Toward realistic virtual surgical simulation: using heuristically parameterized anisotropic mass-spring model to simulate tissue mechanical responses

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***Abstract*—Mass-spring model is a popular method to simulate soft tissues in virtual surgery applications. However, the setting of appropriate model parameters to reproduce the real mechanical responses of the tissues, which is anisotropic in general, remains an issue. This paper presents a hybrid heuristic approach to identify the parameters by incorporating simulated annealing into genetic algorithms. The optimization process is performed with reference to the benchmarks obtained by applying continuum mechanics and the finite element method. Experiments are performed to evaluate the feasibility of the approach.**

Keywords-virtual reality; medical education; mass-spring model; parameterization; simulated annealing; genetic algorithm

I. Introduction

Virtual-reality simulation of operative procedures is a promising approach to facilitate medical education or surgical planning. In these simulators, interactive deformation of virtual tissues is a key feature, as shown in Fig. 1. It requires both real-time and realistic modeling of soft-tissue responses so that the virtual tissues can respond to user's action by changing their shape autonomously. Finite element method (FEM) and mass-spring model (MSM) are the two major physics-based approaches for simulating soft-tissue deformation. They are at the opposite ends of the accuracy-efficacy spectrum, where the former is considered more accurate, and the latter is computationally efficient and thus suitable for real-time interactive simulations.

Although MSM has been successfully applied in many virtual surgery applications [1], the model parameter should be set appropriately in order to simulate the actual tissue responses. Unlike FEM, the parameters in MSM do not have direct relationship with the elastic constants that describe the mechanical properties of materials, e.g. Young's modulus and Poisson's ratio. As a result, the setting of MSM parameters usually resorts to repeated manual tuning until the simulated responses become seemingly realistic. Attempts have been made to obtain the parameters heuristically [2-5] and analytically [6, 7]. In this paper, a hybrid heuristic approach combining simulated annealing (SA) and genetic algorithms (GA) is developed to identify the MSM parameters automatically. Since FEM is parameterized directly with material's elastic constants, deformation simulated by FEM are employed here as the benchmarking reference to identify the MSM parameters. While previous work has been limited to isotropic materials [2-5], this paper concerns the identification of MSM parameters for simulating anisotropic materials, in particular, transversely isotropic and orthotropic materials.

II. Mechanics of Materials

In continuum mechanics, the stress $\boldsymbol{\sigma}$ applied to an elastic material is linearly related to the resulting strain $\boldsymbol{\varepsilon}$ by the generalized 3D Hooke's Law, $\boldsymbol{\varepsilon} = \mathbf{C}\boldsymbol{\sigma}$, where $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are 6×1 column vectors, and $\mathbf{C}$ is the material matrix**,** a 6×6 square matrix describing the mechanical properties of material. Soft tissues are often modeled as being purely isotropic to simplify the mathematical formulation, where $\mathbf{C}$ only depends on two elastic constants, Young's modulus and Poisson's ratio. However, many biological tissues are orthotropic or transversely isotropic [8]. Orthotropic material exhibits different mechanical properties along three orthogonal axes. The corresponding material matrix $\mathbf{C}$ is thus evaluated with three sets of Young's modulus E, Poisson's ratio ν and shear modulus G, totally 9 independent elastic constants. They are E_1, E_2, E_3, ν_{12}, ν_{13}, ν_{23}, and G_{12}, G_{13}, G_{23}, where the subscripts 1, 2 and 3 denote the three orthogonal axes respectively. For transversely isotropic materials, the mechanical properties are the same in a plane and different along the direction perpendicular to that plane. Suppose the properties in the plane containing axis 2 and 3 are the same, and that along axis 1 is different, $\mathbf{C}$ is evaluated with 5 independent constants, i.e. E_1, E_2, ν, ν_{12} and G_{12}, where $\nu = \nu_{23} = \nu_{32}$. Hence, to simulate the deformation of orthotropic and transversely isotropic materials by continuum mechanics and FEM, it is necessary to provide 9 and 5 elastic constants respectively. These constants are available from experiments or literature. In this study, the simulated deformations obtained by continuum mechanics and FEM are used as the benchmarking references for tuning the MSM parameters. The mathematical formulations of FEM are available from standard texts [9].

