Dynamic Routing Model and Solution Methods for Fleet Management with Mobile Technologies

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Abstract

We develop and analyze a mathematical model for dynamic fleet management that captures the characteristics of modern vehicle operations. The model takes into consideration dynamic data such as vehicle locations, travel time, and incoming customer orders. The solution method includes an effective procedure for solving the static problem and an efficient re-optimization procedure for updating the route plan as dynamic information arrives. Computational experiments show that our re-optimization procedure can generate near-optimal solutions.

Keywords: Dynamic vehicle routing; heuristics; mobile technologies

1. INTRODUCTION

With the advances of mobile and information technologies, dynamism has become a key feature of today's logistics operations environment. However, a major limitation of many traditional vehicle management systems is their inability to re-plan transportation operations satisfactorily when facing dynamic data. Motivated by the characteristics of the new communication technologies, such as radio frequency identification (RFID), global positioning systems (GPS), and geographical information systems (GIS), in this paper we develop and analyze a mathematical model for dynamic fleet management that attempts to capture the characteristics of modern vehicle operations.

With the use of RFID, data such as identity, volume, and types of stock keeping units of each customer order can be accurately recorded and tracked. Such technologies can lead to the real-time update of information of new customer orders, including their expected arrival time at the pickup locations, their vehicle capacity consumption, their delivery requirements, due dates, etc. These are translated into dynamic demand data in the vehicle scheduling system. The use of GPS and GIS can provide real-time data on the vehicle locations, spatial distribution of roads and related facilities, and road conditions. GIS can further be used for managing, querying, and analyzing spatial related data for fleet analysis. The real-time data obtained from the GPS/GIS are translated into dynamic data on the vehicle's travel time for each transportation segment, as well as the vehicle's travel time from its current position to its next pickup or drop-off point. The tracking feature of an advanced vehicle scheduling system can also provide dynamic data on the load level of each vehicle. In order to make good use of these dynamic data, a robust vehicle management system needs to either periodically or dynamically update the route plan, while the updating process has to be performed with high efficiency. In this paper, we propose a vehicle routing framework which takes into account the dynamism of data as well as the efficiency of the route-plan updating process.

Our proposed framework requires us to determine a route plan through solving a static vehicle routing problem before the start of the daily operation. It also requires us to make dynamic changes of the routing whenever new information is obtained throughout the day. We develop a mathematical model for such a dynamic vehicle routing framework. This mathematical model includes solving a static problem as well as a problem for dynamic change of the route plan. We propose heuristic solution methods for both the static and dynamic problems. The heuristic for the static problem emphasizes effectiveness. It includes the development of an initial feasible solution using the idea of "seed selection" (see Fisher and Jaikumar 1981), as well as the refinement of the solution via local search techniques and a genetic algorithm. The heuristic for the dynamic problem emphasizes efficiency. The design of our solution methods is therefore customized for the vehicle routing setting where advanced mobile technologies are employed.

Real-time vehicle routing has been studied by a number of researchers. Psaraftis (1995), Gendreau and Potvin (1998), and Ghiani *et al.* (2003) have conducted recent surveys on this topic. More recent developments in this area include the following: Yang *et al.* (2004) and Chen and Xu (2006) have developed mathematical-programming-based algorithms for different variants of the dynamic vehicle routing model. Larsen *et al.* (2002), Attanasio *et al.* (2004), Liao (2004), Van Hemert and La Poutre (2004), Du *et al.* (2005), Montemanni *et al.* (2005), Tang and Hu (2005), Fabri and Recht (2006), and Potvin *et al.* (2006) have developed various rule-based and local search techniques, such as insertion methods, tabu search, ant colony optimization, intra-route improvement, inter-route improvement, etc., for dynamic vehicle routing. Godfrey and Powell (2002a,b), Hu *et al.* (2003), Bent and Van Hentenryck (2004), Larsen *et al.* (2004), and Thomas and White (2004) have used "look-ahead" approaches to tackling the dynamism of real-time routing problems. They try to incorporate probabilistic characteristics of future events into decision making, Mitrovic-Minic *et al.* (2004), Mitrovic-Minic and Laporte (2004), and Branke *et al.* (2005) have analyzed waiting strategies for vehicles that are already under way when they face dynamic customer order arrivals. Fleischmann *et al.* (2004) have presented a planning framework for dynamic vehicle routing. Our research differs from the above-mentioned works in that we focus on the development of a solution method that takes into consideration both the effectiveness of the static route plan and the efficiency of the re-planning procedure, and we consider the dynamic data of the vehicle position, load level of each vehicle, travel time, and the status of the customer orders.

The rest of the paper is organized as follows: In the next section, we describe our problem in detail. The solution method for the static problem is developed in Section 3. In Section 4, the dynamic updating routine is presented and computational results are presented. Some concluding remarks are provided in Section 5.

