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A Modified Sequential Multilateration Scheme and Its Application in Geometric Error Measurement of Rotary Axis

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Abstract

Geometric error compensation becomes an essential technology for improving the accuracy of multi-axes machine tools. Laser tracker has been has been considered as an enabling instrument which can be used to measure the geometric error of machine tools due to its high accurate and feasible measurement capability. This paper presents a modified sequential multilateration method for measuring the geometric errors of rotary axes of the machine tools. The method is developed based on a single laser tracker and three targets fixing on the rotary stage. The coordinates of the three targets are measured based on the modified sequential multilateration method using a single laser tracker when the rotary stage is rotated. Then geometric error components are identified based on established geometric error model which takes the positioning error of the targets as input. The validity of the proposed method is verified through both computer simulation and experiments conducted on a multi-axis machine tool.

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Keywords: Geometric error compensation; multilateration method; laser tracker; machine tools; rotary axis.

1. Introduction

Multi-axes machine tools are widely used in modern productions such as automotive and aerospace. There are many factors affecting the accuracy of the machine tools, among which geometric error of the machine tool components are considered as the one of the main contributors to the machining error. Modern machine tools are geometrically calibrated based on a sequence of the measurement of the geometric error of each single component. Therefore, high accurate measurement methods are necessary in order to determine the geometric error of the machine tools.

Extensive research has been conducted on the measurement of the geometric errors of the machine tools for improving the accuracy of the multi-axis machine tools via geometric error compensation [1-3]. Laser interferometer is a widely used instrument to directly measure the geometric components of the axis of the machine tools associated with prismatic joints, 6D sensors, multi-face mirrors, and so on [4-5]. However, the installation and calibration process is complicated especially for large machine tools and the accuracy of the measurement results heavily depends on the experience of the operators. Calibrated 1D or 2D ball bar are also a widely used device to evaluate the geometric error components of machine tools [6-7]. However, the effectiveness is limited by the size of the ball bar and it can only cover limited workspace of a machine tool.

Comparing with the measurement of geometric error of the linear axis, the measurement of the rotary axis is more complicated due to the reason that it's difficult to determine the error components of rotary axis in a single measurement process and the error components are usually measured separately. For example, laser interferometer [8] can measure angular error and tilt error of the rotary axis with self-centering device, optical components and electronic level, but the displacement errors cannot be measured at the same time. So the error of the rotary axis is normally measured using

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precision sphere or mandrel which is rotated together with the rotary axis, and the displacement sensors are used to measure the displacement errors. [9-10]. Liu et al developed a measuring system using a diffraction grating, a laser diode and position sensitive detectors to measure error motions of an indexing table [11]. Park et al developed an optical measurement system with a laser diode, two position sensitive detectors, beam splitters and a turning mirror to measure the six geometric errors of a rotary axis separately [12]. These methods always require long pre-calibration time and the accuracy is heavily depending on the operators. These methods are also related to the type of machine-tool and the structure should be adjusted greatly with different type of machine tools.

Laser tracker is a large scale 3D coordinate measurement instrument which offers the capability of developing high-efficiency methods for geometric error measurement [13-17], such as multilateration and sequential multilateration methods. These methods are able to efficiently measure all the six geometric error components of linear axis or rotary axis. When using these methods to test the rotary axis, the laser tracker is fixed on the rotary axis, and the target reflector moving with the spindle is used as base station. As a result, the method is not applicable if the size of the table of rotary axis is not adequate to install the laser tracker, and the positioning error of the target reflector induced by the motion of the spindle introduces uncertainty to measurement result. A general strategy based on the point measurement method has developed to identify the all the geometric errors of multi-axis machine tools [14, 17]. However, the systemic positioning errors of the measured points are not considered in this method, which will also introduce uncertainty to measurement result.

As a result, this paper focuses on the research of geometric error measurement of rotary axis by using a modified sequential multilateration scheme. During the measuring procedure, the laser tracker itself is used as based station, and the target reflector is fixed on the table of rotary axis, which is more flexible for testing small rotary axis. The mathematical model of this method is presented, and the experiment is conducted to verify the effectiveness of this method.

