Paired-Relay Selection Schemes for Two-way Relaying with Network Coding

Yunxiang Jiang*, Francis C.M. Lau*, Zeeshan Sattar*, Ivan W. H. Ho*, and Qing F. Zhou†

*Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong

†Communication Engineering Department, School of Computer and Information, HeFei University of Technology (HFUT), China

Email: yunxiang.jiang@connect.polyu.hk, encmlau@polyu.edu.hk, zeeshan.sattar@connect.polyu.hk, ivanwh.ho@polyu.edu.hk, enqfzhou@ieee.org

Abstract

In order to exchange information between two sources in a two-way relaying network with multiple potential relays, most researches focus on two-hop relay system with Single Relay Selection (SRS) scheme. In SRS scheme, only one relay is selected among multiple relays to broadcast information to both sources. Comparing to SRS scheme, we first design a Paired-Relay Selection (PRS) scheme in which a pair of "best" relays broadcast network-coded information to other nodes (source or relay). We also propose an optimal selection algorithm and a suboptimal

This work was presented in part at IEEE Wireless Communication and Networking Conference (WCNC), April, 2013.

Y. Jiang, Francis C. M. Lau, Zeeshan Sattar and Ivan W. H. Ho are with Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Hong Kong, China. (emails:yunxiang.jiang, encmlau@connect.polyu.hk, zeeshan.sattar@connect.polyu.hk).

Qing F. Zhou is with Communication Engineering Department, School of Computer and Information, HeFei University of Technology (HFUT), China. (email:enqfzhou@ieee.org).

algorithm that select the pair of "best" relays in the PRS scheme and we describe how the nodes exchange information in a frame consisting of 4 time-slots. In order to compare these two relay selection schemes, we assume that relays are uniformly distributed in both one-dimensional and two-dimensional space between two sources. Both our analytical and simulation results show that when the path-loss exponent is large and/or there is a sufficient number of relays to choose from, using two relay nodes can provide a lower outage compared with using only one relay node even under the same total transmit power in uniformly distributed relay networks. In addition, in order to reduce the overhead of the PRS scheme, we propose an iterative-PRS (I-PRS) scheme in which the paired-relay is selected in an iterative and opportunistic way. Simulation results show that the I-PRS scheme has nearly the same outage performance as the PRS scheme under time-invariant channels and significantly outperforms the PRS scheme under time-varying channels.

Index Terms

Network coding, paired-relay selection, relay selection, two-way relaying.

I. INTRODUCTION

Over the past decade, various aspects of wireless cooperative channels and networks such as information-theoretic capacity [1], diversity [2], outage performance [3], [4], and cooperative coding [5] have been investigated. Relay selection (RS) [6] is a promising scheme to achieve diversity gain and is easy to be implemented. A lot of work has been done on the issue of RS and the achievable diversity order of RS [7], [8], [9] in cooperative communication systems.

A. Related Work

In [7], Zhao *et al.* have proposed an Optimal Power Allocation (OPA) scheme for the amplify-and-forward (AF) protocol in cooperative communication networks. They have also proposed a criterion for selecting the "best" relay to participate in a transmission. They have proved analytically and also with the help of simulations that the single "best" relay achieves the same maximum diversity order of $(K+1)$ with K relays. They also claimed that Single-Relay-Selection (SRS) scheme and two-hop relay system can achieve the maximum diversity order. Moreover, the SRS scheme achieves a higher instantaneous throughput and lower outage probability than the m-relay scenario. This result, however, is based on the assumption that all channels have equal gain, which might not be realistic in practice. In [8], Jing *et al.* have proposed the idea of multiple-relay selection. They have introduced the idea of relay ordering for selecting more than one relay in the AF protocol. It has been proved that multiple-relay selection using relay ordering achieves full diversity and performs better than single-relay selection in terms of Bit Error Rate (BER). Yet, they impose no constraint on the total transmit power of the cooperative relay network. Therefore, the total transmit power increases as the number of cooperative relays increases. Furthermore, they have assumed that all cooperative relays work in parallel and are perfectly synchronized. In [9], Yi and Kim have proposed an amplify-and-forward relay-ordering (RO) strategy, in which data transmission is achieved using K relays and $(K+1)$ hops. An approximated closed-form end-to-end signal-to-noise ratio (SNR) in a two-relay case has been derived. In [10], RS and RO have been jointly considered in a two-relay system. RS and RO can complement each other because the former is more spectrally efficient while the latter is more energy efficient. The results in [10] have indicated that RO is preferred for severely attenuating channels while RS is preferred for less attenuating channels.

At the same time, network coding is a promising method aiming to improve the throughput of communication systems [11], [12], [13]. In the two-way relaying channel in [14], [15], which is a special case of cooperative communication networks, network coding has been applied. The network consists of two users who communicate with each other with the help of a single relay, and network coding has been shown to improve the network capacity, diversity gain and network efficiency of such a wireless two-way relay network [14], [15]. In [14], a physical-layer network coding method has been presented to improve the network efficiency further. It has been demonstrated that the joint use of relay and network coding not only improves the information transmission efficiency [13], but also reduces the overall power consumption significantly in communication systems [14].

In [12], [16], [17], RS with network coding for two-way relaying has been studied under a multiple-relay scenario. A denoise-and-forward network coding opportunistic relaying scheme has been proposed in

[12] while a decode-and-forward two-way relaying with network coding and opportunistic relay selection has been proposed in [16]. Both schemes have been shown to achieve the full diversity gain.

B. Motivations and Contributions

Most researches in SRS scheme focus on diversity gain and do not consider the influence of path-loss. However, in practice, path-loss is an unavoidable issue in a wireless channel. In [10], it has been claimed that in a system with two relays, RO is preferred to RS in severely attenuating channels or channels with large path-loss factors. In a multiple-relay network, there is no doubt that when two sources are very far away, more relays are needed to forward information rather than a single relay. But it is still an open problem in the number of relays needed and the ordering of the selected relays. In [9], an RO strategy has been proposed for a multi-hop relay network but no theoretical results has been derived. In a two-way relay network, using network coding will save more time slots in a multi-hop than in a dual-hop scenario but the problem becomes more complex. Also multi-hop two-way relay networks are not as well studied as the dual-hop ones. The design of a practical protocol for a multi-hop bidirectional relay under fading conditions is also an open problem. In this paper, in order to evaluate the performance of a multi-hop two-way relay network, we design a protocol to select two relays. Consequently the information exchange between the two sources needs three hops. The main contributions of this paper are described as follows.

- We design a paired-relay selection scheme, in which the "best" pair of relays are selected by a novel Distributed Pair Selection Algorithm (DPaSA). Based on the decode-and-forward (DF) relaying mechanism, the selected relays are used to broadcast network-coded information to other nodes (source or relay) using a network coding scheme. The DPaSA algorithm exploits the conventional max-min criterion and uses it as a basis to select the best pair of relays. We show that this max-min criterion is the best criterion for the decode-and-forward relaying scheme. We also present two different network coding schemes for the selected relays to broadcast their network-coded information.
- In the DPaSA, all relays are involved during the selection of the best pair of relays and the trans-

mission paths. As a result, the computational overhead increases with the number of relays in the system. To reduce the overhead, we propose a Partial-DPaSA (P-DPaSA) algorithm. In the P-DPaSA algorithm, we allow only a limited number of relays with "good" channel conditions to broadcast their information to other nodes. Hence, the overhead can be reduced significantly. We further investigate the tradeoff between computational overhead and outage performance of a generic two-way relaying network.

- We consider both one-dimensional and two-dimensional relay distributions when evaluating the performance of the network. Our outage performance analysis on the proposed paired-relay selection (PRS) scheme shows that, when the path-loss exponent is large and/or there are sufficient relays to choose from, using two relay nodes provides a lower outage than using only one relay node, which is also known as the single relay selection (SRS) scheme. We show that when the number of relays in the network is very large, the outage performance of P-DPaSA is almost the same as that of DPaSA. Yet, P-DPaSA requires a much lower overhead.
- We further propose an iterative paired-relay-selection (I-PRS) scheme, in which the "best" pair of relays are selected in an opportunistic and iterative manner without incurring any overhead. In other words, the I-PRS scheme works similar to an opportunistic SRS scheme (i.e., selection of a single relay in an iterative manner). It is also proved that the I-PRS scheme converges to the PRS scheme after several rounds of iterations. Therefore, the I-PRS scheme provides almost the same performance as the PRS scheme but without any additional overhead. In addition, the I-PRS scheme significantly outperforms the PRS scheme if the channels change dynamically with time.

