Modelling occurrence and duration of building drainage discharge loads from random and intermittent appliance flushes

KW Mui[a], LT Wong[b]

Department of Building Services Engineering, The Hong Kong Polytechnic University, Hong Kong, China.

Emails: [a]behorace@polyu.edu.hk; [b]beltw@polyu.edu.hk

Corresponding author: LT Wong

Email: beltw@polyu.edu.hk, Tel: 852 2766 7783, Fax: 852 2765 7198

Abstract

Building drainage demands are assumed random and intermittent. The existing Hunter-based models have been developed to aid in the sizing of pipes for instantaneous demands but not to predict the occurrence and duration of such demands. This paper proposes a time series model to determine the occurrence and duration of drainage demand which are greater than some predefined flow rates due to a number of appliances that discharge randomly and intermittently with variable flow rates. While the duration is resolved using the time series data for simultaneous demands, the occurrence together with the time-dependent flow rate is quantified via Monte Carlo simulation based on occupant usage patterns and appliance discharge profiles derived from appliance flushes at the base of the main vertical stack. The applicability of this model is demonstrated with example discharges from a number of WC cisterns with reduced flush volume.

Practical application

This study will help designers determine the duration and occurrence of drainage demand due to a number of appliances that discharge randomly and intermittently with variable flow rates.

Keywords

Simultaneous demand model, demand duration, random and intermittent demand, drainage system

List of symbols

A_f	apartment floor area (m^2)
c_1, c_2	arbitrary constant partition time (s)
N_a	hourly demand of an appliance (h^{-1})
N_d	number of continuous demands
N_f	number of hourly continuous demands in $\tau_d \geq \tau_d^*$ (h^{-1})
N_p	number of persons (persons)
N_s	number of iterations
N_t	number of partition times
N_w	number of appliances
O_a	occupant-area ratio (person m^{-2})
q_w	appliance demand (Ls^{-1})
n_a	per person hourly demand (person^{-1} h^{-1})
q_d	stack end demand (Ls^{-1})
$t_{w,1}$, $t_{w,2}$	demand start time and demand end time (s)
t_0, t_∞	period start time and period end time (s)
t_i	partition time (s)

t	time (s)
$U(\hat{t}_{w,1})$	uniformly distributed fractional demand start time (s)
V_w	demand volume (L)

Greek

Δ	a finite change
ϵ	relative error
ϑ	random number between 0 and 1
ϕ	occupant load variation factor
φ	dummy variable
λ_0, $\lambda_0^{'}$, λ_∞ $\lambda_\infty^{'}$	boundary parameters at t_0 and t_∞
τ	time period (s)
τ_0, τ_w	time periods of zero demand and non-zero demand (s)
τ_d	demand period (s)
τ_d^*	demand duration (s)
τ_k	last demand period (s)

Subscripts

i, j, k	of i-th, j-th, k-th values

0, 1, 2, … ∞	of period start time, times 0, 1, 2 … in the period, and period end time
max	of maximum

Superscripts

$\sim$	of distribution
$\frown$	of fraction
*	of constant
$'$	of change

1. Introduction

Building drainage demands in terms of simultaneous water flow rates are assumed to be random and intermittent.[1] Simultaneous drainage demand overload is legitimate in some piping systems for it is a rare episode as a result of intermittent and random appliance usages. In other words, a small "on demand" failure probability of the simultaneous demand (e.g. a demand failure rate not exceeding 1% of the calculated simultaneous discharge flow rates during daily rush hours) is allowed in these systems for sizing a main drainage pipe.[2,3]

Hunter's fixture unit approach provides a straightforward method to evaluate the probable maximum simultaneous demands and is used to calculate the pipe sizes for building drainage systems.[3,4] However, it lacks flexibility when dealing with the variations in appliance types, building types, occupancy levels and usage patterns.[5] Intensive survey studies for the accuracy of appliance usage patterns are therefore required.[6,7] Moreover, this approach may overestimate the simultaneous demand and that leads to oversized drainage pipes.[8] For determining the simultaneous demands from groups of mixed appliances that will not be discharged simultaneously, a time series model can be used.[9,10] The model calculates the simultaneous demands over a period of time instead of at any instant and the time variable is employed in the evaluation of water system performance and associated pressure fluctuations at a discharging stack..[10,11]

Green measures and appliances are being developed to conserve water and energy.[12-16] A reduced or low WC flush volume, however, increases the risk of sewer blockages.[17,18] According to recent full-scale experimental investigations, the time duration of a continuous drainage flow rate can be a significant parameter for sizing the building main slope drainage pipe, which collects appliance discharges at the base of a vertical stack.[19]

The existing Hunter-based models do not resolve the variable of time period and this paper proposes a time series model to determine the occurrence of continuous drainage demands due to a number of appliances that discharge randomly and intermittently with variable flow rates. While the duration is resolved using the time series data for simultaneous demands, the occurrence together with the time-dependent flow rate is quantified via Monte Carlo simulation based on occupant usage patterns and appliance discharge profiles reported in the open literature. The applicability of this model is demonstrated with example discharges from a number of WC cisterns with reduced flush volume.

