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Structural Design in the Post-Effective Length Era

S.L. Chan^a, Y.P. Liu and S.W. Liu

Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, China

Abstract

Before the publication of several new codes for structural design of steel structures in recent years, design of structures is based on the linear analysis and the effective length method, which relies on a non-existent or erroneous assumption of buckling at undeformed geometry. After the publication of the several codes including the Eurocode-3 (2005), LRFD (2005) and the Hong Kong Steel Code (2005), interestingly all in 2005, a new method based on the equilibrium at deformed configuration emerges as a replacement of the old effective length method. In theory, the second-order and advanced analysis can be used for any structural forms but calibration for imperfections against coded and experimental results is essential for safety. Its application to various structural forms and materials has been successful with advantages of this new method manifested. This paper summaries the new design method on two structural forms as transmission line towers and steel, concrete and composite portal frames.

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Keywords: Composite column; second-order analysis; buckling.

1. INTRODUCTION

Since the Euler buckling concept introduced one century ago, the linear analysis and the effective length design method was developed and used for practical design and research reference in the past several decades. The design practiced by most engineers is still based on this concept of undeformed geometry for equilibrium which is obviously incorrect as no deformation leads to no displacement, no strain and no stress which cannot balance the external forces. In actual practice, the method leads to the need of using separate codes for design of members made of different materials, for example, (Eurocode-2 2005) is used for reinforced concrete members, (Eurocode-3 2005) for steel members and (Eurocode-4 2005) for composite members and no code for design of special structural forms like pre-tensioned steel trusses or bamboo scaffolds. This brings much inconvenience and sometimes confusion to the design engineer. This paper aims to propose a practical numerical approach for structures made of various

^a Corresponding author. Tel: (852) 27664484 fax: (852) 23346389
E-mail address: ceslchan@polyu.edu.hk

common building materials with the consideration of geometrical instability and material nonlinearity that all structural members can be designed in a unified way with the section capacity check carried out for all members to insure safety in stability, strength and ductility.

Although the description for the stability check in steel, concrete and composite codes may be different, the requirement for consideration of second-order effects such as $P-\Delta$ and $P-\delta$ effects is conceptually and numerically similar. It is noted that the $P-\delta$ effect and imperfections are still commonly ignored in most previous structural analysis and design software and therefore the tedious member buckling strength design by code is still needed and the software cannot be used for a proper second-order analysis. In this paper, with the use of the pointwise-equilibrium-polynomial (PEP) element (Chan and Zhou, 2000) allowing for initial imperfection in a robust nonlinear incremental-iterative procedure, both the $P-\delta$ effect of individual members and the $P-\Delta$ effect and imperfections of the structural system can be simulated. Thus, a unified design method is developed in the present project and no tedious member design check to various codes is required.

The proposed method has been extended to inelastic analysis by using the plastic hinge approach which is required in plastic design, seismic design, advanced analysis and progressive analysis. To capture the gradual yield behaviour under the interaction of axial and bending effects, the first and full yield surfaces for a hybrid section, which contains steel, reinforcement and concrete materials, are chosen for demonstration. The sectional fibre approach is adopted to calculate the stress resultants of the concrete, the reinforcement and the structural steel. Both the strength reduction and stiffness deterioration can be represented in the proposed method.

In this paper, the basic element formulation for considering the geometric nonlinearity with initial imperfections and plastic hinge formulation are discussed. Finally, two examples are presented for demonstration of the validity, accuracy and advantages of the proposed practical second-order analysis method for practical design.

2. BASIC ASSUMPTIONS AND DEFINITIONS

In the formulation of the beam-column element, the following assumptions are taken: (1) The Euler-Bernoulli hypothesis is valid and warping is neglected; (2) strains are small but the deflection can be large; (3) plane section normal to the centroid axis before deformation remains plane after deformation and normal to the axis; (4) the concept of lumped plasticity is employed, i.e., the yielding of material is assumed to be concentrated at the both ends of beam-column element; (5) loads are conservative and shear distortions are negligible.

3. GEOMETRIC NONLINEARITY

The $P-\Delta$ and $P-\delta$ effects are two principal parameters needed to be considered in the second-order or advanced analysis. In the topic of formulating a design element capturing the $P-\delta$ effect of a member, (Chan and Zhou, 2000) developed several elements with different features to simplify the analysis procedure and make the advanced analysis practicable. In this paper, the pointwise equilibrating polynomial (PEP) element proposed by (Chan and Zhou, 2000) is adopted. The PEP element is capable of modelling the member initial curvature which is mandatory for buckling design in various codes. The basic force-displacement relations in an element are illustrated below and more details about its formulation can be referred to the original papers. The equilibrium condition can be stipulated as follows.

