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Research Article

An EPQ Model for Deteriorating Production System and Items with Rework

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This paper investigates the economic production quantity model jointly considering product deterioration and a deteriorating production system with rework. In this imperfect deteriorating production system, not only does the machine produce defective product but also the machine is subjected to quality deterioration. To be more specific, the defective rate increases at certain time intervals. The defects produced are stored until the end of normal production process. Then they are reworked with extra cost to restore their quality and regarded as perfect product. The main objective is to minimize the total cost per unit product by determining the optimal combination of production run time and backlog quantity. Numerical experiments are carried out to illustrate the behavior of the inventory and show the impact of different parameters on the model. Discussion and conclusions are made at the end of the paper.

1. Introduction

Inventory control and management are crucial to the modern companies in order to survive in the competitive market. Too much inventory causes high inventory cost and low turnover rate, while having extremely low inventory level puts the company at the risk of losing incoming orders and customers [1]. For manufacturing companies, how many products to produce to meet the demand and keep the cost at low level is always a challenge. Economical production quantity (EPQ) is one of the classical problems in inventory control. The aim of this study is to identify the optimal stocking levels and production quantity to minimize the total cost generated. Nowadays the consideration of different demand type, product deterioration, production system reliability, and other uncertainties makes these classical problems even more complicated. In terms of product deterioration, perishable goods such as food, fruit, and drink and other products like electronic devices and metal processing are the good examples and their quality is heavily influenced by the storing condition and length of storage time after production. If the quality does not meet the standard or requirement anymore, either extra cost is needed to recover the quality or the product will be disposed of [2]. The reliability of production system, on the other hand, addresses the quality of production process such as defective rate, machine breakdown, and production speed. Defects can be generated from processing mistakes, setup mistakes, adjustment mistakes, and tools mistakes [3]. Although management techniques such as quality control and the improvement of manufacturing technology have successfully reduced the defect rate to a relatively low level, it is still a problem for companies, especially when the complexity of production is high. Mobile phone industry can be used as a good example for having extra cost caused by defects [4]. Similar to defective rate, the decrease of production and machine breakdown are also the obstacles to achieving a high productivity in a manufacturing system [5]. Maintenance is an effective method that prevents the serious machine breakdown and restores the production quality. But for deterioration within each production cycle, maintenance cannot solve the problem.

In academic world, the EPQ problem has attracted a large amount of researchers in the past 80 years. Goyal and Giri [6] have provided an in-depth overview of the literatures focusing on the deteriorating products. They have clearly defined the cause of product deterioration and categorized the different models used based on demand type, pricing policy, and so forth; interested reader can refer to it for more information. In 1993 Wee [7] first proposed an EPQ model with partial backlog and product deterioration and later he and Law [8] integrated the problem with the consideration of pricing policy in a finite planning horizon. Goyal and Gunasekaran [9] also combined the pricing problem with EPQ problem on deteriorating product. In addition, advertisement frequency is also considered in their model. The model with time-dependent deteriorating rate was developed by Manna and Chaudhuri [10] as an extension of previous works. In their research, not only is the deteriorating rate time-dependent but also the production rate is proportional to demand rate. All the previously mentioned research work assumed a continuous issuing policy to meet the demand of customers. Discontinuous issuing or delivery policy was taken into consideration by Chiu et al. [11]. Discontinuous issuing policy, as stated in their work, was more practical and common in the industry. In their paper, the delivery of products is divided into n installments with a fixed time interval and quantity. Numerical methods were utilized to obtain the optimal solution. Later, they further looked at the discontinuous issuing EPQ problem with partial rework [12]. However in these two papers, the number of shipments is assumed to be known. Cardenas-Barron et al. [13] jointly considered the multidelivery EPQ problem with both lot size and number of deliveries to minimize the inventory cost. Backlog is also an important element in the traditional EPQ problem. Research works such as [14–18] have examined the EPQ problem with full/planned/partial backlog. Interested readers can refer to those papers for more details. Other similar works include Abad [19] and Sarkar [20].

For imperfect manufacturing system, Khouja and Mehrez [21] in 1994 stated that the previous research normally assumed constant production rate and perfect quality. In their work, an EPQ model with variable production rates and imperfect quality was developed where defective rate is the function of production speed. Kim and Hong [22] assumed that the machine changes from in-control to out-control state after a random period and defects are generated in out-control state. Chiu [15] further looked at the problem in which rework of defective products is carried out on the same machine and the impact of reworking on the EPQ model is investigated. Machine breakdown was also taken into consideration. The deterioration of production system was introduced by Lin and Kroll [23] and a time-dependent defective rate was utilized to model the deterioration of machines. In recent studies, random defective rate [24], stochastic breakdown [25], stochastic demand, and maintenance scheduling [26] were also combined with the traditional EPQ problem.