The remainder of this paper is organized as follows. Section II describes the system model. In Section III, we propose the paired-relay selection scheme, and give theoretical analysis and simulation results. The iterative paired-relay selection is proposed in Section IV. Conclusions are drawn in Section VI.

II. SYSTEM MODEL

A. System Model

We consider a two-way relay network in a time-division half-duplex mode. There are two source nodes S_1 and S_2, and a set of K relay nodes, denoted as $\mathcal{R} = \{R_i : i = 1, \ldots, K\}$. We also define $\mathcal{K} = \{1, \ldots, K\}$ as the index set of the relays. We assume that the distance between the two sources is large or that there are obstacles between the sources. Thus, the sources have to exchange their information with the help of one or more relay nodes.

We denote the distance between (i) S_1 and S_2 by D; (ii) S_1 and R_i by $d_{S_1,i}$; (iii) S_2 and R_i by $d_{S_2,i}$; and (iv) R_i and R_j by $d_{i,j}$. We consider path loss between the nodes and we model the channel coefficients as variables following complex Gaussian distributions, i.e., $\mathcal{CN}(0, \Omega)$ where $\Omega = (d/d_0)^{-\alpha}$, α is the path-loss exponent (typically ranging from 2 to 4), d_0 is a reference distance, and d is the distance between two nodes [18]. We assume that all the channels are reciprocal, which means that the channel coefficients are the same in both directions. We further define the complex channel coefficient between (i) S_1 and R_i as $f_i \sim \mathcal{CN}(0, \Omega_{S_1,i})$, (ii) S_2 and R_i as $g_i \sim \mathcal{CN}(0, \Omega_{S_2,i})$, and (iii) R_i and R_j as $h_{i,j} \sim \mathcal{CN}(0, \Omega_{i,j})$; and we assume that the channel is a block fading one, which means that the channel coefficients remain fixed over a channel coherence time. In subsequent sections, unless otherwise stated, all signal transmissions occur within one channel coherence time. It has been proved that $|f_i|$, $|g_i|$ and $|h_{i,j}|$ follow Rayleigh distribution with parameters $\sqrt{\Omega_{S_1,i}}$, $\sqrt{\Omega_{S_2,i}}$ and $\sqrt{\Omega_{i,j}}$, respectively [19]. It has also been proved that $|f_i|^2$, $|g_i|^2$ and $|h_{i,j}|^2$ follow exponential distribution with parameters $\Omega_{S_1,i}$, $\Omega_{S_2,i}$ and $\Omega_{i,j}$, respectively. Moreover, we assume that the noise at all nodes (sources or relays) is complex and identical, and follows a complex Gaussian distribution with zero mean and variance σ^2, i.e., $\mathcal{CN}(0, \sigma^2)$.

B. Review of Single Relay Selection Scheme

Single relay selection (SRS) has been widely studied in cooperative relay networks [6]. When there are multiple relays available, there are several ways to select the "best" relay including the max-min criterion

[2], max-harmonic-mean criterion [6], max-generalized-min criterion [20], nearest-neighbor criterion [21] and max-received-SNR (signal-to-noise-ratio) criterion [22]. For example, in the max-min criterion, the "best" relay R_r is selected using

$$r = \arg\max_{i \in \mathcal{K}} \{\min\{|f_i|^2, |g_i|^2\}\} \tag{1}$$

where f_i and g_i represent the channel coefficients defined in the previous section.

The transmission rate when S_1 broadcasts to R_r and that when S_2 broadcasts to R_r are given, respectively, by

$$T_{S_1,r} = \log_2 \left(1 + \frac{P_{S_1}}{\sigma^2}|f_r|^2\right) \tag{2}$$

and

$$T_{S_2,r} = \log_2 \left(1 + \frac{P_{S_2}}{\sigma^2}|g_r|^2\right). \tag{3}$$

As the relay broadcasts the network-coded data to both sources, the transmission rate of this broadcast channel can be formulated as

$$T_{r,(S_1,S_2)} = \min\left[\log_2\left(1 + \frac{P_r}{\sigma^2}|f_r|^2\right), \log_2\left(1 + \frac{P_r}{\sigma^2}|g_r|^2\right)\right]. \tag{4}$$

Consequently, the end-to-end transmission rate, which is given by the minimum of the transmission rate among all links, can be expressed as

$$T_{\text{SRS}} = \frac{2}{3}\min\left[T_{S_1,r}, T_{S_2,r}, T_{r,(S_1,S_2)}\right]. \tag{5}$$

In the above equation, the coefficient $2/3$ exists because three timeslots have been taken to exchange the two data packets between S_1 and S_2. Finally, the outage probability of the SRS two-way relaying system with network coding can be expressed as

$$P_{\text{out,SRS}} = \Pr[T_{\text{SRS}} < T_0] \tag{6}$$

where T_0 denotes the target end-to-end transmission rate in bits per second per hertz (b/s/Hz). With equal transmit powers for all nodes, i.e., $P_{S_1} = P_{S_2} = P_r = P/3$ where P is the total power, the minimum of the transmission rate among all links, i.e., (5), can be rewritten as

$$T_{\text{SRS}} = \frac{2}{3}\min\left[\log_2\left(1+\frac{P}{3\sigma^2}|f_r|^2\right), \log_2\left(1+\frac{P}{3\sigma^2}|g_r|^2\right)\right]. \tag{7}$$

Hence, the outage probability in (6) can be simplified to

$$P_{\text{out,SRS}} = \Pr(\min\{|f_r|^2, |g_r|^2\} \leq G(T_0, 3)) \tag{8}$$

where $G(T_0, l) = \frac{l\sigma^2}{P}(2^{lT_0}-1)$ and l represents the total number of transmission timeslots in one complete exchange. $P_{\text{out,SRS}}$ can be expressed as [19]

$$P_{\text{out,SRS}} = \prod_{i=1}^{K}(1-e^{-G(R_0,3)(1/\Omega_{S_1,i}+1/\Omega_{S_2,i})}). \tag{9}$$

III. Paired-Relay Selection Scheme

In this section, we consider a paired-relay selection (PRS) scheme, in which a pair of relays is selected to help the exchange of information between S_1 and S_2. We first propose an optimum selection algorithm. Then, in order to reduce the complexity and overhead of the optimum algorithm, we propose a suboptimum selection algorithm. Furthermore, we simulate the outage performance of the two algorithms.

A. Selection Scheme and Information Exchange

1) Selection Schemes: We apply the max-min criterion when selecting the best path between the two sources. Hence, we will select the best pair of relays denoted by $\{R_{p1}, R_{p2}\}$ where

$$(p_1, p_2) = \underset{(i,j):i,j\in\mathcal{K},i\neq j}{\arg\max}\{\min\{|f_i|^2, |g_j|^2, |h_{i,j}|^2\}\}. \tag{10}$$

To efficiently select the best pair of relays, we propose an optimum decentralized protocol named as Distributed Pair Selection Algorithm (DPaSA). The details of this algorithm are shown in ***Algorithm 1***.