Figure 1. Interactive deformation of virtual tissues.

III. Optimizing MSM against FEM

A. Heuristic Optimization

The objective here is to determine a set of optimum parameters for MSM such that the simulated deformation is close to the reference obtained by FEM. This is achieved by tuning the MSM parameters until the difference is minimized. The basic steps are as follows.

1. Obtain reference deformation using FEM (benchmark).
2. Under the same conditions in Step 1, use MSM to simulate deformation with a certain set of parameters.
3. Compute the difference in deformation between the benchmark in Step 1 and the result in Step 2.
4. Obtain a new set of parameters based on the difference computed in Step 3.
5. Use the new parameters in Step 4, repeat Step 2 to Step 5 until the difference computed in Step 3 is minimized and the corresponding set of parameters is the optimum solution.

The set of model parameters in Step 2 and Step 4 is referred to as a *solution* to the problem, and the difference in Step 3 is the *cost* of optimization and computed with a *cost function f*. Heuristic techniques have been used to minimize the cost by tuning the parameters automatically. These include SA which mimics the thermal annealing process of metals, and GA which imitates the crossover and mutation processes of chromosomes in the evolution of species. Details about SA and GA can be obtained from standard texts [10, 11].

An early work applied SA to identify the model parameters of a 2D mass-spring array [3]. The reference was obtained from the analytical solution to static deformation of a square plate due to shear and tensile stress. Later work primarily employed GA to set the MSM parameters, and the reference deformation was obtained numerically by applying FEM and continuum mechanics to model elastic materials [2, 4, 5]. While GA is becoming a common heuristic approach to identify MSM parameters, it is well-known for being poor in hill-climbing that an immediate solution could be trapped by a local minimum, leading to premature convergence. The random nature of crossover and mutation operations may also result in inferior offspring. Besides, a large population of solutions is required in GA. Hybrid algorithms combining SA with GA have been proposed and shown to be advantageous over pure GA [12, 13]. In addition to the reduction in population size, improvement in hill-climbing performance at the later stages is achieved by allowing for high mutation probability (usually 1% in pure GA). Higher reliability and consistency are also demonstrated [14]. This hybrid heuristic approach is thus capitalized upon in this paper to develop an efficient algorithm for identifying the MSM parameters of anisotropic materials.

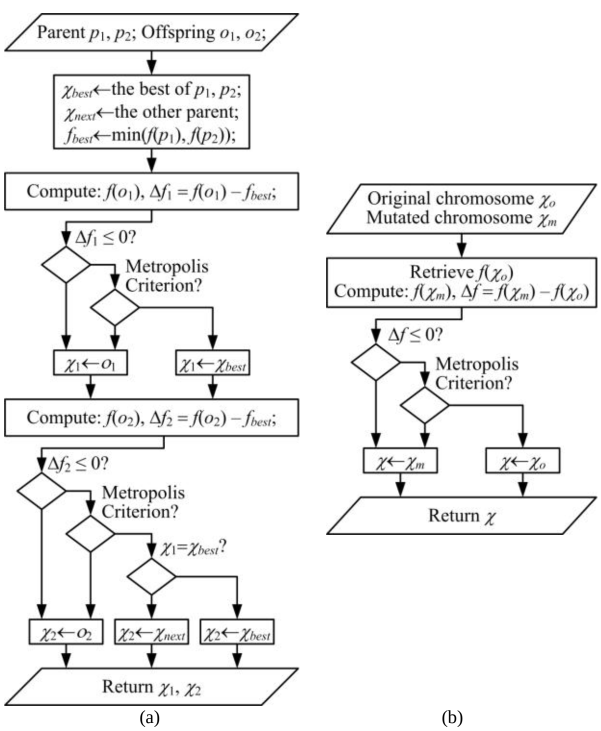

Figure 2. (a) Modified crossover algorithm XSA, (b) modified mutation algorithm MSA.