2. Demand model

Operation of an appliance (i.e. the stack end demand) in a time period τ (s) which starts at time t_0 (s) and ends at time t_∞ (s) can be expressed by the sum of time periods of non-zero demands $\tau_{w,i}$ (s) and zero demands $\tau_{0,i}$ (s) for $i = 1,2,\dots,N_d$, where N_d is the number of demands of an appliance in the time period τ (say one hour) and τ_k (s) is the last demand period (for closure) as shown in Figure 1(a).

$$\tau = t_\infty - t_0 = \tau_{0,1} + \tau_{w,1} + \tau_{0,2} + \tau_{w,2} + \cdots + \tau_{0,i} + \tau_{w,i} + \cdots + \tau_{0,N_d} + \tau_{w,N_d} + \tau_k \qquad \dots (1)$$

The time periods are defined by the appliance demand start times $t_{w,1,i}$ (s) and the appliance demand end times $t_{w,2,i}$ (s).

$$\tau_{0,1} = t_{w,1,1} - t_0 \,;\, \tau_{0,i} = t_{w,1,i} - t_{w,2,i-1};\, \tau_{w,i} = t_{w,2,i} - t_{w,1,i};\, \tau_k = t_\infty - t_{w,2,N_d} \qquad \dots (2)$$

The demand start times $t_{w,1,i}$ (s) are given by the fractional demand start times $\hat{t}_{w,1,i} \in [0,1]$,

$$t_{w,1,i} = \hat{t}_{w,1,i}\tau \qquad \ldots (3)$$

These fractional demand start times are assumed randomly distributed in τ and can be determined via Monte Carlo simulations using a uniformly distributed fractional demand start time $U(\hat{t}_{w,1})$ (s) and Equation (4) below, where $\vartheta \in [0,1]$ is a random number between 0 and 1 taken from a pseudo-random number set generated by a prime modulus multiplicative linear congruential generator.[20] The pseudo set was demonstrated to be applicable in a number of engineering applications.[21,22]

$$\vartheta = \int_{-\infty}^{\hat{t}^*_{w,1}} U(\hat{t}_{w,1}) d\hat{t}_{w,1} \qquad \ldots (4)$$

If the last demand end time is beyond the time period τ, i.e., $t_{w,2,N_d} > t_\infty$ as presented in Figure 1(b), then when τ starts at $\tau_{w,0}$ (s), Equation (1) can be written as,

$$\tau = \tau_{w,0} + \tau_{0,1} + \tau_{w,1} + \tau_{0,2} + \tau_{w,2} + \cdots + \tau_{0,i} + \tau_{w,i} + \cdots + \tau_{0,N_d} + \tau_{w,N_d} - \tau_k \qquad \ldots (5)$$

Equation (2) thus becomes,

$$\tau_{0,1} = t_{w,1,1} - t_{w,2,0};\ \tau_{w,0} = t_{w,2,0} - t_0 = \tau_k = t_{w,2,N_d} - t_\infty \qquad \ldots (6)$$

Appliance demands q_w (Ls^{-1}) in the time period are expressed by,

$$q_w(t \in \tau) = \begin{cases} 0 & ;\ t \in \tau_0 \\ \tilde{q}_w(t) & ;\ t \in \tau_w \end{cases} \qquad \ldots (7)$$

The drainage demand of a building is the demand at the end of the building main discharge stack. Figure 2 exhibits two example demand flow rates $\tilde{q}_w$ (Ls^{-1}) – one from a 9L WC cistern and the other from a 6L WC cistern – experimentally measured at the end of an 8m vertical drainage stack (100mm in diameter).[23] These two examples have the same demand duration only at reference flow rate = 0 Ls^{-1}. The demand profile of either a single flush or a combination of flushes is assumed at the end of the stack when determining the probable simultaneous demand. It can be seen that for a demand volume V_w (L) of 9L and a maximum flow rate of 1.46 Ls^{-1}, the discharge period is 12s.

$$V_w = \int_{\tau_w} \tilde{q}_w(t)\, dt \qquad \ldots (8)$$

The probable simultaneous stack end demand q_d (Ls^{-1}) at time t (s) in the time period τ (s) is approximated by the sum of simultaneous discharge flow rates from a group of connected appliances $q_{w,i}$ (Ls^{-1}) for $i = 1,2,\ldots,N_w$, where N_w is the number of appliances,

$$q_d(t) \approx \sum_{i=1}^{N_w} q_{w,i}(t) \qquad \ldots (9)$$

2.1 Occupant demand

The hourly demand N_a (h^{-1}) of an appliance type k is given by the following equation, where $n_{a,k}$ ($person^{-1}\ h^{-1}$) is the per person hourly demand of k,

$$N_{a,k}(t) = n_{a,k} N_p(t) \quad \ldots (10)$$

Number of persons at a time $N_p(t)$ (persons) is expressed through an occupant load variation factor $\phi(t)$,[24]

$$N_p(t) = N_{p,max} \phi(t) \quad \ldots (11)$$

The hourly occupant load profiles of government and non-government funded residential buildings in Hong Kong were created from interview surveys and the occupant load variation factor was determined by the occupant load at a time as a percentage of the maximum occupant load.

The maximum occupant load $N_{p,max}$ (persons) is a design parameter for an appliance serving an apartment floor area A_f (m^2). The occupant-area ratio O_a ($person\ m^{-2}$) is determined by,[25]

$$O_a = \frac{N_{p,max}}{A_f} \quad \ldots (12)$$

2.2 Demand duration and occurrence

Continuous demands are demands with flow rates greater than a predefined constant flow rate at the end of the stack q_d^* (Ls^{-1}) over the demand periods τ_d (s) as illustrated in Figure 3 and can be expressed by Equation (13) below, where t_i (s) is a partition time and N_t is the number of partition times t_i within the time period τ (s).

$$q_d(t) - q_d^* = 0;\ t = t_i;\ i = 1,2,\ldots,N_t \qquad \ldots (13)$$

The chosen constant flow rate is a minimum flow rate required to reduce the risk of blockages in drainage pipes. In numerical simulations, a time series of demand in each second of an hour is obtained by,

$$q_d(t) = [q_d(t=0), q_d(1), q_d(2), q_d(3) \ldots q_d(t=3600)] \qquad \ldots (14)$$