2. PROBLEM DESCRIPTION

We consider a directed transportation network $G = (N \cup \{0\}, A)$, where $N = \{1, 2, \ldots, n\}$ is the set of customer locations, 0 is the central depot of vehicles, and A is the set of arcs. For any arc $i \to j \in A$, let τ_{ij} denote the normal (minimum) travel time from node i to node j. The actual travel time between these two nodes, denoted as t_{ij}, will be updated dynamically over time, where $t_{ij} \geq \tau_{ij}$. There are m identical vehicles available, where each of them has a capacity K. Each customer order is represented by d_{ij}, which is the amount of goods to be picked up at node i and delivered to node j, where $d_{ij} = 0$ if such an order does not exist. For simplicity, we assume that every node is either a pickup node of exactly one customer order or a delivery node of exactly one customer order, but not both. In reality, if there are two or more pickups/deliveries taking place at the same location i, then we will transform the problem into our model by duplicating node i into multiple nodes and assigning a node to each pickup/delivery requirement. Also, if there is a node which is neither a pickup node nor a delivery node, then we can eliminate it from the network and update the t_{ij} values accordingly. Hence, at any point in time, if there are q customer orders to be satisfied, then network G contains $n = 2q + 1$ nodes (i.e., one pickup node and one delivery node for each order, plus a node 0 for the central depot). Once a customer order is delivered, the pickup and delivery nodes of that order will be removed from the network.

For each customer order d_{ij}, there is a given pair of time windows $[r_{ij}^P, s_{ij}^P]$ and $[r_{ij}^D, s_{ij}^D]$, where parameter r_{ij}^P is the earliest pickup time of order d_{ij} at node i, parameter s_{ij}^P is the latest pickup time of order d_{ij} at node i, parameter r_{ij}^D is the earliest delivery time of order d_{ij} at node j, and parameter s_{ij}^D is the latest delivery time of order d_{ij} at node j. We assume that each customer order has to be picked up and delivered by a single vehicle (i.e., order splitting is not allowed). The vehicle has to arrive at the pickup location of an order no later than the latest pickup time. In case it arrives at the pickup location before the earliest pickup time, it can wait at that location until the goods are ready for pickup. Similarly, the vehicle has to arrive at the delivery location no later than the latest delivery time of the order. Waiting is allowed in case it arrives at the delivery location early. All vehicles are initially located at the central depot, and they have to return to the depot after they finish their pickup and delivery tasks. We assume that the loading and unloading time of goods is negligible at the pickup and delivery locations.

In reality, the earliest pickup time r_{ij}^P represents the ready time of the order at the pickup location. For example, the order may be arriving at the pickup location via another transportation mode. However, with the real-time order-tracking capability of the RFID-based system, the ready time of this order at its pickup location can be determined well in advance. The latest pickup time s_{ij}^P represents the deadline for the order pickup. If there is no such deadline, then we set $s_{ij}^P = +\infty$. The latest delivery time s_{ij}^D represents the deadline for the arrival of the goods at the delivery location as specified by the customer. The earliest delivery time r_{ij}^D is the earliest possible time that the customer would like the goods to arrive at the delivery point. If there is no such requirement, then we set $r_{ij}^D = 0$.

Our problem is to determine a set of m vehicle routes such that (i) the total amount of goods carried by each vehicle is no more than K at any point in time; (ii) the pickup of an order must precede its delivery; (iii) the time-window constraints of all customer orders are not violated; and (iv) the total travel time (excluding idle time) of the m vehicles is minimized. The total vehicle travel time is chosen as our objective as an approximation of the total cost of operation, which includes fuel cost, driver cost, equipment depreciation, and other intangible costs. In fact, it is not difficult to modify our solution method to handle other cost-minimization objectives.

The implementation of our dynamic pickup and delivery routing system is done as follows: At the beginning of a day, the system determines a route plan based on the normal travel times and demand data available on hand. This requires the determination of a solution to the "static" pickup and delivery vehicle routing problem with time windows. The system may spend a substantial amount of time on determining the route plan. Hence, we develop a sophisticated heuristic procedure for solving this static problem. The details of the heuristic procedure are provided in Section 3. Dynamic data arrive throughout the course of the day. They include the change in traffic conditions captured by the GPS, which results in a change of travel time t_{ij}, as well as the arrivals of new customer orders. When new travel time data arrive, the system will update and re-optimize the route plan. When a new customer order arrives, the system will also attempt to update the route plan immediately to determine if the order should be accepted (the order will be rejected if no feasible route plan can be identified). Once the new customer order is accepted, the system will re-optimize the route plan. This kind of re-optimization must be done quickly with minimal computational time, say within 30 seconds. In particular, when a new customer order arrives, a "near real-time" response is more desirable. Therefore, we develop an efficient procedure for the re-optimization. The details of the procedure are presented in Section 4.

3. SOLUTION METHOD FOR THE STATIC PROBLEM

The heuristic solution procedure for the static problem is divided into two phases. In the first phase, an initial population of feasible solutions is generated. This is done by constructing a population of initial feasible solutions via a "seed selection" approach (Section 3.1), followed by an application of a solution refinement procedure (Section 3.2) to improve each initial feasible solution. In the second phase, the solution population is improved by a genetic search routine, and a final solution is selected (Section 3.3).

3.1 Initial Feasible Solution Construction

Each initial feasible solution is constructed based on the idea of “seed selection” (see Fisher and Jaikumar 1981). The details are given as follows:

(1) Randomly select m' seed nodes sequentially from those nodes that are farthest from the depot (say those nodes that are of at least α time units away from the depot), where m' is a predetermined number no greater than m. For each selected node i, if it is less than, say, β time units away from one of the selected nodes, then we deselect node i. Here, α and β are constants with values specified in advance.