B. Integrating SA into GA

In the proposed approach, standard GA is employed as the skeleton of the heuristic optimization process, where SA comes into play as an extra step to screen the offspring generated by crossover or mutation before they are accepted as new chromosomes in the next generation. To this end, the Metropolis Criterion in SA is embedded into the crossover and mutation operations. The modified crossover algorithm, XSA, is shown in Fig. 2(a). After the two offspring chromosomes o_1 and o_2 are created by mating a pair of parent chromosomes p_1 and p_2, each individual offspring is compared with the best of the two parents based on the Metropolis Criterion. Suppose p_1 is better than p_2, i.e. $f(p_1) < f(p_2)$, or in terms of fitness ϕ, $\phi(p_1) > \phi(p_2)$. If the fitness of the offspring is higher than $\phi(p_1)$, the offspring is directly accepted by the new population. Otherwise, the offspring may still be accepted if the Metropolis Criterion is satisfied, or it is discarded and replaced by its parent.

On the other hand, SA is incorporated in a similar way to give the modified mutation algorithm MSA. As shown in Fig. 2(b), after mutation is performed, the mutated chromosome χ_m and the original chromosome χ_0 are compared. If χ_m fits better, i.e. $\phi(\chi_m) > \phi(\chi_0)$, it will be accepted and put into the new population of the next generation. Conversely, if χ_0 has better fitness, there is still a chance of accepting χ_m with a probability of $\exp(-\Delta f/T)$, otherwise χ_m will be rejected, reverting to the original chromosome χ_0. The overall algorithm that incorporates SA as an additional screening step in GA is shown in Fig. 3.

IV. Hybrid Heruistic Optimization

The hybrid parameter identification algorithm presented above has been applied to obtain the MSM parameters for simulating the deformation of orthotropic and transversely isotropic materials. The parameters required to model these two types of materials using FEM and MSM are given in Table I. For MSM, d denotes the nodal damping and k's are

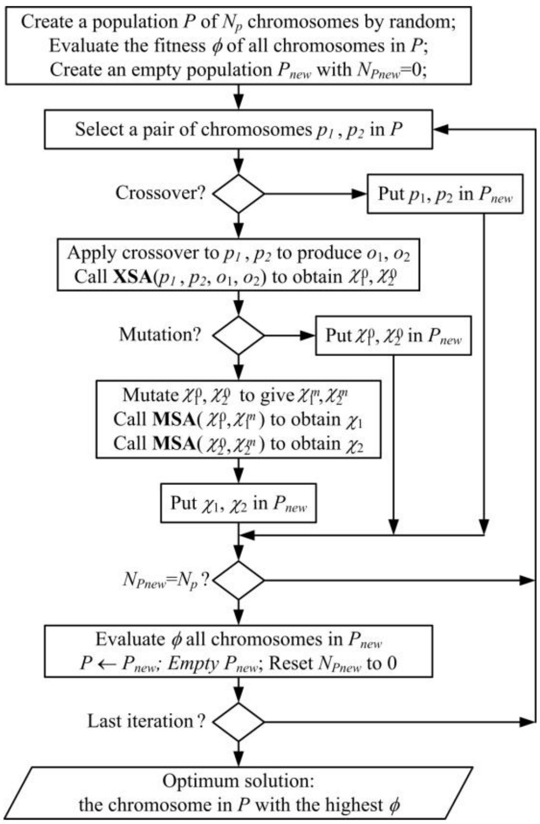

Figure 3. The hybrid algorithm merging GA with SA.

the spring constants along the principal axes. The MSM parameters are unknown and to be identified by the hybrid algorithm.

