Simultaneous Multi-frequency Topological Edge Modes between One-dimensional Photonic Crystals

KA HEI CHOI,[1] C. W. LING,[1] K. F. LEE, [1] Y. H. TSANG,[1] KIN HUNG FUNG[1],*

[1]Department of Applied Physics, The Hong Kong Polytechnic University, Hong Kong, China
*Corresponding author: khfung@polyu.edu.hk

We show theoretically that, in the limit of weak dispersion, one-dimensional binary centrosymmetric photonic crystals can support topological edge modes in all photonic band gaps. By analyzing their bulk band topology, these "harmonic" topological edge modes can be designed in a way that they exist at all photonic band gaps opened at the center of the Brillouin Zone, or at all gaps opened at the zone boundaries, or both. The results may suggest a new approach to achieve robust multi-frequency coupled modes for applications in nonlinear photonics, such as frequency up-conversion.

OCIS codes: (230.5298) Photonic Crystals; (350.1370) Berry's phase; (190.2620) Harmonic generation and mixing;

The study of topological properties in condensed matter physics emerges from the discovery of quantum hall effect, where topological protected states exist at the edge of a crystal and are robust against small perturbations [1–6]. Recent studies have discovered that these topological properties exist also in photonic systems [7–13] and acoustic systems [14]. Rigorous mathematical treatments on making analogies between electronic and photonic systems have also been proposed by Lein and De Nittis [15–17].

As a geometric phase initially obtained from electron Bloch state, Zak phase is considered to be a Z_2 topological invariant (π or 0) for classifying bands of one-dimensional (1-D) electronic systems with inversion symmetry [18,19]. By making analogies, Zak phase can also be obtained in a centrosymmetric 1-D photonic crystal (PhC) [20]. Zak phase describes the bulk band topology of the photonic band structure and determines the existence of edge modes. Recently, it has been shown that Zak phase is also related to the reflection phase and surface impedance [20].

Photonic crystals can be designed to confine light in small volumes for enhancement of light-matter interaction [21–25]. For example, by introducing a defect layer into a perfect 1-D PhC, it can support a resonant state for confining light within the defect region [26–30]. This provides an opportunity to design devices for nonlinear photonic applications [31]. Instead of introducing a defect layer, recent advances in topological photonics offers a new perspective to create resonant states (edge modes). Topological approach guarantees the creation of edge modes for a range of dielectric properties and structure of 1-D PCs, which has the benefit of producing robust topological edge modes for enhancing light-matter interaction.

To enhance frequency conversion in nonlinear optics, it is important to enhance the light localization at both excitation and emission frequencies. Since topological edge modes can be supported in PhCs at more than one frequency, it could be useful for supporting enhancement at both excitation and emission frequencies. Based on the topological principles stated in Ref. [20], we further propose in this Letter that two combined 1-D PhCs can be specially designed to have bulk band topologies that guarantee the existence of "harmonic" topological edge modes under the limit of weak dispersion. We also show that it is able to choose whether these modes should exist at all band gaps opened at the center of the first Brillouin Zone , or at all band gaps opened at the zone boundaries, or both. These topologically protected edge modes at multiple frequencies could be robust against weak nonlinearity and localized at the same area, allowing additional enhancement of nonlinear effects such as frequency conversion.

We first consider a fundamental 1-D centrosymmetric PhC (PhC X in Fig. 1). It is created by stacking alternating layers of A and B with the normal of their planes parallel to the z-axis, which is also the direction of electromagnetic wave propagation. For a centrosymmetric unit cell, a single layer of A is sandwiched between half layer of B. The period (Λ) of an unit cell is given by $\Lambda = d_A + d_B$, where d_A and d_B are the thickness of a single dielectric layer of A and B. In this paper, the optical path length of each unit cell is specified by Eq. (1) for the purpose of creating a topological non-trivial 1-D PhC [20],

$$\alpha\Lambda = n_A d_A + n_B d_B, \qquad (1)$$

where n_A and n_B are the refractive indexes of the non-dispersive dielectric layer A and B, α is the ratio of optical path length of a unit cell to its period. For simplicity, we first assume that $n_B = 1.5$ and every dielectric medium has negligible dispersion effect and magnetic response. Layer B is considered as the "reference" medium because its dielectric property will always remain unchanged in all of our simulations.

