的动态模型，并进行了动力学仿真，仿真结果具有明显的规律性，从而为行星传动技术的研究提供一种正确的理论分析方法。

关键词　行星齿轮　传动效率　键合图　仿真

引言

行星齿轮传动广泛地应用于现代机械传动设备中[1]，本文应用行星轮系的图画表示法与基本回路方法，对行星齿轮变速器进行了运动分析与效率计算；采用键合图理论，建立了系统的动态模型，并进行了动力学仿真，从而为行星传动技术的研究提供一种正确的理论分析方法。

1　行星齿轮系统的运动分析

本文要讨论的行星齿轮系统是一种汽车自动变速器，机构简图如图 1a 所示。根据 Lam，K. T. 提出的图画转换方法[2]，可得到图 1b 所示的齿轮系图画。

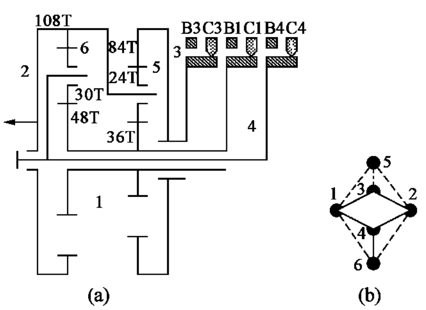

图 1　行星齿轮自动变速器及其图画

1.1　基本回路运动方程式[2]

基本回路是由 3 点、1 个齿轮副边和 2 个回副对边（或回副对多边形）所构成。若 ω_i，ω_j 与 ω_k 分别为基本回路中对应 3 个构件 i，j 与 k 的转速，且 i 与 j 是一对啮合齿轮，基本回路方程式可写成

$$\omega_i - \gamma_{ij}\omega_j + (\gamma_{ij} - 1)\omega_k = 0 \quad (1)$$

式中，$\gamma_{ij} = \pm z_j / z_i$，$z_i$ 和 z_j 分别表示齿轮 i 与 j 的齿数，正负号分别表示齿轮副为内啮合或外啮合齿轮副。

由图 1b 知该图画有 4 个基本回路，分别是 1－6－4，1－5－2，2－6－4，3－5－2，按式（1）列出基本回路方程式，分别为

$$\begin{cases} \omega_1 - \gamma_{61}\omega_6 + (\gamma_{61} - 1)\omega_4 = 0 \\ \omega_1 - \gamma_{51}\omega_5 + (\gamma_{51} - 1)\omega_2 = 0 \\ \omega_2 - \gamma_{62}\omega_6 + (\gamma_{62} - 1)\omega_4 = 0 \\ \omega_3 - \gamma_{53}\omega_5 + (\gamma_{53} - 1)\omega_2 = 0 \end{cases} \quad (2)$$

式中，$\gamma_{61} = -z_6/z_1$；$\gamma_{51} = -z_5/z_1$；$\gamma_{62} = z_6/z_2$；$\gamma_{53} = z_5/z_3$，其中各齿轮的齿数如图 1 中标注。

1.2　行星齿轮系统的传动比

如图所示当制动组件 $B4$ 作用（即组件 4 的转速为零 $\omega_4 = 0$），离合器与组件 3 接合（$C3$ 作用），是该变速器的一个档位。依据 4 个基本回路方程式，因 $\omega_4 = 0$，可得矩阵式

$$\begin{bmatrix} 1 & 0 & 0 & -r_{61} \\ 1 & r_{51}-1 & -r_{51} & 0 \\ 0 & 1 & 0 & -r_{62} \\ 0 & r_{53}-1 & -r_{53} & 0 \end{bmatrix} \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_5 \\ \omega_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -\omega_3 \end{Bmatrix}$$

代入具体数值求解得到

小。而如 $(x + yy')' = 0$，媒介齿条齿廓曲率中心必须处处落在节线上，意味着两共轭齿廓曲率相同。齿面啮合点邻域间隙的 3 阶参数由 $(x + yy')''$ 表出。而如媒介齿条形状同时满足 $(x + yy')' = 0$ 和 $(x + yy')'' = 0$，齿面啮合点邻域间隙的 3 阶参数为 0，齿面啮合点邻域间隙为 4 阶无穷小。齿面啮合点邻域间隙的 4 阶参数由 $(x + yy')'''$ 表出。从逻辑上看很有规律。本文还远不完善，只是抛砖引玉以期引起学术界重视。