TABLE I. MECHANCAL PARAMETERS

Material	FEM	MSM
Orthotropic	$E_1, E_2, E_3, \nu_{12}, \nu_{13}, \nu_{23}, G_{12}, G_{13}, G_{23}$	k_1, k_2, k_3, d
Trans. isotropic	$E_1, E_2, \nu_{12}, \nu, G_{12}$.	k_1, k_2, d

A. *Crossover and Mutation*

In the algorithm, the MSM parameters d and k's are regarded as the genes. The quadruple (k_1, k_2, k_3, d) and ordered triple (k_1, k_2, d) are respectively the general representation of a chromosome for orthotropic and transversely isotropic materials. A two-point crossover operation is implemented for mating a chromosome pair, where the starting and end points of the crossover are selected at random. A gene in the offspring is created by blending the two corresponding genes from each parent by linear combination. Furthermore, the two parent genes are blended in random proportion to give the new gene. For example, suppose two-point crossover is performed on the pair of chromosomes (k_{11}, k_{21}, k_{31}, d_1) and (k_{12}, k_{22}, k_{32}, d_2), and the randomly picked crossover points are the third and fourth genes, the two offspring will be (k_{11}, k_{21}, k_{3+}, d_+) and (k_{12}, k_{22}, k_{3-}, d_-), with $k_{3+}=\alpha_1 k_{31}+(1-\alpha_1)k_{32}$, $d_+=\alpha_2 d_1+(1-\alpha_2)d_2$, $k_{3-}=\alpha_3 k_{31}+(1-\alpha_3)k_{32}$ and $d_-=\alpha_4 d_1+(1-\alpha_4)d_2$. The blending parameters α_1, α_2, α_3 and α_4 in the above expressions are random numbers between 0 and 1, so that the genes in the offspring carry some information from both parents. On the other hand, mutation is implemented by randomly picking one of the genes from a given chromosome and replacing it with a new value generated at random.

A slow cooling schedule [15] is adopted in the algorithm. The annealing temperature is reduced by one temperature step whenever the population evolves into the next generation. That is, at each generation, SA is applied to crossover and mutation at the same temperature.

B. *Cost Function*

The cost function $f(\chi)$ of the algorithm is the sum of the Euclidean distance between the nodes obtained with the MSM and FEM. Mathematically, the cost function is expressed as

$$f(\chi)=\sum_{\mathbf{F}=\mathbf{F}_1}^{\mathbf{F}_m}\sum_{i=1}^{N}\left|\mathbf{x}_i^{MSM}(\chi,\mathbf{F})-\mathbf{x}_i^{FEM}(\mathbf{F})\right|, \quad (1)$$

where $\mathbf{x}_i^{MSM}$ and $\mathbf{x}_i^{FEM}$ are the position vectors of node i in the MSM and FEM respectively; N is the number of nodes; and $\mathbf{F}_1$, $\mathbf{F}_2$,..., $\mathbf{F}_m$ refer to m sets of nodal forces applied to the object. Each set of nodal forces corresponds to a specific configuration of the experiment, defining the magnitude and direction of the forces exerted on the nodes as well as the boundary conditions applied. The fitness function $\phi(\chi)$ in GA is defined as the reciprocal of the cost function $f(\chi)$. $\mathbf{x}_i^{FEM}$ is obtained by using static FEM with the known elastic constants of materials when equilibrium is reached. $f(\chi)$ is minimized by tuning the MSM parameters encapsulated in chromosome χ using the hybrid algorithm.

C. *Parameter Identification*

The proposed algorithm has been tested with cubic deformable objects made of orthotropic and transversely isotropic materials, denoted respectively by A and B. The density of the materials is set to be 1040 kgm^{-3}. Both deformable objects have a size of 0.1^3 m^3, and are modeled with 512 nodes. Besides, 1344 springs and 343 hexahedrons are used respectively in the MSM and FEM. The elastic constants required by the FEM are given in Table II.