To understand how we produce topological edge modes periodically, we provide a brief review on the topological properties in a 1-D PhC. For

each photonic pass band, its topological phase is characterized by the quantized Zak phase of π or 0, which is given by [20],

$$\theta_m^{\text{Zak}} = \int_{-\frac{\pi}{\Lambda}}^{\frac{\pi}{\Lambda}} \left[i \int_{\text{unit cell}} \varepsilon(z) u_{m,K}^*(z) \partial_K u_{m,K}(z) dz \right] dK,$$

(2)

where $\varepsilon(z)$ is the function of permittivity in space, and $u_{m,K}(z)$ represents the Bloch eigenfunction of electric field at the mth photonic pass band with a Bloch wavevector K.

Since the reflection phase of a 1-D PhC is a physical observable for Zak phase, the existence of a topological edge mode can be predicted from finding the total sum of Zak phase below the mth band gap ($\sum_i^m \theta_i^{\text{Zak}}$) [20,32]. Suppose there are two 1-D PhCs, X and Y. When the sum $\sum_i^m \theta_i^{\text{Zak}}$ of X is different from that of Y by $(2l + 1)\pi$, where l is an integer, a boundary formed by combining X and Y will support topological edge mode(s) within this band gap [20]. The above X and Y can be found by changing the dielectric compositions of a PhC continuously but keeping the optical path length of the unit cells unchanged [see Eq. (1)]. For example, by changing the refractive index n_A of X continuously, some band gaps may eventually close and then re-opened. It is a topological phase transition since the Zak phases of the pass bands just below and above this gap changes by π simultaneously. This important topological phenomenon will change $\sum_i^m \theta_i^{\text{Zak}}$ by π [20]. This means that if the n_A of X is tuned just before topological phase transition and the n_C of Y is just after the transition, the combined structure (Fig. 1) will support a topological edge mode within this band crossing gap due to their π difference in $\sum_i^m \theta_i^{\text{Zak}}$.

The principle above gives us a hint to create multiple numbers of topological edge modes simultaneously. It tells us that if there is a 1-D PhC with multiple band gaps crossing simultaneously, we can design a PhC interface which supports topological edge modes in all these crossing gaps. These PhCs with multiple band crossing can be found using the band crossing condition, $(n_A d_A)/(n_B d_B) = s_1/s_2$, where s_1 and s_2 are integers, and band crossing occurs at the $(s_1 + s_2)^{\text{th}}$ band gap [20].

The band crossing condition suggests that various combinations of PhCs of different parameters will have different types of "harmonic" topological edge modes. First, let us consider the case of $s_1 = s_2$ (i.e., $n_A d_A = n_B d_B$). As $s_1 = s_2$, $s_1 + s_2$ is always an even number, which suggests that when $n_A d_A = n_B d_B$, the number of band crossings is maximized and the crossings will all occur at every band gap that opens at zone center. To avoid confusion, this critical refractive index of n_A at $s_1 = s_2$ is denoted as n_α. From the condition $n_\alpha d_\alpha = n_B d_B$ and Eq. (1), it can be shown that $2/\alpha = (1/n_\alpha) + (1/n_B)$. As an example, we set $n_B = 1.5$ and $\alpha = 1.9$ for our 1-D PhCs, which gives $n_\alpha = 2.59$. Using the arguments above, topological edge modes can be generated at every zone-center gap by choosing some refractive indexes such that $n_A < n_\alpha$ and $n_C > n_\alpha$. In this case, the sum $\sum_i^m \theta_i^{\text{Zak}}$ for every zone-center gap of X is different from Y by π.
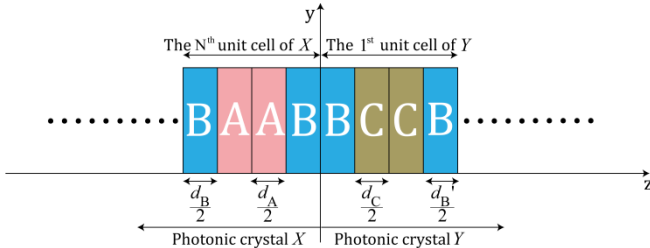

Fig. 1. Proposed 1-D photonic crystal model constructed by connecting two 1-D photonic crystals X and Y (X+Y), where X and Y contain N unit cells. Thicknesses are given by Eq. 1. In each unit cell, a single layer of A (or C) is sandwiched between half layers of B.