2) Reduced-Overhead Selection Scheme: In the DPaSA algorithm, all relays that can decode the training data from Source S_1 correctly will broadcasts its own identity and the value $|f_i|^2$ to all other relays. The number of such relays is given by the size of Δ_1 and is expected to increase with the total number of relays in the network, i.e., K. If K is large, much time will be spent by relays in Δ_1 to broadcast their

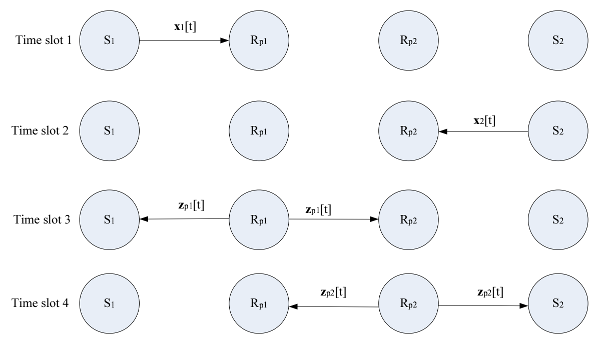


Fig. 1. Information exchange between the nodes within each frame.

information. To reduce the computational complexity and overhead, we further propose a sub-optimum protocol called Partial-DPaSA (P-DPaSA). The P-DPaSA protocol is very similar to the DPaSA protocol, except that only $M \leq K$ relays in Δ_1 possessing the smallest back-off timer (equivalently largest $|f_i|^2$) are allowed to broadcast their information. The only required change to DPaSA is therefore to limit the number relays in Δ_1 that can broadcast their data to $M \leq K$ in Step 2 of ***Algorithm 1***.

3) Information Exchange: After the "best" pair of relays $\{R_{p1}, R_{p2}\}$ has been determined, the information exchange between the two sources is arranged into frames, each of which consists of 4 timeslots. In the following, we give two different network-coding schemes by which the sources can complete their information exchange in a frame.

Type-I Network-Coding Scheme: Referring to Fig. 1, we consider the tth frame. In the first timeslot, S_1 transmits its data $\mathbf{x}_1[t]$ to R_{p1}. In the second timeslot, S_2 transmits its data $\mathbf{x}_2[t]$ to R_{p2}. In the third

Algorithm 1 Distributed Pair Selection Algorithm (DPaSA)

1: Source S_1 broadcasts its training data and all the relays listen to the training data.

2: We denote the set of relays that can decode the training data correctly by Δ_1. Each R_i in Δ_1 estimates the channel coefficient f_i and computes a back-off timer using $\lambda/|f_i|^2$ where λ is a constant [6]. When the timer expires, R_i broadcasts its own identity and the value $|f_i|^2$ to all other relays. Assuming that no two relays in Δ_1 have the same back-off timer values, the relays in Δ_1 broadcast one after another. All relays listen to the data broadcasted from relays in the set Δ_1.

3: We denote the set of relays that can decode correctly the data from any relay in Δ_1 by Δ_2. If R_j in Δ_2 can decode correctly the data from R_i, it estimates the channel coefficient $h_{i,j}$ and calculates $\min\{|f_i|^2, |h_{i,j}|^2\}$. After all the broadcasts from Δ_1 are completed, R_j selects its "best" partner $R_{i(j)}$ based on the max-min criterion, i.e., $i(j) = \arg\max_{i\in\mathcal{K}_1}\{\min\{|f_i|^2, |h_{i,j}|^2\}\}$ where $\mathcal{K}_1$ is the index set of relays in Δ_1. Each R_j in Δ_2 also stores the intermediate value $|\hat{h}_j|^2 = \min\{|f_{i(j)}|^2, |h_{i(j),j}|^2\}$.

4: Source S_2 broadcasts its training data and all the relays in Δ_2 listen to the training data.

5: Each R_j in Δ_2 estimates the channel coefficient g_j and calculates the max-min function $\max\{\min\{|g_j|^2, |\hat{h}_j|^2\}\}$. Each R_j further computes a back-off timer using $\lambda/\max\{\min\{|g_j|^2, |\hat{h}_j|^2\}\}$ and the "best" relay is the one with the smallest back-off timer. In other words, the "best" relay $R_{j(S)}$ is selected opportunistically based on the max-min criterion, i.e., $j(S) = \arg\max_{j\in\mathcal{K}_2}\{\min\{|g_j|^2, |\hat{h}_j|^2\}\}$ where $\mathcal{K}_2$ is the index set of relays in Δ_2. Moreover, when the timer of the "best" relay expires, the relay broadcasts its own identity and identity of "best" partner $i(j)$ to Source S_2 and "best" partner $R_{i(j)}$. Then other relays stop their operations.

6: S_2 receives the message from $R_{j(S)}$ and obtains the identities of the "best" paired-relay. $R_{i(j)}$ receives the message and further relays the message to S_1.

timeslot, R_{p1} broadcasts the XORed data $\mathbf{z}_{p1}[t] = \mathbf{x}_1[t] \oplus \mathbf{z}_{p2}[t-1]$ to S_1 and R_{p2}, where $\mathbf{z}_{p2}[t-1]$ denotes the data broadcasted by R_{p2} during the $(t-1)$th frame. Then, in the fourth timeslot, R_{p2} broadcasts the XORed data $\mathbf{z}_{p2}[t] = \mathbf{x}_2[t] \oplus \mathbf{z}_{p1}[t]$ to S_2 and R_{p1}. Based on the above transmission scheme, S_1 can decode the data from S_2 using

$$\mathbf{x}_2[t] = \mathbf{z}_{p1}[t+1] \oplus \mathbf{x}_1[t+1] \oplus \mathbf{z}_{p1}[t] \tag{11}$$

while S_2 decodes the data from S_1 using

$$\mathbf{x}_1[t] = \mathbf{z}_{p2}[t] \oplus \mathbf{x}_2[t] \oplus \mathbf{z}_{p2}[t-1]. \tag{12}$$

In this scheme, each relay broadcasts the network-coded version of the *two* incoming packets. Moreover, each source decodes the information from the other source based on *two* incoming packets and one of its own transmitted packets.

Type-II Network-Coding Scheme: Referring to Fig. 1 again, we consider the tth frame. In the first timeslot, S_1 broadcasts its data $\mathbf{x}_1[t]$ to R_{p1}. In the second timeslot, S_2 broadcasts its data $\mathbf{x}_2[t]$ to R_{p2}. In the third timeslot, R_{p1} broadcasts the XORed data $\mathbf{z}_{p1}[t] = \mathbf{x}_1[t] \oplus \mathbf{z}_{p2}[t-1] \oplus \mathbf{x}_1[t-1]$ to S_1 and R_{p2}, where $\mathbf{z}_{p2}[t-1]$ denotes the data broadcasted by R_{p2} during the $(t-1)$th frame. Then, in the fourth timeslot, R_{p2} broadcasts the XORed data $\mathbf{z}_{p2}[t] = \mathbf{x}_2[t] \oplus \mathbf{z}_{p1}[t] \oplus \mathbf{x}_2[t-1]$ to S_2 and R_{p1}. Based on the expressions of $\mathbf{z}_{p1}[t]$ and $\mathbf{z}_{p2}[t]$, we can readily show that $\mathbf{z}_{p1}[t] = \mathbf{x}_1[t] \oplus \mathbf{x}_2[t-1]$ and $\mathbf{z}_{p2}[t] = \mathbf{x}_1[t] \oplus \mathbf{x}_2[t]$. Then, based on the above transmission scheme, S_1 can decode the data from S_2 using,

$$\mathbf{x}_2[t] = \mathbf{z}_{p1}[t+1] \oplus \mathbf{x}_1[t+1] \tag{13}$$

while S_2 decodes the data from S_1 using

$$\mathbf{x}_1[t] = \mathbf{z}_{p2}[t] \oplus \mathbf{x}_2[t]. \tag{14}$$

In this scheme, each relay broadcasts the network-coded version of the *three* incoming packets. Moreover, each source decodes the information from the other source based on *one* incoming packet and one of its own transmitted packets.

Compared with the Type-I scheme, the Type-II scheme requires more complex computations at the relays but simpler computations at the sources. Note also that the two given network-coding schemes are not exhaustive. There are other network-coding schemes that can allow the two sources to exchange information completely via the pair of relays in 4 timeslots.

Note that R_{p1} and R_{p2} has the probability of receiving information from S_2 and S_1, respectively such that they can utilize Maximal Ratio Combine (MRC) or some other techniques to exploit higher transmission rate. However, in this paper, we only consider the relay using the instantaneous decoding method, i.e., receiving and directly decoding, which is simple and fast.

