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Quantitative Analysis of the Measurement Uncertainty in Form Characterization of Freeform Surfaces based on Monte Carlo Simulation

Mingjun Ren^{a, b*}, ChiFai Cheung^b, Lingbao Kong^b, Sujuan Wang^a

^aGuangdong Provincial Key Laboratory of Micro-nano Manufacturing Technology and Equipment, School of Electromechanical Engineering, Guangdong University of Technology, Guangzhou, China

^bPartner State Key Laboratory of Ultra-precision Machining Technology, Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Kowloon, Hong

* Corresponding author. Tel.: +852-27666574; fax: +852-27647657. E-mail address: mj.ren@polyu.edu.hk

Abstract

Freeform surfaces possessing no symmetry in rotation are widely used in many fields such as space optics for their superior optical properties. Due to their geometric complexities, the growth of application of these surfaces in precision industries is still hindered by a lack of definitive methodologies for traceable measurement of manufactured freeform surfaces. This paper presents a method for quantitative analysis of the measurement uncertainty in the form characterization of freeform surfaces. The study starts from developing a method for the form characterization of freeform surfaces, and the associated uncertainties are evaluated based on Monte Carlo simulation by quantitatively analyzing the uncertainty induced by the sampling strategy and the evaluation method. To integrate the effect of the workpiece form deviation, a profile simulation method is developed based on fractional Brownian motion, which can be used to generate random surface form error with given magnitude. Based on computer simulation, mathematical relationships between the magnitude of the critical errors and the resulting uncertainties are identified so that an estimation of uncertainty can be given for the measured surface parameters in a specific measurement. A case study is conducted to demonstrate the effectiveness of the proposed study which provides a better understanding of the associated uncertainty in the form characterization of freeform surfaces.

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Keywords: Surface measurement; freeform surface; form characterization, uncertainty analysis; Monte Carlo method.

1. Introduction

With the requirement for ever higher performance of the parts, extensive research has been conducted on their surfaces in order to produce high-value products. An emerging consequence is the use of freeform surfaces for their superior optical and mechanical properties. Freeform surfaces are considered as complex surfaces which possess no symmetry in rotation and translation [1, 2]. These surfaces are commonly found in many fields ranging from the space optics to biomedical and geographical data processing applications for enhancing the performance of the components in functionality and size reduction [3]. However, the growth of the application of these surfaces in precision industries is still hindered by a

lack of definitive methodologies for the traceable measurement of manufactured freeform surfaces.

The state-of-the-art measurement instruments are able to extract the data from a freeform surface with sub-micrometer accuracy, such as ultra-precision coordinate measuring machines [3]. The evaluation of the uncertainty associated in the measured surface parameters is however difficult due to many uncertainty contributors involved in the freeform surface measurement including the hardware of the measuring instruments, the form deviation of the workpiece, the adopted sampling strategy, and the fitting and evaluation algorithm, etc [4]. Uncertainty analysis is indispensable for conformance test of workpiece with specifications [5]. This becomes increasingly crucial as more and more companies strive to maintain traceability to expand the interchangeability in the

global market. The use of calibrated artefact [6, 7] and the Monte Carlo simulation [8, 9] are two commonly used approaches in the evaluation of the measurement uncertainty. As described in 15530-3 [10], the use of a calibrated artefact allows for very reliable uncertainty evaluations. However, this method lies in limited application of one type of freeform artefact for a specific freeform surface with similar shape. Monte Carlo simulation, as presented in the supplement of ISO GUM [11], is widely used in the uncertainty evaluation of coordinate measurement. It allows integrating many task specific uncertainty influences for complex measurement processes.