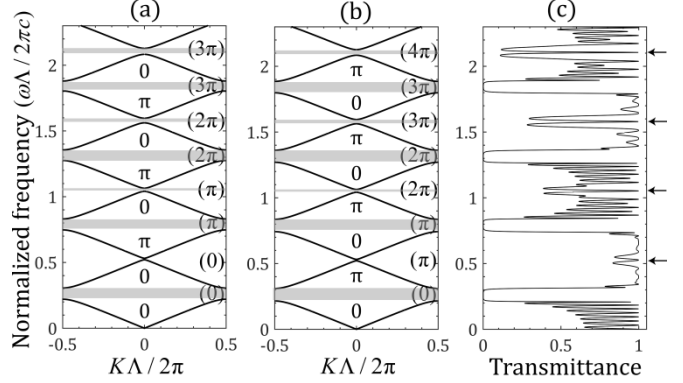

Fig. 2. (a) Photonic band structure of X, where $n_A = 2.51, n_B = 1.5$ and $\alpha = 1.9$. The Zak phase of each pass band is printed at the center of the pass band. Grey regions denote band gaps, with numbers in parentheses are the sum of the Zak phases below the band gap ($\sum_i^m \theta_i^{\text{Zak}}$). (b) Photonic band structure of Y, where $n_C = 2.67, n_B = 1.5$ and $\alpha = 1.9$. (c) Transmission spectrum of X+Y photonic crystals, where the number of units $N_X = 4$ and $N_Y = 4$. Black arrows indicate there are topological edge states.

Figs. 2(a) and 2(b) shows the photonic band structure of X and Y, which is plotted from the dispersion given by [33],

$$\cos(K\Lambda) = \cos(k_A d_A)\cos(k_B d_B) + \left(\frac{Z_A}{Z_B} + \frac{Z_B}{Z_A}\right)\sin(k_A d_A)\sin(k_B d_B),$$

(3)

where K is the Bloch wavevector, $Z_i = (\mu_i/\varepsilon_i)^{\frac{1}{2}}$ is the impedance of the i-th medium. In the figures, we used relative permittivity instead of refractive index to describe the dielectric property of our 1-D PhCs. The critical refractive index n_α is represented in relative permittivity by $\varepsilon_\alpha = n_\alpha^2 = 6.71$ while the designed refractive indices are given by $\varepsilon_A = n_A^2 = 6.31$ and $\varepsilon_C = n_C^2 = 7.11$. Each photonic pass band is labelled by the Zak phase of its own, which is obtained from finding a special set of frequencies ω_Z by $\sin(\omega_Z n_B d_B/c) = 0$ [20], where c is the speed of light. This special set of frequencies determines the Zak phases of the pass bands. When ω_Z is within a pass band, the Zak phase of this pass band is π, otherwise, it is 0 [20].

From the simulation results, the topological edge modes can be observed in the transmission spectrum and verified by the bulk band topology of X and Y. In Figs. 2(a) and 2(b), the Zak phase of each pass band is labelled at the center of their own band. The grey region represents photonic band gaps, while the numbers in parentheses on the right hand side are the $\sum_i^m \theta_i^{\text{Zak}}$ of the band gap. Due to the combination of these unique bulk band topologies (Figs. 2(a) and 2(b)), topological edge modes will all exist in zone-center band gaps and they are shown in the transmission spectrum of X+Y PhCs calculated using the Transfer Matrix Method [33] (See Fig. 2(c)). Theoretically, transmission should be low within the band gaps due to the absence of Bloch state. However, there are resonant transmission peaks (indicated by tiny black arrows) in some band gaps, which are due to the resonances associated with the edge modes. On the other hand, for photonic band gaps that open at the zone boundaries, $\sum_i^m \theta_i^{\text{Zak}}$ of these

band gaps are all identical in Figs. 2(a) and 2(b), and thus there will be no resonance transmission peak exists within these band gaps, which is consistent with our previous discussions.