This demand time series is used to determine continuous demands. Some examples of continuous demands are highlighted in Figure 3. A demand illustrated below is considered as continuous and sustains for a time period of $(c_2 - c_1 + 1)$ seconds in the simulations, where c_1 and c_2 are respectively the start time and end time of the continuous demand in the time period,

$$\begin{cases} q_d(c_1 - 1) < q_d^* \\ q_d(c_1), q_d(c_1+1), q_d(c_1+2), q_d(c_1+3), \ldots, q_d(c_1+c_2) > q_d^* \\ q_d(c_1+c_2+1) < q_d^* \end{cases}$$

$$\ldots (15)$$

Figure 4 shows the boundary parameters λ_0, λ_0', λ_∞ and λ_∞' at times $t = t_0$ and $t = t_\infty$. There should be a total of 16 (i.e. 4×4) sets of boundary conditions. The non-zero parameters λ' are determined only when λ=0.

$$\lambda_0 = q_d(t_0) - q_d^*;\ \lambda_0' = \frac{q_d(t_1) - q_d(t_0)}{2} \qquad \ldots (16)$$

$$\lambda_\infty = q_d(t_\infty) - q_d^*;\ \lambda_\infty' = \frac{q_d(t_\infty) - q_d(N_t)}{2} \qquad \ldots (17)$$

$\tau_{d,j}$ (s), the duration of continuous demands, is defined by the partition times t_i (s) for $j = 1,2,\ldots,N_d$, where N_d is the number of continuous demands,

$$\tau_{d,j} = t_i - t_{i-1} = \begin{cases} t_{2j} - t_{2j-1} & ;\lambda_0 \begin{cases} < 0 \\ = 0;\ \lambda_0' < 0 \end{cases} \\ t_{2j-1} - t_{2(j-1)} & ;\lambda_0 \begin{cases} > 0 \\ = 0;\ \lambda_0' > 0 \end{cases} \end{cases} \qquad \ldots (18)$$

The number of continuous demands N_d is given by,

$$N_d = \begin{cases} \frac{N_t}{2} & ;\lambda_0 \begin{cases} < 0 \\ = 0;\ \lambda_0' < 0 \end{cases} & ;\lambda_\infty \begin{cases} < 0 \\ = 0;\ \lambda_\infty' > 0 \end{cases} \\ \frac{N_t + 1}{2} & ; \begin{cases} \lambda_0 \begin{cases} < 0 \\ = 0;\ \lambda_0' < 0 \end{cases} \\ \lambda_0 \begin{cases} > 0 \\ = 0;\ \lambda_0' > 0 \end{cases} \end{cases} & ; \begin{matrix} \lambda_\infty \begin{cases} > 0 \\ = 0;\ \lambda_\infty' < 0 \end{cases} \\ \lambda_\infty \begin{cases} < 0 \\ = 0;\ \lambda_\infty' > 0 \end{cases} \end{matrix} \\ \frac{N_t}{2} + 1 & ;\lambda_0 \begin{cases} > 0 \\ = 0;\ \lambda_0' > 0 \end{cases} & ;\lambda_\infty \begin{cases} > 0 \\ = 0;\ \lambda_\infty' < 0 \end{cases} \end{cases} \qquad \ldots (19)$$

Occurrence of the number of hourly continuous demands N_f over a minimum duration τ_d^* (s) within the time period is expressed by Equation (20) as follows, where $\tilde{\tau}_d$ (s) is the probability density function of the demand durations,

$$N_f = N_d \left(1 - \int_0^{\tau_d^*} \tilde{\tau}_d dt\right) \quad \ldots (20)$$

Alternatively, for $i = 1,2,\ldots,N_s$ where N_s is the number of iterations, the occurrence of N_f can be approximated by,

$$N_f = \frac{\sum_{i=1}^{N_s} N_{d,i}}{N_s} \quad \ldots (21)$$

3. Simulation examples

3.1 Sample output

Open literature data reveal that the time-dependent flow rate at the end of a stack is related to random appliance discharges which are usually attributed to an average discharge frequency within the demand period.[19,23] For demonstration, this study applied the model to some residential WCs with flush volumes V_w of 6L and 9L.[26] Assuming one WC discharge per hour in the off-peak period ($N_a = 1$) and three discharges per hour in the peak period ($N_a = 3$), and setting the simulation period to one hour (τ = 3600s), drainage demands at an interval of one second were evaluated for a number of WCs, i.e. $N_{w,j}$ = 20 to 100.

Sensitivity of the simulation results can be tested against the number of iterations. The maximum number of iterations selected was $N_{s,max}$ = 200K due to limitations on computer resources. Both the demand average and variance were examined for the sensitivity. As pictorialized in Figure 5, the relative errors ϵ determined by Equation (22), where φ is a dummy variable, were 5% for 1K iterations and about 0.1% for 10K to 100K iterations.

$$\epsilon = 1 - \left| \frac{\varphi_{N_s}}{\varphi_{N_{s,max}}} \right| \quad \ldots (22)$$

Example hourly demand patterns are shown in Figure 6. They were sensitive to the hourly demand of WC and the number of WCs installed. Tripling the number of flushes would increase both the number of demands and the maximum simultaneous flow rate in the period. There were obviously a number of non-simultaneous demands, and the maximum simultaneous demands decreased with the reduced flush volume (i.e. from 9L to 6L). The demands, the time portions of simultaneous discharges and the maximum flow rates all increased with the increasing number of appliances as well as hourly flushes.

It has been demonstrated that a continuous flow of 1.5 Ls^{-1} to 2 Ls^{-1} discharging from the stack is sufficient to remove test specimens from slope drainage pipes.[19] Therefore, constant demands q_d^* of 1.5 and 2 Ls^{-1} were used to exemplify the number of hourly continuous demands N_f in the demand durations τ_d^* = 6, 10 and 14s. The results as graphed in Figure 7 showed a significant decrease in the expected number of continuous demands as the flush volume was reduced from 9L to 6L. Taking appliance number N_W = 100 and hourly flushes per WC N_a = 3, the decrease of number of continuous demands greater than q_d^* = 2 Ls^{-1} and sustained for

τ_d^* = 10s, was from 2 h^{-1} to 0.19 h^{-1} or from 6.45 h^{-1} to 1.48 h^{-1} for q_d^* = 1.5 Ls^{-1}.