Much research has been conducted on the calibration and uncertainty analysis of the coordinate measuring instruments as well as the post data processing techniques [3]. However, research on uncertainty analysis in post data processing mostly focuses on simple geometry [12], such as circles, spheres, and cylinders. Different from conventional simple surfaces, the form characterization of freeform surfaces is more complex since they cannot be represented by a universal equation. Although much research work has been undertaken on the form characterization of freeform surfaces [3, 13, 14], most of the studies failed to clearly state the uncertainty associated in the measurement results. Some researchers have represented freeform surfaces using several simple geometries, such as spheres and cylinders, and hence the problem is transformed to the analysis of each piece of the used simple geometry [6]. Recently, a freeform artefact was developed by National Physical Laboratory (NPL) for the verification of non-contact measuring systems [15]. However, the form accuracy of the artefact is still at micrometer level and the artefact is mainly used for the verification of the performance of optical coordinate measuring instruments. The uncertainty analysis model presented in ISO standard document [16] is also difficult to be applied to the measurement of freeform surfaces since the form characterization of freeform surfaces is highly non-linear complex process and the uncertainty propagation changes with different surface geometry being measured.

As a result, this paper aims to analyze and estimate the measurement uncertainty in the form characterization of freeform surfaces with the guidance of the latest ISO Guide to the Expression of Uncertainty in Measurement [11]. The analysis will integrate the influence of sampling strategy and the evaluation method by taking the error of the measuring instruments and the workpiece form error as the error sources. Based on computer simulation, mathematical relationships between the critical uncertainty contributors and resulting uncertainties are identified so that estimation of uncertainty can be made for a specific measurement. The effectiveness of the proposed study is also experimental verified on a case study.

2. Form characterization of freeform surfaces

The form characterization of a machined freeform surfaces is performed by determining the deviation of the surface from corresponding theoretically designed surface. Hence the first step of the form characterization is the use of the precision coordinate measuring instruments to capture the geometry of

the machined surface by measuring a set of points on it. The number and the distribution of the measured points are determined by adopted sampling strategy. In the second step, the measured surface is orientated to the designed surface based on some surface matching methods, e.g. the least square based best fitting method [13], and the form error of the measured surface is evaluated by calculating the deviation of the measured surface from the design surface. At the end, required surface parameters, such as peak-to-valley height and root-mean-square, are calculated based on the evaluated form deviation. Fig. 1 shows the operation of the form characterization process of the freeform surfaces.

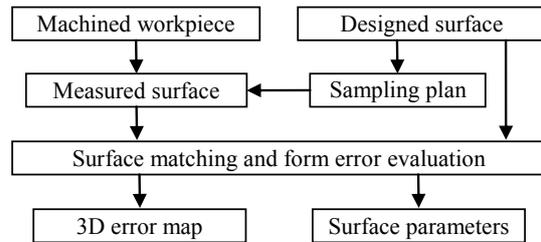


Fig. 1: Schematic diagram of the form characterization of freeform surfaces

One of the key steps in the form characterization process is the surface matching between the measured surface and the designed surface. The surface matching is to search for an optimal position and attitude for the measured surface relative to the design surface so that they align with each other as close as possible. The process can be expressed by Eq. (1) as follows [14]:

$$F = \min \sum_{i=1}^N |P_i - T(\mathbf{m})Q_i|^2 \quad (1)$$

where Q_i is the measured point; P_i is the corresponding point on the design surface, $T(\mathbf{m})$ is a coordinate transformation matrix used to align the measured points on the design surface; \mathbf{m} is a spatial vector containing three translational offsets and three rotational angles.

A residual vector $\mathbf{R} \in \mathfrak{R}^{3N \times 1}$ is defined as

$$R_k = \begin{cases} px_i - qx_i & k = 3i - 2 \\ py_i - qy_i & k = 3i - 1 \\ pz_i - qz_i & k = 3i \end{cases} \quad i = 1, 2, \dots, N \quad (2)$$

where $[px_i, py_i, pz_i]$ is components of P_i ; $[qx_i, qy_i, qz_i]$ is components of $T(\mathbf{m})Q_i$. Then the following preserve for the local minimum,

$$\frac{\partial F}{\partial \mathbf{m}} = 2 \left(\frac{\partial \mathbf{R}}{\partial \mathbf{m}} \right)^T \mathbf{R} = 0 \quad (3)$$