2. Point measurement based on sequential multilateration method

Traditional multilateration method requires four laser tracker to set up a measuring system, as shown in Fig. 1. S_1, S_2, S_3, and S_4 denote the location of the four laser tracker respectively. A reference coordinate system is established by setting the origin at the S_1. The x-axis passes through S_2, and S_3 lies on the x-y coordinate plane, and the z-axis is perpendicular to the x-y coordinate plane. The 3D coordinates of S_1, S_2 and S_3 are denoted as (0, 0, 0), (x_{S2}, 0, 0), and (x_{S3}, y_{S3}, 0), respectively. Thus, all the coordinates will be determined under this reference coordinate system in the measurement. The forth laser tracker at S_4 is used to provide redundant information for coordinate evaluation.

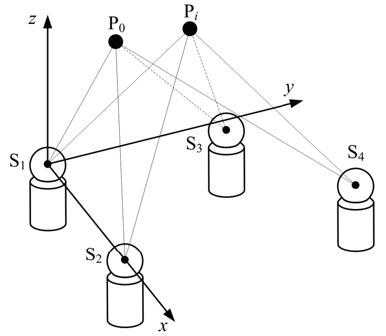

Fig. 1 Schematic diagram of the measurement principle of the multilateration method

Suppose the coordinates of an arbitrary point P_0 is (x_0, y_0, z_0), then the following preserve:

$$\begin{cases}(x_0-0)^2+(y_0-0)^2+(z_0-0)^2=l_1^2\\(x_0-x_{S2})^2+(y_0-0)^2+(z_0-0)^2=l_2^2\\(x_0-x_{S3})^2+(y_0-y_{S3})^2+(z_0-0)^2=l_3^2\\(x_0-x_{S4})^2+(y_0-y_{S4})^2+(z_0-z_{S4})^2=l_4^2\end{cases} \quad (1)$$

where l_j is the absolute distance between point P_0 and the jth laser tracker, j = 1, 2, 3, 4. The P_0 is then moved to a position P_i, then the following preserve:

$$\begin{cases}(x_i-0)^2+(y_i-0)^2+(z_i-0)^2=(l_1+\Delta l_{i1})^2\\(x_i-x_{S2})^2+(y_i-0)^2+(z_i-0)^2=(l_2+\Delta l_{i2})^2\\(x_i-x_{S3})^2+(y_i-y_{S3})^2+(z_i-0)^2=(l_3+\Delta l_{i3})^2\\(x_i-x_{S4})^2+(y_i-y_{S4})^2+(z_i-z_{S4})^2=(l_4+\Delta l_{i4})^2\end{cases} \quad (2)$$

where (x_i, y_i, z_i) is the new coordinate of the P_0 after movement; where Δl_{ij} is the incremental distance of l_j when P0 is moved. Considering that, a laser tracker has much better accuracy in measuring the incremental distance than absolute distance, only the incremental distance Δl_{ij} is considered as known value in the present study. Suppose the P_0 is moved n time, then there will be 4(n+1) equations with (3n+13) unknown parameters. When the P_0 is moved no less than 9 times, i.e. $n \geq 9$, the established equations can be used to determine all the coordinates of the involved points and the inertial parameters of the measuring system. Nonlinear least square optimization can be used to determine all the unknowns if $n > 9$.

There are a total of four laser trackers that should used in the measuring system, which makes the system expensive. As a result, sequential multilateration scheme is proposed. That is, instead of four laser trackers, a laser tracker is sequentially fixed on S_1, S_2, S_3, and S_4 to realize the multilateration algorithm. Hence, the movement of the measured point should be repeated when the laser tracker is moved from a position to another position. As a result, the repeatability of position of the measured point in the whole measurement process will affect the accuracy of the proposed method. The main purpose of the proposed measuring system is to identify the geometric error of the machine tool which is a kind of systematic error. Hence the repeatability of the position of the measured point is considered having random statistics and can be averaged by repeated measurement.

3. Identification of geometric error of rotary axis

Generally, a rotary axis has six geometric error components as shown in Fig. 2, where θ is the angular displacement; $\delta_x(\theta)$ and $\delta_y(\theta)$ are the radial displacement errors; $\delta_z(\theta)$ is the axial displacement error; $\varepsilon_x(\theta)$ and $\varepsilon_y(\theta)$ are the tilt errors; $\varepsilon_z(\theta)$ is the angular displacement error. All the error components are functions of the angular displacement θ. By using the homogeneous transformation matrix, the volumetric error of a point P_0 (x_0, y_0, z_0) on the rotary axis can be given as follows:

$$\begin{bmatrix} \Delta_x(\theta) \\ \Delta_y(\theta) \\ \Delta_z(\theta) \\ 0 \end{bmatrix} = \begin{bmatrix} -\varepsilon_z(\theta)\mathrm{s}\theta & -\varepsilon_z(\theta)\mathrm{c}\theta & \varepsilon_y(\theta) & \delta_x(\theta) \\ \varepsilon_z(\theta)\mathrm{c}\theta & -\varepsilon_z(\theta)\mathrm{s}\theta & -\varepsilon_x(\theta) & \delta_y(\theta) \\ -\varepsilon_y(\theta)\mathrm{c}\theta+\varepsilon_x(\theta)\mathrm{s}\theta & \varepsilon_y(\theta)\mathrm{s}\theta+\varepsilon_x(\theta)\mathrm{c}\theta & 0 & \delta_z(\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} \quad (3)$$

where s and c are the abbreviation of sine and cosine function.

To identify the error components, a three point method is used in the present study [17]. Three targets are mounted on the rotary stage and a point on each target, i.e. a total of three points P, Q, and K are served as fiducials which are used to identify the geometric error of the rotary axis when it rotates. Two coordinate system are established for the ease of the evaluation, including a reference coordinate frame denoted as *oxyz*, and an embedded coordinate frame on the rotary axis denoted as $o_C x_C y_C z_C$. The initial coordinates of P_0, Q_0 and K_0 in $o_C x_C y_C z_C$ are denoted as (x_{CP0}, y_{CP0}, z_{CP0}), (x_{CQ0}, y_{CQ0}, z_{CQ0}) and (x_{CK0}, y_{CK0}, z_{CK0}) respectively. It is emphasized that the three points should not be colinear and should not possess same height along Z axis in $o_C x_C y_C z_C$.

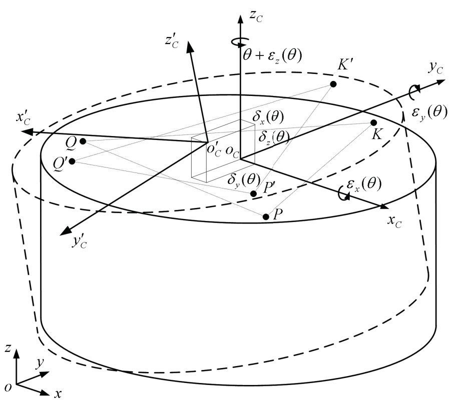

Fig. 2 Geometric error components measurement of C-axis

The targets are rotated along with the rotary axis step by step, and the position of the target points at each step are collected by the methods presented in Section 2 as shown in Fig. 3. Taking point P as an example, the actual coordinates collected by the laser tracker is denoted as P_i (x_{CPi}, y_{CPi}, z_{CPi}) $i=1,\dots N$, where N is the number of collected points. The positioning errors of target point P_i in *i*th step i.e. (Δ_{xPi}, Δ_{yPi}, Δ_{zPi}) can be obtained by Eq. (4) in reference coordinate frame as follows:

$$\begin{bmatrix} \Delta_{xPi} \\ \Delta_{yPi} \\ \Delta_{zPi} \\ 0 \end{bmatrix} = \begin{bmatrix} x_{CP0}c\theta_i - y_{CP0}s\theta_i + x_C \\ x_{CP0}s\theta_i + y_{CP0}c\theta_i + y_C \\ z_{CP0} + z_C \\ 1 \end{bmatrix} - \begin{bmatrix} x_{CPi} \\ y_{CPi} \\ z_{CPi} \\ 1 \end{bmatrix} \quad (4)$$

where (x_c, y_c, z_c) is the coordinate of the origin of the $o_C x_C y_C z_C$ in the reference coordinate frame.

Point Q and K can get the similar equation as Eq. (4) with the same procedure as follows:

$$\begin{bmatrix} \Delta_{xQi} \\ \Delta_{yQi} \\ \Delta_{zQi} \\ 0 \end{bmatrix} = \begin{bmatrix} x_{CQ0}c\theta_i - y_{CQ0}s\theta_i + x_C \\ x_{CQ0}s\theta_i + y_{CQ0}c\theta_i + y_C \\ z_{CQ0} + z_C \\ 1 \end{bmatrix} - \begin{bmatrix} x_{CQi} \\ y_{CQi} \\ z_{CQi} \\ 1 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} \Delta_{xKi} \\ \Delta_{yKi} \\ \Delta_{zKi} \\ 0 \end{bmatrix} = \begin{bmatrix} x_{CK0}c\theta_i - y_{CK0}s\theta_i + x_C \\ x_{CK0}s\theta_i + y_{CK0}c\theta_i + y_C \\ z_{CK0} + z_C \\ 1 \end{bmatrix} - \begin{bmatrix} x_{CKi} \\ y_{CKi} \\ z_{CKi} \\ 1 \end{bmatrix} \quad (6)$$

