Quantifying Supply Chain Ineffectiveness Under Uncoordinated Pricing Decisions

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Abstract

We analyze a multiple-stage supply chain model of a seasonal product with pricing decisions. We develop closed-form expressions for the optimal expected profits of different stages. The results enable us to quantify the loss of supply chain profits if uncoordinated pricing decisions are made by supply chain agents.

Keywords: Supply chain management; newsvendor model; pricing; coordination

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1 Introduction

It is well known that when different parties in the supply chain make their ordering, pricing, and inventory holding decisions independently, the overall supply chain profit suffers from poor performance. In the supply chain management literature, numerous studies have been reported to address this issue. Coordination mechanisms such as revenue-sharing contracts, buyback contracts, volume discounts, etc. have been analyzed by many researchers for resolving the problem (see, for example, [3]). However, it is important to quantify the deficiency of the uncoordinated decisions so that the potential benefits obtained from a coordinated supply chain can be better understood.

In this paper we analyze a multiple-stage supply chain model of a seasonal product with pricing decisions made independently by each party. The retailer is facing a price-sensitive uncertain demand. Through extending the newsvendor model with pricing decisions, we develop a closed-form expression for the optimal expected profit of the supply chain, and we compare the total expected profit with that of an integrated system with centralized decisions.

Whitin [13] was the first to develop and analyze a newsvendor model with price decisions. Since then, numerous developments and applications of the newsvendor model with pricing considerations have appeared in the literature (see [5, 11] for recent surveys on newsvendor models with pricing decisions). Some of these works have extended the newsvendor model with pricing to a two-stage supply chain setting and discussed various coordination issues. These include the work on returns policies [6, 8, 10], the work on supply contracts and manufacturer's incentives [2, 4, 12], the work on supply chain pricing under risk aversion [1], among others. However, unlike these papers, we consider a multiple-stage supply chain and focus on the quantification of the ineffectiveness of decentralized supply-chain pricing decisions. For the case with two stages, our result is similar to some of the results in Wang *et al.* [12]. However, Wang *et al.*'s model differs from ours in that they consider a two-stage supply chain with a consignment sales arrangement.

Recently, Majumder and Srinivasan [9] have studied the impact of the position of the contract leader on the performance of a multi-stage supply chain. Similar to our work, they consider price-sensitive demand, derive the decentralized supply chain solution for their model, and compare it with the centralized solution. Unlike our work, they assume a deterministic linear demand function, and they focus on the implications of contract leadership. For other published works that address coordination issues in multi-stage supply chains, see, for example, [3, 7] and the references therein.

Our model is defined as follows: We consider an n-stage supply chain with a manufacturer, a retailer, and $n-2$ other parties such as distributer, wholesaler, etc. in between ($n \geq 2$). There is a single seasonal product facing an uncertain and price-sensitive demand. The manufacturer produces each unit of the product at a cost of c. It has to determine the unit price p_1 of the product to offer to its immediate downstream customer, so as to maximize its expected profit in anticipation of the reaction of the other supply chain parties after it has announced the price p_1. Once the price p_1 is given, the next stage of the supply chain has to determine the unit price p_2 to offer to its downstream customer so as to maximize its expected profit in anticipation of the reaction of the downstream supply chain parties. The same logic applies to the other parties along the supply chain. The last stage of the supply chain, i.e. the retailer, has to determine the unit price p to offer to the end customers as well as the order quantity q. Thus, the retailer's problem is a price-dependent newsvendor problem. We refer this supply chain system to as the "decentralized" system. We denote the manufacturer as M, the retailer as R, and the other parties as $P_2, P_3, \ldots, P_{n-1}$ as shown in Figure 1. For notational convenience, we denote $P_1 \equiv M$, $P_n \equiv R$, and $p_0 \equiv c$. The following assumptions are made:

- End customer's demand is given as $D_b(p)\varepsilon$, where ε is a nonnegative random variable with a general probability distribution and $D_b(p) = ap^{-b}$ $(a > 0;\ b > 1)$.

- Unsold items bear no salvage value or disposal cost.

