AN EFFICIENT ALGORITHM FOR THE EXTRACTION OF A EUCLIDEAN SKELETON *

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ABSTRACT

The skeleton is essential for general shape representation but the discrete representation of an image presents a lot of problems that may influence the process of skeleton extraction. Some of the methods are memory-intensive and computationally intensive, and require a complex data structure. In this paper, we propose a fast, efficient and accurate skeletonization method for the extraction of a well-connected Euclidean skeleton based on a signed sequential Euclidean distance map. A connectivity criterion that can be used to determine whether a given pixel inside an object is a skeleton point is proposed. The criterion is based on a set of points along the object boundary, which are the nearest contour points to the pixel under consideration and its 8 neighbors. The extracted skeleton is of single-pixel width without requiring a linking algorithm or iteration process. Experiments show that the runtime of our algorithm is faster than those of using the distance transformation and is linearly proportional to the number of pixels of an image.

1. INTRODUCTION

The skeleton is essential for general shape representation. It is a useful means of shape description [1] in different areas, such as content-based image retrieval systems, character recognition systems, circuit board inspection systems, as well as biomedical imagery for shape analysis. The extracted skeleton can be used as a feature to represent the original shape as it has a more compact representation. In real-time image processing, a fast skeletonization algorithm is necessary.

Due to the importance of skeletonization, many approaches have been proposed throughout the past decades. Most of the skeletonization algorithms can be simply classified into two essential types. The first type is referred to as thinning algorithms, such as shape thinning [2] and the wave grassfire transform [3]. These algorithms iteratively remove border points, or move to the inner parts of an object in determining an object's skeleton. However, the iterative process is a time-consuming operation and requires some terminating criteria. In addition, the uniqueness of the extracted skeleton may be dependent on the initial conditions provided. The second type of algorithm is based on the medial-axis transform, as introduced by Blum [1]. Examples include the line skeleton [4,5], Voronoi skeleton [6,7], morphological transform [8], and maximal disk method [9,10]. As these algorithms search the set of centers and the corresponding radii of the maximal disks contained in an object, they can usually preserve all the information about the object and allow the reconstruction of the object. Although medial axis transform is a very useful tool and yields an intuitively pleasing skeleton, its direct implementation is usually prohibitively computational and hard to compute. Moreover, due to the use of discrete space, the extracted skeletons are sensitive to local variations and noise, and there is no guarantee that the extracted median-axis can preserve the original object's topology. In addition, these algorithms may produce redundant points on determination of the skeleton, and are memory-intensive and require a complex data structure.

An abundant amount of work on skeletonization and its application has been conducted, in which a profound theoretical background of the skeleton from different aspects has been provided. The properties of the skeleton, including its thickness, connectivity and reconstructability, have been investigated. In this paper, we propose a new skeletonization algorithm, which is fast, efficient and accurate, to extract a well-connected Euclidean skeleton based on a signed sequential Euclidean distance map. By using the connectivity criterion proposed in this paper, a skeleton point can be determined efficiently and independently by considering a set of points along the object's boundary, which are the nearest contour points to the pixel under consideration and its 8 neighbors.

2. SKELETON BASED ON THE MAXIMAL DISK

Suppose that the contour C of an object in an image is represented by a continuous closed curve. Inside the contour C, the planar shape F represents the content of the object. The corresponding skeleton S can be determined, as shown in Fig. 1. According to Blum's definition [1], the skeleton S of a planar shape is defined as the locus of the centers of the maximal disks contained inside the planar shape F. The medial axis transform is defined as follows:

Definition 1: The medial axis transform is the set of ordered pairs of the centers and radii of maximal disks in the planar shape F. That is,

$$SK(F) = \{(\boldsymbol{p},r) \in F \mid B(\boldsymbol{p},r) \in MaxDisk(F)\} \qquad (1)$$

where $MaxDisk(F)$ is the set of all maximal disks in the planar shape F and $B(\boldsymbol{p},r)$ is a disk with radius r centered at the point $\boldsymbol{p}$.

From Fig. 1, it can be observed that each of the skeleton points is associated with at least two boundary points whose

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respective distances to the skeleton point are the shortest, except for the end points of the skeleton. These boundary points, q_1 and q_2, divide the contour into two separate segments. Consider a maximal disk centered at the point p with radius r, the object's contour and the maximal disk touch each other at the points q_1 and q_2. These two points divide the contour C into two segments, A and B. If there exists at least one point along segment A and along segment B outside the maximal disk with a certain distance, the point p will be declared a skeleton point.

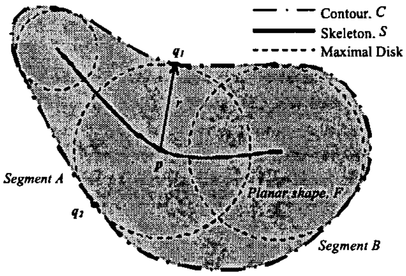


Fig. 1. The definition of a skeleton.