B. Transmission Rate and Outage Probability

The transmission rate when S_1 broadcasts to R_{p1} and that when S_2 broadcasts to R_{p2} are given by, respectively,

$$T_{S_1,p1} = \log_2\left(1 + \frac{P_{S_1}}{\sigma^2}|f_{p1}|^2\right) \tag{15}$$

and

$$T_{S_2,p2} = \log_2\left(1 + \frac{P_{S_2}}{\sigma^2}|g_{p2}|^2\right). \tag{16}$$

Further, the transmission rate when R_{p1} broadcasts to S_1 and R_{p2} equals

$$\begin{aligned} T_{p1,(S_1,p2)} \quad = \quad \min\Bigg[&\log_2\left(1 + \frac{P_{p1}}{\sigma^2}|f_{p1}|^2\right), \\ &\log_2\left(1 + \frac{P_{p1}}{\sigma^2}|h_{p1,p2}|^2\right)\Bigg] \end{aligned} \tag{17}$$

while that when R_{p2} broadcasts to S_2 and R_{p1} is computed from

$$\begin{aligned} T_{p2,(S_2,p1)} \quad = \quad \min\Bigg[&\log_2\left(1 + \frac{P_{p2}}{\sigma^2}|g_{p2}|^2\right), \\ &\log_2\left(1 + \frac{P_{p2}}{\sigma^2}|h_{p1,p2}|^2\right)\Bigg]. \end{aligned} \tag{18}$$

Therefore, the end-to-end transmission rate for the proposed paired-relay selection (PRS) scheme equals

$$T_{\text{PRS}} = \frac{2}{4}\min\{T_{S_1,p1}, T_{S_2,p2}, T_{p1,(S_1,p2)}, T_{p2,(S_2,p1)}\} \tag{19}$$

where the coefficient $2/4$ exists because four timeslots are used to exchange the two data packets transmission between S_1 and S_2. Consequently, the outage probability of this two-way relaying system with network coding can be expressed as

$$P_{\text{out,PRS}} = \Pr[T_{\text{PRS}} < T_0]. \tag{20}$$

Equal Transmit Power: Suppose the transmit powers of all nodes are the same and are denoted by $P/4$. Using the results in (15) to (18), (19) is simplified to

$$T_{\text{PRS}} = \frac{2}{4}\min\left[\log_2\left(1+\frac{P}{4\sigma^2}|f_{p1}|^2\right), \log_2\left(1+\frac{P}{4\sigma^2}|g_{p2}|^2\right),\right. \left.\log_2\left(1+\frac{P}{4\sigma^2}|h_{p1,p2}|^2\right)\right] \tag{21}$$

and the outage probability in (20) can be simplified to

$$\begin{aligned} &P_{\text{out,PRS}} \\ &= \Pr[\min\{|f_{p1}|^2, |g_{p2}|^2, |h_{p1,p2}|^2\} < G(T_0,4)] \\ &= \Pr[\max_{(i,j):i,j\in\mathcal{K},i\neq j}\min\{|f_i|^2, |g_j|^2, |h_{i,j}|^2\} < G(T_0,4)]. \end{aligned} \tag{22}$$

For simplicity, we will use G to denote $G(T_0, 4)$ in the remaining part of the paper.

Theorem 1: When there are only two relays in the network, i.e., $K = 2$, the exact closed-form of the end-to-end outage probability $P^{(2)}_{\text{out,PRS}}$ equals

$$P^{(2)}_{\text{out,PRS}} = 1 - e^{-G/\Theta_{1,2}} - e^{-G/\Theta_{2,1}} + e^{-G/\Phi_{1,2}} \tag{23}$$

where $\Theta_{1,2} = (1/\Omega_{S_1,1} + 1/\Omega_{1,2} + 1/\Omega_{S_2,2})^{-1}$; $\Theta_{2,1} = (1/\Omega_{S_1,2} + 1/\Omega_{1,2} + 1/\Omega_{S_2,1})^{-1}$; and $\Phi_{1,2} = (1/\Omega_{S_1,1} + 1/\Omega_{1,2} + 1/\Omega_{S_2,2} + 1/\Omega_{S_1,2} + 1/\Omega_{S_2,1})^{-1}$.

Proof: Please see Appendix A.

When $K > 2$, the end-to-end paths between the sources may not be independent of one another. Considering the paths $S_1 \leftrightarrow R_{i'} \leftrightarrow R_j \leftrightarrow S_2$ where $j \in \mathcal{K}$ and $j \neq i'$, it is obvious that the paths are not independent of one another because they share the common link between S_1 and $R_{i'}$, i.e., $S_1 \leftrightarrow R_{i'}$. Unlike in the SRS, the paths in the PRS are too complex to analyze when $K > 2$.

Therefore, in this paper, we do not intend to derive the exact closed-form outage probability of PRS when $K > 2$ for random distribution of relays. Instead, we have derived an empirical outage probability, which is expressed as

TABLE I

VALUES OF THE PARAMETERS USED IN THE EMPIRICAL OUTAGE PROBABILITY WHEN THE PATH-LOSS EXPONENT α EQUALS 3.

$\alpha = 3$	$K = 3$	$K = 5$	$K = 8$	$K = 20$
k	1.1	1.1	1.1	1.1
θ	0.008	0.008	0.008	0.01
m	1.8	4.2	6.8	15.2

TABLE II

VALUES OF THE PARAMETERS USED IN THE EMPIRICAL OUTAGE PROBABILITY WHEN THE PATH-LOSS EXPONENT α EQUALS 2.

$\alpha = 2$	$K = 3$	$K = 5$	$K = 8$	$K = 20$
k	1.1	1.1	1.1	1.1
θ	0.05	0.05	0.05	0.05
m	1.8	3.5	5.7	17.5

$$P_{\text{out,PRS}} \approx 1 - \left(1 - \left(\Gamma(k, \frac{G}{\theta})/\Gamma(k)\right)^m\right)^3 \tag{24}$$

where $\Gamma(k) = \int_0^\infty e^{-t} t^{k-1} dt$ is the gamma function; $\Gamma(k, \frac{x}{\theta}) = \int_0^{x/\theta} t^{k-1} e^{-t} dt$ is an incomplete gamma function [19]. Moreover, the parameters k, θ and m are defined in Table I and Table II when the path-loss exponent α equals 3 and 2, respectively. Due to shortage of space, details of the derivation are omitted here but are readily available at http://www.eie.polyu.edu.hk/∼encmlau/proof.pdf.

C. Comparison between PRS and SRS

We assume that the total transmit powers P are the same in both PRS and SRS schemes. Moreover, in each of the PRS and SRS schemes, all nodes transmit with the same power. Recall that the outage probabilities of PRS and SRS schemes under such circumstances are given by

$$P_{\text{out,PRS}} = \Pr[\min\{|f_{p1}|^2, |g_{p2}|^2, |h_{p1,p2}|^2\} < G(T_0, 4)] \tag{25}$$

$$P_{\text{out,SRS}} = \Pr[\min\{|f_r|^2, |g_r|^2\} \leq G(T_0, 3)] \tag{26}$$

where $G(T_0, l) = \frac{l\sigma^2}{P}(2^{lT_0} - 1)$ and l represents the total number of transmission timeslots in one complete exchange.

When both P and T_0 (assumed positive) are fixed, $G(T_0, l)$ is an increasing function of l, implying that the outage probabilities increases with the total number of transmission timeslots in one complete exchange. This is due to the transmit power per node (i.e., P/l) decreasing with l. In this aspect, the PRS scheme does not perform as well as the SRS scheme.

On the other hand, the distances between nodes in the PRS scheme are shorter compared with those in SRS scheme, particularly when there is a large number of relays to choose from. As a result, the path losses in the PRS scheme are smaller than those in SRS scheme. In other words, it is almost guaranteed that

$$\min\{|f_{p1}|^2, |g_{p2}|^2, |h_{p1,p2}|^2\} > \min\{|f_r|^2, |g_r|^2\}. \tag{27}$$

In this aspect, the PRS scheme outperforms the SRS scheme.