The extant literatures show that majority of the papers have only considered either rework and imperfect system or the product deterioration. The few integrated studies can be found in the work of Teunter and Flapper [27], Inderfurth et al. [28], and Tai [29]. In the first paper, the rework and production were assumed to use the same machine and the defective rate follows a given probability distribution. The production quantity was desired to be determined to maximize the profit obtained. While in the second one, the model proposed in [28] generalized the problem, assuming the demand for good items is limited and the demand will be entirely satisfied. And also closed form results were obtained by utilizing constant and deterministic defective and deteriorating rate, while Tai [29] added inspection errors into the model with deteriorating items and imperfect production. In addition, he also investigated the effect of selling imperfect product to customer. However in all of the three studies mentioned above, the defective rate is regarded as stationary and it would not change along with time. In addition, backlog is not considered in the first two models. Hence as the main contribution of this paper, we model a single machine, single product EPQ problem with the joint consideration of deterioration of product and production systems with rework. Defective rate is assumed to be time-dependent and backlog is allowed in the model. The effects of production deterioration on the system are investigated and analyzed.

The rest of the paper is arranged as follows. Section 2 introduces all the notations and assumptions used in this paper, while the detailed mathematical model is presented and elaborated in Section 3. Numerical experiments are included in Section 4 with sensitivity analysis followed by the conclusions in Section 5.

2. Notations and Assumptions

In this research, a production system with single machine and single product is modelled. In the system, the machine can conduct both production and rework process. The product is subjected to quality deterioration and the machine is assumed to be imperfect and deteriorating in terms of an increasing defective rate. The detailed assumptions made and the notations used in this paper are shown in the following subsections.

2.1. Notations

α_i: the defective rate of time period $i * \theta$;

θ: constant length between each change of defective rate (hour);

δ: deteriorating ratio;

μ: demand rate (unit/time);

p: production rate (unit/time);

p_r: rework rate (unit/time);

M: the total number of intervals θ in normal production period;

N: the total number of intervals θ in backlog period;

C_{hp}: holding cost of perfect products (\$/unit/time);

C_{hi}: holding cost of imperfect products (\$/unit/time);

C_{dc}: deteriorating cost (\$/unit);

C_p: production cost (\$/unit);

C_{pr}: rework cost (\$/unit);

C_b: penalty cost for backlog (\$/unit/time);

C_s: fixed setup cost (\$/cycle);

B: backlog quantity (unit);

Q: economic production quantity (unit);

I_s: the inventory level at the end of normal production period (unit);

I_m: the inventory level at the end of rework process (unit);

I_im: the inventory level of imperfect product (unit).

2.2. Assumptions

(1) Unsatisfied order will be backlogged and the backlog will be fulfilled at the beginning of the cycle.

(2) During the production period, the defective items are produced at a constant rate α_i in the time interval $[(i-1)\theta, i\theta)$, $i \in \mathbb{N}$. We assume that the production system is deteriorating in the sense that the defective rate increases at time $i \times \theta$, $i \in \mathbb{N}$. Hence we have $\alpha_1 < \alpha_2 < \alpha_3 < \cdots$. Through this paper, we consider a linear relation $\alpha_i = i \times \gamma$, where γ is a constant.

(3) To reduce the complexity of the cost function, the normal production run time T_1 and the length of backlog period T_2 are assumed to be the integer multiple of θ since the value of θ is small.

(4) Maintenance is carried out after the whole production period, so at the beginning of each cycle, the defective rate is minimized.

(5) The imperfect products are reworked after the normal production process with an extra rework cost and the rework is assumed to be perfect.

(6) The deterioration only occurs to perfect products with a constant rate δ.

(7) The deteriorated products are disposed of with cost.

(8) Demand rate μ is known and constant.

3. Mathematical Modelling

According to the assumptions and description of the production system, a mathematical model for the system has been formulated. The behaviour of the inventory level in one production cycle is shown in Figures 1 and 2 for perfect and imperfect products, respectively. As illustrated in Figure 1, the inventory level starts from backlog B and the backlogged orders are satisfied first. During the first time interval θ, the defective rate is maintained as α_1. And at the end of the time interval, the defective rate increases to α_2. After the backlog is made up, the production will be continued until the desired economical production quantity is achieved. All the imperfect products are reworked in T_3 and the whole production process is completed. In T_4 and T_5, the stocks are consumed by demand and backlog is generated during T_5. Similarly in Figure 2, the total amount of imperfect product piles up in periods T_1 and T_2 due to the defects produced. The gradient increases along with the rise of defective rate.

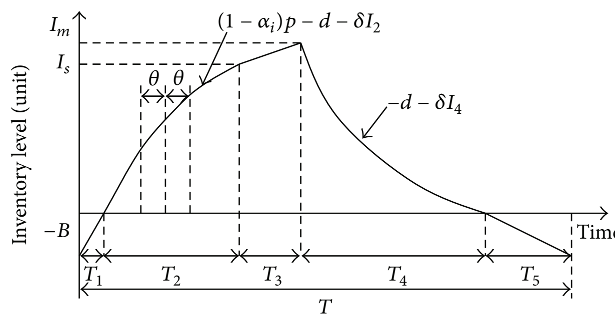


Figure 1: Inventory level of perfect items.