Other than adjusting dielectric properties of our 1-D PhCs, the unit cell structure also influences the corresponding Zak phase. We can manipulate the bulk band topology of PhCs by shifting the origins of their unit cells. A 1-D centrosymmetric PhC with inversion symmetry always possesses of two inversion centers [18,20]. If the origin of a unit cell is shifted from one inversion center to the another one, the Zak phase of each photonic pass band will change from 0 to π (or from π to 0) [20]. Based on this approach, we can demonstrate that topological edge states can be created not only periodically at every zone-center band gap, but also periodically at every zone-boundary band gap.

Suppose there are two PhCs, X and X′ [Fig. 3(b)], where both of them have the same dielectric compositions as X in Fig. 3(a) but different unit cell origins. Due to the shift of origin, every Zak phase in X will be different from X′ by π. As a result, the bulk band topology of X′ will be similar to Y [Fig. 2(b)], with an exception for the 0th pass band because the Zak phase of the 0th pass band is determined only by $\exp\left(i\theta_0^{\text{Zak}}\right) = \text{sgn}\left[1 - \frac{\varepsilon_A \mu_B}{\varepsilon_B \mu_A}\right]$ [20]. This additional Zak phase difference of π between X and X′ in the 0th pass band turns out to be significantly influential. We can conclude that when X and Y has a difference of π in $\sum_i^m \theta_i^{\text{Zak}}$, X and X′ will have a difference of 2π in $\sum_i^m \theta_i^{\text{Zak}}$; on the other hand, if X and Y have identical $\sum_i^m \theta_i^{\text{Zak}}$, X and X′ will have a difference of π in $\sum_i^m \theta_i^{\text{Zak}}$. This implies that shifting the origin of the unit cell without changing its dielectric properties can produce periodical topological edge modes at every zone-boundary band gap. Whenever there is a topological edge mode in the zone-center band gap of X+Y, the mode will be absent from the zone-center band gap of X+X′. On the opposite side, as topological edge modes are absence from the zone-boundary band gaps of X+Y, these modes will appear in the zone-boundary gap of X+X′. This phenomenon is shown in the transmission spectrum of X+X′ in Fig. 3(e). The resonant transmission peaks resulted from topological edge modes are found to exist periodically at each zone-boundary gap instead of the zone-center gap, which are indicated by black arrows.

Another significant consequence due to the Zak phase change of π at the 0th pass band is the creation of periodical topological edge mode at both zone center gaps and zone boundary gaps. This can be done by considering another set of PhCs, X and Y′ [Fig. 3(c)], where Y′ have the same dielectric property of Y but a different. Now, consider the Zak phase of Y′ after both modifications: With the dielectric property of Y, the Zak phase of each pass band will be different from X by π except the 0th pass band. Then, modifying Y to Y′ will change the Zak phase of each pass band by π again, but this time including the 0th pass band. As a result, except the 0th pass band, the Zak phase of each pass band in Y′ has changed by π twice, i.e. 2π. However, at the 0th pass band, the Zak phase only changes by π once because the modification of dielectric properties does not influence the Zak phase at the 0th pass band. Accordingly, for X and Y′, the differences of $\sum_i^m \theta_i^{\text{Zak}}$ of every band gap are 3π, excepted that the first band gap has the differences of π only. As we have described before, this implied that topological edge modes will exist at every zone-center and zone-boundary band gaps. We can find these topological edge modes in the transmission spectrum of X+Y′ in Fig. 3(f), where resonant transmission peaks are observed in both zone-center and zone-boundary band gaps.
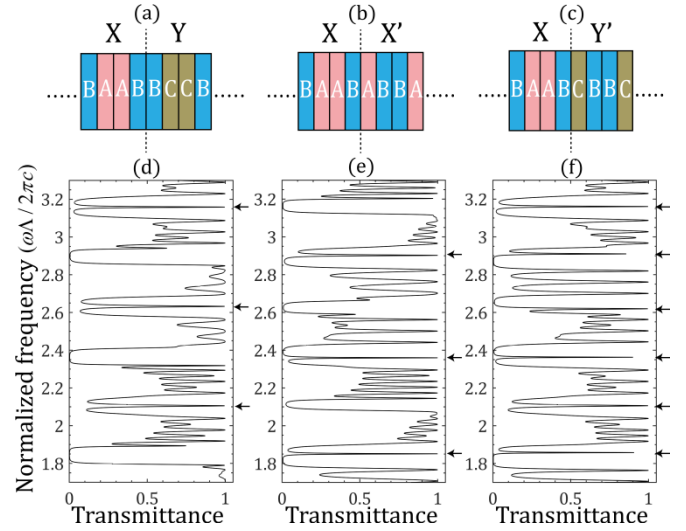