Eq. (3) is expanded with the Taylor series,

$$\frac{\partial F}{\partial \mathbf{m}} = 2 \left(\frac{\partial \mathbf{R}}{\partial \mathbf{m}} \right)^T \mathbf{R} + 2 \left[\left(\frac{\partial \mathbf{R}}{\partial \mathbf{m}} \right)^T \frac{\partial \mathbf{R}}{\partial \mathbf{m}} + \mathbf{R}^T \frac{\partial^2 \mathbf{R}}{\partial^2 \mathbf{m}} \right] (\mathbf{m}^* - \mathbf{m}) + \left[O(\mathbf{m}^* - \mathbf{m})^2 \right] = 0 \quad (4)$$

By ignoring the higher order terms in Eq. (4), the Newton method (Piegl and Tiller, 1997) can be used iteratively updates the solution by

$$\delta \mathbf{m} = -(\mathbf{J}^T \mathbf{J} + \mathbf{S})^{-1} \mathbf{J}^T \mathbf{R} \quad (5)$$

where $\mathbf{J} = \frac{\partial \mathbf{R}}{\partial \mathbf{m}}$ is $3N \times 6$ Jacobian matrix; \mathbf{S} is 6×6 matrix

$$\text{with } S_{i,j} = \mathbf{R}^T \frac{\partial^2 \mathbf{R}}{\partial m_i \partial m_j}.$$

In real measurement, both \mathbf{P}_i and $\mathbf{T}(\mathbf{m})$ are unknown. Hence, Eq. (1) is iteratively solved in two steps. In the first step, the measured points \mathbf{Q}_i is projected on the design surface to determine the \mathbf{P}_i , and the established correspondence pairs are used to determine the $\mathbf{T}(\mathbf{m})$. The correspondence pairs can be further refined by moving the measured points \mathbf{Q}_i in a new position. Hence a new the correspondence is established by projecting each moved measured point on the design surface and the projection is considered as the new correspondence pair of that point. The newly established correspondence is then used to estimate the new coordinate transformation matrix. This process is continued upon reaching the desired accuracy.

3. Quantitative analysis of uncertainty based on Monte Carlo simulation

The profile of a workpiece for a given design surface S can be simulated as follows:

$$\mathbf{RS}_i = \mathbf{S}_i + \bar{\mathbf{n}}_i E f m_i \quad (6)$$

where \mathbf{RS}_i is a point on the workpiece; \mathbf{S}_i is the corresponding point of \mathbf{RS}_i on designed surface; $\bar{\mathbf{n}}_i$ is the normal vector of the S at point \mathbf{S}_i ; $E f m_i$ is the form error of the workpiece at point \mathbf{RS}_i . Hence the measured coordinate at point \mathbf{RS}_i can be represented as follows:

$$\mathbf{Q}_i = \mathbf{T}(\mathbf{m}_0) \mathbf{RS}_i + \mathbf{Ems}_i = \mathbf{T}(\mathbf{m}_0) (\mathbf{S}_i + \bar{\mathbf{n}}_i E f m_i) + \mathbf{Ems}_i \quad (7)$$

where \mathbf{Ems}_i is the instrument error; $\mathbf{T}(\mathbf{m}_0)$ is a coordinate transformation matrix which used to indicate the misalignment of the coordinate frames between the measuring instrument and the designed surface. The Eq. (6) and Eq. (7) are incorporated into Eq. (1), and form error $E f m_i$ can be determined by minimizing (8) based on the Eq. (2)-Eq. (5) as given in Section 2.

$$F(\mathbf{m}) = \min \sum_{i=1}^N \left| \mathbf{P}_i - \mathbf{T}(\mathbf{m}) (\mathbf{T}(\mathbf{m}_0) (\mathbf{S}_i + \bar{\mathbf{n}}_i E f m_i) + \mathbf{Ems}_i) \right|^2 \quad (8)$$