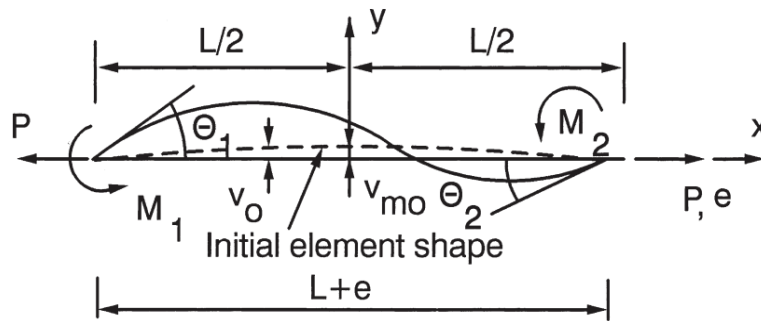


Figure 1 The basic forces vs. displacements relations in an elastic element

$$EI \ddot{v} = P(v + v_0) + \frac{M_1 + M_2}{L} \left(\frac{L}{2} + x \right) - M_1 \quad (1)$$

in which E is the Young's modulus of elasticity, I the second moment of area, L the member length, v the lateral displacement due to applied loads, v_0 the initial member deflection, P the axial force, and M_1 and M_2 the nodal end moments. A superdot represents a differentiation with respect to the distance x along an element.

The secant stiffness matrix, which relates the equilibrium equations between forces, moments, displacements and rotations, can be obtained by the energy principle. For incremental-iterative nonlinear procedure, the tangent stiffness matrix which relates the incremental forces, moments to rotations and displacements is needed and can be formed by the second variation of the total potential energy functional.

The remarkable advantage of proposed advanced analysis by PEP element is its automatic computation of primary linear and secondary non-linear stresses such that the assumption of K-factor or effective length factor is avoided. The influence on member stiffness in the presence of axial load is also allowed for in the stress computation and analysis. The first global eigenvalue buckling mode shape is used to determine the direction of local member imperfection and the global frame imperfection due to out-of-plumbness which is normally taken in the range from 1/1000 to 1/200 of the building height H . The member initial imperfection can be obtained from codes such as Table 5.1 in the (Eurocode-3, 2005) and Table 6.1 in the (HKSC, 2005) and the imperfections of HKSC are adopted herein as they are more consistent with the buckling curves.

4. MATERIAL NONLINEARITY

For inelastic analysis, it is necessary to assume a yield function to monitor the gradual plastification of a section. This refined plastic hinge approach introduced by (Chan and Chui, 1997) is revised and adopted in this study. The nodal rotations of a deformed PEP element with pseudo-springs at the end nodes are shown in Figure 2.

The zero-length spring elements belonging to the internal degrees of freedom of beam-column element can be eliminated by a standard static condense procedure such that the size of the element stiffness matrix will not be increased. The final incremental stiffness relationships of the hybrid element can also be formulated.

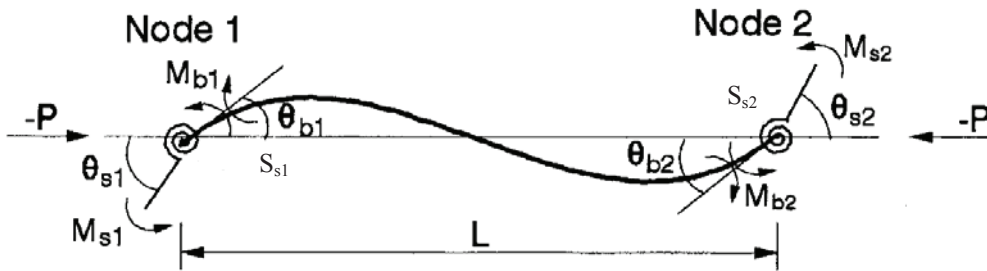


Figure 2 Plasticity is considered by end-section springs

$$\begin{pmatrix} \Delta P \\ \Delta_e M_1 \\ \Delta_e M_2 \end{pmatrix} = \begin{bmatrix} EA/L & 0 & 0 \\ 0 & S_1 - S_1^2(K_{22} + S_2)/\beta & S_1 S_2 K_{12}/\beta \\ 0 & S_1 S_2 K_{21}/\beta & S_1 - S_2^2(K_{11} + S_1)/\beta \end{bmatrix} \begin{pmatrix} \Delta L \\ \Delta_e \theta_1 \\ \Delta_e \theta_2 \end{pmatrix} \quad (2)$$

in which S_1 and S_2 are the stiffness of the end springs, K_{ij} are the flexural stiffness of the PEP element considering the presence of axial force, ΔP is the axial force increment, $\Delta_e M_1$ and $\Delta_e M_2$ are the incremental nodal moments at the junctions between the spring and the global node and between the beam and the spring, ΔL is the axial deformation increment, $\Delta_e \theta_1$ and $\Delta_e \theta_2$ are the incremental nodal rotations corresponding to these moments, and

$$\beta = \begin{vmatrix} K_{11} + S_1 & K_{12} \\ K_{21} & K_{22} + S_2 \end{vmatrix} = (K_{11} + S_1)(K_{22} + S_2) - K_{12}K_{21} > 0 \quad (3)$$