Substituting Eq. (4) ~ Eq. (6) to Eq. (3) and formulating the equations in matrix format, then following preserves:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & z_{CP0} & -\hat{y}_{CP} \\ 0 & 1 & 0 & -z_{CP0} & 0 & \hat{x}_{CP} \\ 0 & 0 & 1 & \hat{y}_{CP} & -\hat{x}_{CP} & 0 \\ 1 & 0 & 0 & 0 & z_{CQ0} & -\hat{y}_{CQ} \\ 0 & 1 & 0 & -z_{CQ0} & 0 & \hat{x}_{CQ} \\ 0 & 0 & 1 & \hat{y}_{CQ} & -\hat{x}_{CQ} & 0 \\ 1 & 0 & 0 & 0 & z_{CK0} & -\hat{y}_{CK} \\ 0 & 1 & 0 & -z_{CK0} & 0 & \hat{x}_{CK} \\ 0 & 0 & 1 & \hat{y}_{CK} & -\hat{x}_{CK} & 0 \end{bmatrix} \begin{bmatrix} \delta_{xt}(\theta) \\ \delta_{yt}(\theta) \\ \delta_{zt}(\theta) \\ \varepsilon_{xt}(\theta) \\ \varepsilon_{yt}(\theta) \\ \varepsilon_{zt}(\theta) \end{bmatrix} = \begin{bmatrix} \Delta_{Pxt} \\ \Delta_{Pyt} \\ \Delta_{Pzt} \\ \Delta_{Qxt} \\ \Delta_{Qyt} \\ \Delta_{Qzt} \\ \Delta_{Kxt} \\ \Delta_{Kyt} \\ \Delta_{Kzt} \end{bmatrix} \quad (7)$$

where

$$\begin{aligned} \hat{x}_{C*} &= x_{C*0}c\theta_i - y_{C*0}s\theta \\ \hat{y}_{C*} &= x_{C*0}s\theta_i + y_{C*0}c\theta \\ \Delta_{*xi} &= x_{C*i} - \hat{x}_{C*} - x_C \qquad , \quad * = P,\ K,\ Q \\ \Delta_{*yi} &= y_{C*i} - \hat{y}_{C*} - y_C \\ \Delta_{*zi} &= z_{C*i} - z_{C0*} - z_C \end{aligned}$$

All the error components can be evaluated by Eq. (7) based on non-linear optimization.

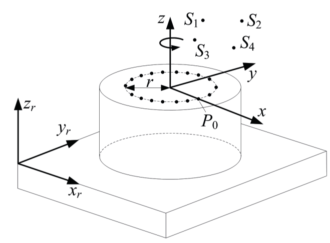

Fig.3: Rotary axis test by using sequential multilateration

4. Simulation and experiments

To demonstrate the validity of the proposed method, several experiments were conducted by using a commercial FARO laser tracker based on the proposed method. The incremental length measurement accuracy of the laser tracker has been approved to possess accuracy better than 0.334μm [13, 17]. The experiment is divided into two parts, including the verification of the point measurement accuracy based on computer simulation, and the performance verification of the proposed method in identification of the geometric error of a rotary stage of a machine tool.

4.1 Verification of the point measurement accuracy of the modified sequential multilateration

Monte Carlo method based computer simulation is conducted to evaluate the measurement uncertainty of the sequential multilateration method in point coordinate measurement. All the measured points are located in the measurement volume of a measuring system with an optimal arrangement as shown in Fig.4.

According to the 3σ criterion, the normal distribution is assigned to these incremental lengths with expectation equal to zero and standard deviation equal to 0.269μm. The statistical distribution of the positioning errors of the measured points is assumed to be normal distributions within [0, 3μm]. The simulation is repeated 10000 times. Finally, the measurement error of point measurement can be estimated.

The simulation results are summarized in table 1, in which the uncertainties of *x*-coordinates, *y*-coordinates and *z*-coordinates are quantified by standard deviation with respect to the number of repeat measurement in each Monte Carlo trial. It is also found that the uncertainty is remarkably improved after averaging the measurement results by repeating the experiment 10 times. The simulation results indicate that sequential multilateration method can meet the accuracy requirement of geometric error measurement of machine tools.