The above multiplicative demand model with iso-elastic demand has been widely adopted in the literature (see, for example, [11]). Without loss of generality, we assume $a = 1$. Parameter b represents the price-elasticity index of the expected demand. In this study, we focus on price-elastic products with $b > 1$. We let $f(x)$ and $F(x)$ denote the probability density function and cumulative distribution function, respectively, of ε.

In the next section, we consider the optimal decision of each supply chain party in the decentralized system, and we develop a closed-form expression for the optimal expected profit of the supply chain. In section 3, we analyze the situation where there exists a centralized decision maker, and then we compare the expected total profit of the decentralized system with that of the centralized system. Section 4 concludes the paper and provides a comparison between our model and Wang *et al.*'s [12] model with consignment contracts.

2 Optimal Decisions of the Decentralized System

We first consider the retailer's decision. Following Petruzzi and Dada [11], we denote $z = q{\cdot}p^b$ and $\Lambda(z) = \int_0^z F(x)dx = \int_0^z (z-x)f(x)dx$. The retailer's expected profit is given as

$$\Pi_R(p, z) = p \cdot E\big[\min\{q, D(p)\varepsilon\}\big] - p_{n-1}q$$

$$= p^{-b}\Big\{p \cdot E\big[\min\{z, \varepsilon\}\big] - p_{n-1}z\Big\}$$

$$= p^{-b}\Big\{p\big[z - \Lambda(z)\big] - p_{n-1}z\Big\}.$$

This implies that

$$\frac{\partial \Pi_R(p,z)}{\partial p} = -(b-1)p^{-b}\big[z - \Lambda(z)\big] + bp^{-b-1}p_{n-1}z$$

and

$$\frac{\partial \Pi_R(p,z)}{\partial z} = p^{-b+1}\big[1 - F(z)\big] - p^{-b}p_{n-1}.$$

Setting these partial derivatives to zero, we have

$$p^* = \frac{bp_{n-1}z^*}{(b-1)\big[z^* - \Lambda(z^*)\big]} \tag{1}$$

and

$$p^* = \frac{p_{n-1}}{1 - F(z^*)}, \tag{2}$$

where p^* and z^* denote the optimal values of p and z, respectively. Note that the optimal value of q is given as $q^* = z^*(p^*)^{-b}$. Combining (1) and (2), we obtain

$$\frac{bz^*}{(b-1)\big[z^* - \Lambda(z^*)\big]} = \frac{1}{1 - F(z^*)}. \tag{3}$$

Equation (3) implies that z^* is independent of p_{n-1}. Let Π_R^* denote the maximum expected profit of the retailer. By (2) and (3), we have

$$\Pi_R^* = \Pi_R(p^*, z^*) = \frac{1}{b-1}\big[1 - F(z^*)\big]^b z^* p_{n-1}^{-b+1}.$$

Next, we consider the decisions of $P_1, P_2, \ldots, P_{n-1}$. The following theorem states the optimal pricing decisions of these $n-1$ parties and their corresponding optimal expected profits.

Theorem 1 *For $i = 1, 2, \ldots, n-1$, the expected profit of P_i is maximized when $p_i^* = \frac{b}{b-1} p_{i-1}$, and the maximum expected profit of P_i is given as*

$$\Pi^*_{P_i} = \frac{1}{b-1}\big[1 - F(z^*)\big]^b z^* \Big(\frac{b}{b-1}\Big)^{-(n-1)b+i-1} c^{-b+1}.$$

Proof We first prove that

$$p_i^* = \frac{b}{b-1} p_{i-1} \tag{4}$$

for $i = 1, 2, \ldots, n-1$. For the case where $i = n-1$, the expected profit of P_{n-1} is given as

$$\begin{aligned}
\Pi_{P_{n-1}}(p_{n-1}) &= (p_{n-1} - p_{n-2})q^* \\
&= (p^*)^{-b}(p_{n-1} - p_{n-2})z^* \\
&= \big[1 - F(z^*)\big]^b z^* p_{n-1}^{-b}(p_{n-1} - p_{n-2}) \qquad \text{(by (2))}.
\end{aligned}$$