The section spring stiffness, S , can be calculated by the following equation,

$$S = \frac{6EI}{L} \left(\frac{M_{pr} - M}{M - M_{er}} + \rho \right) \quad (4)$$

where EI is the flexural constant, L is the member length and M_{er} and M_{pr} are respectively the first and full yield moments reduced due to the presence of axial force and the ρ represents a strain-hardening parameter. From this equation, the section stiffness varies from infinity to a small strain-hardening value which represents three sectional stages, i.e., elastic, partially plastic and fully plastic with strain-hardening stages.

5. DESIGN OF TRANSMISSION TOWER

The tower of type RCS-90 SUSPENSION TOWER as shown in Figure 3 is selected here for demonstration. The height of the tower is 110 m and grade S405 and S245 steels have been used in the design. The second-order effect due to axial force is considered by the use of curved and initially imperfect stability function element with initial curvature set to the code requirement as one-300th of the length between nodes.

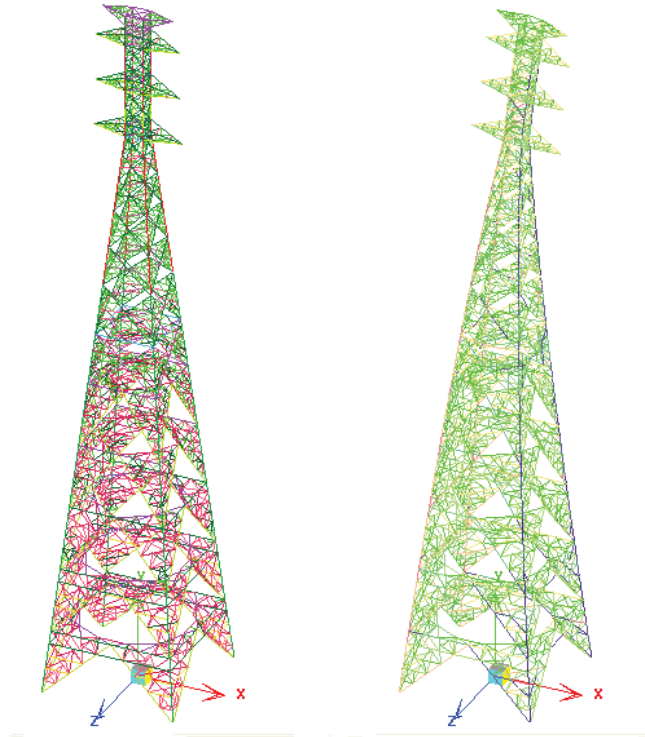


Figure 3: Model of RCS-90 SUSPENSION TOWER before and after deformation

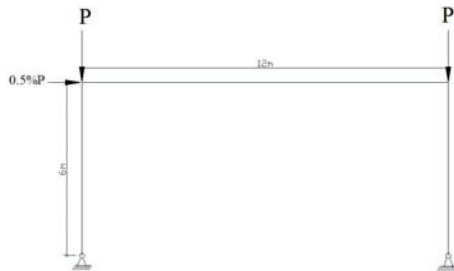


Figure 4: Completed tower in Myanmar

Different load cases are assumed in the analysis and they include wind at different angles, self-weight and load from hanging conductors. The case for broken cable is also considered as it introduces a large twist to the structure. The main legs are assumed continuous whereas the bracing members are assumed pinned at their ends. All support conditions are pinned to the foundation. A 250 year return wind speed is considered in the design. For starred angle sections, section properties about geometric axes are used despite the principal axes are diagonal to the geometric axes. The completed tower is shown in Figure 4 above with the user of the program NIDA standing in front of the tower.

Material Properties:

Concrete : C45 , $f_{cu} = 45 \text{ N/mm}^2$, $\gamma_m = 1.5$
 Steel section: S355, $f_y = 355 \text{ N/mm}^2$, $\gamma_m = 1.0$
 Reinforcement: R460, $f_y = 460 \text{ N/mm}^2$, $\gamma_m = 1.15$



CASE	Beam	Column
1		
2		
3		

Figure 5 Geometry of portal frame and section properties

6. ANALYSIS AND DESIGN OF PORTALS OF DIFFERENT MATERIALS

In this example, a simple portal frame with three types of sections, i.e., bare steel, reinforced-concrete and composite, will be studied. The geometrical layout and the section properties are shown in Figure 5 below.