3.2 Simulation of main drainage demands for typical residential drainage stacks

Based on the model input parameters exhibited in Table 1, further simulations were performed for typical residential drainage systems both medium (N_w = 60) and large (N_w = 120). The expected numbers of hourly continuous demands N_f were determined at the hours 0400, 0800 and 2100 on any day for the minimum, morning peak and evening peak demands respectively. The number of hourly flushes was determined using Equations (10) to (12). It is noted that tripling the per person hourly demand of WC or the occupant load variation factor would result in tripling the hourly demands at the end of the stack. The probability density functions of the hourly demand of a WC at times $t = 0400$, 0800 and 2100 can be expressed by,

$$P(N_a \geq N_a^*) = \int_0^{N_a^*} \widetilde{N}_a dN_a \qquad \ldots (23)$$

The corresponding hourly demands of an appliance found at t were N_a^* = 1.8, 2.2 and 3.6 h^{-1} at $P = 0.01$, and N_a^* = 2.4, 2.9 and 4.6 at $P = 0.001$, respectively.

Figure 8 illustrates the simulated percentile stack end demands q_d (Ls^{-1}) at P > 0.5, which are given by,

$$P(q_d \leq q_d^*) = \int_0^{q_d^*} \tilde{q}_d^* dq_d \qquad \ldots (24)$$

Taking a reference flow rate not exceeding the flow rates by 1% of the probability given in Equation (24),[8,26] the design flow rates q_d^* (Ls^{-1}) at times $t =$ 0400, 0800 and 2100 are: 2.56, 2.85 and 3.13 Ls^{-1} when N_w = 60 or 2.9, 3.5 and 3.95 Ls^{-1} when N_w = 120 for $V_w =$ 6L; and 3.84, 4.27 and 4.69 Ls^{-1} when N_w= 60 or 4.35, 5.24 and 5.92 Ls^{-1} when N_w = 120 for $V_w =$ 9L, respectively.

Figure 9 shows the simulated demand scenarios for N_f. Doubling the number of WCs from 60 to 120 would increase the demand duration by up to 5s at a predefined flow rate $q_d^* =$ 2 Ls^{-1}, whereas reducing the WC cistern flush volume from 9L to 6L would decrease the demand duration by at least 5s.

4. Conclusion

This paper describes a time series model for determining the duration and occurrence of drainage demand due to a number of appliances that discharge randomly and intermittently with variable flow rates. Demand durations in peak and off-peak periods were simulated for 20 to 100 residential WCs with flush volumes of 6L and 9L. A number of zero demand periods were observed and the expected number of continuous demands over a demand period was low even for a stack serving 100 WCs. Furthermore, doubling the number of WCs from 60 to 120 would increase the demand duration by up to 5s whereas reducing the WC cistern flush volume from 9L to 6L would decrease the demand duration by at least 5s. The proposed model can be a useful tool for evaluating the corresponding reduction of the continuous demands, while further experimental data is required for verification.

5. Acknowledgment

The work described in this paper was partly funded by the Research Grant Council of the Hong Kong Special Administrative Region, China (PolyU5337/09E).

6. References

1. Galowin LS. "Hunter" fixture units development. Proceedings of the 34th CIBW062 International Symposium on Water Supply and Drainage for Buildings, 8-10 September, The Hong Kong Polytechnic University, Hong Kong, 2008: 58-80.

2. Bull LC. Simultaneous demand from a number of draw-off points. Journal of The Institution of Heating and Ventilating Engineers, 1956; 2: 445-452.

3. Plumbing Services Design Guide. The Institute of Plumbing, Essex, 2002.

4. Konen TP, Goncalves OM. Summary of mathematical models for the design of water distribution systems within buildings. Proceedings of the 20th CIBW062 International Symposium on Water Supply and Drainage for Buildings, 1993: paper no. 1993024.

5. Jack LB. Overview of investigations addressing the issue of low water use sanitary appliances and associated drainage network design. Building Services Engineering Research and Technology 2000; 21(2): 139-44.

6. Friedler E, Butler D, Brown DM. Domestic WC usage patterns. Building and Environment 1996; 31(4): 385-92.

7. Wong LT, Mui KW. Drainage demands of domestic washrooms in Hong Kong. Building Services Engineering Research and Technology 2009; 30(2): 121-33.

8. Mui KW, Wong LT. A comparison between the fixture unit approach and Monte Carlo simulation for designing water distribution systems in high-rise buildings. Water SA 2011; 37(1): 109-14.

9. Carson W. From averages to time series. Proceedings of the CIBW062 International Symposium on Water Supply and Drainage for Buildings, 1979: paper no. 5.

10. Wong LT, Mui KW. Stochastic modelling of water demand by domestic washrooms in residential tower blocks. Water and Environment Journal 2008; 22(2): 125-30.

11. Liao WJ, Mui KW, Cheng CL, Wong LT, He KC. Air pressure fluctuations of drainage stacks at a high-rise office building. Indoor and Built Environment 2011; 20(4): 412-419. .

12. Cheng CL. Evaluating water conservation measures for Green Building in Taiwan. Building and Environment 2003; 38(3): 369-79.

13. Oliveira LH, Goncalves OM, Uchida C. Performance evaluation of dual-flush WC cistern in a multifamily building in Brazil. Building Services Engineering Research and Technology 2009; 30(1): 27-36.

14. Berndtsson JC. Experiences from the implementation of a urine separation system: Goals, planning, reality. Building and Environment 2006; 41(4): 427-37.