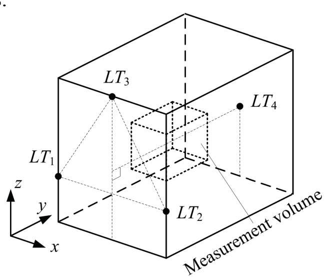

Fig. 4 The optimal arrangement of sequential multilateration

Table 1 Measuring errors obtained by simulation

Number of repeat measurement in each trial	Measurement uncertainty/μm		
	x	*y*	*z*
1	1.17	1.23	1.09
10	0.29	0.32	0.32

4.2 Experiment of geometric error measurement of rotary axis

A rotary stage of a machine tool is tested as shown in Fig. 5. Three target reflectors P, K and Q are fixed on the rotary stage. The laser tracker is fixed on a certain station. The stage rotates step by step with spacing 20°. At each step, the distances between the target reflectors are measured by laser tracker. The measurement is repeated four times by fixing the laser tracker at four different stations. Then the coordinate sequences of these three target reflectors and positioning errors can be determined by using the sequential multilateration algorithm based on the acquired distance data. The initial coordinates of P, K and Q are P_0 (-105.8046, 135.3414, 0), Q_0 (128.8513, 87.8158, 39.9797), K_0 (24.9026, -110.8575, 15.2662) in the rotary stage coordinate frame, and the positioning errors of each target points at each step can be simultaneously obtained. Subsequently, the six geometric error components of C-axis can be evaluated based Eq. (7). The experimental results are illustrated in Fig. 6.

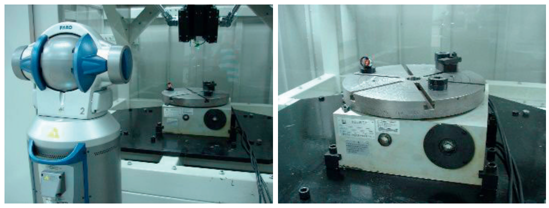

Fig. 5 Experiment of geometric error measurement of C-axis

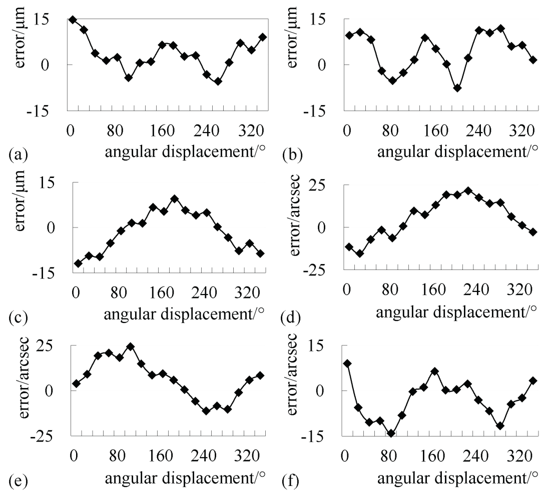

Fig. 6 Experimental results of geometric error measurement of C-axis. (a) radial displacement error $\delta_x(\theta)$, (b) radial displacement error $\delta_y(\theta)$, (c) axial displacement error $\delta_z(\theta)$, (d) tilt error $\varepsilon_x(\theta)$, (e) tilt error $\varepsilon_y(\theta)$, (e) angular error $\varepsilon_z(\theta)$.

To verify the effectiveness of the proposed method, the results are compared with that is obtained by the method presented in ref. 14. It found that both of the two methods generates similar results in evaluation of the six error components. However, the repeatability of the proposed method is much better than that of the method presented ref. 14, taking the error component $\delta_x(\theta)$ as an example as shown in Fig. 7. This is due to the reason that, instead of the laser tracker, the proposed method fixes the measured target reflectors on the rotary stage, which makes the measurement flexible for testing large-sized or small rotary axis. Furthermore, the proposed method is an absolute sef-calibration process, which can avoid that the uncertainty caused by the position of the base stations. In addition, the proposed method use all the measured data in the evaluation of the geometric error components based on nonlinear optimization.