Eq. (8) clearly demonstrates that, except the instrument error, there are several factors will affect the form characterization results in terms of surface matching variance. Due to the existence of the form deviation and the instrument error, the measured surface will never perfectly match the designed surface which would cause coordinate misalignment between them. The surface matching error can be caused by many factors including the number and the distribution of the measured points, i.e. the sampling strategy, the instrument errors, and the form error of the measured surface itself. It is demonstrated in some previous research [13] that the error of the surface matching will vary with change of the topology of the form error. In this study, the form error of the measured surface is considered as a kind of unknown systematic error, and the effect of which will be integrated into the uncertainty analysis.

The magnitude of the form error of a freeform surface is commonly described by the offset of the two envelope surfaces which include the measured surface, as shown in Fig. 2. For a given magnitude, form errors with the same magnitude but with different topology can be superimposed onto the design surface. When the number of the random surfaces is sufficiently large, as shown in Fig. 2, the set of the random form errors is able to simulate any topology of the measured surface included in the two envelope surfaces. Hence the set of the random form errors can be used to quantitatively analyze the uncertainty which is contributed by the form error with given magnitude. In this study, the random form error is simulated by fractal surface which is generated by fractional Brownian motion (FBM) [13].

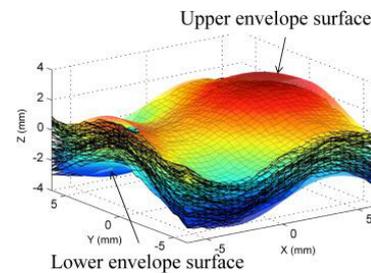


Fig. 2: Simulated measured surfaces superimposed by a set of random form errors with the same magnitude

Monte Carlo simulation can be used to estimate the uncertainty of the measured surface parameters by taking the error of the measuring instruments and the form error of the workpiece as input (JCGM, 2008). Eq. (8) is used as the model function in the simulation. In each Monte Carlo trial, a realistic machined surface is generated based on Eq. (6). Then, a set of discrete points is extracted from the machined surface based on utilized sampling strategy, and the measured points is simulated by superimposing the instrument errors on the extracted points based on Eq. (7). The measured points are used to determine the form error of the machined surface based on the developed form characterization method. The

number of the Monte Carlo trials is determined iteratively upon reaching desired accuracy. Fig. 3 shows the operations for each Monte Carlo trial. The results can be used to determine a confidence interval of the surface parameters with preferred confidence level for a given magnitude of workpiece form deviation.

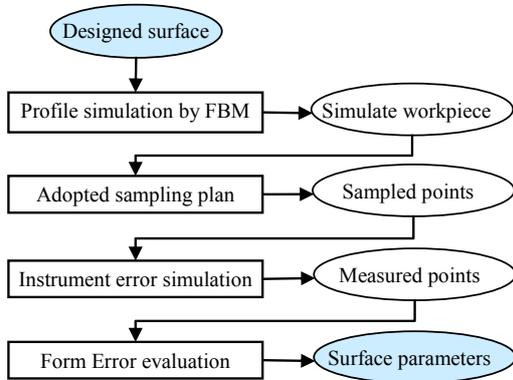


Fig. 3: Operations for each Monte Carlo trial

4. Case study

The effectiveness of the proposed method is verified on a ultra-precision freeform mould insert of a bifocal optical lens. The produced workpiece is measured by a coordinate measuring machine (Zeiss Prismo navigator). Uniform sampling strategy is adopted with spacing 2 mm in both X and Y directions. Fig. 4a and Fig. 4b shows the designed surface and measured surface. The form of the measured surface is characterized by the method presented in Section 2. Fig. 5 shows the evaluated form deviation. The peak-to-valley height is found to be 2.02µm.