15. Willis RM, Stewart RA, Panuwatwanich K, Jones S, Kyriakides A. Alarming visual display monitors affecting shower end use water and energy conservation in Australian residential households. Resources. Conservation and Recycling 2010; 54(12): 1117-27.

16. Concalves OM. Influence of the users behavior in determining the water flow-rates in water supply systems in buildings. Proceedings of the CIBW062 International Symposium on Water Supply and Drainage for Buildings, 1989: paper no. 1.

17. McDougall JA, Wakelin RHM. The influence of flush volume and branch drain cross-section on deformable solid transport in attenuating flows. Building Services Engineering Research and Technology 2007; 28(1) 7-22.

18. Gormley M, Campbell DP. Modelling water reduction effects: method and implications for horizontal drainage. Building and Research Information 2006; 34(2): 131-44.

19. Cheng CL, He KC, Liao WJ, Hsieh CH. Research of main drain system and solid transportation performance in existing buildings. The 36th CIBW062 International Symposium of Water Supply and Drainage for Buildings, 8-10 November, Sydney, Australia, 2010: 214-226.

20. Park SK, Miller KW. Random number generators: good ones are hard to find. Communications of the ACM 1988; 31(10): 1192-201.

21. Yuen WW, Chow WK. A Monte Carlo Approach for the Layout Design of Thermal Fire Detection System. Fire Technology 2005; 41(2): 93-104.

22. Wong LT, Mui KW, Guan Y. Shower water heat recovery in high-rise residential buildings of Hong Kong. Applied Energy 2010; 87(2): 703-9.

23. Wise AFE, Swaffield JA. Water, sanitary and waste services for buildings, 5th Ed. London: Butterworth Heinemann, 2002.

24. Wong LT, Mui KW. An occupant load survey for residential buildings in Hong Kong. International journal for Housing Science and its Applications 2006; 30(3): 195-204.

25. Wong LT, Mui KW. An epistemic analysis of residential occupant load factor, Proceedings of Zhejiang-Hong Kong Joint Symposium 2007 – Innovative Building Design and Technology-Challenges of Climate Change, 6-7 Jul, Hangzhou, China, 2007: 38-44.

26. Wong LT, Mui KW. Modeling Water Consumption and Flow Rates for Flushing Water Systems in High-Rise Residential Buildings in Hong Kong. Building and Environment 2007; 42(5): 2024-2034.

Table 1: Model input parameters

Input parameter	Values
Occupant-area ratio O_a (person m^{-2})	0.096 (0.038)
Apartment floor area A_f (m^2)	43.75
Occupant load variation factor ϕ at time t = 0400/0800/2100	0.9621 (0.0505)
	0.5175 (0.0851)
	0.8921 (0.0417)
Per person hourly demand of an appliance n_a (h^{-1}) at time t = 0400/0800/2100	0.1214 (0.0967)
	0.3610 (0.1416)
	0.3404 (0.1507)
Appliance demand $\tilde{q}_w(t)$	See Figure 2
Demand volume V_w (L)	6, 9
Stack end demand cut-off for continuity q_d^* (Ls^{-1})	2
Number of appliances N_w	60, 120
Number of iterations N_s	200000

Standard deviations in brackets.

Figure 1: Demand time series of an appliance

Figure 2: Discharge flow rates at the end of a stack connected to 9L and 6L WC cisterns

Figure 3: Continuous demands within a period $\tau = [t_0, t_\infty]$ **(s)**

Figure 4: Boundary conditions at t_0 **and** t_∞

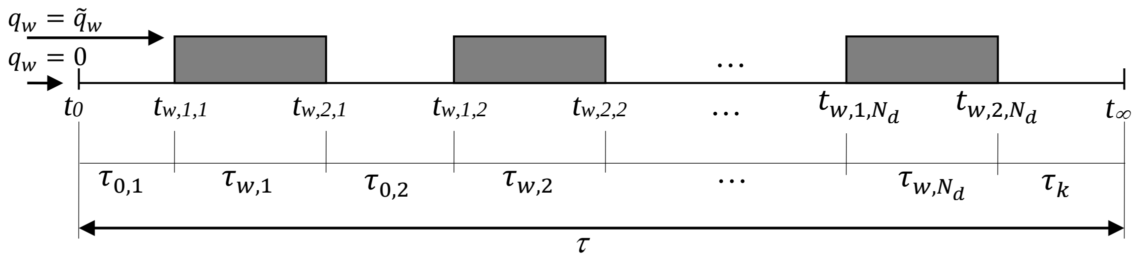
Figure 5: Demands at the end of a stack

Figure 6: Examples of hourly demand patterns

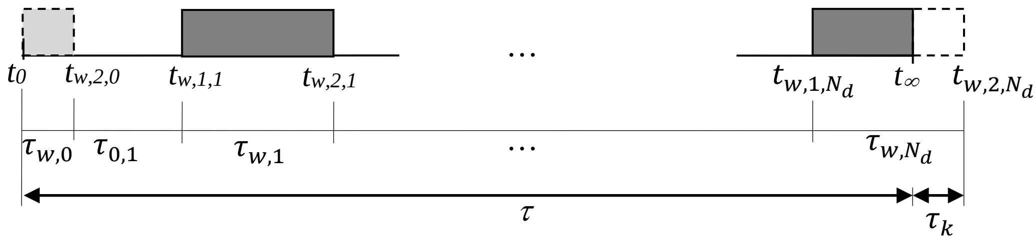
Figure 7: Expected no. of hourly continuous demands $N_f\left(q_d^*\right)$**, (a)** N_a=1, V_w=9; **(b)** N_a=3, V_w=9; **(c)** N_a=1, V_w=6; **(d)** N_a=3, V_w=6

Figure 8: Stack end demands q_d^***, (a)** N_w=60; **(b)** N_w=120

Figure 9: Simulated demand scenarios for $N_f(q_d^* = 2)$**, (a)** N_w=60; **(b)** N_w=120