(2) For each selected seed node i, if i is a pickup node, we create a vehicle route beginning at node 0, followed by node i, followed by one of the corresponding delivery nodes (i.e., the nodes to which the goods picked up at i are delivered), and then followed by node 0. On the other hand, if i is a delivery node, we create a vehicle route beginning at node 0, followed by one of the corresponding pickup nodes of i, followed by node i, and then followed by node 0.

(3) Finally, the remaining customer orders are included in the route plan. For each of these customer orders, we insert its pickup and delivery nodes into the closest possible positions of the same route. This is done by trying all possible positions for the insertion and selecting the feasible insertion point with the smallest increase in total travel time. Note that an insertion point is feasible only if both the time-window and capacity constraints of every node remain satisfied after the insertion.

After the initial route plan construction, if there are some unusually short routes, say routes that contain less than 1/4 of the average number of nodes in a route, then those short routes are very much under capacity. In such a case, we try merging (or eliminating) some of them by the following procedure:

(i) *Merging two short routes into one:* Under normal conditions, merging two short routes can obtain a significant saving in total travel time. One way to perform this merging is by linking one route (say the one where the first node to be visited by the vehicle is farthest away from the depot) to the other so that the last node of the preceding route is followed immediately by the first node of the other (see Figure 1, case A). There is no capacity constraint violation resulting from this merger. However, time window constraint satisfaction needs to be checked. We may also consider the merging of two short routes by inserting one of these short routes in between two consecutive nodes on the other route. It may help to avoid time window constraint violation, but the vehicle may have to be checked for overloading at the insertion point. We can only be sure of substantial saving if these two routes to be merged are sufficiently close to each other, or if there is a considerably wide gap between these two consecutive nodes (see Figure 1, case B).

(ii) *Eliminating unusually short routes:* If the route plan contains some unusually short routes (say routes with no more than three pairs of pickup and delivery nodes), we transfer all the pairs of pickup-delivery nodes to the other routes, provided that all constraints are satisfied and that the total travel time is reduced after this transfer.

3.2 A Solution Refinement Procedure

The purpose of the refinement procedure is to improve the initial feasible solution generated by the method described in Section 3.1. The procedure includes two parts. These two parts are applied alternately and iteratively until a limit on the computational time (say 1 minute for a network up to 400 nodes) is reached:

(1) *Refinement within each route:* This is done by transferring a node from its current position to another position on the vehicle route. The position that this node is being transferred to is selected in such a way that: (i) if the node is a pickup node, then it must precede the corresponding delivery node in the new vehicle route; (ii) if the node is a delivery node, then it must be preceded by the corresponding pickup node in the new vehicle route; (iii) the new route plan must satisfy all time-window constraints; (iv) the new route plan must satisfy the vehicle capacity constraint; and (v) the improvement in total travel time is maximized (among all the feasible choices). Such a refinement technique is known as "Or-opt" (see Golden and Stewart 1985).

(2) *Refinement between two routes:* This is done by transferring a customer order (i.e., a pair of pickup and delivery nodes) from one vehicle route to another. Again, this is done in such a way that all pickup and delivery precedence constraints, time-window constraints, and vehicle capacity constraint are not violated, and the selection of the locations for this pair of pickup and delivery nodes is made in such a way that the improvement in total travel time is maximized.

3.3 Solution Enhancement by Genetic Search Method

Since the problem size of our model is usually quite large in practice, construction of a limited number of feasible solutions followed by subsequent refinements may not be sufficient to attain global optimality, as the total number of possible assignments is almost inexhaustible.

Hence, the application of a genetic search method may help in getting a very good solution, and hopefully close to the true optimum. We now present a Genetic Algorithm (GA), which takes the population of feasible solutions generated by the first phase as input and iteratively improves it. The efficiency of the GA is enhanced by some well-proven modifications (see Cheung *et al.* 2001 and Cheung 2005). The following is a description of the modified GA:

(1) *Encoding method:* We represent a chromosome by a list of customer orders along with their vehicle route assignment. That is, a chromosome is denoted as $\Delta = \{(i, j, \delta_{ij}) \mid d_{ij} > 0\}$, where $\delta_{ij} = k$ if customer order d_{ij} is served by the kth vehicle route. We refer to each element (i, j, δ_{ij}) as a "gene" and δ_{ij} as the value of the gene. For example, consider the problem instance shown in Figure 2(a). The route assignment corresponding to chromosome A depicted in Figure 2(b) is {(1,2,1), (3,6,2), (4,5,2), (7,8,1), (9,10,2), (11,12,2)}.

(2) *Fitness evaluation:* The fitness of a chromosome is determined as follows: We heuristically construct each vehicle tour. This is done by first selecting the farthest node as the seed node and then applying steps (2) and (3) of the seed selection algorithm (Section 3.1) to construct a feasible tour. This is followed by step (1) of the solution refinement procedure (Section 3.2). Then, the fitness of the chromosome is given by the total travel time of all vehicles in the constructed solution. For example, consider chromosome A as shown in Figures 2(b) and 2(c). The initial route plan of this chromosome is obtained by constructing a feasible route plan with three routes choosing nodes 2, 6, and 5 as seeds. The resulting route plan, which includes routes $0 \to 1 \to 2 \to 7 \to 8 \to 0$ and $0 \to 3 \to 4 \to 5 \to 6 \to 9 \to 10 \to 11 \to 12 \to 0$, is obtained through merging two shortest routes. Route 1 has a length of 432 (minutes), and route 2 has a length of 452 (minutes). In this case, the fitness value of the chromosome is 884.