Fig. 3. (a) The photonic crystal structure of X + Y, (b) X + X′, and (c) X + Y′. (d) The transmission spectrum of X + Y, (e) X + X′, and (f) X + Y′. Each photonic crystals above are composed by stacking 4 unit cells. Parameters for each type of layers (A, B, or C) are identical to that of Fig. 2. The topological edge modes of each band gap are also indicated by black arrows.

From our previous discussions, it seems that creating periodical topological edge modes in X+Y requires the freedom of dielectric properties. However, what it requires are only two materials with close refractive indexes ($n_A - n_C$ is small compared with n_α) and the design of thicknesses. Suppose that our 1-D PhCs in Fig. 1 has no flexibility of choosing their dielectric properties, such that n_B, n_A, and n_C are fixed. To design X+Y with periodical topological edge modes, we know that the thickness of our PhCs must be given by Eq. (1). As $n_A - n_C$ is small, we can assume that the critical index n_α is the average of n_A and n_C. With n_α and n_B, we can obtain α by the relation $2/\alpha = (1/n_\alpha) + (1/n_B)$, and hence we know the thickness d_A, d_C, d_B, and d_B' for creating periodical topological edge modes at zone-center gaps. Therefore, as long as $n_A - n_C$ is small, periodical topological edge modes at zone-center gap can always be produced by thickness adjustment.

Nevertheless, it should also be noted that the value of $n_A - n_C$ determines an upper frequency limits of periodical topological edge modes. These modes are not guaranteed at high frequencies because small change of dielectric property can induce rapid change of the bulk band topology at high frequencies. Still, the upper frequency limit is pretty high for application purposes, which is at $\omega\Lambda/2\pi c \sim 5$ for our case and can be extended by reducing the value of $n_A - n_C$.

The simultaneous topological edge modes proposed in this Letter may be used to enhance nonlinear light–matter interactions such as frequency up-conversion. Some nonlinear optical applications utilizing localized defect modes in 1D PhC have been proposed [21–25]. While previous works usually consider enhancement at single-frequency such as excitation frequency, it should be noted that enhancement at emission frequency also plays an important role. To demonstrate how the multi-frequency topological edge modes can be used to enhance nonlinear optical processes, we show the normalized intensity pattern inside the PhC of X+Y′ in Fig. 4. First, we note that, for every edge mode, there are strongly localized fields near the interface between X and Y′. The enhancement at the edge mode frequencies in Fig. 4 are about 20 to 30 while the enhancement region has a two-dimensional (2D) surface area of beam size of the incident light in plane wave approximation. The

enhancement can increase exponentially with increasing number of units. This result suggests that the interface region is suitable for inserting single or few layers of thin or 2D materials for enhancing frequency up-conversion optical processes.
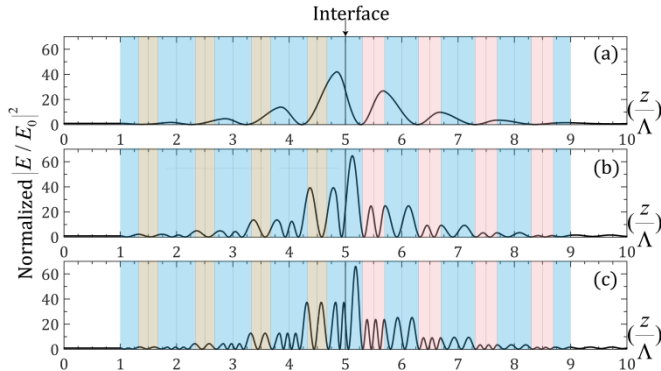