(a)



(b)

Figure 1: Demand time series of an appliance

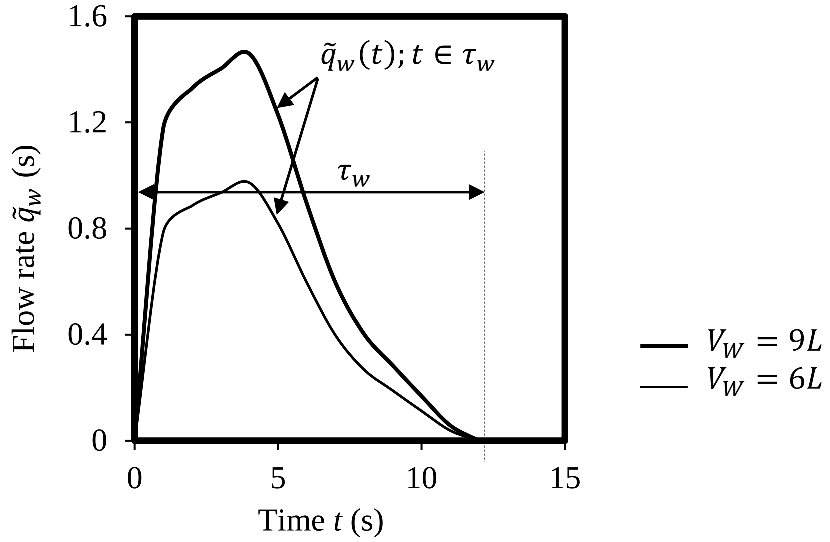


Figure 2: Discharge flow rates at the end of a stack connected to 9L and 6L WC cisterns

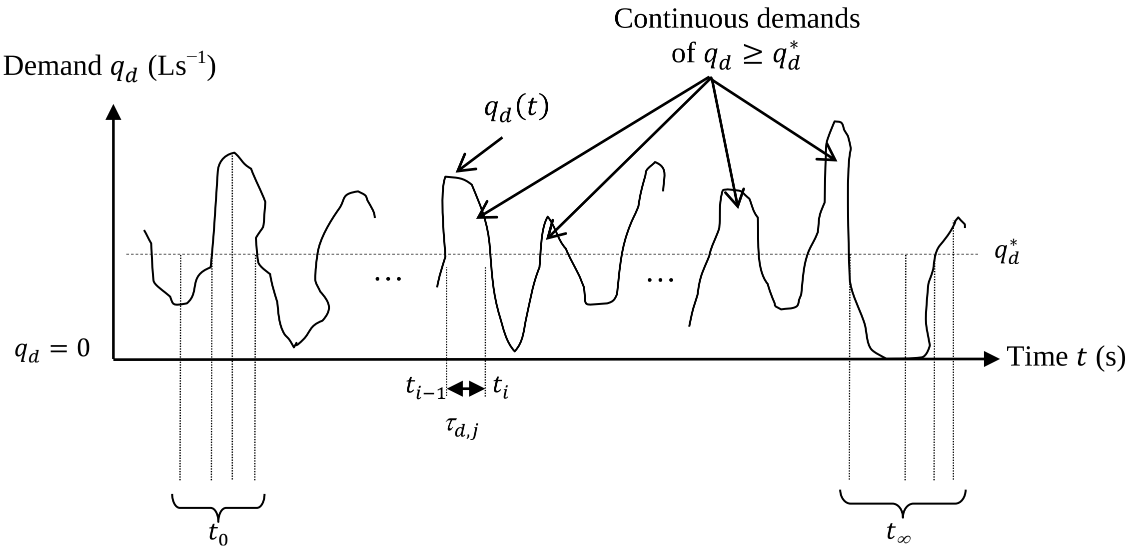


Figure 3: Continuous demands within a period $\tau = [t_0, t_\infty]$ (s)