(3) *The uniform crossover operations:* The crossover operation involves exchanging gene values in a subset of randomly selected entries between a pair of chromosomes. This is done as follows: First, a pair of chromosomes is randomly selected. Second, a positive integer $h \in \{1, 2, \ldots, q\}$ is randomly generated and h entries are randomly selected, where q is the number of customer orders. We swap the gene values between this pair of chromosomes at all of the selected entries. These two newly produced solutions (chromosomes) are evaluated and compared with the fitness values of their parents. If the fitness value of one of these new chromosomes produced are found to be better than both of its parents, we discard the parents and replace them by their offspring. Otherwise, we keep the parents. To avoid redundancy and to enhance the effectiveness of the procedure,

we will perform this crossover operation only if the corresponding gene values of at least two of the selected entries are distinct. For example, suppose that the pair of chromosomes selected are chromosome A = {(1,2,1), (3,6,2), (4,5,2), (7,8,1), (9,10,2), (11,12,2)} and chromosome B = {(1,2,1), (3,6,2), (4,5,3), (7,8,2), (9,10,1), (11,12,3)} as shown in Figures 2(b) and 2(c). Suppose $h = 2$ and that the fourth and fifth entries are selected. Then, the offspring are chromosome C = {(1,2,1), (3,6,2), (4,5,2), (7,8,2), (9,10,1), (11,12,2)} and chromosome D = {(1,2,1), (3,6,2), (4,5,3), (7,8,1), (9,10,2), (11,12,3)}. Since the first offspring is better than both parents (see Figure 2(c)), we replace both parents by the two offspring. In our genetic search procedure, we repeat the crossover operation for a pre-specified number of pairs of chromosome in the population. Here in our example as shown in Figure 2, the best solution is chromosome C with the fitness value found to be 851.

(4) *The uniform mutation with adaptive rate:* The uniform mutation step changes the values of a random subset of the genes in the chromosome. For a selected mutation rate $\mu \in (0,1]$, the mutation is done as follows: For $i = 1,2,\ldots,q$, we change the value of gene i of chromosome Δ to a new value with probability μ, where the new value is chosen randomly between 1 and q, and make no change to gene i with probability $1-\mu$. Thus, the expected number of genes that get "mutated" is μq. After the mutation operation is applied to a chromosome, the mutated chromosome will replace the original one if it has a better fitness value. For instance, if we take Δ as chromosome D = {(1,2,1), (3,6,2), (4,5,3), (7,8,1), (9,10,2), (11,12,3)} in our illustrative example, and suppose that the third and sixth genes of this chromosome are selected for mutation and that these two genes both get a new random value of 2, then we change these two genes to (4,5,2) and (11,12,2), respectively, and the mutated chromosome Δ' becomes {(1,2,1), (3,6,2), (4,5,2), (7,8,1), (9,10,2), (11,12,2)}. Normally, we replace Δ by Δ' only if Δ' has a better fitness value. The mutation operation is applied to every new chromosome produced by the crossover operation. It may help improve its fitness value, but it tends to be very disruptive as well (e.g., the above two genes may be mutated to three other equally probable combinations of values distinct from (4,5,2) and (11,12,2); also, different gene subset combinations may be selected leading to lower fitness values). An appropriate rate of uniform mutation will allow those strings to be mutated to have a higher probability of improving their fitness value. When the search is close to the optimal point, most of the chromosomes in the population will be very similar to the optimal chromosome. At that stage, only changes of gene value at very few chromosome entries are necessary to transform the chromosomes into the optimal one. Thus, we adaptively reduce the mutation rate when this scenario

occurs. Specifically, we gradually reduce the mutation rate, for instance, from 0.2 to 0.1 after each 100 generations.

(5) *The selection scheme and termination criteria:* The selection of chromosomes for an improved population after each generation (i.e., after all the crossover and mutation operations) is made according to the steps described in (3) and (4). It is also important to update the solution with the best fitness value at each generation. The iteration will stop after a pre-specified number of generations, while this number can be determined through some trial runs with examples.

3.4 A Test Instance

To test the above solution method, we randomly generate a problem instance with the following parameter setting: $n = 200$; $K = 20$; $m = 12$; nodes $\{0,1,2,\ldots,n\}$ are generated uniformly and independently within a unit square (in practice, we can always rescale the distance unit so that the size of the square is equal to one); travel time t_{ij} and t_{ji} are both equal to the Euclidean distance between node i and node j for $i, j = 0,1,2,\ldots,n$ (here we assume, without loss of generality, that the speed of the vehicles is equal to one); for $k = 1,2,\ldots,100$, node k is the pickup node and node $k+100$ is the delivery node of the kth customer order; demand $d_{k,k+100}$ is an integer uniformly distributed between 1 and 10; each r_{ij}^P is generated uniformly between 0 and 5; each s_{ij}^P is set equal to $r_{ij}^P + 2$; each r_{ij}^D is set equal to $r_{ij}^P + d_{ij} + 0.5$; and each s_{ij}^D is set equal to $r_{ij}^D + 2$. We set $m' = m$, $\alpha = 0.5$, and $\beta = 0.1$.