The benchmarking references are obtained with the FEM by applying forces respectively along the three principal axes in such a way that the forces are exerted on the nodes on one face while the opposite face is kept fixed throughout the deformation simulation. Fig. 4 shows the reference deformation computed for the orthotropic object A. In addition, constant forces with magnitudes 1, 5 and 10 N are also applied respectively to the objects along each principal axis. Hence, for each object, nine sets of reference data (i.e. $m = 9$ in (1)) are obtained by the FEM and used as the benchmark, each containing the position of nodes after deformable simulation is performed with the force configurations and boundary conditions described above. On

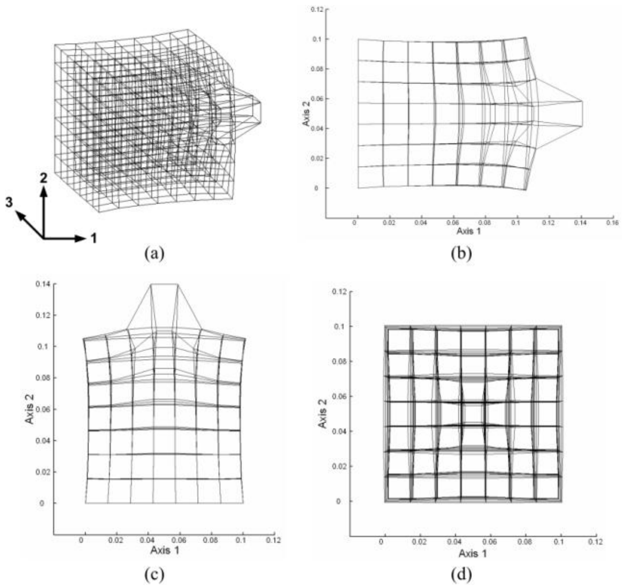

Figure 4. Reference FEM deformation of object *A*: (a) perspective view; orthographic views with forces applied along (b) Axis 1, (c) Axis 2 and (d) Axis 3.

the other hand, the MSM parameters identified heuristically by the hybrid algorithm are used to simulate the deformation under the same force configurations and boundary conditions. By benchmarking against each set of reference data, the cost function and fitness are evaluated by using (1). A population of 50 chromosomes is allowed to evolve for 100 generations in the genetic algorithms. The MSM parameters identified are shown in Table III.

The optimization errors, i.e. the discrepancy between the optimized MSM and the FEM benchmark, are measured by calculating the Euclidean distance of the nodes in MSM and FEM, normalized to the rest length of spring in the MSM. The average and maximum nodal position errors along the three principal axes under the specified force configurations are given in Table IV and Table V. The minimum nodal position error is zero since the positions of some nodes in both models are identical. The average error is in the range of 0.54% to 6.78% for object A and 0.71% to 10.2% for object B, which will not produce noticeable visual difference. Nevertheless, it is found that in the specified range of external forces, discrepancy increases with the magnitude of applied forces. This is because the underlying principles of MSM and FEM are different. The large deformation resulting from strong external forces exposes more conspicuously the inherent difference between them, making it more difficult for the MSM to match the FEM reference. Besides, the error is smallest along the axis where the Young's modulus is largest, i.e. Axis 3 of object A and Axis 1 of object B (see Table IV and Table V). Since Young's modulus is a measure of stiffness, the extent of deformation along the principal axis with higher stiffness is smaller under the same external forces and MSM is able to approximate FEM more closely along those axes, thus resulting in small optimization error.

The convergence of the proposed hybrid method has been studied by comparing with the pure GA and pure SA approach. A population of 50 chromosomes is allowed to evolve for 100 generations in both the hybrid and the pure GA approach. The settings of the genetic algorithm in these two approaches are identical. The pure SA approach is implemented by allowing annealing to proceed for 50 iterations at each temperature for 100 temperature steps. At each temperature step, the average and standard deviation of fitness are calculated with the fitness values of the 50 iterations. Similarly, the settings of the annealing parameters in the hybrid and the pure SA approach are the same. The results are shown in Fig. 5 and Fig. 6, which demonstrate the advantage of the hybrid approach (GA+SA) that asymptotic convergence can be attained much sooner even when a high mutation probability of 20% is applied. The fluctuation of the average and standard deviation of fitness in the hybrid approach is also significantly less than that of the pure GA or pure SA approach, which is attributed to the use of the Metropolis Criterion as an extra screening step. This finding is in agreement with research work on heuristic algorithms that incorporate SA into GA [14].