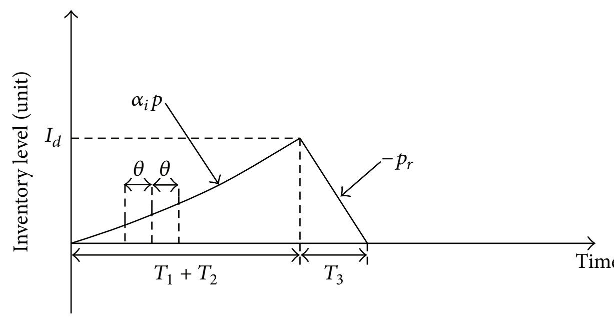


Figure 2: Inventory level of imperfect items.

So in general, the inventory level can be represented with the following equations.

For backlog period $0 \le t_1 \le T_1$,

$$I_1'(t_1) = (1-\alpha_i)\,p - \mu, \quad (i-1)\,\theta \le t_1 \le i\theta. \tag{1}$$

For $i = 1$,

$$I_1(t_1) = ((1-\alpha_1)\,p - \mu)\,t_1 - B, \quad 0 \le t_1 \le \theta. \tag{2}$$

Assume $\lambda_i = (1-\alpha_i)p - \mu$ for simplification and this will be used throughout the paper:

$$I_1(\theta) = \lambda_1\theta - B. \tag{3}$$

For $i = 2$, $I_2(0) = I_1(\theta) = \lambda_1\theta - B$,

$$I_1(t_1) = \lambda_2 t_1 + \lambda_1\theta - B, \quad \theta \le t_1 \le 2\theta. \tag{4}$$

Similarly, the general inventory function in backlog period can be calculated as follows:

$$I_1(t_1) = \begin{cases} \lambda_1 t_1 - B & 0 \le t_1 \le \theta \\ \lambda_2(t_1 - \theta) + \lambda_1\theta - B & \theta \le t_1 \le 2\theta \\ \vdots & \\ \lambda_N(t_1 - (N-1)\,\theta) + \sum_{i=1}^{N-1}\lambda_i\theta - B & (N-1)\,\theta \le t_1 \le T_1. \end{cases} \tag{5}$$

For surplus stage, for the first period θ, the inventory function is

$$I_2'(t_2) = \lambda_N - \delta I_2(t_2), \quad 0 \le t_2 \le \theta,$$
$$I_2(t_2) = \frac{\lambda_N}{\delta}(1 - \exp(-\delta t_2)) \quad 0 \le t_2 \le \theta. \tag{6}$$

So for general surplus inventory function can be obtained:

$$I_2(t_2) = \begin{cases} \frac{\lambda_N}{\delta}(1 - \exp(-\delta t_2)) & 0 \le t_2 \le \theta \\ \frac{\lambda_N - \lambda_{N+1}}{\delta}\exp(-\delta t_2) - \frac{\lambda_N}{\delta}\exp(-\delta t_2) + \frac{\lambda_N + 1}{\delta} & \theta \le t_2 \le 2\theta \\ \vdots \\ \sum_{j=1}^{M-N}\frac{\lambda_{N+j-1} - \lambda_{N+j}}{\delta} \cdot \exp(-\delta((i-j)\theta) + t_2) - \frac{\lambda_N}{\delta}\exp(-\delta((M-N-1)\theta + t_2)) + \frac{\lambda_M}{\delta} & (M-1)\theta \le t_2 \le T_2. \end{cases} \tag{7}$$

The total production quantity can be calculated as

$$pM\theta = Q. \tag{8}$$

Hence the inventory level at the end of normal production $I_s = I_2(T_2)$ equals

$$I_s = \sum_{j=1}^{M-N}\frac{\lambda_{N+j-1} - \lambda_{N+j}}{\delta} \cdot \exp(-\delta((M-N-j+1)\theta)) - \frac{\lambda_N}{\delta}\exp(-\delta((M-N)\theta)) + \frac{\lambda_M}{\delta}. \tag{9}$$

After the normal production process, the slope of inventory level can be represented with

$$I_3'(t_3) = (p_r - \mu) - \delta I_3(t_3), \quad 0 \le t_3 \le T_3, \tag{10}$$

$$I_4'(t_4) = -\mu - \delta I_4(t_4), \quad 0 \le t_4 \le T_4, \tag{11}$$

$$I_5'(t_5) = -\mu, \quad 0 \le t_5 \le T_5. \tag{12}$$

According to the boundary conditions $I_5(T_5) = -B$, $I_2(T_2) = I_3(0) = I_s$, $I_3(T_3) = I_4(0) = I_m$, and $I_4(T_4) = I_5(0) = 0$, the inventory level can be obtained:

$$I_3(t_3) = \left(I_s - \frac{p_r - \mu}{\delta}\right)\exp(-\delta t_3) + \frac{p_r - \mu}{\delta}, \quad 0 \le t_3 \le T_3,$$
$$I_4(t_4) = \left(I_m + \frac{\mu}{\delta}\right)\exp(-\delta t_4) - \frac{\mu}{\delta}, \quad 0 \le t_4 \le T_4,$$
$$I_5(t_5) = -\mu t_5, \quad 0 \le t_5 \le T_5. \tag{13}$$