With these input data, we run our solution procedure for 4 randomly generated problem instances of the same scale, each on a 2 GHz machine for 5 minutes (including the time spent on initial feasible solution construction, solution refinement, and the execution of the modified GA). We obtain the best solution with a total vehicle travel time of 156.20 units representing a reduction close to 20% of the initial feasible solution and the worst solution is 166.82 which is a reduction of 10% of the initial feasible solution. (The average reduction is over 15%.) The solution value is plotted against the running time, and the result is shown in Figure 3. We observe that the heuristic solution value in all these instances converge to within 1% of the final solution after 210 seconds of computational time.

4. DYNAMIC CHANGE OF ROUTE PLAN AS NEW INFORMATION ARRIVES

As mentioned earlier, dynamic data arrive throughout the course of the day, and re-optimization of the route plan will be performed efficiently when new travel time data or new customer orders arrive. Let t be a time epoch at which new data arrive. The following are the crucial data required in order to perform the re-optimization:

(i) the position of each vehicle at time t and the vehicle's travel time to the next node;

(ii) the load level of each vehicle at time t;

(iii) the status of each customer order at time t; and

(iv) the travel time of each arc at time t.

We now describe how the re-optimization is performed heuristically. We first consider the case in which a new customer order has arrived. In this case, we create a new pickup node i and a new delivery node j for this new order, and the new customer order is denoted as d_{ij}. We attempt to add this new order to an existing route. We need to determine the best possible insertion points for i and j. To do that, we search through all existing routes and all possible pairs of insertion points, check the feasibility of each possible insertion, and evaluate the impact of the insertion. If the number of vehicle routes in the current solution is less than m, then one of the candidates in this search is obtained by creating a new route that serves only this new customer order. This step is followed by the application of the refinement procedure described in Section 3.2 to further improve the solution. Next, we consider the case in which new travel time data have arrived. In this case, if the new travel time does not cause infeasibility of the current route plan, then we simply apply the refinement procedure described in Section 3.2 to improve the solution. If the new travel time does cause infeasibility of the current route plan, then remove those infeasible orders and reinsert them into the route plan (i.e., treat them as newly arrived orders). When the refinement procedure is applied upon arrival of new information, a very short computational time limit (say within 30 seconds) is set so that the system can update the route plan efficiently.

We conduct computational experiments using randomly generated data to test the effectiveness of the re-optimization procedure for dynamically changing data. For each test instance I, we first solve the static problem using the method presented in Section 3. We then generate a set of new customer orders and updated travel times, and we denote this updated instance as I'. We determine the re-optimized route plan for I' using the method presented above. We let $Z(I')$ denote the total travel time of the solution generated by this approach. Next, we solve instance I' directly as a static problem using the method presented in Section 3 and denote the total travel time of the resulting solution as $Z^*(I')$. We measure the ineffectiveness of our re-optimization procedure by $R = [Z(I') - Z^*(I')]/Z^*(I') \times 100\%$ when it is applied to problem instance I'.

In the computational study, n is set to 50, 100, 200, 400 and K is set to 15, 20, 30. The number of vehicles available, m, is set equal to 12. The size of each time window (denoted as λ) is set 2, 4, and 6. Thus, there are a total of $4 \times 3 \times 3 = 36$ test instances. Similar to the test instance presented in Section 3.4, nodes $\{0,1,2,\ldots,n\}$ are generated uniformly and independently within a unit square. Travel time t_{ij} and t_{ji} are both equal to the Euclidean distance between node i and node j ($i, j = 0,1,2,\ldots,n$). For $k = 1,2,\ldots,100$, node k is the pickup node and node $k+100$ is the delivery node of the kth customer order. Demand $d_{k,k+100}$ is an integer uniformly distributed between 1 and 10. Each r_{ij}^P is generated uniformly between 0 and 5. Each s_{ij}^P is set equal to $r_{ij}^P + \lambda$; each r_{ij}^D is set equal to $r_{ij}^P + d_{ij} + 0.5$; and each s_{ij}^D is set equal to $r_{ij}^D + \lambda$. We set $m' = 0.8m$, $\alpha = 0.75$, and $\beta = 0.25$. The initial route plan is obtained by solving the static problem with a 5-minute computational time limit.

Table 1 summarizes the results of the computational experiments. For example, consider the case with $n = 50$ and $K = 15$. In our experiments, there are 4 new customer orders to be inserted into the existing route plan. When the re-optimization procedure is applied, the total travel time of the solution increases from 48.25 to 59.04. However, if we solve the problem with the new customer orders directly as a static problem, the total travel time of the solution becomes 56.50. Hence, the ineffectiveness of the re-optimization procedure in this case is 4.49%.

The ineffectiveness of the re-optimization procedure is normally below 10% for instances with $n < 200$, and below 5% for instances with $n \geq 200$. For the smaller networks, scenarios with the lowest ineffectiveness appear in those cases where the vehicles have the smallest capacity ($K = 15$), while for the larger networks, scenarios with the lowest ineffectiveness appear in those cases with medium capacity vehicles ($K = 20$). This can be explained as follows: Recall that the number of available vehicles and average demand are fixed for all the test cases. In those test instances with smaller networks, the vehicles are under-loaded most of the time, and the pre-merging and GA enhancement routines that help to improve the initial solution are not available in the dynamic updating process. Thus, the re-optimization procedure is less effective for smaller networks, particularly for those cases with large vehicle capacities.

We also observe that the effectiveness of the dynamic re-optimization procedure tends to increase as the sizes of the time windows increase. This is because larger time windows allow more flexibility for inserting new customer orders into the existing route plan. In fact, we have also tested our solution method on problem instances with smaller time windows (i.e., $\lambda < 2$). However, when the time windows are too tight, the solution procedure frequently rejects newly arrived orders, which makes it difficult to compare the solutions generated by the two approaches.