TABLE II. MECHANCAL PARAMETERS

Object *A*								
E_1	E_2	E_3	ν_{12}	ν_{13}	ν_{23}	G_{12}	G_{13}	G_{23}
179	188	228	0.26	0.30	0.31	57.1	65.8	71.1
Object *B*								
E_1	E_2		ν_{12}		ν_{23}	G_{12}		
170	115		0.46		0.58	33		

TABLE III. OPTIMIZATION RESULTS

MSM parameters	Object *A*	Object *B*
k_1	968.2	735.5
k_2	1010.0	679.9
k_3	1137.7	--
d	795.3	537.2

TABLE IV. OPTIMIZATION ERROR (%) OF OBJECT A

	Axis 1		***Axis 2***		***Axis 3***	
Force	***Avg.***	***Max.***	***Avg.***	***Max.***	***Avg.***	***Max.***
1 N	0.67	2.08	0.64	1.97	0.54	1.62
5 N	3.39	10.3	3.22	9.85	2.71	8.10
10 N	6.78	20.6	6.44	19.6	5.14	16.1

TABLE V. OPTIMIZATION ERROR (%) OF OBJECT B

	Axis 1		***Axis 2/3***	
Force	***Average***	***Max.***	***Average***	***Max.***
1 N	0.71	2.14	1.03	2.87
5 N	3.55	10.6	5.15	14.2
10 N	7.07	21.9	10.2	28.4

V. CONCLUSION

This paper reports a study to develop a hybrid heuristic method for identifying MSM parameters by benchmarking

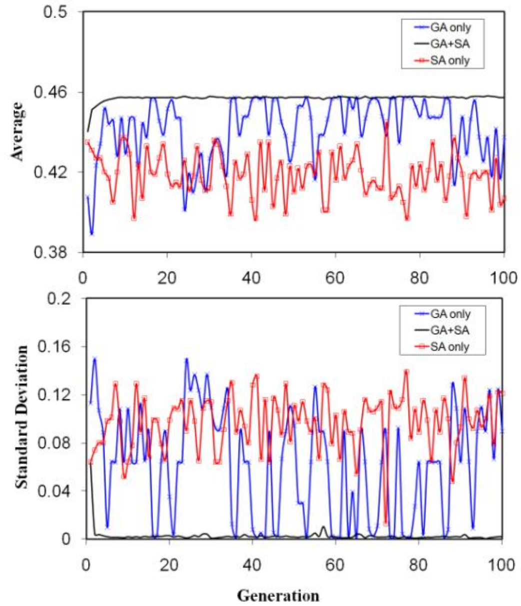

Figure 5. Evolution of the average and standard deviation of fitness for the orthotropic object A.

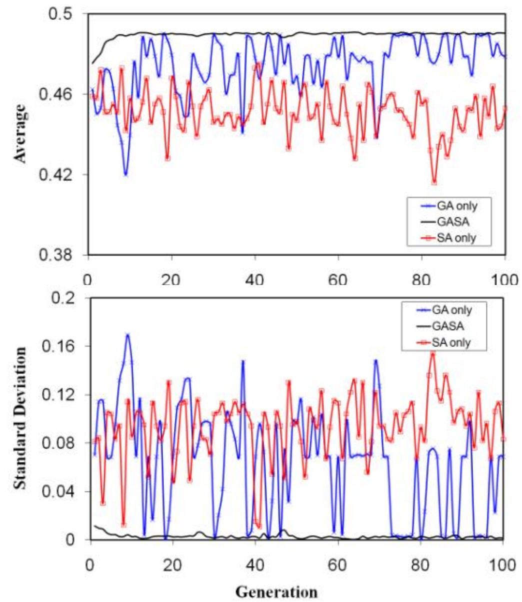

Figure 6. Evolution of the average and standard deviation of fitness for the transversely orthotropic object B.