Fig. 4. Intensity profile for plane-wave excited topological edge mode in X + Y' photonic crystals, where the $|E|$ is normalized by the amplitude of the incident wave, $|E_0|$. A plane wave enters from the right hand side of the photonic crystal. The colored regions represent the dielectric composition of X and Y', while the uncolored regions are an air medium. It shows the topological edge modes at (a) the 1st, (b) the 2nd, and (c) the 3rd zone-boundary band gap.

In conclusion, we showed how 1-D binary PhCs can be designed to create topological edge modes in any photonic band gap by manipulating their bulk band topology. We have designed a PhC interface supporting periodical topological edge modes that exist at all zone-center gaps, or at all zone-boundary gaps, or both. The edge modes are topologically robust and could be maintained in weak perturbations such as weak nonlinearities. The result may suggest a new approach to achieve robust multi-frequency coupled modes for applications in nonlinear photonics, such as frequency up-conversion, through simultaneous enhancement at both excitation and emission frequencies.

References

1. M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
2. D. Xiao, M.-C. Chang, and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2012).
3. J. E. Avron, D. Osadchy, and R. Seiler, Phys. Today **56**, 38 (2003).
4. X.-L. Qi, Science **338**, 1550–1551 (2012).
5. S. Ryu and Y. Hatsugai, Phys. Rev. Lett. **89**, 077002 (2002).
6. M. Kohmoto, Ann. Phys. (N. Y). **160**, 343 (1985).
7. W. Chen, S. Jiang, X. Chen, B. Zhu, L. Zhou, J. Dong, and C. T. Chan, Nat. Commun. **5**, 5782 (2014).
8. F. D. M. Haldane and S. Raghu, Phys. Rev. Lett. **100**, 013904 (2008).
9. L. Lu, J. D. Joannopoulos, and M. Soljacicć, Nat. Photonics **8**, 821 (2014).
10. S. Raghu and F. D. M. Haldane, Phys. Rev. A **78**, 033834 (2008).
11. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljacicćć, Phys. Rev. Lett. **100**, 013905 (2008).
12. Z. Wang, Y. Chong, J. D. Joannopoulos, and M. Soljacicć, Nature **461**, 772 (2009).
13. J. Mei, Y. Wu, C. T. Chan, and Z. Zhang, Phys. Rev. B **86**, 035141 (2012).
14. M. Xiao, G. Ma, Z. Yang, P. Sheng, Z. Q. Zhang, and C. T. Chan, Nat. Phys. **11**, 240 (2015).
15. G. De Nittis and M. Lein, arXiv:1502.07235v1 [math-ph] (2015).
16. G. De Nittis and M. Lein, Commun. Math. Phys. **332**, 221 (2014).
17. G. De Nittis and M. Lein, Ann. Phys. (N. Y). **350**, 568 (2014).
18. J. Zak, Phys. Rev. Lett. **62**, 2747 (1989).
19. M. Atala, M. Aidelsburger, J. T. Barreiro, D. Abanin, T. Kitagawa, E. Demler, and I. Bloch, Nat. Phys. **9**, 795 (2013).
20. M. Xiao, Z. Q. Zhang, and C. T. Chan, Phys. Rev. X **4**, 021017 (2014).
21. T. Hattori, N. Tsurumachi, and H. Nakatsuka, J. Opt. Soc. Am. B **14**, 348 (1997).
22. G. J. Schneider and G. H. Watson, Appl. Phys. Lett. **83**, 5350 (2003).
23. N. Tsurumachi, S. Yamashita, N. Muroi, T. Fuji, T. Hattori, and H. Nakatsuka, Jpn. J. Appl. Phys. **38**, 6302 (1999).
24. N. Tsurumachi, M. Abe, M. Arakawa, T. Yoda, T. Hattori, J. Qi, Y. Masumoto, and H. Nakatsuka, Jpn. J. Appl. Phys. **38**, L1400 (1999).
25. J. Y. Ye, M. Ishikawa, Y. Yamane, N. Tsurumachi, and H. Nakatsuka, Appl. Phys. Lett. **75**, 3605 (1999).
26. J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals: Molding the Flow of Light* (Princeton University Press, 2011).
27. K. Sakoda, *Optical Properties of Photonic Crystals* (Springer Science & Business Media, 2004).
28. T. C. King and C. J. Wu, Phys. E Low-dimensional Syst. Nanostructures **69**, 39 (2015).
29. H. Jiang, H. Chen, H. Li, Y. Zhang, and S. Zhu, Appl. Phys. Lett. **83**, 5386 (2003).
30. I. L. Lyubchanskii, N. N. Dadoenkova, A. E. Zabolotin, Y. P. Lee, and T. Rasing, J. Opt. A Pure Appl. Opt. **11**, 114014 (2009).
31. M. Soljačić and J. D. Joannopoulos, Nat. Mater. **3**, 211 (2004).
32. W. S. Gao, M. Xiao, C. T. Chan, and W. Y. Tam, Opt. Lett. **40**, 5259 (2015).
33. A. Yariv and P. Yeh, *Optical Waves in Crystals: Propagation and Control of Laser Radiation* (John Wiley and Sons, Inc., New York, NY, 1983).