3. THE CRITERION FOR A SKELETON POINT

3.1. The width of the skeleton

An ideal skeleton is connected and has zero-width. A continuous boundary will produce a path-connected skeleton [11]. However, in real applications, the contour points and the skeleton points must be located at the pixel grids; this induces a lot of discrete problems. The skeleton may not pass through the pixel exactly. Hence, in practice, the skeleton has a non-zero width and all the pixels passed through by the idea skeleton will be considered to be skeleton pixels. Consider a skeleton point p, the corresponding maximal disk touches the boundary at points q_1 and q_2, as shown in Fig. 2. Points q_1 and q_2 are the two nearest contour points with respect to point p. The distance between points q_1 and q_2 is denoted as D. Suppose that the true skeleton point p lies midway between the two adjacent points p_1 and p_2. The width of the skeleton can then be represented by a line segment $\overline{p_1p_2}$, which is parallel to the line $\overline{q_1q_2}$ and perpendicular to the direction of the skeleton. Due to the width of the skeleton, two values, r_1 and r_2, which are the distances of $|p_1q_1|$ or $|p_2q_2|$, and $|p_2q_1|$ or $|p_1q_2|$, can be obtained with the condition that $r_2 \geq r_1$. By using the Cosine Law and $\angle q_1q_2p_1 = \angle q_1p_2p_1$, we have:

$$\frac{D^2+r_2^2-r_1^2}{2Dr_2}=\frac{w^2+r_2^2-r_1^2}{2wr_2}$$

$$\Rightarrow w=\frac{r_2^2-r_1^2}{D}=\frac{(r_2-r_1)(r_2+r_1)}{D} \qquad (2)$$

According to the above equation, the width of a skeleton is therefore proportional to the difference between the two local shortest distances and the radius of the maximal disk, and is inversely proportional to the value D, where D is the distance between the two nearest contour points q_1 and q_2. Consequently, a skeleton of non-zero even width can be obtained if the following criterion is satisfied:

$$w=\frac{r_2^2-r_1^2}{D}\leq \delta \qquad (3)$$

where δ is defined as a threshold to determine the maximum width of a skeleton and w represents the corresponding width of the skeleton at a particular point.

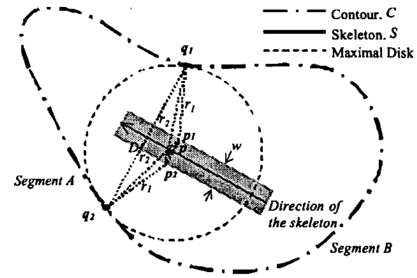
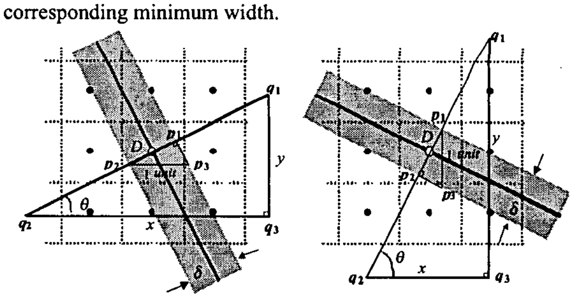


Fig. 2. The width of a skeleton.

For the point p_1, the first shortest distance r_1 is $|p_1q_1|$ while the second shortest distance r_2 is $|p_1q_2|$. Similarly, for the point p_2, the first shortest distance r_1 is $|p_2q_2|$ while the second shortest distance r_2 is $|p_2q_1|$. For the point lying midway between the points p_1 and p_2, the two shortest distances are equal, so the value of w is equal to zero, which is the exact position of the skeleton point. Any point p along the line $\overline{p_1p_2}$ is also a skeleton point. Therefore, the criterion, as shown in equation (3), can be used to determine whether or not a point is a skeleton point with a given δ

3.2. Connectivity of a skeleton under the square-coordinate systems

The use of different coordinate systems, such as hexagonal, polar or square, will make the corresponding skeleton have a different minimum width in different orientations. Considering the square coordinate system, the minimum width of a skeleton depends on its direction that is perpendicular to the line $\overline{q_1q_2}$. Figure 3 illustrates the effect of different orientations of a skeleton on the corresponding minimum width.



(a) $x > y$ (b) $x < y$

Fig. 3. The extracted skeleton under the square-coordinate system.