Since z^* is independent of p_{n-1}, this function is maximized when $p_{n-1} = bp_{n-2}/(b-1)$. Thus, equation (4) holds when $i = n-1$. Next, by induction, suppose equation (4) holds for $i = j+1, j+2, \ldots, n-1$, where $1 \le j \le n-2$. Then

$$p^*_{n-1} = \frac{b}{b-1} \cdot p^*_{n-2} = \Big(\frac{b}{b-1}\Big)^2 p^*_{n-3} = \cdots = \Big(\frac{b}{b-1}\Big)^{n-j-2} p^*_{j+1} = \Big(\frac{b}{b-1}\Big)^{n-j-1} p_j.$$

Hence,

$$\begin{aligned}
\Pi_{P_j}(p_j) &= (p_j - p_{j-1})q^* \\
&= (p^*)^{-b}(p_j - p_{j-1})z^* \\
&= \big[1 - F(z^*)\big]^b z^* (p^*_{n-1})^{-b}(p_j - p_{j-1}) \qquad \text{(by (2))} \\
&= \big[1 - F(z^*)\big]^b z^* \Big(\frac{b}{b-1}\Big)^{-(n-j-1)b} p_j^{-b}(p_j - p_{j-1}),
\end{aligned} \tag{5}$$

which is maximized when $p_j = bp_{j-1}/(b-1)$. Thus, equation (4) also holds for $i = j$. Therefore, equation (4) holds for $i = 1, 2, \ldots, n-1$.

By (4) and (5), the maximum expected profit of P_i is

$$\begin{aligned}
\Pi^*_{P_i} &= \big[1 - F(z^*)\big]^b z^* \left(\frac{b}{b-1}\right)^{-(n-i-1)b} (p^*_i)^{-b}(p^*_i - p^*_{i-1}) \\
&= \frac{1}{b-1}\big[1 - F(z^*)\big]^b z^* \left(\frac{b}{b-1}\right)^{-(n-i)b} (p^*_{i-1})^{-b+1} \\
&= \frac{1}{b-1}\big[1 - F(z^*)\big]^b z^* \left(\frac{b}{b-1}\right)^{-(n-i)b} \left(\frac{b}{b-1}\right)^{-(i-1)(b-1)} p_0^{-b+1} \\
&= \frac{1}{b-1}\big[1 - F(z^*)\big]^b z^* \left(\frac{b}{b-1}\right)^{-(n-1)b+i-1} c^{-b+1}.
\end{aligned}$$

This completes the proof of the theorem. ∎

By Theorem 1, we can determine the maximum expected total profit of the decentralized system:

$$\begin{aligned}
\Pi^* &= \sum_{i=1}^{n} \Pi^*_{P_i} \\
&= \sum_{i=1}^{n} \frac{1}{b-1}\big[1 - F(z^*)\big]^b z^* \left(\frac{b}{b-1}\right)^{-(n-1)b+i-1} c^{-b+1} \\
&= \big[1 - F(z^*)\big]^b z^* \left(\frac{b}{b-1}\right)^{-(n-1)b} \left[\left(\frac{b}{b-1}\right)^{n} - 1\right] c^{-b+1}. \qquad (6)
\end{aligned}$$

3 Comparing the Centralized and Decentralized Systems

In this section, we first analyze the situation where there exists a centralized decision maker for determining p and q so as to maximize the expected systemwide total profit. We perform this analysis so that we can compare the total profits of the centralized and decentralized systems. We let $\hat{p}$, $\hat{q}$, and $\hat{z}$ denote the values of p, q, and z, respectively, in the centralized system. The centralized system's expected profit, which includes the profits of both the manufacturer and the retailer, is

$$\begin{aligned}\hat{\Pi}(\hat{p}, \hat{z}) &= \hat{p} \cdot E\big[\min\{\hat{q}, D(\hat{p})\varepsilon\}\big] - c\hat{q} \\ &= \hat{p}^{-b}\Big\{\hat{p} \cdot E\big[\min\{\hat{z}, \varepsilon\}\big] - c\hat{z}\Big\} \\ &= \hat{p}^{-b}\Big\{\hat{p}\big[\hat{z} - \Lambda(\hat{z})\big] - c\hat{z}\Big\}.\end{aligned}$$