Full References

1. M. Z. Hasan and C. L. Kane, "*Colloquium* : Topological insulators," Rev. Mod. Phys. **82**, 3045–3067 (2010).
2. D. Xiao, M.-C. Chang, and Q. Niu, "Berry Phase Effects on Electronic Properties," Rev. Mod. Phys. **82**, 1959–2007 (2012).
3. J. E. Avron, D. Osadchy, and R. Seiler, "A topological look at the quantum Hall effect," Phys. Today **56**, 38 (2003).
4. X.-L. Qi, "Symmetry Meets Topology," Science (80-.). **338**, 1550–1551 (2012).
5. S. Ryu and Y. Hatsugai, "Topological Origin of Zero-Energy Edge States in Particle-Hole Symmetric Systems," Phys. Rev. Lett. **89**, 077002 (2002).
6. M. Kohmoto, "Topological invariant and the quantization of the Hall conductance," Ann. Phys. (N. Y). **160**, 343–354 (1985).
7. W. Chen, S. Jiang, X. Chen, B. Zhu, L. Zhou, J. Dong, and C. T. Chan, "Experimental realization of photonic topological insulator in a uniaxial metacrystal waveguide," Nat. Commun. **5**, 5782 (2014).
8. F. D. M. Haldane and S. Raghu, "Possible realization of directional optical waveguides in photonic crystals with broken time-reversal symmetry," Phys. Rev. Lett. **100**, 013904 (2008).
9. L. Lu, J. D. Joannopoulos, and M. Soljacicć, "Topological photonics," Nat. Photonics **8**, 821–829 (2014).
10. S. Raghu and F. D. M. Haldane, "Analogs of quantum-Hall-effect edge states in photonic crystals," Phys. Rev. A **78**, 033834 (2008).
11. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljacicćć, "Reflection-Free One-Way Edge Modes in a Gyromagnetic Photonic Crystal," Phys. Rev. Lett. **100**, 013905 (2008).
12. Z. Wang, Y. Chong, J. D. Joannopoulos, and M. Soljacicć, "Observation of unidirectional backscattering-immune topological electromagnetic states," Nature **461**, 772–775 (2009).

13. J. Mei, Y. Wu, C. T. Chan, and Z. Zhang, "First-principles study of Dirac and Dirac-like cones in phononic and photonic crystals," Phys. Rev. B **86**, 035141 (2012).

14. M. Xiao, G. Ma, Z. Yang, P. Sheng, Z. Q. Zhang, and C. T. Chan, "Geometric phase and band inversion in periodic acoustic systems," Nat. Phys. **11**, 240–244 (2015).