The maximum inventory level I_m equals

$$I_m = \left(I_s - \frac{p_r - \mu}{\delta}\right)\exp(-\delta T_3) + \frac{p_r - \mu}{\delta}, \tag{14}$$

and from (11) and $I_4(T_4) = 0$

$$I_m = \frac{\mu}{\delta}(\exp(\delta T_4) - 1). \tag{15}$$

We can also find

$$\sum_{i=1}^{N}\lambda_i\theta = dT_5 = B. \tag{16}$$

For imperfect product, the inventory function in normal production time is

$$I_{im}(t_{im}) = \sum_{1}^{i-1}\alpha_i\theta + \alpha_i t, \quad (i-1)\theta \le t_{im} \le i\theta,$$

$$I_{im}(t_{im}) = \begin{cases} \alpha_1 t_{im} & 0 \le t_{im} \le \theta \\ \alpha_1\theta + \alpha t_{im} & \theta \le t_{im} \le 2\theta \\ \vdots \\ \sum_{i=1}^{M-1}\alpha_i\theta + \alpha_M t_{im} & (M-1)\theta \le t_{im} \le T_2 + T_1. \end{cases} \tag{17}$$

The maximum number of imperfect products is

$$I_d = \sum_{i=1}^{M}\alpha_i p\theta = p_r T_3. \tag{18}$$

With respect to total cost per unit product, it consists of 6 parts: holding cost for both perfect and imperfect products, backlog cost, deterioration cost, cost of production and rework, and lastly the fixed setup cost for each cycle run. And we aim to minimize the value of total cost per unit product:

$$\text{TC} = \frac{(\text{PHC} + \text{IHC} + \text{BC} + \text{DC} + \text{PRC} + C_s)}{Q}, \tag{19}$$

whereas the holding cost of perfect product is

$$\text{PHC} = C_{\text{hp}}\left[\int_0^{T_2} I_2(t_2)\,dt_2 + \int_0^{T_3} I_3(t_3)\,dt_3 + \int_0^{T_4} I_4(t_4)\,dt_4\right]. \tag{20}$$

The holding cost of imperfect products is

$$\text{IHC} = C_{\text{hi}}\left[\int_0^{T_1+T_2} I_{im}(t_{im})\,dt + \int_0^{T_3} I_3(t_3)\,dt_3\right]. \tag{21}$$

The backlog cost is

$$\text{BC} = -C_{\text{bc}}\left[\int_0^{T_1} I_1(t_1)\,dt_1 + \int_0^{T_5} I_5(t_5)\,dt_5\right]. \tag{22}$$

The deteriorating cost is

$$\begin{aligned} \mathrm{DC} &= C_{\mathrm{dc}}\left[\frac{1}{2}\lambda_N\theta + \sum_{i=N+1}^{M-1}\lambda_i\theta + \frac{1}{2}\lambda_M\theta - I_s\right] \\ &\quad + C_{\mathrm{dc}}\left[(p_r-\mu)T_3 - (I_m - I_s)\right] \\ &\quad + C_{\mathrm{dc}}\left[I_m - dT_4\right]. \end{aligned} \tag{23}$$

The production and rework cost are

$$\mathrm{PRC} = C_pQ + C_rp_rT_3. \tag{24}$$

By substituting all the inventory functions into the cost equation, we get the following.

The perfect product inventory holding cost PHC is

$$\begin{aligned} &\mathrm{PHC} \\ &= C_{\mathrm{hp}}\sum_{i=1}^{M-N}\int_0^\theta\left[\sum_{j=1}^{i}\frac{\lambda_{N+j-1}-\lambda_{N+j}}{\delta}\exp\left(-\delta\left((i-j)\theta\right)+t_2\right)\right. \\ &\qquad - \frac{\lambda_N}{\delta}\exp\left(-\delta\left((i-1)\theta+t_2\right)\right) \\ &\qquad \left. + \frac{\lambda_{N+i}}{\delta}\right]dt \\ &\quad + C_{\mathrm{hp}}\left[\left(I_s - \frac{p_r-\mu}{\delta}\right)\left(\frac{1-\exp(-\delta T_3)}{\delta}\right) + \frac{(pr-\mu)T_3}{\delta}\right] \\ &\quad + C_{\mathrm{hp}}\left[\left(I_m - \frac{\mu}{\delta}\right)\left(\frac{1-\exp(-\delta T_4)}{\delta}\right) - \frac{\mu T_4}{\delta}\right]. \end{aligned} \tag{25}$$

The holding cost for imperfect product is

$$\mathrm{IHC} = C_{\mathrm{hi}}\left[\sum_{i=1}^{M}\left(\sum_{j=1}^{i}\alpha_j\theta^2 + \frac{1}{2}\alpha_i\theta^2\right) + \frac{1}{2}p_rT_3^2\right]. \tag{26}$$

The backlog cost is

$$\begin{aligned} \mathrm{BC} &= -C_{\mathrm{bc}}\left[\int_0^\theta(\lambda_1t - B)\,dt\right] \\ &\quad - C_{\mathrm{bc}}\sum_{i=1}^{N}\left[\int_0^\theta\left(\left(\sum_{j=1}^{i-1}\lambda_i\theta - B\right)\theta + \lambda_it\right)dt\right] \\ &\quad - C_{\mathrm{bc}}\left[-\frac{1}{2}\mu T_5^2\right]. \end{aligned} \tag{27}$$