the reference obtained by continuum mechanics and FEM. Instead of simple elastic materials, transversely isotropic and orthotropic properties are considered. With the FEM references, a heuristic optimization approach is developed by incorporating SA into GA to identify the MSM parameters automatically. Results demonstrate that this method exhibits better convergence over pure GA approach. Using the parameters identified by the hybrid approach to set the mass-spring models, the virtual tissues and organs are able to deform autonomously in a way similar to that achieved by the mathematically more accurate FEM but at the speed of the computationally efficient MSM. The discrepancy is visually not noticeable when virtual tissues are subjected to moderate forces, which is adequate for medical education to embrace virtual reality for procedural training or for applications where rigorous numerical accuracy is not necessary, e.g. medical palpation training and virtual surgery simulation involving small deformations. The presented method provides a feasible approach to automate the setting of parameters in mass-spring models for simulating deformable objects in the virtual environment.

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REFERENCES

[1] H. Delingette, "Toward realistic soft-tissue modeling in medical simulation," Proc. IEEE, vol. 86, pp. 512-523, 1988.

[2] G. Bianchi, B. Solenthaler, G. Székely, and M. Harders, "Simultaneous topology and stiffness identification for mass-spring models based on FEM reference deformations," Proc. Medical Image Computing and Computer-Assisted Intervention, 2004, pp. 293-301.

[3] O. Deussen, L. Kobbelt, and P. Tucke, "Using simulated annealing to obtain good nodal approximations of deformable bodies," Proc. Eurographics Workshop on Computer Animation and Simulation 1995, pp. 30-43.

[4] J. Mosegaard, "Parameter optimization for the behaviour of elastic models over time," Proc. Medicine Meets Virtual Reality, 2004, pp. 253-255.

[5] G. Bianchi, M. Harders, and G. Székely, "Mesh topology identification for mass-spring models," Proc. Medical Image Computing and Computer-Assisted Intervention, 2003, pp. 50-58.

[6] A. Van Gelder, "Approximate simulation of elastic membranes by triangulated spring meshes," J. Graphics Tools, vol. 3, pp. 21-42, 1998.

[7] A. Lloyd, G. Szekely, and M. Harders, "Identification of spring parameters for deformable object simulation," IEEE Trans. Visualization and Computer Graphics, vol. 13, pp. 1081-1094, 2007.

[8] W. Maurel, Y. Wu, N. Thalmann, and D. Thalmann, Biomechanical Models for Soft Tissue Simulation. Berlin: Springer-Verlag, 1988.

[9] O. C. Zienkiewicz and R. L. Taylor, The Finite Element Method, 6th ed. Oxford ; New York: Elsevier/Butterworth-Heinemann, 2005.

[10] E. H. L. Aarts and J. Korst, Simulated annealing and Boltzmann machines: a stochastic approach to combinatorial optimization and neural computing. Chichester, England ; New York: Wiley, 1989.

[11] D. E. Goldberg, Genetic algorithms in search, optimization, and machine learning. Reading, Mass.: Addison-Wesley Pub. Co., 1989.

[12] D. E. Goldberg, "A note on Boltzmann tournament selection for genetic algorithms and population-oriented simulated annealing," Complex Systems, vol. 4, pp. 445-460, 1990.

[13] D. J. Sirag and P. T. Weisser, "Toward a unified thermodynamic genetic operator," Proc. 2nd Int'l Conf. on Genetic Algorithms, 1987, pp. 116-122.

[14] D. Adler, "Genetic algorithms and simulated annealing: A marriage proposal," IEEE Int'l Conf. Neural Networks, 1993, pp. 1104 -1109.

[15] M. Lundy and A. Mees, "Convergence of an annealing algorithm," Mathematical Programming, vol. 34, pp. 111-124, 1986.