This implies that

$$\frac{\partial \hat{\Pi}(\hat{p}, \hat{z})}{\partial \hat{p}} = -(b-1)\hat{p}^{-b}\big[\hat{z} - \Lambda(\hat{z})\big] + b\hat{p}^{-b-1}c\hat{z}$$

and

$$\frac{\partial \hat{\Pi}(\hat{p}, \hat{z})}{\partial \hat{z}} = \hat{p}^{-b+1}\big[1 - F(\hat{z})\big] - \hat{p}^{-b}c.$$

Setting these partial derivatives to zero, we have

$$\hat{p}^* = \frac{bc\hat{z}^*}{(b-1)\big[\hat{z}^* - \Lambda(\hat{z}^*)\big]} \tag{7}$$

and

$$\hat{p}^* = \frac{c}{1 - F(\hat{z}^*)}, \tag{8}$$

where $\hat{p}^*$ and $\hat{z}^*$ denote the optimal values of $\hat{p}$ and $\hat{z}$, respectively. Combining (7) and (8), we obtain

$$\frac{b\hat{z}^*}{(b-1)\big[\hat{z}^* - \Lambda(\hat{z}^*)\big]} = \frac{1}{1 - F(\hat{z}^*)}. \tag{9}$$

From (3) and (9), we have $\hat{z}^* = z^*$. Hence, the maximum expected profit of the centralized system is given as

$$\begin{aligned}\hat{\Pi}^* &= \hat{\Pi}(\hat{p}^*, \hat{z}^*) \\ &= (\hat{p}^*)^{-b}\Big\{\hat{p}^*\big[\hat{z}^* - \Lambda(\hat{z}^*)\big] - c\hat{z}^*\Big\} \\ &= \frac{1}{b-1}\big[1 - F(\hat{z}^*)\big]^b \hat{z}^* c^{-b+1} \qquad \text{(by (8) and (9))} \\ &= \frac{1}{b-1}\big[1 - F(z^*)\big]^b z^* c^{-b+1}. \end{aligned} \tag{10}$$

Next, we analyze the performance of the decentralized system by comparing its expected total profit with that of the centralized counterpart. The percentage loss of channel profit due to decentralization decisions is

$$\Delta_n(b) = \frac{\hat{\Pi}^* - \Pi^*}{\hat{\Pi}^*}.$$

Substituting (6) and (10) into this equation and simplifying, we have

$$\Delta_n(b) = \left\{1 - (b-1)\Big(\frac{b}{b-1}\Big)^{-(n-1)b}\Big[\Big(\frac{b}{b-1}\Big)^n - 1\Big]\right\} \times 100\%.$$

The following theorem provides some important properties of the function $\Delta_n(b)$.

Theorem 2 *For any integer $n \geq 2$, (i) the function $\Delta_n(b)$ is increasing in b for $b > 1$, and (ii) $\lim_{b\to\infty} \Delta_n(b) = 1 - ne^{-(n-1)}$.*

Proof Define

$$\eta(\lambda) = \frac{2(1-\lambda)}{1+\lambda} + \log \lambda$$

for $0 < \lambda \leq 1$. Note that

$$\eta'(\lambda) = \frac{(1-\lambda)^2}{\lambda(1+\lambda)^2} > 0$$

for $0 < \lambda < 1$. Thus, η is strictly increasing on $(0, 1)$. This, together with the fact that $\eta(1) = 0$, implies that $\eta(\lambda) < 0$ for $0 < \lambda < 1$. Hence, for any $\mu \in (0, 1)$ and $x > \frac{1}{2}$, we have $\eta(\mu^{2x}) < 0$, that is,

$$\frac{2(1-\mu^{2x})}{1+\mu^{2x}} + \log \mu^{2x} < 0. \tag{11}$$

Define

$$\theta_\mu(x) = \frac{\mu^{-x} - \mu^x}{x}$$

for $x > \frac{1}{2}$, where $0 < \mu < 1$. Then

$$\begin{aligned} \theta'_\mu(x) &= -\frac{\mu^{-x} + \mu^x}{2x^2}\Big[\frac{2(1-\mu^{2x})}{1+\mu^{2x}} + \log \mu^{2x}\Big] \\ &> 0 \qquad \text{(by (11))}. \end{aligned}$$