Suppose that the coordinates of the two nearest contour points q_1 and q_2 are (x_1, y_1) and (x_2, y_2), respectively. Since the direction of the line segment $\overline{q_1q_2}$ is perpendicular to the direction of the skeleton, the minimum width δ of the skeleton can be determined by the deviation of the line segment $\overline{q_1q_2}$. If the horizontal deviation x is greater than the vertical deviation y of the line segment $\overline{q_1q_2}$, as shown in Fig. 3(a), the minimum width δ of the skeleton can be obtained as x/D by considering the two similar triangles, $\Delta q_1q_2q_3$ and $\Delta p_3p_2p_1$. Similarly, if the horizontal deviation x is less than the vertical deviation y of the line segment $\overline{q_1q_2}$, as shown in Fig. 3(b), the minimum width δ of the skeleton can be obtained as y/D by considering the two similar triangles, $\Delta q_1q_2q_3$ and $\Delta p_1p_3p_2$. The minimum width δ of the skeleton can therefore be set as follows:

$$\delta = \frac{\max(x,y)}{D} \tag{4}$$

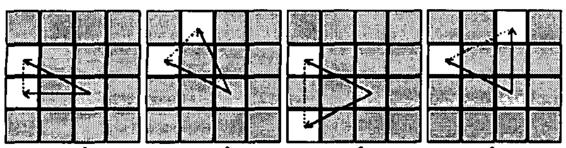
where $x = abs(x_2\text{-}x_1)$ and $y = abs(y_2\text{-}y_1)$, and D is the distance between the two nearest contour points q_1 and q_2.

A skeleton of non-zero width with threshold δ is illustrated in Fig. 3. The thick solid line represents the idea skeleton. All these pixels passed over by this idea skeleton are considered to be a skeleton point. Therefore, the connectivity criterion for the square-coordinate system can be obtained based on equations (3) and (4) as follows:

$$\frac{r_2^2 - r_1^2}{\max(x,y)} \le 1 \quad \text{and} \quad D^2 \ge \rho \tag{5}$$

where $x = abs(x_2\text{-}x_1)$ and $y = abs(y_2\text{-}y_1)$, and ρ is a non-zero threshold used to determine the minimum distance between the two nearest contour points.

As the skeleton points are determined based on the connectivity criterion, simple contour segments that are either in a horizontal/vertical or a diagonal direction will cause some points to be mistaken for skeleton points. These points can be removed by simply determining the corresponding segment type. Fig. 4 illustrates the values of the square distances D^2 between the two nearest contour points. The pixel under consideration is represented by a shaded square, while the corresponding two nearest contour points are represented by white squares.



(a) D^2=1 (b) D^2=2 (c) D^2=4 (d) D^2=5

Fig. 4. The different types of boundary segments.

4. THE SKELETONIZATION ALGORITHM

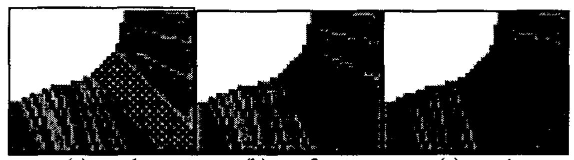
To determine whether a pixel point is a skeleton point, the corresponding nearest contour point for each of the 8 neighboring points will be determined, and the connectivity criterion will then be applied to this set of 8 contour point pairs. The relative positions of the nearest contour points for each pixel can be obtained by using the signed sequential Euclidean distance (8SSED) map. The nearest contour points for each of the 8-neighboring points of the point under consideration can be obtained by subtracting the relative position of its nearest contour point from the relative position of its neighborhoods. The nearest contour points of the pixel under consideration and one of its 8 neighbors then form a point pair. If any one of the point pairs satisfies the connectivity criterion, the pixel can be declared a skeleton point. Otherwise, if all the 8 point pairs fail to fulfill the connectivity criterion, the pixel is not a skeleton point.

5. EXPERIMENTAL RESULTS

In the experiment, the shapes in an image can be extracted by using a contour extraction method called the adaptive snake method [12] or any edge follower method. Based on the extracted contours, the skeletons of the shapes are extracted using our proposed algorithm. The skeletonization performance and the complexity of our proposed algorithm are evaluated in this section. The effect of the threshold values ρ on the extraction of a skeleton will be illustrated. Then, the complexity of our proposed algorithm is compared to some maximal disk extraction algorithms. The experiments were conducted on a Pentium II 400MHz PC.

5.1. The effect of the threshold ρ

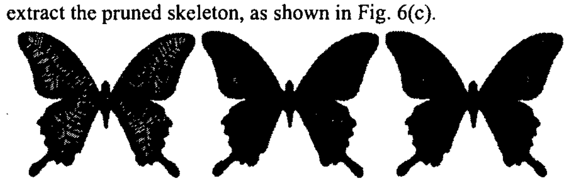
Figure 5 illustrates the effect of different threshold values ρ on a simple contour segment. When the value of the threshold ρ increases, a smaller number of skeleton points will be extracted as a result there being fewer branches in the skeleton. For example, the extracted branches of a skeleton corresponding to a simple contour segment (i.e. diagonal segments), as shown in Fig. 5(a) and (b), are removed by increasing the threshold ρ to 2. If the threshold ρ is increased to 4, more branches of the skeleton are removed, as shown in Fig. 5(c).