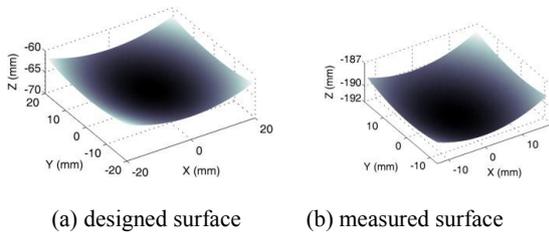


Fig. 4. Designed and measured bifocal optical lens

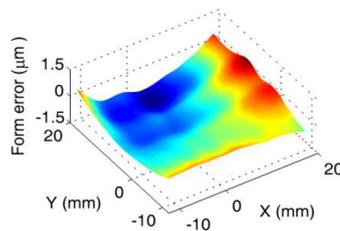


Fig. 5. Evaluated form error of the measured surface

The uncertainty of the measured peak-to-valley height is analyzed based on the method presented in Section 3. A total

of 12 cases studies are conducted based on the established uncertainty analysis model with difference sampling density and difference magnitudes of workpiece deviation, as shown in Table 1. A total of 1500 Monte Carlo trails are used for uncertainty analysis. For this, fractional Brownian motion is used to generated 1500 random form errors for each case study with given magnitude. The error of the measuring instrument is given as follows: X axis: $u=0.5 \mu\text{m}$ (uniform); Y axis: $u=0.5 \mu\text{m}$ (uniform); Z axis: $u=0.6 \mu\text{m}$ (1σ , normal).

Table 1: Cases studies with different sampling density and magnitude of form deviation

SD\MFD	Magnitude of form deviation(MFD)(µm)			
Sampling density (SD) (mm)	2 \ 1	2 \ 1.5	2 \ 2	2 \ 2.5

Fig. 6 shows the uncertainty of the peak-to-valley (PV) height with respect to different magnitude of the form error of the measured surface. It can be seen from the results that the uncertainty of the PV increases along with the increase of the magnitude of the form error of the measured surface. The results are as might be expected intuition, and can be used establish a relationship between the magnitude of the uncertainty contributors and the resulting uncertainty of the evaluated surface parameters by spline interpolation. According to this relationship, for the measured surface as presented in this case study, the uncertainty of the evaluated peak-to-valley height is estimated to be 0.49 µm.

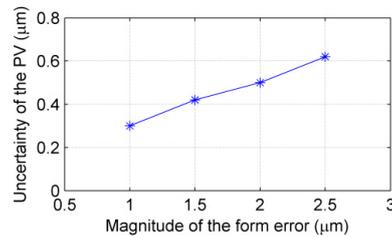


Fig. 6: the uncertainty of the peak-to-valley height with respect to different magnitude of the form error

5. Discussion

Due to the variety of the geometry of the freeform surfaces, the proposed method should be carried out task specific analysis of the uncertainty for given freeform surface with given sampling strategy. Based on computer simulation, a mathematical relationship between the workpiece form deviation and the resulting uncertainty can be established as presented in Fig. 6. This relationship can be used to evaluate the uncertainty in actual measurement based on the PV of the measured surface. The proposed method can be incorporated into the software of the measuring instruments, such as coordinate measuring machines and profilometry, for helping the operators to select appropriate sampling strategies with better understanding of their uncertainties in the form characterization of freeform surfaces.

6. Conclusion

With the explosive growth of application of freeform surfaces, traceable measurement of freeform surfaces becomes a cutting edge problem in the field of surface metrology. This paper presents a study on the uncertainty estimation for the form characterization of freeform surfaces. A form characterization model is established for freeform surfaces and the associated uncertainties were analyzed based on Monte Carlo simulation by integrating the influence of the error of the measuring instruments, the selected sampling strategy, and the form deviation of the workpiece. The simulation results can be used to give an estimation of the uncertainty for a specific measurement. The proposed study will provide an important means for reliable quality control of the form characterization of freeform surfaces and shed some light on the contribution to the future standardization of freeform surface measurement.

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