Thus, $\theta_\mu(x)$ is strictly increasing when $x > \frac{1}{2}$. This implies that when $b > 1$, the function

$$\gamma_b(n) = \frac{(\frac{b-1}{b})^{-n/2} - (\frac{b-1}{b})^{n/2}}{n/2}$$

is strictly increasing for $n \geq 1$. This, together with the property that $\gamma_b(n) > 0$, implies that $\gamma_b^2(1) < \gamma_b^2(n)$ for $n > 1$. In other words,

$$\frac{(\frac{b-1}{b})^{-1} + (\frac{b-1}{b}) - 2}{1/4} < \frac{(\frac{b-1}{b})^{-n} + (\frac{b-1}{b})^{n} - 2}{n^2/4},$$

or equivalently,

$$\Big(\frac{b-1}{b}\Big)^{n-1} + \Big(\frac{b-1}{b}\Big)^{n+1} - 2\Big(\frac{b-1}{b}\Big)^{n} < \frac{1}{n^2}\Big[1 + \Big(\frac{b-1}{b}\Big)^{2n} - 2\Big(\frac{b-1}{b}\Big)^{n}\Big].$$

After rearranging terms, we have

$$\frac{n\big[1 - (\frac{b-1}{b})^{n-1}\big]}{(n-1)\big[1 - (\frac{b-1}{b})^{n}\big]} > \frac{(n+1)\big[1 - (\frac{b-1}{b})^{n}\big]}{n\big[1 - (\frac{b-1}{b})^{n+1}\big]}. \tag{12}$$

Define

$$\beta_b(n) = \frac{n\big[1 - (\frac{b-1}{b})^{n-1}\big]}{(n-1)\big[1 - (\frac{b-1}{b})^{n}\big]}.$$

Equation (12) implies that $\beta_b(n)$ is strictly decreasing in n, for any $b > 1$ and $n = 2, 3, \ldots$. Let $\alpha(b) = \frac{1}{b} \cdot \beta_b(2) + \log\big(\frac{b-1}{b}\big) = \frac{2}{2b-1} + \log\big(\frac{b-1}{b}\big)$. Then, $\lim_{b\to\infty} \alpha(b) = 0$ and

$$\alpha'(b) = \frac{1}{b(b-1)(2b-1)^2} > 0$$

for $b > 1$. This implies that $\alpha(b) < 0$ for $b > 1$, which in turn implies that

$$\beta_b(2) < -b \log\Big(\frac{b-1}{b}\Big).$$

Hence,

$$\beta_b(n) < -b \log\Big(\frac{b-1}{b}\Big)$$

for $n = 2, 3, \ldots$. Differentiating $\Delta_n(b)$ with respect to b, we have

$$\begin{aligned}\Delta'(b) &= -\left(\frac{b-1}{b}\right)^{(n-1)(b-1)}\left\{\left[1-\left(\frac{b-1}{b}\right)^{n-1}\right]n+\left[1-\left(\frac{b-1}{b}\right)^{n}\right](n-1)b\log\left(\frac{b-1}{b}\right)\right\} \\ &= -\left(\frac{b-1}{b}\right)^{(n-1)(b-1)}\left[1-\left(\frac{b-1}{b}\right)^{n}\right](n-1)\left[\beta_b(n)+b\log\left(\frac{b-1}{b}\right)\right] > 0.\end{aligned}$$

Therefore, $\Delta_n(b)$ is increasing when $b > 1$.