The relative performances of the PRS scheme and the SRS scheme hence depend on the path loss model, the number of relays and their spatial distribution. If the path loss exponent is large (i.e., severe path loss) and there are more relays to choose from, the improvement in path loss can compensate for the reduction in the transmit power per node. Under such circumstances, the PRS scheme will outperform the SRS scheme.

Note also that if network coding is not applied, the number of timeslots (i.e., l) required to complete one exchange between the sources will be 6 and 4 for the PRS scheme and SRS scheme, respectively. We can therefore observe that applying network coding improves the performance of both schemes. Yet the improvement to the PRS scheme (reduced from 6 to 4) is relatively larger compared with the SRS scheme (reduced from 4 to 3). In Table III, we list the characteristics of the PRS and SRS transmission schemes with and without network coding.

TABLE III

COMPARISON AMONG DIFFERENT TRANSMISSION SCHEMES. PRS SCHEME WITH NETWORK CODING (PRS-NC), PRS SCHEME WITHOUT NETWORK CODING (PRS-NNC), SRS SCHEME WITH NETWORK CODING (SRS-NC), AND SRS SCHEME WITHOUT NETWORK CODING (SRS-NNC)

	PRS-NC	PRS-NNC	SRS-NC	SRS-NNC
Path loss per link	low	low	high	high
No. of timeslots required	4	6	3	4
Transmit power per node	$P/4$	$P/6$	$P/3$	$P/4$

D. Simulation Results

In this section, we present and compare the simulation, analytical and empirical results. We assume that the distance D between the two sources equals $10d_0$ (d_0 is the reference distance defined in Sect. II). We also assume that the end-to-end transmission rate T_0 equals 2 b/s/Hz. To ensure a fair comparison, the total transmit powers of the SRS and PRS schemes are set to be identical and are equal to P. We define the SNR as the total transmit power over noise power, i.e., P/σ^2.

For each set of SNR and K and for a given relay distribution function, 100 relay distributions are realized. Moreover, in each realization, 100000 different channel conditions are simulated to evaluate the average outage probability.

1) Relay distribution in one dimension: We assume that the K relays are uniformly distributed on the straight line connecting the two sources. For $K = 2$ relays, we plot the simulated outage probabilities and the theoretical ones (based on (9) and (23)) in Fig. 2 when the path-loss exponent α equals 2 and 3. The curves indicate that the theoretical results closely match with the simulation results in this 1-D case. We can also observe that when there are only 2 relays, the PRS scheme is outperformed by the SRS scheme.

For $K = 3, 5, 8$ and 20 relays, we plot the simulated outage probabilities, the theoretical ones (based on (9)) and the empirical ones (based on (24)) in Fig. 3 when the path-loss exponent α equals 2 and 3. The results show that when the path-loss exponent is small ($\alpha = 2$), the PRS scheme (i) produces a better outage probability than the SRS scheme when $K = 20$; (ii) is outperformed by the SRS scheme when

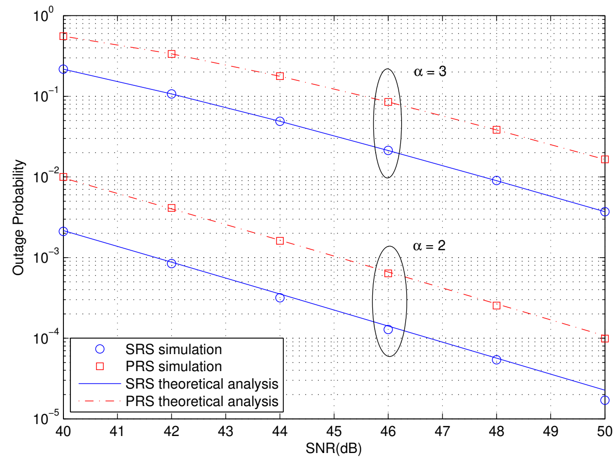
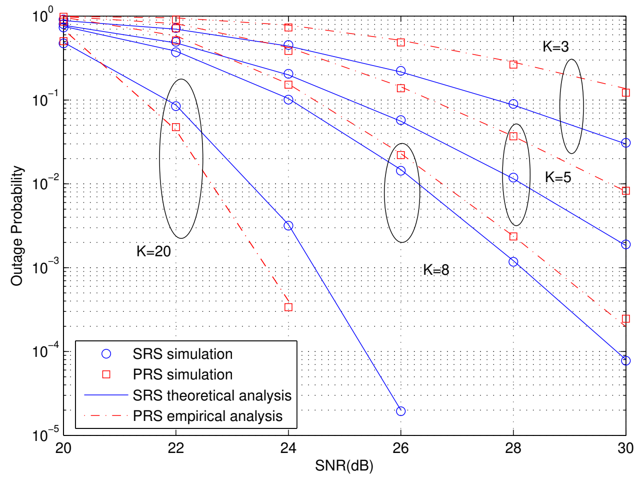


Fig. 2. Outage probability against SNR(dB) for the SRS and PRS schemes in a one-dimensional relay-distribution setting. Number of relays $K = 2$. Path-loss exponent $\alpha = 2$ and 3. The transmit power of each node in the SRS scheme and the PRS scheme equals $P/3$ and $P/4$, respectively. Theoretical results are plotted using lines and simulated ones are shown with symbols.

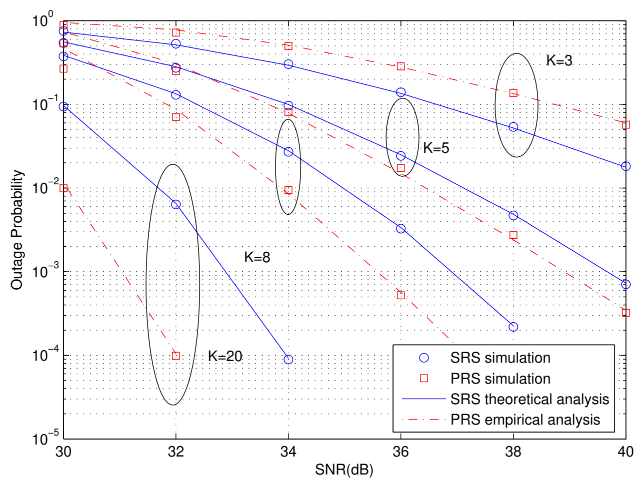
$K = 3, 5, 8$. However, when the path-loss exponent is larger ($\alpha = 3$), the PRS scheme outperforms the SRS scheme when $K \geq 5$. We can also see that the empirical results for the PRS scheme match with the simulation results in this 1-D case.

2) Relay distribution in two dimensions: We consider the case when the relays are uniformly distributed in a circle with a radius of $D/2$ and centre at the mid-point between the two sources. The results in Fig. 4 show the PRS scheme (i) produces a better outage probability than the SRS scheme when $K = 8, 20$; (ii) is outperformed by the SRS scheme when $K = 3, 5$. Our simulation results have verified our analysis in Sect. III-C that the PRS scheme gives a lower outage probability than the SRS scheme when the path loss is severe and there is a large number of relays to choose from.

When the number of relays is large, we can use the P-DPaSA algorithm to save some overhead when selecting the paired-relay (refer to Section III-A1). We consider the case when the number of relays in the network is $K = 30$. We also allow only M relays in Δ_1 possessing the smallest back-off timer to broadcast their information where $M = 3, 5, 8, 15$ and 30. (When $M = K$, P-DPaSA has the same performance as DPaSA). Figure 5 plots the simulated outage probability when the path-loss exponent $\alpha = 3$. The results indicate that under the same PA mechanism, the PRS scheme based on $K = 30$ (i)



(a)



(b)

Fig. 3. Outage probability against SNR(dB) for the SRS and PRS schemes in a one-dimensional relay-distribution setting. The transmit power of each node in the SRS scheme and the PRS scheme equals $P/3$ and $P/4$, respectively. Number of relays $K = 3, 5, 8, 20$. Theoretical results are plotted using solid lines, empirical results are plotted using dashed lines and simulated ones are shown with symbols. Path-loss exponent (a) $\alpha = 2$; (b) $\alpha = 3$.

produces a better outage probability than the SRS scheme when $M = 8, 15, 30$; (ii) has nearly the same outage performance with SRS scheme when $M = 5$; and (iii) is outperformed by the SRS scheme when $M = 3$. In summary, we show that the P-DPaSA algorithm is feasible and effective even when a small percentage of relays are allowed to broadcast their information in the selection process.