The deteriorating cost is

$$\begin{aligned} \mathrm{DC} &= C_{\mathrm{dc}}\left[\left(\sum_{i=N+1}^{M}\lambda_i\theta\right) - I_s + (p_r-\mu)T_3\right. \\ &\qquad \left. - (I_m - I_s) + I_m - dT_4\right]. \end{aligned} \tag{28}$$

In order to minimize the total cost per product, the optimal combination of M and N is desired to be determined. So the relationship among M, N, and other variables such as time T_i and backlog B is essential to solve the problem. First of all, according to the assumption, T_1 and T_2 can be represented by

$$\begin{aligned} T_1 &= \theta N, \\ T_2 &= \theta(M-N). \end{aligned} \tag{29}$$

Also since the defective rate is linearly increasing with a constant rate γ, we have

$$\alpha_i = \gamma * i \quad \text{for } i \in [0, M]. \tag{30}$$

Hence

$$\begin{aligned} \lambda_i &= p - \mu - \gamma p i, \\ \lambda_i - \lambda_{i+1} &= \gamma p. \end{aligned} \tag{31}$$

According to (18) and (30), we could have

$$T_3 = \frac{\gamma p\theta(M+1)^2}{2p_r}. \tag{32}$$

Substituting (30) into (9), we could obtain the following equation:

$$\begin{aligned} I_s &= \frac{\gamma p}{\delta}\sum_{j=1}^{M-N}\exp\left(\left(-\delta(M-N-j+1)\theta\right) + \frac{1}{2}\theta\right) \\ &\quad - \frac{(1-\gamma N)p - \mu}{\delta}\exp\left(-\delta(M-N)\theta\right) \\ &\quad + \frac{(1-\gamma M)p-\mu}{\delta}. \end{aligned} \tag{33}$$

The above expression can be simplified by using the Taylor series approximation under the assumptions that $\delta(M-N)$, δT_3, and δT_4 are small. This approach can also be found in other research work on deteriorating product such as [7, 29]. So,

$$\begin{aligned} &\exp\left(-\delta(M-N-j+1)\theta\right) \\ &\quad \approx 1 - \delta(M-N-j+1)\theta + \frac{1}{2}\left(\delta(M-N-j+1)\theta\right)^2, \end{aligned} \tag{34}$$

$$\exp(-\delta T_3) \approx 1 - \delta T_3 + \frac{1}{2}(\delta T_3)^2, \tag{35}$$

$$\exp(-\delta T_4) \approx 1 - \delta T_4 + \frac{1}{2}(\delta T_4)^2. \tag{36}$$

We could simplify the equation of I_s and I_m based on (33) and (14):

$$\begin{aligned} I_s &= \theta(M-N)\left(p - \mu - \frac{\gamma p}{2}(M+N+1)\right), \\ I_m &= \theta(M-N)\left(p - \mu - \frac{\gamma p}{2}(M+N+1)\right) \\ &\quad \cdot \left(1 - \frac{\delta\gamma p\theta M^2}{2p_r}\right) + (p_r - \mu)\frac{\gamma p\theta M^2}{2p_r}. \end{aligned} \tag{37}$$

Since I_m can also be represented with (15), T_4 can also be determined with (15) and (36)

$$T_4 = \frac{1}{\mu}\left[\theta(M-N)\left(p-\mu-\frac{\gamma p}{2}(M+N+1)\right)\cdot\left(1-\frac{\delta\gamma p\theta M^2}{2p_r}\right)+(p_r-\mu)\frac{\gamma p\theta M^2}{2p_r}\right]. \quad (38)$$

According to (16) B can be expressed as

$$B = \left(p-d-\frac{1}{2}\gamma pN\right)\theta N,$$
$$T_5 = \frac{(p-d-(1/2)\gamma pN)\theta N}{\mu}. \quad (39)$$

Deteriorating cost is reduced to

$$\mathrm{DC} = C_{\mathrm{dc}}\left[(p-\mu)(M-N)\theta-\gamma p\theta\frac{(M-N)(M-N-1)}{2}+p_rT_3-dT_4\right]. \quad (40)$$

Backlog cost is

$$\mathrm{BC} = -C_b\left[\frac{1}{2}(p-\mu)N^2\theta^2-\frac{1}{12}\gamma p\theta^2(N-1)(2N+5)N-B\theta N-\frac{1}{2}dT5^2\right]. \quad (41)$$