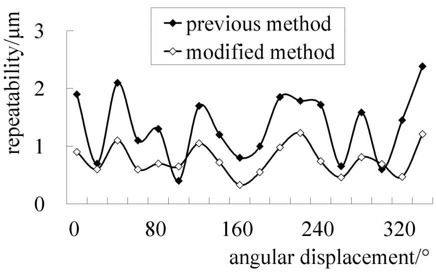

Fig. 7 Experimental results of geometric error measurement of C-axis

5. Conclusion

A modified sequential multilateration scheme is developed for geometric error measurement of rotary axis in this paper. Different to previous methods, the method fixes the measured target points on the table of rotary axis and considers the laser tracker's stations as base station in multilateration algorithm. As a result, the proposed modified scheme is more flexible for testing large-sized or small rotary axis. In addition, the proposed method is an absolute self-calibration process, which can avoid that the uncertainty of the positioning error of the base station which would introduce uncertainty to the measurement results. The mathematical model of this scheme is presented and simulation is conducted to confirm that the accuracy of coordinate measurement can meet the requirement of calibrating machine tools. The experiments indicate that this modified sequential multilateration scheme can effectively measure all the six geometric error components of rotary axis.

References

[1] Cui G W, Lu Y, Gao D, Yao Y X. A novel error compensation implementing strategy and realizing on Siemens 840D CNC systems. International Journal of Advanced Manufacturing Technology, 2012; 61: 595–608.

[2] Guo Q, Sun Y W, Guo D M. Analytical modeling of geometric errors induced by cutter runout and tool path optimization for five-axis flank machining. Science China – Technological Sciences, 2011; 54: 3180-3190.

[3] Cheng Q, Liu G B, Liu Z F, Xuan D S, Chang W F. An identification approach for key geometric error Sources of machine tool based on sensitivity analysis. Chinese Journal of Mechanical Engineering, 2012; 48: 171-179.

[4] Lau K, Ma Q, Chu X, Liu Y, Olson S.An advanced 6-degreeof-freedom laser system for quick CNC machine and CMM error mapping and compensation. WIT Transactions on Engineering Sciences, 1999; 23: 421–434.

[5] Castro H F F. A method for evaluating spindle rotation errors of machine tools using a laser interferometer. Measurement, 2008; 41: 526–537.

[6] Bringmann B, Kung A, Knapp W. A measuring artefact for true 3D machine testing and calibration. CIRP Annals - Manufacturing Technology, 2005; 54: 471-474.

[7] Trapet E, Aguilar Martin J, Yague J, et al. Self-centering probes with parallel kinematics to verify machine-tools. Precision Engineering, 2006; 30: 165-179.

[8] Schwenke H, Knapp W, Haitjema H, Weckenmann A, Schmitt R, Delbressine F. Geometric error measurement and compensation of machines - an update. Annals of the CIRP, 2008; 57: 660–675.

[9] Huang C X, Cao Q X, Fu Z, Leng C T. Submicron radial runout error compensation method for the turntable in a wafer prealigner. Chinese Journal of Mechanical Engineering, 2008; 44: 123-127.

[10] Castro H F F. A method for evaluating spindle rotation errors of machine tools using a laser interferometer. Measurement, 2008; 41: 526~537.

[11] Liu C H, Jywe W Y, Shyu L H , Chen C J. Application of a diffraction grating and position sensitive detectors to the measurement of error motion and angular indexing of an indexing table. Precision Engineering, 2005; 29: 440-448.

[12] Park S R, Hoang T K, Yang S H. A new optical measurement system for determining the geometrical errors of rotary axis of a 5-axis miniaturized machine tool. Journal of Mechanical Science Technology, 2010; 24: 175-179.

[13] Umetsu K, Furutnani R, Osawa S, Takatsuji T, Kurosawa T. Geometric calibration of a coordinate measuring machine using a laser tracking system. Measurement Science Technology, 2005; 16: 2466-2472.

[14] Wang J D, Guo J J, Zhang G X, Guo B A, Wang H J. The technical method of geometric error measurement for multi-axis NC machine tool by laser tracker. Measurement Science Technology, 2012; 23: 045003.

[15] Aguado S, Samper D, Santolaria J, Aguilar J J. Identification strategy of error parameter in volumetric error compensation of machine tool based on laser tracker measurements. International Journal of Machine Tools & Manufacture, 2012; 53: 160-169.

[16] Schwenke H, Schmitt R, Jatzkowski P, Warmann C. On-the-fly calibration of linear and rotary axes of machine tools and CMMs using a tracking interferometer. CIRP Annals – Manufacturing Technology, 2009; 58: 477-480.

[17] Zhang Z J, Hu H. A general strategy for geometric error identification of multi-axis machine tools based on point measurement. International Journal of Advanced Manufacturing Technology, 2013; 69: 1483-1497.