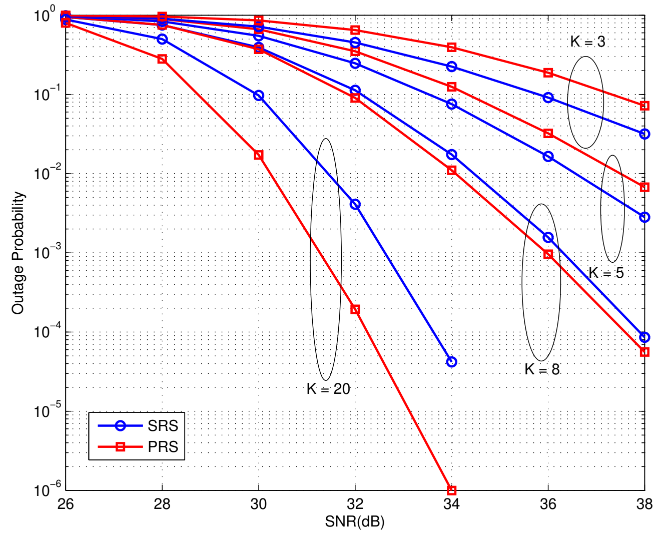


Fig. 4. Simulated outage probability against SNR(dB) for the SRS and PRS schemes in a two-dimensional relay-distribution setting. Number of relays $K = 3, 5, 8, 20$. Path-loss exponent $\alpha = 3$.

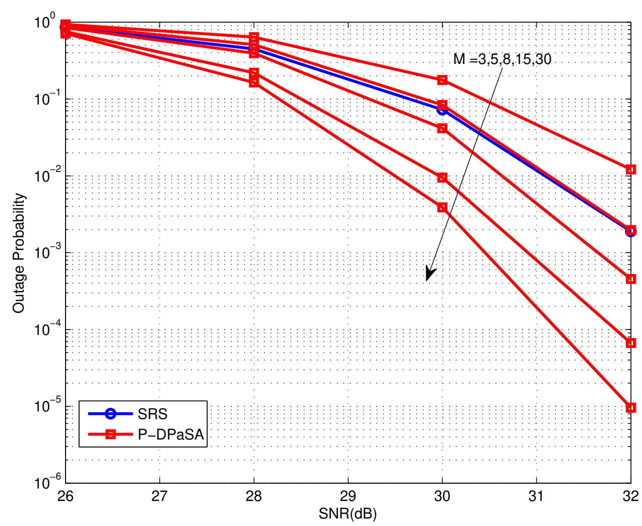


Fig. 5. Simulated outage probability against SNR(dB) for the SRS and PRS schemes on a two-dimensional relay-distribution setting. Path-loss exponent $\alpha = 3$. Number of used relays $M = 3, 5, 8, 15, 30$ for P-DPaSA and the total number of relays $K = 30$.

IV. Iterative Paired-Relay Selection

In both the DPaSA and P-DPaSA algorithms, some overhead and hence time is needed for selecting the "best" paired-relay. After the "best" paired-relay has been selected, the outage performance is optimal if the channel coefficients remain the same. In a practical environment, however, the channel coefficients change from time to time. If the channel coefficients vary too rapidly, the DPaSA and P-DPaSA algorithms may not be effective in selecting the "best" paired-relay. To overcome this issue, we propose an iterative-PRS (I-PRS) scheme in which **no pre-selection process or overhead** is needed. Moreover, the relay selection process is performed in a dynamic and opportunistic manner.

A. Selection Scheme and information exchange

We continue to use the max-min criterion as the basis to select relays in the I-PRS algorithm. The information exchange between the two sources is also arranged into frames, each of which consists of 4 timeslots. We assume that the channel coefficients remain the same within each frame. We consider the tth frame and we denote $R_{p1}^{(t)}$ and $R_{p2}^{(t)}$ as the selected relays in the tth frame. In the first timeslot, S_1 broadcasts its data $\mathbf{x}_1[t]$ to all the relays. In the second timeslot, S_2 broadcasts its data $\mathbf{x}_2[t]$ to all the relays. In the third timeslot, the selected $R_{p1}^{(t)}$ broadcasts the network-coded data $\mathbf{z}_{p1}[t] = \mathbf{x}_1[t] \oplus \mathbf{z}_{p2}[t-1]$ to S_1 and other relays, where $\mathbf{z}_{p2}[t-1]$ denotes the data broadcasted by $R_{p2}^{(t-1)}$ during the $(t-1)$th frame. Then, in the fourth timeslot, the selected $R_{p2}^{(t)}$ broadcasts the XORed data $\mathbf{z}_{p2}[t] = \mathbf{x}_2[t] \oplus \mathbf{z}_{p1}[t]$ to S_2 and other relays. According to (11) and (12), respectively, S_1 and S_2 can decode the data from the other source.

In the above information exchange process, the relays $R_{p1}^{(t)}$ and $R_{p2}^{(t)}$ in the tth frame are selected based on the max-min criterion, i.e.,

$$p_1^{(t)} = \arg\max_{i \in \mathcal{K}, i \neq p_2^{(t-1)}} \min\{|f_i^{(t)}|^2, |h_{i,p_2}^{(t-1)}|^2\} \tag{28}$$

$$p_2^{(t)} = \arg\max_{j \in \mathcal{K}, j \neq p_1^{(t)}} \min\{|g_j^{(t)}|^2, |h_{p_1,j}^{(t)}|^2\} \tag{29}$$

where $p_1^{(t)}$ and $p_2^{(t)}$ are the indices of $R_{p1}^{(t)}$ and $R_{p2}^{(t)}$, respectively. Note that in general we use the superscript "$^{(t)}$" to indicate the tth frame. For example, $f_i^{(t)}$ in (28) represents the channel coefficient f_i during the tth frame. In the following, we describe the iterative paired-relay selection process.

In the fourth timeslot of the $(t-1)$th frame, after $R_{p2}^{(t-1)}$ has broadcasted the network-coded information, each of the other relays R_i ($i \neq p_2^{(t-1)}$) listens to the broadcast and estimates the channel condition between $R_{p2}^{(t-1)}$ and itself, i.e. $|h_{i,p_2}^{(t-1)}|^2$. In the first two timeslots of the tth frame, each of the relays R_j

$(j = 1, 2, \ldots, K)$ listens to the broadcasts from S_1 and S_2 and estimates the channel coefficients $|f_j^{(t)}|^2$ and $|g_j^{(t)}|^2$. Then, in the third timeslot of the tth frame, each relay R_i $(i \neq p_2^{(t-1)})$ computes a back-off timer using $\frac{\lambda}{\min\{|f_i^{(t)}|^2, |h_{i,p_2}^{(t-1)}|^2\}}$. The relay whose back-off timer first expired declares itself as the selected relay $R_{p1}^{(t)}$ and transmits the network-coded packet immediately. Each of the other relays R_j $(j \neq p_1^{(t)})$ listens and estimates the channel coefficient $|h_{p_1,j}^{(t)}|^2$. Similarly, in the fourth timeslot, each relay R_j $(j \neq p_1^{(t)})$ sets up a back-off timer using $\frac{\lambda}{\min\{|g_j^{(t)}|^2, |h_{p_1,j}^{(t)}|^2\}}$. The relay whose back-off timer first expired declares itself as the selected relay $R_{p2}^{(t)}$ and transmits the network-coded packet. Since the paired-relay is selected iteratively and opportunistically with no feedback or overhead, we call this scheme Iterative PRS (I-PRS).

B. Convergence Analysis and Performance Evaluation

Based on the above description, the I-PRS scheme is performed in an iterative way. In this subsection, we provide an insightful analysis on this algorithm.

Lemma 1: Suppose the best paired-relay is denoted by $(R_{p_1^*}, R_{p_2^*})$. If $R_{p_1^*}$ or $R_{p_2^*}$ is selected in the current step, the other best relay, i.e., $R_{p_2^*}$ /$R_{p_1^*}$, would be selected in the next step. The proof is trivial and is omitted.