Holding cost for perfect product is

$$\begin{aligned}\mathrm{PHC} = C_{\mathrm{hp}}\Big[&\frac{1}{2}(p-\mu-\gamma pN)\theta^2\\ &\cdot((M-N)(M-N+1)-1)\\ &-\frac{1}{12}\gamma p\theta^2(2M-2N+1)\\ &\cdot(M-N)(M-N+1)\\ &+I_s\left(T_3-\frac{1}{2}\delta T_3^2\right)+\frac{1}{2}(p_r-\mu)T_3^2+\frac{1}{2}dT_4^2\Big],\end{aligned}$$

$$\mathrm{IHC} = C_{\mathrm{hi}}\left[\frac{1}{12}\gamma p\theta^2M(M+1)(2M+3)+\frac{1}{2}\gamma p\theta M^2T_3-\frac{1}{2}pT_3^2\right]. \quad (42)$$

Lastly by substituting $T_1, T_2, T_3, T_4, T_5, B$, and Q with M and N, the cost function can be further reduced. The overall total cost per unit product is obtained. To be noticed, the items with second or higher order δ and γ have been removed in order to simplify the equation:

$$\begin{aligned}\mathrm{TC} = {}&A_1+\frac{A_2}{M}+A_3M+A_4M^2+A_5M^3+A_6\frac{N^3}{M}\\ &+A_7N^2+A_8\frac{N^2}{M}+A_9N^2M\\ &+A_{10}N+A_{11}\frac{N}{M}+A_{12}NM+A_{13}NM^2,\end{aligned} \quad (43)$$

where

$$\begin{aligned}A_1 &= C_p+\frac{C_{\mathrm{hp}}\theta}{2}\left(\theta-\frac{\mu}{p}-\frac{\gamma}{6}\right)+\frac{C_{\mathrm{hi}}\gamma\theta}{4},\\
A_2 &= \frac{C_s}{p\theta}-\frac{C_{\mathrm{hp}}\theta}{2}\left(1-\frac{\mu}{p}\right),\\
A_3 &= \frac{C_r\gamma}{2}+\frac{5C_{\mathrm{hi}}\gamma\theta}{12}+\frac{C_{\mathrm{hp}}\theta}{2}\left(\frac{\gamma}{2}-\frac{p\gamma}{\mu}-\frac{1}{2}+\frac{p}{2\mu}\right),\\
A_4 &= \frac{C_{\mathrm{hi}}\gamma\theta}{6}-\frac{C_{\mathrm{hp}}\gamma\theta}{6}-\frac{C_{\mathrm{dc}}\mu\gamma\delta\theta}{2p_r}(\mu-p),\\
A_5 &= \frac{C_{\mathrm{hp}}\gamma\delta\theta^2}{p_r}\left(p-\frac{\mu}{2}-\frac{p^2}{2\mu}\right),\\
A_6 &= \frac{\gamma\theta}{2}\left(\frac{1}{3}-\frac{p}{\mu}\right)(C_b+C_{\mathrm{hp}}),\\
A_7 &= \frac{C_{\mathrm{hp}}p\gamma\theta}{2\mu},\\
A_8 &= C_{\mathrm{dc}}\gamma+\frac{C_b\theta}{2}\left(\frac{\gamma}{2}-1+\frac{p}{\mu}\right)\\
&\quad+\frac{C_{\mathrm{hp}}\theta}{2}\left(\frac{p}{\mu}+\frac{3\gamma}{2}-\frac{p\gamma}{\mu}-1\right),\\
A_9 &= \frac{C_{\mathrm{hp}}\gamma\delta\theta^2}{p_r}\left(p-\frac{\mu}{2}-\frac{p^2}{\mu}\right),\\
A_{10} &= C_{\mathrm{dc}}\gamma+C_{\mathrm{hp}}\theta\left(\frac{p}{\mu}-1\right)(\gamma-1),\\
A_{11} &= \frac{C_{\mathrm{hp}}\theta}{2}\left(\frac{\mu}{p}+\frac{7\gamma}{6}-1\right)-\frac{5C_b\gamma\theta}{12},\\
A_{12} &= \frac{C_{\mathrm{dc}}\gamma\delta\theta}{2p_r}(\mu-p),\\
A_{13} &= \frac{C_{\mathrm{hp}}\gamma\delta\theta^2}{p_r}\left(\mu-p+\frac{p^2}{\mu}\right).\end{aligned} \quad (44)$$

For optimum values of $TC(N,M)$, we make $\partial TC(N,M)/\partial N = 0$ and $\partial TC(N,M)/\partial M = 0$ which is equivalent to

$$3A_6\frac{N^2}{M}+2A_7N+2A_8\frac{N}{M}+2A_9NM+A_{10}+A_{11}\frac{1}{M}+A_{12}M+A_{13}M^2 = 0,$$

$$A_3 - \frac{A2}{M^2} + 2A_4M + 3A_5M^2 + 2A_12N + A_{11}\frac{N}{M^2} + 2A_{13}MN + A_9N^2 - A_*\frac{N^2}{M^2} - A_6\frac{N^3}{M^2} = 0. \quad (45)$$

The corresponding Hessian matrix is shown below:

$$H = \begin{pmatrix} H_1 & H_2 \\ H_2 & H_3 \end{pmatrix}, \quad (46)$$

where

$$H_1 = 2A_4 + \frac{2A_2}{M^3} + 6A_5M + 2A_13N + \frac{2A_{11}N}{M^3} + \frac{2A_8N^2}{M^3} + \frac{2A_6N^3}{M^3},$$

$$H_2 = A_{12} - \frac{A_{11}}{M^2} + 2A_{13}M + 2A_9N - \frac{2A_8N}{M^2} - \frac{3A_6N^2}{M^2}$$

$$H_3 = 2A_7 + \frac{2A_8}{M} + 2A_9M + \frac{6A_6N}{M}. \quad (47)$$

According to the optimum condition if the second order partial derivative for M and N is positive, then the hessian matrix is positive definite and the minimum total cost can be found. However, due to the complexity of the cost function, an explicit solution cannot be obtained. Instead, we will use numerical examples to illustrate that the cost function is convex and the optimum result is also determined accordingly.