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收稿日期：20050927　　收修改稿日期：20051103
作者简介：邹昌平（1963－），男，重庆市人，博士，教授，机械学院副院长

ABSTRACTS & KEY WORDS

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Investigation on the Static Stiffness of Parallel Kinematic Machine

Abstract The static stiffness theory of a parallel kinematic machine (PKM) is still in progress. To obtain the static stiffness of a PKM, the statics of a PKM is researched first and then the energy method is introduced. Based on the outcome of the PKM' s statics, the magnitude of the PKM' s deforming is obtained using the unit payload method, so the static stiffness of a five DOF PKM is gained. The method provides the theory foundation of a PKM' s static stiffness and its design method.

Key words: Static stiffness Statics algorithm Parallel kinematic machine

Simulation of Lubricant Flow and Heat Transfer in Lubrication System of Vehicle Gearing

Abstract A steady model of flow and heat transfer in vehicle gearing lubrication system has been developed. And a lubricant flow model of vehicle gearing was established by using one— dimensional incompressible flow equations. The heat transfer mechanisms within the lubricating oil circuit were studied, and the heat transfer simulation model was developed with heat network method. An exemplary simulation has been carried out for the lubrication system of a tracked armored vehicle gearing, pressures, flow rates and temperatures in lubricating system were calculated, and the simulation results were compared to test data with good agreement. This study shows that the simulation model can be used as a theoretical analysis means for studying lubricating oil flow performance of vehicle gearings, as well as providing an useful tool.

Key words: Lubrication system Flow Heat transfer Simulation

Study on Simulation Analysis and Experiment of Cutting Temperature During High— speed Dry Flying Cutter Cutting

Abstract According to the knowledge of Heat Transfer and Metal Cutting Theory, high— speed dry gear milling by flying cutter to simulate the gear hobbing process is adopted. The milling temperature model of milling by flying cutter is established based on metal cutting and diathermanous theory. The multiple—factor cutting temperature calculation formulas are derived out, and combined with the flying cutter high —speed dry cutting experiment, then it is contrasted the experimental data to the theoretical data, the relationship and the changing rule among main cutting parameter is studied. The exactness of the theoretical simulation model is verified. The foundation is established for further studying the cutting temperature of high— speed dry gear hobbing and its' cutting mechanism.

Key words: High— speed Dry Cutting temperature Hobbing

Theory of High Degree Contact Gear Profile

Abstract In view of the high power transmission gears in practical application, a new kind profile with the best load capacity both in contact and bending strength is presented. This is a kind of gearing with equal meshing curvature over the whole tooth surface. The forming principle of this tooth is analyzed.

Key words: Gear High degree contact Media rank

A Method of the Motion Analysis and Dynamic Simulation for Planetary Gear Trains

Abstract Based on graph theory and the methodology of fundamental circuits for gear trains, the motion analysis and the mechanical efficiency of the planetary gear trains were introduced. Then the dynamic model of the planetary gear system was set up, and the dynamic simulation was established. The results of this work could be used as a correct theoretical tool to analyze the planetary gear trains and the transmissions.

Key words: Planetary gear Mechanical efficiency Bond graph Simulation

Modeling and Software' s Realization of Torsion Vibration for the Tree Construction Transmission of Vehicle

Abstract Structure of vehicle transmission is the type of tree constructed by link. The linear array created related to the vehicle transmission properties is employed here realizing easy data' s storage and getting method. The torsion vibration with damp calculation modeling method and software' s realization' s method, combined by the Matlab program language realized in lab. The symbolic variable and simplifying step by step were employed such that the computing capacity was enlarged.

Key words: Torsion vibration Vehicle transmission Binary tree Matlab GUI

Dynamics Modeling and Calculation of Planar Flexible Multi— link Manipulators with Material Damping