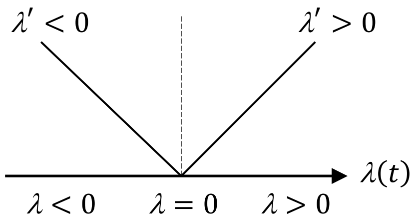


Figure 4: Boundary conditions at t_0 and t_∞

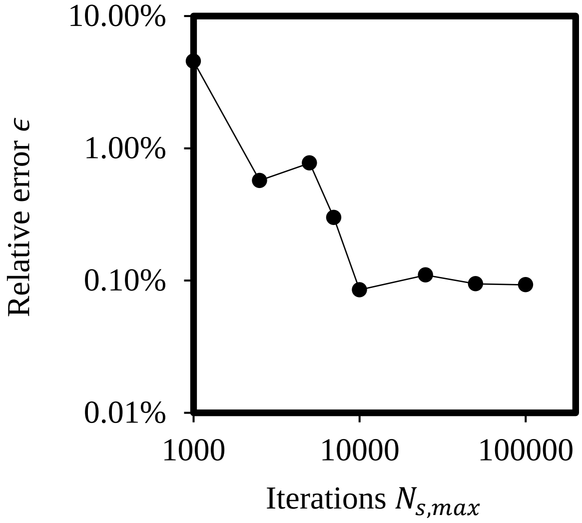
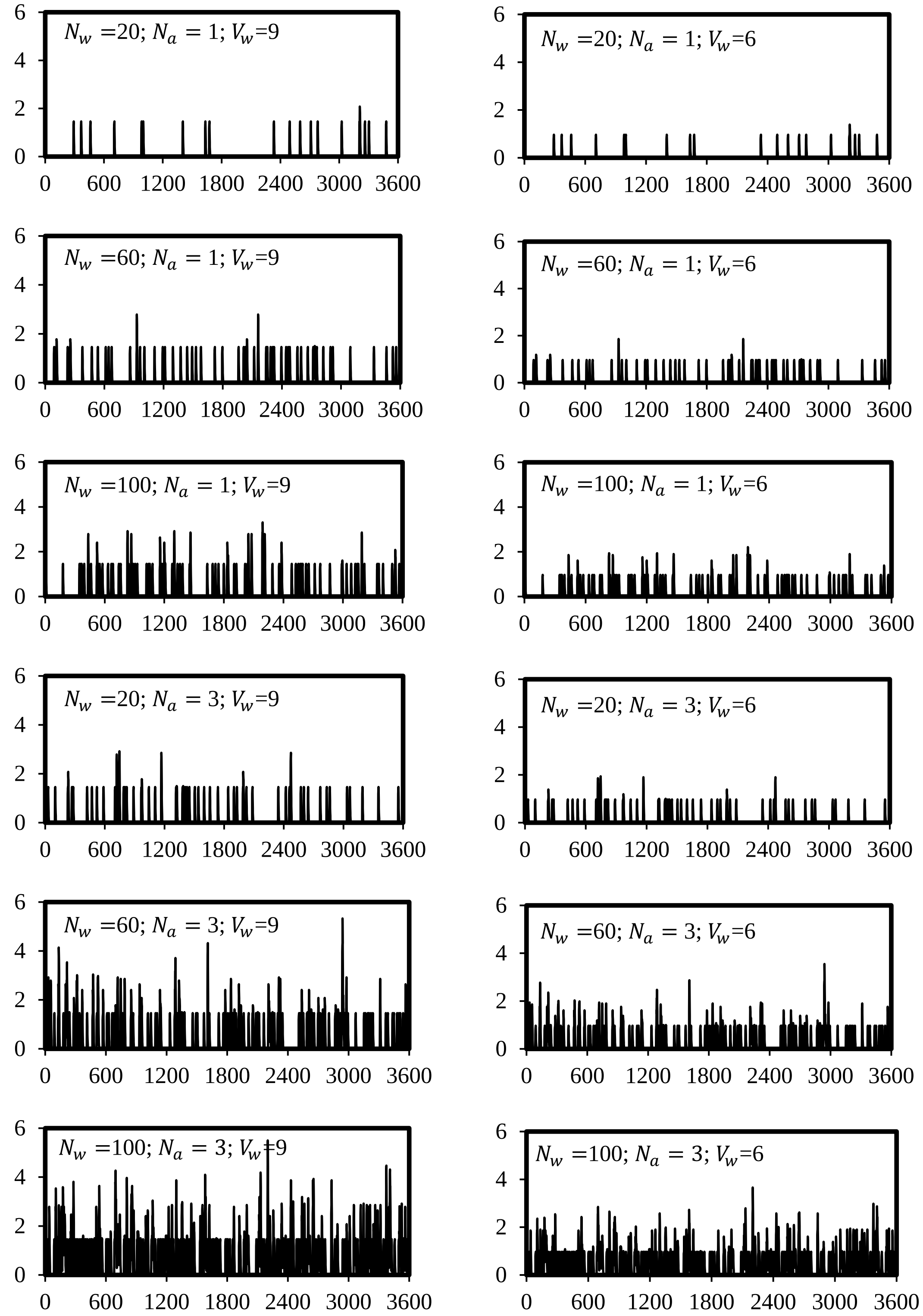
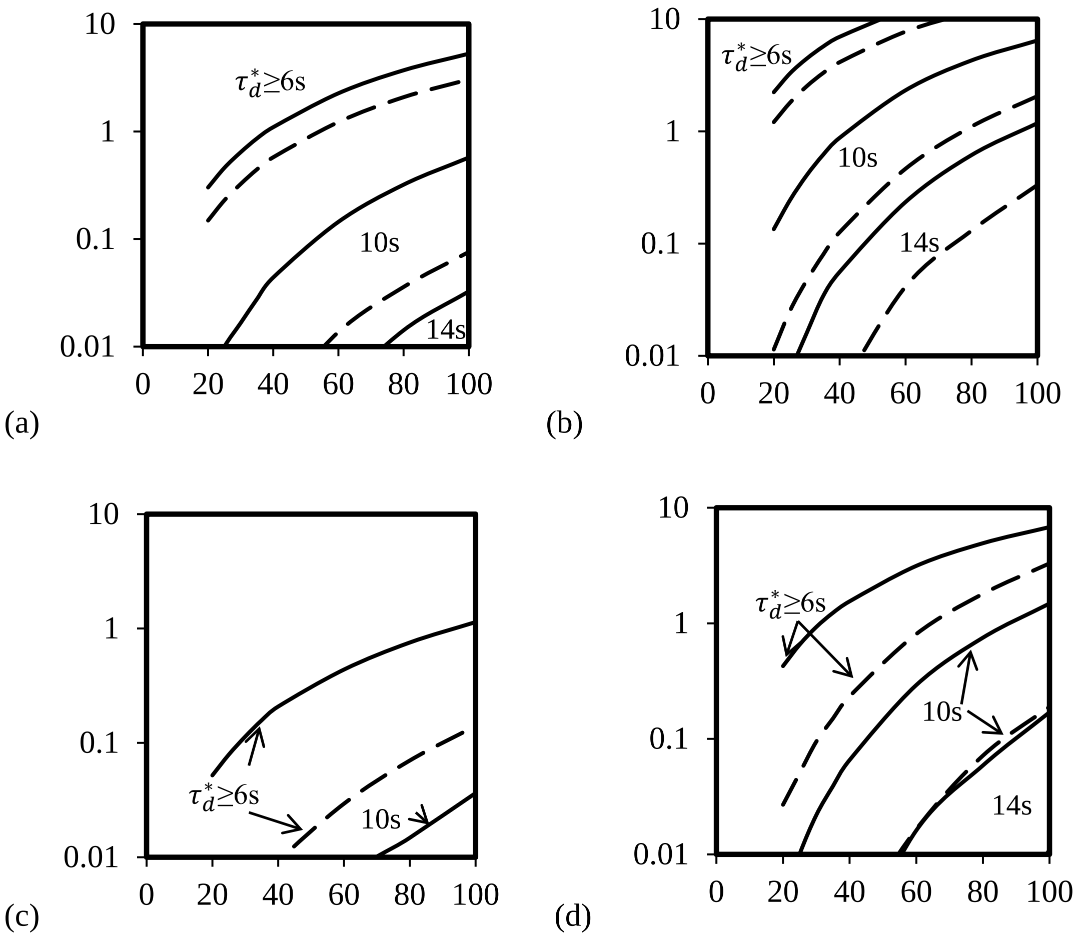