4. Numerical Experiment

Numerical examples and sensitivity are conducted in the following section. To be more specific, first of all, the plots generated for cost function are used to demonstrate that the cost function is convex. And then sensitivity analysis indicates the impact of different parameters on the overall inventory performance.

4.1. Numerical Examples. In these examples, the values of the parameters are assumed as follows: $\gamma = 0.01$, $\theta = 0.01$, $\delta = 0.1$, $C_{\text{hp}} = \$40$, $C_{\text{hi}} = \$30$, $C_p = \$100$; $C_r = \$40$, $C_b = \$60$, $C_{\text{dc}} = \$60$, $\mu = 100$, $p = 600$, and $p_r = 300$. The optimum combination of N and M is $(N^*, M^*) = (4, 10)$.

As shown in Figure 3, the cost function shows its convexity and the optimum pair of (N^*, M^*) is the lowest point, while, in Figures 4(a) and 4(b), the convexity is more clear when M and N are fixed at its optimum value, respectively. Hence the optimal T_1^*, T_2^*, T_3^*, T_4^*, T_5^*, and T^* are calculated as

$$T_1^* = 0.04, \quad T_2^* = 0.06,$$
$$T_3^* = 0.01, \quad T_4^* = 0.293, \quad (48)$$
$$T_5^* = 0.195.$$

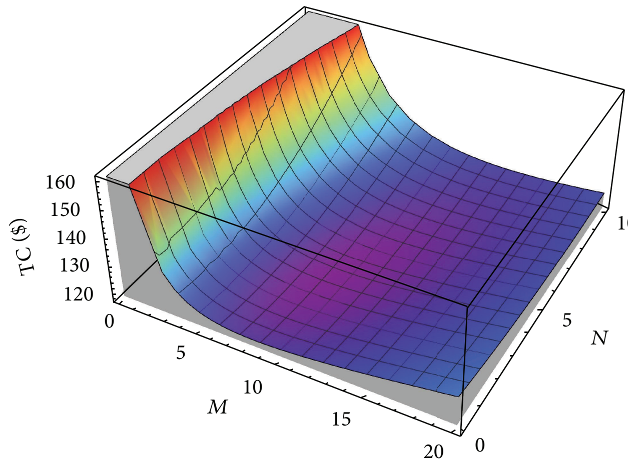


Figure 3: The plot of total cost per unit product against M and N.

The optimal production quantity and backlog quantity are determined as

$$Q^* \approx 60, \qquad B^* \approx 20. \quad (49)$$

The optimal total cost per unit product, the optimum production run time, and the optimal cycle time are

$$\text{TC}^* = 118.479, \qquad T_1^* + T_2^* = 0.1, \qquad T^* = 0.598. \quad (50)$$

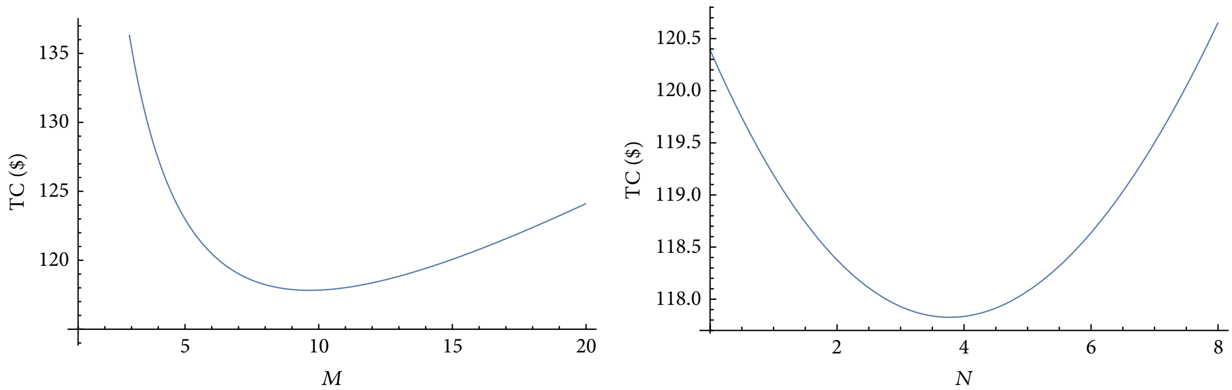
4.2. Sensitivity Analysis. The sensitivity analysis for the parameters is conducted and the results and discussion are listed down below. As shown in Tables 1 and 2, for each of the parameters, four more experiments are carried out with the changes of −50%, −25%, 25%, and 50%. The corresponding optimal values of total cost per unit product, M and N, are presented in the tables. To be noticed, the values of M and N are interpreted as the production run time and backlog quantity, respectively, in the discussion since they have positive relationship.