Corollary 1: In order for the proposed I-PRS scheme to converge to the best paired-relay, a necessary and sufficient condition is that one of the relays in the pair is selected in a certain iteration.

Proof: The necessary condition is directly obtained from the definition of the best paired-relay where the sufficient condition is easily derived from **Lemma 1**.

Corollary 2: The proposed I-PRS scheme may not converge to the best paired-relay.

Proof: We prove by providing an example. Suppose there exists four relays, i.e., R_1, R_2, R_3 and R_4, in the network and the channel parameters are given as follows[1].

- $|f_i|^2 = \{0.8, 0.1, 0.2, 0.6\}$ for $i = 1, 2, 3, 4$
- $|g_j|^2 = \{0.1, 0.8, 0.6, 0.2\}$ for $j = 1, 2, 3, 4$
- $|h_{i,j}|^2 = \{0, 0.3, 0.1, 0.1; \quad 0.3, 0, 0.1, 0.1; \quad 0.1, 0.1, 0, 0.4; \quad 0.1, 0.1, 0.4, 0\}$ for $i, j = 1, 2, 3, 4$

[1]For simplicity, we use a notation that $|h_{i,i}|^2 = 0$ to avoid the self-pairing.

The initial selected relay is R_1 based on the maximum value of $|f_i|^2 (= 0.8)$. Then, according to (29), R_2 is selected in the next step because

$$\min\{|g_j|^2, |h_{1,j}|^2\} = \{0, 0.3, 0.1, 0.1\} \tag{30}$$

and

$$\max_{j \in \mathcal{K}} \min\{|g_j|^2, |h_{1,j}|^2\} = 0.3. \tag{31}$$

Subsequently, according to (28), R_1 is selected again in the next step because

$$\min\{|f_i|^2, |h_{i,2}|^2\} = \{0.3, 0, 0.1, 0.1\} \tag{32}$$

and

$$\max_{i \in \mathcal{K}} \min\{|f_i|^2, |h_{i,2}|^2\} = 0.3. \tag{33}$$

Thus, the I-PRS scheme converges to the paired-relay (R_1, R_2) which achieves

$$\min\{|f_1|^2, |g_2|^2 |h_{1,2}|^2\} = 0.3. \tag{34}$$

However, it can be observed from the channel parameters that the best paired-relay should be (R_4, R_3) which can achieve

$$\min\{|f_4|^2, |g_3|^2 |h_{4,3}|^2\} = 0.4. \tag{35}$$

As a result, the I-PRS scheme may not converge to the best paired-relay.

There also exists other scenarios in which the I-PRS scheme does not converge to the best pair-relay. For example, the I-PRS scheme may be trapped in a cycle consisting of several relays. Due to the limited space, we do not provide explicit examples here. Nonetheless, we can draw the conclusion that the convergence of the I-PRS scheme to the best paired-relay (called global convergence) cannot be guaranteed. It can also be shown that with no more than K iterations, the I-PRS scheme will converge to a pair-relay (best or not) or be trapped in a cycle.

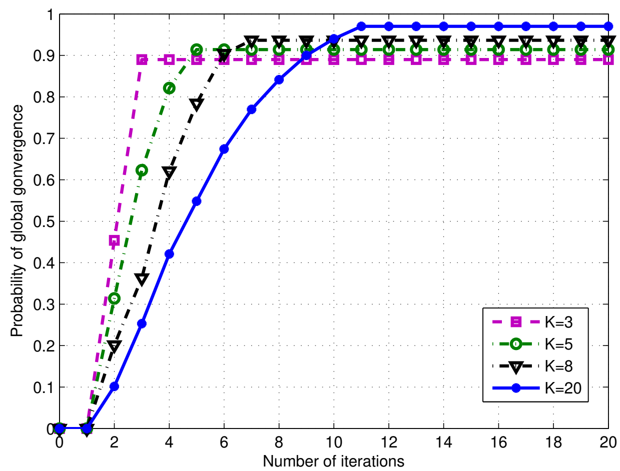


Fig. 6. Probability of global convergence of the I-PRS scheme versus the number of iterations. The number of relays is $K = 3, 5, 8, 20$.

C. Simulation Results

We assume that the relays are distributed in the same two-dimensional setting as in Section III-D2. The number of relays equals $K = 3, 5, 8, 20$ and the path-loss exponent equals $\alpha = 3$. First we simulate the outage probability of the I-PRS scheme for time-invariant channels. Figure 6 plots the probability of global convergence of the I-PRS scheme versus the number of iterations. The results show that the I-PRS scheme converges to the best paired-relay with a probability of 90% to 96%. Moreover, the probability increases with the number of iterations and the number of relays K. Since 100% is not achieved, the results have verified our analytical findings that the I-PRS scheme may not achieve the global convergence. The curves in Fig. 6 also confirm that the I-PRS scheme takes no more than K iterations when it converges to the best paired-relay. Figure 7 plots the results together with those of the PRS. The results show that the I-PRS scheme provides almost the same outage performance as the PRS scheme even though the global optimum cannot be guaranteed.

Next we study the performance of the I-PRS scheme under time-varying channels. We assume that the channel coefficients change every β timeslots and we set $\beta = 4$ and 20. We plot the simulated outage probability of the I-PRS scheme under time-varying channels in Fig. 8. We also plot the results of the PRS scheme for comparison. As the PRS scheme does not select the pair-relay dynamically accordingly to the

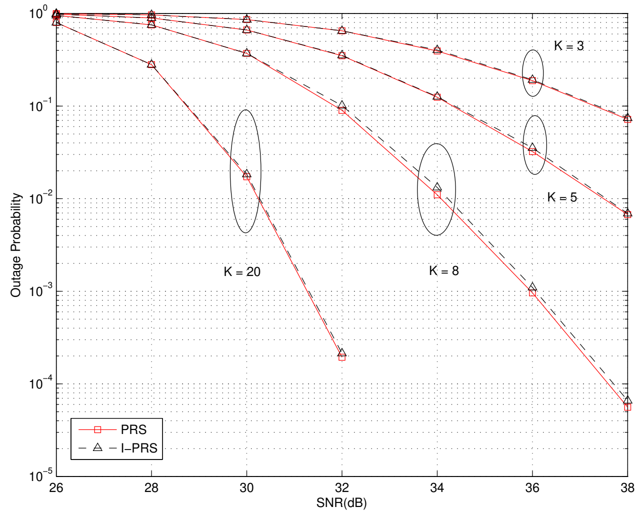


Fig. 7. Simulated outage probability against SNR(dB) for the I-PRS and PRS schemes for the two-dimensional relay-distribution setting. Number of relays $K = 3, 5, 8, 20$. Path-loss exponent $\alpha = 3$. The channels are time-invariant.

changing channel conditions, it gives very high outage probability. As for the I-PRS scheme, the pair-relay is selected iteratively and may change from frame-to-frame. The results show that the I-PRS scheme can achieve much lower outage probability than PRS in time-varying channels. Moreover, the I-PRS scheme gives a better outage performance in a slowly-changing channel than a fast-changing one. As we have seen in Figure 6, the I-PRS scheme has a higher probability of converging to the best paired-relay when the number of iterations increases (under constant channel conditions). Thus, a fast time-varying environment (e.g., channel parameters changed every 4 timeslots) will certainly degrade the performance the I-PRS scheme because sufficient time for the scheme to converge to the best paired-relay has not been provided.

V. Conclusion and Future Work

In this paper, we have proposed a Distributed Pair Selection Algorithm (DPaSA) that selects a pair of "best" relays to broadcast network-coded information to other nodes (source or relay) in a two-way relaying network. Moreover, the network contains multiple potential relays that are randomly distributed in a one-dimensional or two-dimensional space between the sources.