Figure 5: Demands at the end of a stack



x-axis: time t (s); y-axis: demand q_d (Ls^{-1})

Figure 6: Examples of hourly demand patterns



x-axis: Number of WCs installed N_w; y-axis: No. of hourly continuous demands N_f

$q_d^* \geq 1.5$ Ls^{-1}

$q_d^* \geq 2.0$ Ls^{-1}

Figure 7: Expected no. of hourly continuous demands N_f $\left(q_d^*\right)$,

(a) N_a=1, V_w=9; **(b)** N_a=3, V_w=9; **(c)** N_a =1, V_w =6; **(d)** N_a =3, V_w =6

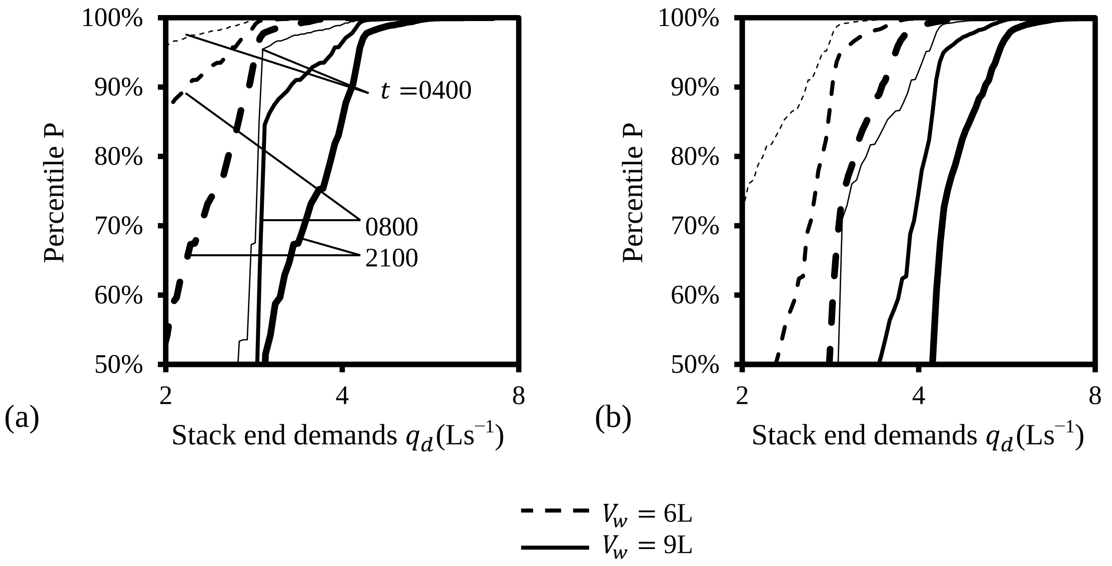


Figure 8: Stack end demands q_d^*, (a) N_w=60; **(b)** N_w=120

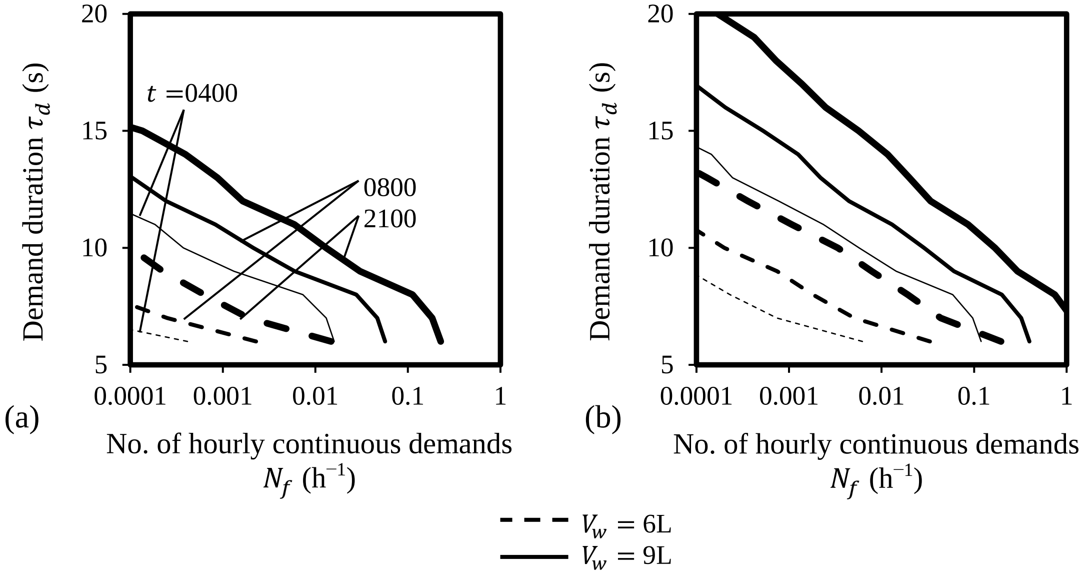


Figure 9: Simulated demand scenarios for $N_f(q_d^* = 2)$**, (a)** N_w=60; **(b)** N_w=120