As discussed, initial imperfections such as local member initial curvature and global frame imperfection should be considered in a practical second-order analysis. Here, the initial member imperfections of $1/300$ and $1/400$ of member length are assumed for columns and beams respectively, while the global imperfection is taken as $1/500$ of building height. The equivalent axial stiffness $(EA)_e$ and $(EI)_e$ are calculated by the following equations.

$$(EA)_e = E_c A_c + E_s A_s + E_r A_r \quad (5)$$

$$(EI)_e = EI + E_s I_s + 0.5 E_c I_c \quad (6)$$

The load-deflection curves of the top of right column for three different types of frame obtained from the proposed second-order inelastic analysis are plotted in Figure 6.

From Figure 6, it can be seen that ultimate load resistance of composite frame is much higher than the other two frames and the bare steel frame shows lowest load resistance. This figure also indicates that both steel and composite frames experience excellent ductility while reinforced concrete frame shows relative poor ductility with maximum lateral deflection at about 175mm.

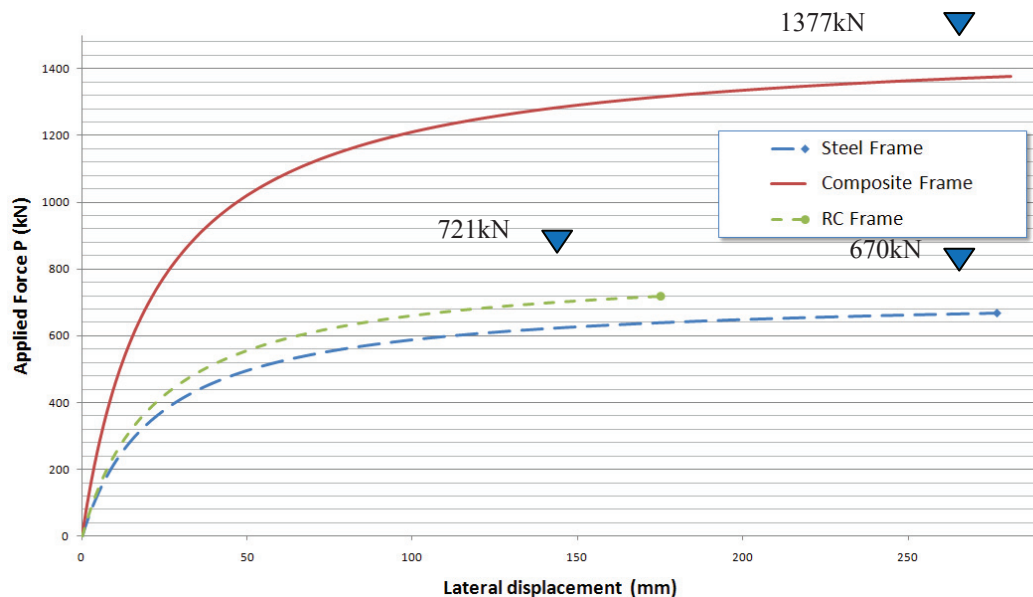


Figure 6 Load-deflection curves of portal frames

7. CONCLUSIONS

The second-order elastic and plastic analysis allowing imperfections at frame and member levels are presented and applied to design of two common structural forms namely as transmission line towers and composite frames. The method has also been used in design of many other structural forms as scaffolds, space frames, building frames and others of which the details are reported elsewhere. It is quite evident that the era of design by simulation has arrived and the effective length method is becoming an obsolete approach which has too many limitations for contemporary structural design. For this reason, many codes have abandoned its use and engineers and researchers should modify their design approach and research direction accordingly.

8. ACKNOWLEDGEMENT

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References

- [1] AISC Load and resistance factor design specification for structural steel buildings, American Institute of Steel Construction Chicago, 2005
- [2] Chan, S.L. and Chui, P.P.T. (1997), A generalized design-based elasto-plastic analysis of steel frames by section assemblage concept, *Journal of Engineering Structures*, vol.19, no.8, pp. 628-636.

- [3] Chan, S.L., and Zhou, Z.H. (2000), “Non-linear integrated design and analysis of skeletal structures by 1 element per member”, *Engineering Structures*, vol.22, pp. 246-257.
- [4] Code of practice for structural uses of steel 2005, Buildings Department, Hong Kong.
- [5] Design of composite steel and concrete structures (2005), The European Committee for Standardization, ed. Eurocode 4, DD ENV 1994-1-1:2004.
- [6] Design of concrete structures (1999), The European Committee for Standardization, ed. Eurocode 2 (DD ENV 1992-1-1:1992).
- [7] Design of steel structures (2005), The European Committee for Standardization, ed. Eurocode 3 , DD ENV 1993-1-3:2006.
- [8] Euler, L. (1910), “Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes, Additamentum I, De curvis elasticis, Lausanne et Geneve (1744), vertaald door H., Linsenbarth opgenomen in, Ostwald’s Klassiker der exakten Wissenschaften, deel 175, Leipzig.