For the key parameters, consider the following.

(i) Both the parameters θ and γ, which are related to the imperfect production system, have significant impact on the total cost per unit product. The increment of θ reduces the cost, while, for γ, the lower the value is, the lower the cost is. It means that slow deterioration of production quality or relatively steady defective rate helps cut down the total cost per unit product. In addition, high γ shortens production run time which is represented by M.

(ii) The deterioration of product has positive relationship with the total cost per unit product. But when compared with θ and γ, its influence on the overall system is relatively smaller.

(iii) The total cost per unit product is especially sensitive to the changes of demand rate μ. With the rise of μ, the cost drops rapidly while the total production run time $M\theta$ increases instead. The value of M is heavily influenced by the production rate. During



(a) TC against M

(b) TC against M

FIGURE 4: Total cost per unit product against M and N.

TABLE 1: The sensitivity analysis for different key parameters.

Parameter	Optimal values	Changes −50%	−25%	0%	25%	50%
θ	TC^*	120.68	118.824	117.835	117.16	116.765
	M^*	16	12	10	8	7
	N^*	5	4	4	3	3
γ	TC^*	116.116	116.977	117.835	118.569	119.327
	M^*	10	10	10	9	9
	N^*	4	4	4	3	3
δ	TC^*	117.824	117.829	117.835	117.84	117.846
	M^*	10	10	10	10	10
	N^*	4	4	4	4	4
μ	TC^*	124.166	120.033	117.835	116.399	115.43
	M^*	8	9	10	10	10
	N^*	4	3	4	4	3
p	TC^*	119.664	118.479	117.835	117.563	117.193
	M^*	17	12	10	9	6
	N^*	4	4	4	3	2
p_r	TC^*	117.817	117.832	117.835	117.841	117.842
	M^*	10	10	10	10	10
	N^*	4	4	4	4	4

the increase of production rate, M declines from 17 to as low as 6. On the contrary, the influence brought by remanufacturing rate is not obvious.

For the cost parameters, consider the following.

(i) High holding cost C_{hp} causes the increase of backlog quantity and decrease of holding quantity; meanwhile the value of backlog quantity drops significantly along with the increase of backlog cost C_b.

(ii) The total cost per unit product is affected by the production cost to a large extent. But it does not change the production run time and backlog quantity.

(iii) Both the production run time and the backlog quantity grow with higher setup cost C_s. The total cost per unit product is also sensitive to the changes of setup cost.

5. Conclusions

In this paper, a modified EPQ model with rework and backlog has been proposed. Compared with the existing works, the deterioration of product and production process is taken into account at the same time which is the main contribution to this research field. To model the deterioration of production process, we assume that the defective rate increases at constant intervals. Defective products are reworked at the end of normal production process and the rework is viewed as perfect process. In order to minimize the total cost per unit product, the optimal pair of the total number

Table 2: The sensitivity analysis for different unit costs.

Unit cost	Optimal values	Changes				
		−50%	−25%	0%	25%	50%
C_{hp}	TC^*	115.054	116.687	117.835	118.593	119.234
	M^*	11	10	10	9	9
	N^*	2	3	4	4	5
C_{hi}	TC^*	117.764	117.819	117.835	117.851	117.866
	M^*	10	10	10	10	10
	N^*	4	4	4	4	4
C_{dc}	TC^*	117.07	117.421	117.835	118.035	118.42
	M^*	10	10	10	10	9
	N^*	4	4	4	3	3
C_p	TC^*	67.803	92.8032	117.835	142.898	171.433
	M^*	10	10	10	10	10
	N^*	4	4	4	4	4
C_{pr}	TC^*	116.803	117.303	117.835	118.303	118.753
	M^*	10	10	10	10	9
	N^*	4	4	4	4	4
C_b	TC^*	116.11	117.19	117.835	118.202	118.515
	M^*	10	10	10	9	9
	N^*	7	5	4	3	2
C_s	TC^*	111.898	114.556	117.835	119.908	121.802
	M^*	7	9	10	11	11
	N^*	3	3	4	4	4

of intervals θ in normal period M and in backlog period N is determined. Due to the high complexity of the cost function, we cannot prove the convexity of the function in the analytical way. Instead numerical experiments are carried out to illustrate the convexity of the cost function and to find the optimal solution. The impact of all the different parameters on the system is provided and summarized in Sensitivity Analysis. In terms of future research, the current model can be extended in several directions. First of all, at present the system deterioration rate is assumed to be linear in this research work. So a more general exploration of other types of deterioration rate can be done to test the performance of the proposed method. Secondly, machine breakdown can be added into the model to make the model more practical, in which machines subject to failures and production process are stopped once the machine is broken. And also the model can be generalized into both multiproduction and rework periods model and multiproduct/multimachine model. Lastly stochastic demand and partial backlog can also be introduced into the current work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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