Overall, the effectiveness of our dynamic re-optimization procedure is quite high, which demonstrates that our proposed framework can handle the dynamic data and update the route plan efficiently without significantly giving up on the effectiveness of the solution.

5. CONCLUSIONS

We have proposed a framework for dynamic vehicle routing with time windows, pickups, and deliveries. This framework is suitable for today's fleet management systems with data collected and updated via advanced mobile technologies. It includes a GA-based procedure for solving the static vehicle routing problem, which enables the managers to develop their daily route plans, and a quick heuristic procedure for dynamic updates of the vehicle routes as new data arrive. The procedure for the static problem includes some simple but effective procedures including pre-merging of routes, insertions, and refinement routines, and is followed by a modified genetic search scheme. The careful design of this scheme allows us to obtain very good solutions within minutes, as verified by our test-runs on randomly generated instances. This is also crucial for practical applications in daily third party logistics operations where the dispatching of vehicles has to be done shortly after the work hour begins. There are well-known Integer Programming approaches that give an almost exact solution to such a model, but they may require much longer computational time for large-size problem instances. For the dynamic-update module, the quick heuristic procedure can generate good updated solutions in seconds, and the effectiveness of the heuristic is also demonstrated computationally.

One interesting future research topic is to improve the current dynamic route change procedure so as to increase its effectiveness for problems with tight time windows. Another possible future research direction is to extend the current solution method to solve more general dynamic routing problems, such as problems with loading and unloading time of goods, problems with order splitting allowed, problems involving the transfer of goods between delivery vehicles, and problems involving the coordination between warehousing and vehicle dispatching.

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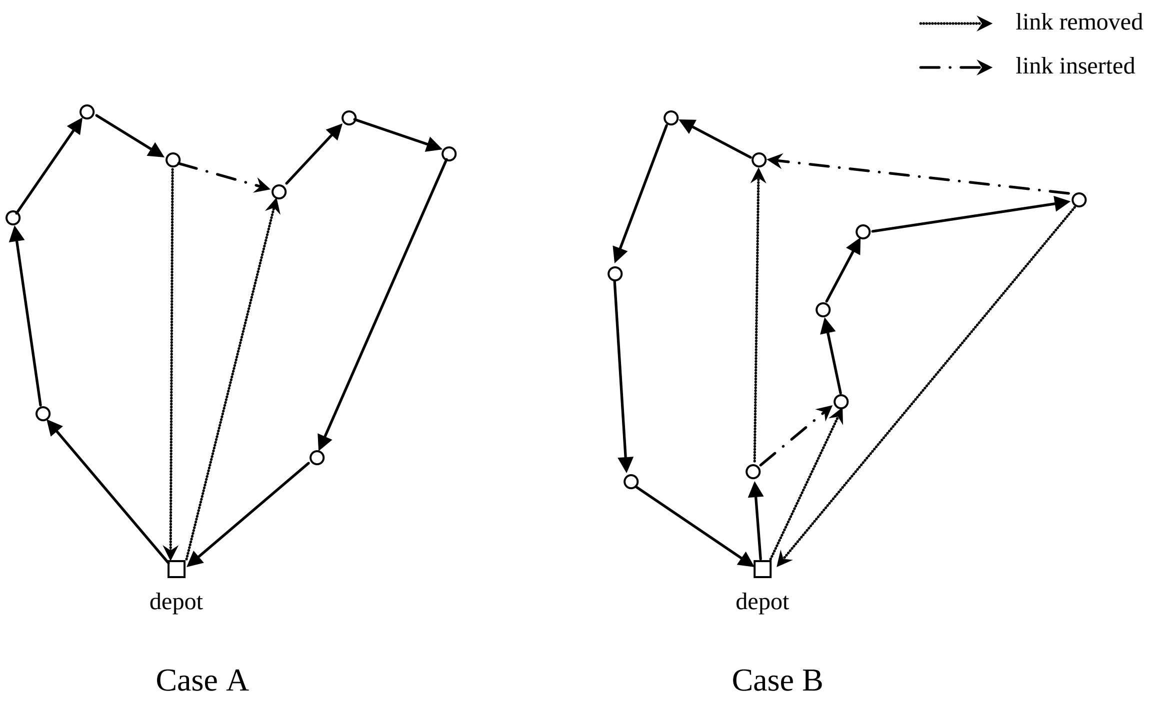