15. G. De Nittis and M. Lein, "Derivation of Ray Optics Equations in Photonic Crystals Via a Semiclassical Limit," arXiv:1502.07235v1 [math-ph] (2015).

16. G. De Nittis and M. Lein, "Effective Light Dynamics in Perturbed Photonic Crystals," Commun. Math. Phys. **332**, 221–260 (2014).

17. G. De Nittis and M. Lein, "On the role of symmetries in the theory of photonic crystals," Ann. Phys. (N. Y). **350**, 568–587 (2014).

18. J. Zak, "Berrys phase for energy bands in solids," Phys. Rev. Lett. **62**, 2747–2750 (1989).

19. M. Atala, M. Aidelsburger, J. T. Barreiro, D. Abanin, T. Kitagawa, E. Demler, and I. Bloch, "Direct measurement of the Zak phase in topological Bloch bands," Nat. Phys. **9**, 795–800 (2013).

20. M. Xiao, Z. Q. Zhang, and C. T. Chan, "Surface Impedance and Bulk Band Geometric Phases in One-Dimensional Systems," Phys. Rev. X **4**, 021017 (2014).

21. T. Hattori, N. Tsurumachi, and H. Nakatsuka, "Analysis of optical nonlinearity by defect states in one-dimensional photonic crystals," J. Opt. Soc. Am. B **14**, 348–355 (1997).

22. G. J. Schneider and G. H. Watson, "Nonlinear optical spectroscopy in one-dimensional photonic crystals," Appl. Phys. Lett. **83**, 5350–5352 (2003).

23. N. Tsurumachi, S. Yamashita, N. Muroi, T. Fuji, T. Hattori, and H. Nakatsuka, "Enhancement of Nonlinear Optical Effect in One-Dimensional Photonic Crystal Structures," Jpn. J. Appl. Phys. **38**, 6302–6308 (1999).

24. N. Tsurumachi, M. Abe, M. Arakawa, T. Yoda, T. Hattori, J. Qi, Y. Masumoto, and H. Nakatsuka, "Time Response of One-Dimensional Photonic Crystals with a Defect Layer Made of Semiconductor Quantum Dots," Jpn. J. Appl. Phys. **38**, L1400–L1402 (1999).

25. J. Y. Ye, M. Ishikawa, Y. Yamane, N. Tsurumachi, and H. Nakatsuka, "Enhancement of two-photon excited fluorescence using one-dimensional photonic crystals," Appl. Phys. Lett. **75**, 3605–3607 (1999).

26. J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals: Molding the Flow of Light* (Princeton University Press, 2011).

27. K. Sakoda, *Optical Properties of Photonic Crystals* (Springer Science & Business Media, 2004).

28. T.-C. King and C.-J. Wu, "Properties of defect modes in one-dimensional symmetric defective photonic crystals," Phys. E Low-dimensional Syst. Nanostructures **69**, 39–46 (2015).

29. H. Jiang, H. Chen, H. Li, Y. Zhang, and S. Zhu, "Omnidirectional gap and defect mode of one-dimensional photonic crystals containing negative-index materials," Appl. Phys. Lett. **83**, 5386–5388 (2003).

30. I. L. Lyubchanskii, N. N. Dadoenkova, A. E. Zabolotin, Y. P. Lee, and T. Rasing, "A one-dimensional photonic crystal with a superconducting defect layer," J. Opt. A Pure Appl. Opt. **11**, 114014 (2009).

31. M. Soljacicić and J. D. Joannopoulos, "Enhancement of nonlinear effects using photonic crystals," Nat. Mater. **3**, 211–219 (2004).

32. W. S. Gao, M. Xiao, C. T. Chan, and W. Y. Tam, "Determination of Zak phase by reflection phase in 1D photonic crystals," Opt. Lett. **40**, 5259–5262 (2015).

33. A. Yariv and P. Yeh, *Optical Waves in Crystals: Propagation and Control of Laser Radiation* (John Wiley and Sons, Inc.,New York, NY, 1983).