Assuming the same total transmit power, the proposed Paired-Relay Selection (PRS) scheme outperforms Single Relay Selection (SRS) scheme in terms of outage when the path-loss exponent between the nodes

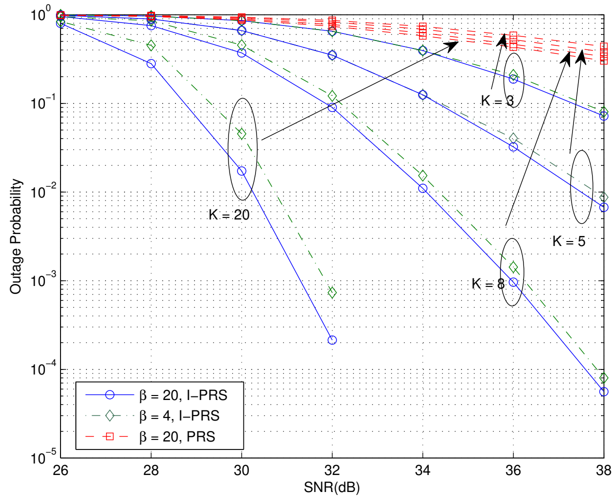


Fig. 8. Simulated outage probability against SNR(dB) for the I-PRS and PRS schemes on two-dimensional relay distribution setting. Number of relays $K = 3, 5, 8, 20$. Path-loss exponent $\alpha = 3$. Channel coefficients change every $\beta = 4, 20$ timeslots.

is large and/or there is a sufficient number of potential relays in the network.

In addition, to reduce the broadcasting overhead, we have proposed a partial-DPaSA (P-DPaSA) algorithm that allows a limited number of relays with "good" channel conditions to broadcast their information to other nodes. The performances of both the DPaSA and P-DPaSA algorithms are evaluated through theoretical analysis, empirical functions and extensive simulations. It has been shown that PRS under P-DPaSA can outperform SRS when there is a sufficient number of potential relays.

Finally, we have considered an iterative-PRS (I-PRS) scheme with further reduced overhead for time-varying channels. Our simulation results indicate that the I-PRS scheme has almost the same outage performance as the PRS scheme if the channel condition does not change, and has significantly outperformed the PRS scheme if the channel condition changes dynamically.

In the future, we aim to analyze the capacity of two-way relay channels under different relaying strategies. We will also explore introducing physical-layer network coding (PNC) [14] to the PRS scheme so as to further reduce the transmission time required.

APPENDIX A

PROOF OF THEOREM 1

When there are only two relays R_1 and R_2, the possible paths between the sources S_1 and S_2 are (i) $S_1 \leftrightarrow R_1 \leftrightarrow R_2 \leftrightarrow S_2$ and (ii) $S_1 \leftrightarrow R_2 \leftrightarrow R_1 \leftrightarrow S_2$. These two paths are not independent of each other because of the common link between R_1 and R_2, i.e., $R_1 \leftrightarrow R_2$ or $R_2 \leftrightarrow R_1$. Consequently, to derive the outage probability, we only need to consider 5 independent links out of the 6 links. By using "success cases", we can easily obtained that

$$P_{\text{out,PRS}}^{(2)} = \Pr[|h_{1,2}|^2 < G] + \Pr[|h_{1,2}|^2 > G, \max_{i,j=1,2;i\neq j} \min\{|f_i|^2, |g_j|^2\} < G] \tag{36}$$

where the second term can be easily derived as

$$\begin{aligned} &\Pr[|h_{1,2}|^2 > G, \max_{i,j=1,2;i\neq j} \min\{|f_i|^2, |g_j|^2\} \\ &= \Pr[|h_{1,2}|^2 > G](1 - Pr[|f_1|^2 > G]Pr[|g_2|^2 > G]) \\ &\quad \times (1 - Pr[|f_2|^2 > G]Pr[|g_1|^2 > G]) \end{aligned} \tag{37}$$

Finally, the results (23) is proved by simple calculation.

REFERENCES

[1] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *IEEE Trans. Inf. Theory*, vol. 47, no. 5, pp. 748–767, Jan. 2004.

[2] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploring cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[3] H. Suraweera, D. Michalopoulos, and G. Karagiannidis, "Semi-blind amplify-and-forward with partial relay selection," *Electronics letters*, vol. 45, no. 6, pp. 317–319, July 2009.

[4] T. Duong and V. N. Q. Bao, "Performance analysis of selection decode-and-forward relay networks," *Electronics Letters*, vol. 44, no. 20, pp. 1206–1207, July 2008.

[5] A. H. Bastami and A. Olfat, "Optimal snr-based selection relaying scheme in multi-relay cooperative networks with distributed space-time coding," *IET Communications*, vol. 4, no. 6, pp. 619–630, Apr. 2010.

[6] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, “A simple cooperative diversity method based on network path selection,” *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.

[7] Y. Zhao, R. Adve, and T. Lim, “Improving amplify-and-forward relay networks: optimal power allocation versus selection,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 3114–3123, Aug. 2007.

[8] Y. Jing and H. Jafarkhani, “Single and multiple relay selection schemes and their achievable diversity orders,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1414–1423, Mar. 2009.

[9] Z. Yi and I.-M. Kim, “Relay ordering in a multi-hop cooperative diversity network,” *IEEE Trans. Commun.*, vol. 57, pp. 2590–2596, Sep. 2009.

[10] M. Ju, I.-M. Kim, and D. I. Kim, “Joint Relay Selection and Relay Ordering for DF-Based Cooperative Relay Networks,” *IEEE Trans. Commun.*, vol. 60, no. 4, pp. 908–915, Apr. 2012.

[11] S. Chen, Z. Zhang, R. Yin, X. Chen, and W. Wang, “Concatenated channel-and-network coding scheme for two-path successive relay network,” *IET Communications*, vol. 7, no. 1, pp. 82–90, Mar. 2013.

[12] X. Jia, L. Yang, H. Fu, B. Feng, and Y. Qi, “Two-way denoise-and-forward network coding opportunistic relaying with jointing adaptive modulation relay selection criterions,” *IET Communications*, vol. 6, no. 2, pp. 194–202, July 2012.

[13] C. Chen, L. Bai, B. Wu, and J. Choi, “Relay selection and beamforming for cooperative bi-directional transmissions with physical layer network coding,” *IET Communications*, vol. 5, no. 14, pp. 2059–2067, July 2011.

[14] S. Zhang, S. C. Liew, and P. P. Lam, “Hot topic: physical-layer network coding,” in *in Proc. ACM MobiCom’06*, pp. 358–365, Los Angeles, USA, 2006.

[15] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Médard, and J. Crowcroft, “Xors in the air: practical wireless network coding,” in *Proc. ACM SigComm’06*, pp. 243–254, Pisa, Italy, 2006.

[16] Q. F. Zhou, Y. Li, F. C. M. Lau, and B. Vucetic, “Decode-and-forward two-way relaying with network coding and opportunistic relay selection,” *IEEE Trans. Commun.*, vol. 58, no. 11, pp. 3070–3076, Nov. 2010.

[17] Y. Li, R. H. Louie, and B. Vucetic, “Relay selection with network coding in two-way relay channels,” *IEEE Transactions on Vehicular Technology*, vol. 59, no. 9, pp. 4489–4499, Aug. 2010.

[18] Z. Ren, G. Wang, Q. Chen, and H. Li, “Modelling and Simulation of Rayleigh Fading, Path Loss, and Shadowing Fading for Wireless Mobile Networks,” *In Simulation Modelling Practice and Theory (Elsevier)*, Feb. 2011.

[19] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. New York: Academic, 7 ed., 2007.

[20] X. Chen, Q. F. Zhou, T. W. Siu, and F. C. M. Lau, “Asymptotic analysis of opportunistic relaying based on the maximum-generalized-mean selection criterion,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1050–1057, April 2011.

[21] A. K. Sadek, Z. Han, and K. J. R. Liu, “A distributed relay-assignment algorithm for cooperative communications in wireless networks,” in *Proc. IEEE Int. Conf. Commun.*, pp. 1592–1597, Istanbul, Turkey, Jun. 2006.

[22] E. Koyuncu, Y. Jing, and H. Jafarkhani, “Beamforming in wireless relay networks with quantized feedback,” *IEEE J. Selected Areas Commun.*, vol. 26, pp. 1429–1439, Oct. 2008.