Figure 1: Merging and eliminating short routes

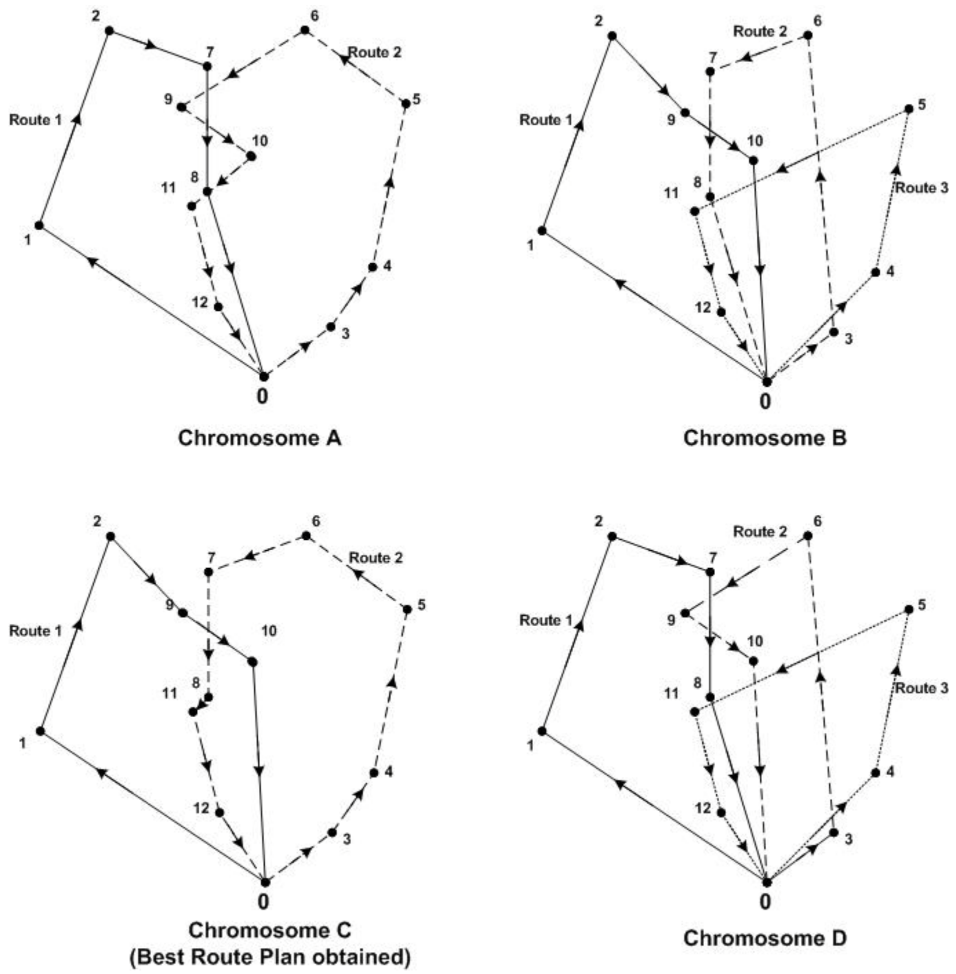
n = 13; m = 3; K = 5; all vehicles leave the depot at 08:00; t_{ij}'s are in minutes

t_{ij}		j												
		0	1	2	3	4	5	6	7	8	9	10	11	12
	0		130	182	40	74	148	167	151	93	135	107	89	40
	1	130		99										
	2	182	99						50		50			
	3	40				35		143						
	4	74			35		80							
	5	148				80		60					114	
i	6	167			143		60		50		70			
	7	151		50				50		60				
	8	93							60				10	
	9	135		50				70				40		
	10	107									40		37	
	11	89					114			10		37		50
	12	40											50	

d_{ij}		j												
		0	1	2	3	4	5	6	7	8	9	10	11	12
	0													
	1			5										
	2													
	3							3						
	4						2							
	5													
i	6													
	7									4				
	8													
	9											5		
	10													
	11													5
	12													

Node	Time window
1	$[r_{12}^P, s_{12}^P]$ = [10:00, 11:00]
2	$[r_{12}^D, s_{12}^D]$ = [11:30, 12:30]
3	$[r_{36}^P, s_{36}^P]$ = [08:30, 09:00]
4	$[r_{45}^P, s_{45}^P]$ = [09:00, 10:00]
5	$[r_{45}^D, s_{45}^D]$ = [10:30, 11:30]
6	$[r_{36}^D, s_{36}^D]$ = [11:00, 12:00]
7	$[r_{78}^P, s_{78}^P]$ = [11:45, 13:15]
8	$[r_{78}^D, s_{78}^D]$ = [12:45, 14:00]
9	$[r_{9,10}^P, s_{9,10}^P]$ = [12:00, 13:00]
10	$[r_{9,10}^D, s_{9,10}^D]$ = [12:30, 13:30]
11	$[r_{11,12}^P, s_{11,12}^P]$ = [12:15, 14:15]
12	$[r_{11,12}^D, s_{11,12}^D]$ = [13:15, 15:00]

(a) problem data



(b) crossover operations

Parents:

4^{th} entry 5^{th} entry

A: {(1,2,1), (3,6,2), (4,5,2), (7,8,1), (9,10,2), (11,12,2)}

- route 1: 0 → 1 → 2 → 7→ 8 → 0
- route 2: 0 → 3 → 4 → 5 → 6 → 9 → 10 → 11 → 12 → 0
- fitness value = 432 + 452 = 884

4^{th} entry 5^{th} entry

B: {(1,2,1), (3,6,2), (4,5,3), (7,8,2), (9,10,1), (11,12,3)}

- route 1: 0 → 1→ 2 → 9→ 10 → 0
- route 2: 0 → 3 → 6 → 7 → 8 → 0
- route 3: 0 → 4 → 5 → 11 → 12 → 0
- fitness value = 426 + 386 + 358 = 1170

Offspring:

C: {(1,2,1), (3,6,2), (4,5,2), (7,8,2), (9,10,1), (11,12,2)}

- route 1: 0 → 1 → 2 → 9 → 10 → 0
- route 2: 0 → 3 → 4 → 5 → 6 → 7 → 8 → 11 → 12 → 0
- fitness value = 426 + 425 = 851