(a) $\rho = 1$ (b) $\rho = 2$ (c) $\rho = 4$

Fig. 5. The extracted skeletons using different threshold values.

Also, the effect of different threshold values ρ on the skeleton of an object is illustrated in Fig. 6. The whole skeleton extracted by using our proposed algorithm is shown in Fig. 6(a), which uses the threshold value $\rho = 4$. The effect of increasing the threshold ρ is similar to that of pruning. However, this pruning procedure may cause the extracted skeleton to disconnect from some spurious points, as shown in Fig. 6(b), because there is no guarantee of extracting a connected skeleton for a larger value of ρ. Since many pruning methods [6,7] have been proposed, we can apply one of them after we have applied our proposed algorithm to extract the Euclidean skeleton. Alternatively, we can use a larger threshold ρ as a simple pruning method to extract the pruned skeleton, as shown in Fig. 6(c).



(a) $\rho = 4$ (b) $\rho = 50$ (c) $\rho = 150$

Fig. 6. The skeletons using different thresholds ρ with an image of size 320×240.

5.2. The computational requirements of the algorithms

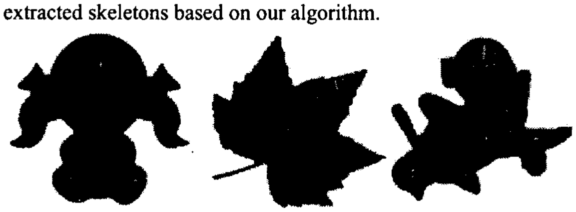
In this section, we will investigate the computational complexity of our proposed method. As mentioned previously, there are many approaches to extracting a skeleton, and it is impossible to compare our proposed method with each of the existing methods. In this paper, we compare our algorithm to the following existing methods, namely, method 2, a neighborhood algorithm followed by an exhaustive algorithm [10]; method 3, a modification of method 2 with the use of an exhaustive algorithm followed by a bounding box-based algorithm [10]; and method 4, a method based on a criterion with a residual distance [13]. The method 1, the signed sequential Euclidean distance map (8SSED) [14], was used in all the methods to be compared.

Table 1. Runtimes of different methods with different image sizes.

Runtime (sec)		#1	#2	#3	#4	#5
1	320×240	0.07	5.67	5.54	4.38	0.10
	640×480	0.28	65.97	58.33	34.28	0.16
	1280×960	1.15	-	-	-	0.65
2	320×240	0.07	5.16	5.02	3.25	0.10
	640×480	0.28	61.13	55.34	28.61	0.15
	1280×960	1.15	-	-	-	0.59
3	320×240	0.07	5.06	5.57	3.77	0.12
	640×480	0.28	28.61	28.61	29.30	0.18
	1280×960	1.15	-	-	-	0.570

#1: 8SSED process
#2: Neighborhood + Exhaustive Method
#3: Neighborhood + Bounding Box-based + Exhaustive Method
#4: Criteria of a skeleton
#5: Proposed Method

Both methods 2 and 3 use the neighborhood algorithm to eliminate most of the points in the planar shape by means of the inclusion test procedure [10]. In order to determine the true maximal disks, the exhaustive algorithm is used to screen out the rest of the non-CMDs. By applying the bounding box-based algorithm before the exhaustive algorithm, the computational time for method 3 can be reduced and a smaller number of false CMDs will be generated. Both methods 2 and 3 can be used to reconstruct the original contour exactly, and so be used as a means for compression, but they require a linking algorithm [10] to obtain a connected skeleton. Method 4 can extract a pruned connected skeleton directly without using a linking algorithm. As shown in Table 1, the runtimes for methods 2, 3 and 4 are longer than that for the proposed method. We can observe that the runtimes required by the three methods increase tremendously when the image size increases. For our proposed method, the runtime is linearly proportional to the number of pixels in the planar shape. Figure 7 illustrates some of the extracted skeletons based on our algorithm.



(a) Example 1 (b) Example 2 (c) Example 3

Fig. 7. The skeletons based on our proposed algorithm with an image of size 640×480.

6. CONCLUSIONS

With the use of the connectivity criterion proposed in this paper, an accurate, simple and efficient algorithm for the extraction of a well-connected Euclidean skeleton is devised with the use of the signed sequential Euclidean distance map. The nearest contour points of the pixel under consideration and its 8 neighbors are generated to form a set of 8 point pairs, which are then used to determine whether the pixel is a skeleton point. This method can generate a connected Euclidean skeleton. The complexity of this algorithm is linearly proportional to the number of the pixels in an image.

7. REFERENCES

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