D: {(1,2,1), (3,6,2), (4,5,3), (7,8,1), (9,10,2), (11,12,3)}

- route 1: 0 → 1 → 2 → 7 → 8 → 0
- route 2: 0 → 3 → 6 → 9 → 10 → 0
- route 3: 0 → 4 → 5 →11 → 12 → 0
- fitness value = 432 + 400 + 358 = 1190

(c) corresponding chromosomes and fitness values

Figure 2: A numerical example

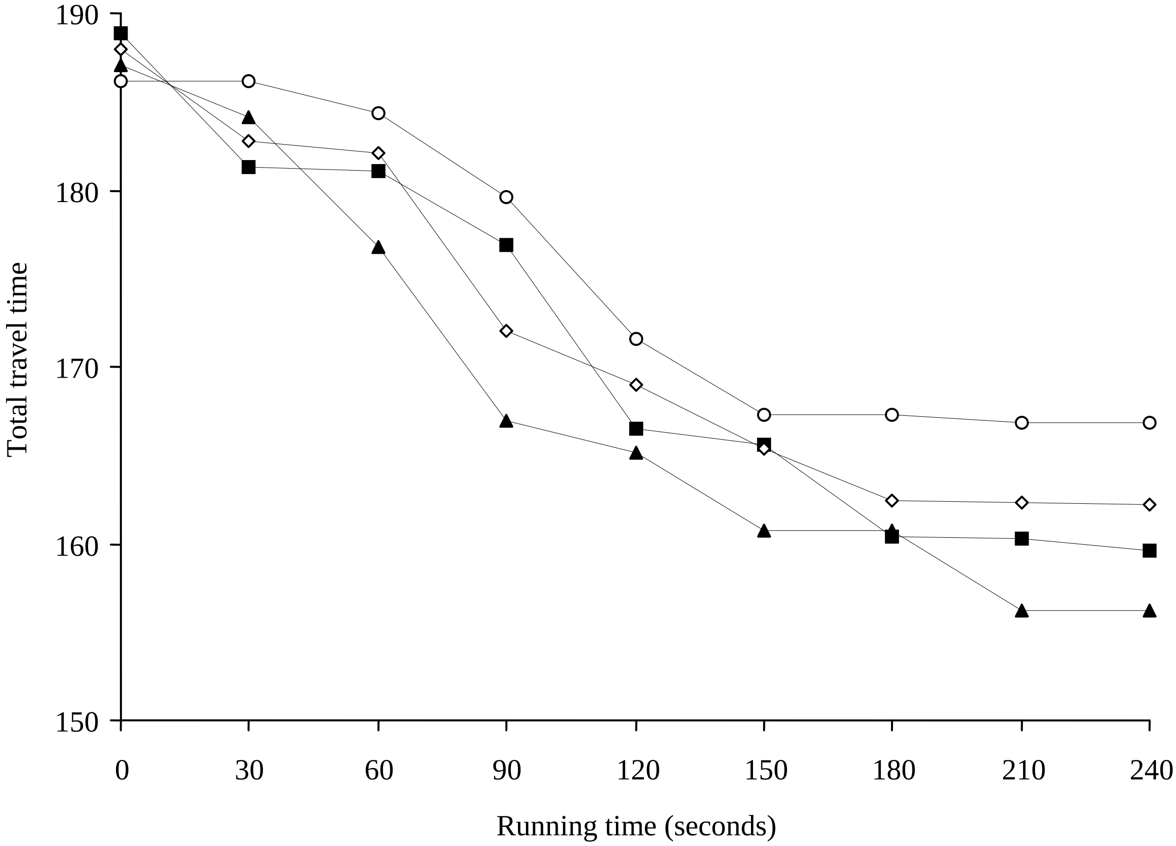

190
180
170
160
150
Total travel time
0
30
60
90
120
150
180
210
240
Running time (seconds)

Table 1. Computational results

	$\lambda = 2$	$\lambda = 4$	$\lambda = 6$
$n = 50$, $K = 15$	$R =$ 4.49%	$R =$ 1.30%	$R =$ 0.57%
$n = 50$, $K = 20$	$R =$10.08%	$R =$11.00%	$R =$ 2.00%
$n = 50$, $K = 30$	$R =$12.90%	$R =$11.80%	$R =$ 6.10%
$n = 100$, $K = 15$	$R =$ 5.85%	$R =$ 5.15%	$R =$ 3.28%
$n = 100$, $K = 20$	$R =$11.00%	$R =$ 9.20%	$R =$ 1.97%
$n = 100$, $K = 30$	$R =$ 5.99%	$R =$ 1.79%	$R =$ 3.45%
$n = 200$, $K = 15$	$R =$ 1.75%	$R =$ 1.36%	$R =$ 3.60%
$n = 200$, $K = 20$	$R =$ 0.50%	$R =$ 2.60%	$R =$ 1.01%
$n = 200$, $K = 30$	$R =$ 5.38%	$R =$ 4.49%	$R =$ 0.57%
$n = 400$, $K = 15$	$R =$ 0.64%	$R =$ 0.36%	$R =$ 0.08%
$n = 400$, $K = 20$	$R =$ 0.05%	$R =$ 0.03%	$R =$ 0.20%
$n = 400$, $K = 30$	$R =$ 0.62%	$R =$ 0.59%	$R =$ 2.93%