Note that

$$\lim_{b\to\infty}(b-1)\left[\left(\frac{b}{b-1}\right)^{n}-1\right] = \lim_{b\to\infty}\frac{(\frac{b}{b-1})^{n}-1}{\frac{b}{b-1}-1} = n$$

and

$$\lim_{b\to\infty}\left(\frac{b}{b-1}\right)^{-(n-1)b} = \left[\lim_{b\to\infty}\left(\frac{b}{b-1}\right)\cdot\lim_{b\to\infty}\left(1+\frac{1}{b-1}\right)^{b-1}\right]^{-(n-1)} = \big(1\cdot e\big)^{-(n-1)} = e^{-(n-1)}.$$

Therefore, $\lim_{b\to\infty}\Delta_n(b) = 1 - ne^{-(n-1)}$. ∎

Theorem 2 implies that the percentage loss of channel profit caused by decentralized decisions increases as the price elasticity increases, but it is bounded from above by $[1 - ne^{-(n-1)}] \times 100\%$. Figure 2 depicts the function $\Delta_n(b)$ for different values of n. As shown in this figure, when the price-elasticity index b is large, the percentage loss of channel profit is quite close to the upper bound. Furthermore, we observe that the loss in channel profit is seriously affected by the number of stages in the supply chain.

4 Concluding Remarks

We have analyzed a multiple-stage supply chain model with pricing decisions made independently by each party, while the retailer is facing a price-sensitive uncertain demand. Our model allows us to quantify the loss in channel profit caused by uncoordinated pricing decisions. The percentage loss of channel profit depends critically on the number of stages in the supply chain and the price elasticity. For a given number of supply chain parties, an upper bound exists on the percentage profit loss of channel profit.

Note that for a two-stage supply chain, the upper bound on the percentage loss of channel profit is $1-(2/e) \approx 26.4\%$, which is identical to channel profit loss in Wang *et al.*'s [12] model. In fact, for the special case of $n = 2$, our mathematical analysis becomes essentially the same as that in [12]. However, our model addresses the problem where the retailer is responsible for setting the unit selling price of the product and determining the order quantity, while in Wang *et al.*'s model the manufacturer chooses the selling price and the production quantity under a consignment sales arrangement.

Consider an extension of Wang *et al.*'s model to an n-stage supply chain ($n \geq 3$), where the manufacturer decides on the retail price and retains ownership of the goods until they are sold at the retailer. Figure 3 depicts such a system. Here, intermediate party P_i ($i = 2, 3, \ldots, n-1$) serves as a middleman who obtains the consignment contract from its upstream party and transfers it to its downstream party while keeping a portion of the revenue as its own profit. We assume that the operating costs of the intermediate parties and the retailer are zero. The manufacturer's expected profit is given as

$$\Pi_M(p, z) = (p - p_2) \cdot E\big[\min\{q, D(p)\varepsilon\}\big] - cq = p^{-b}\Big\{p(1-\xi)\big[z - \Lambda(z)\big] - cz\Big\},$$

where $z = q \cdot p^b$ and $\xi = p_2/p$. The profit of P_i ($i = 2, 3, \ldots, n$) is given as

$$\Pi_{P_i}(p_i) = (p_i - p_{i+1})E\big[\min\{q^*, D(p^*)\varepsilon\}\big],$$

where $p_{n+1} \equiv 0$. Following the method presented in Section 2, we can show that the expected profit of P_i is maximized when

$$p_i^* + \frac{c}{1-F(z^*)} = \frac{b}{b-1}\left[p_{i+1} + \frac{c}{1-F(z^*)}\right]$$

and that the optimal expected profit of P_i is

$$\Pi_{P_i}^* = \frac{1}{b-1}\big[1-F(z^*)\big]^b z^* \Big(\frac{b}{b-1}\Big)^{-(n-1)b+n-i+1} c^{-b+1},$$

for $i = 2, 3, \ldots, n$. The optimal expected profit of the manufacturer is

$$\Pi_{P_1}^* = \Pi_M^* = \frac{1}{b-1}\big[1-F(z^*)\big]^b z^* \Big(\frac{b}{b-1}\Big)^{-(n-1)b} c^{-b+1}.$$

Hence, the optimal expected total profit of this decentralized supply chain, $\sum_{i=1}^n \Pi_{P_i}^*$, is identical to that presented in equation (6). This implies that the percentage loss of channel profit shown in Figure 2 also applies to such a decentralized n-stage consignment model. However, unlike the multiple-stage supply chain shown in Figure 1, it is less likely that such an n-stage consignment model ($n \geq 3$) exists in reality, since a manufacturer usually signs a consignment contract directly with the retailer without going through intermediate parties.

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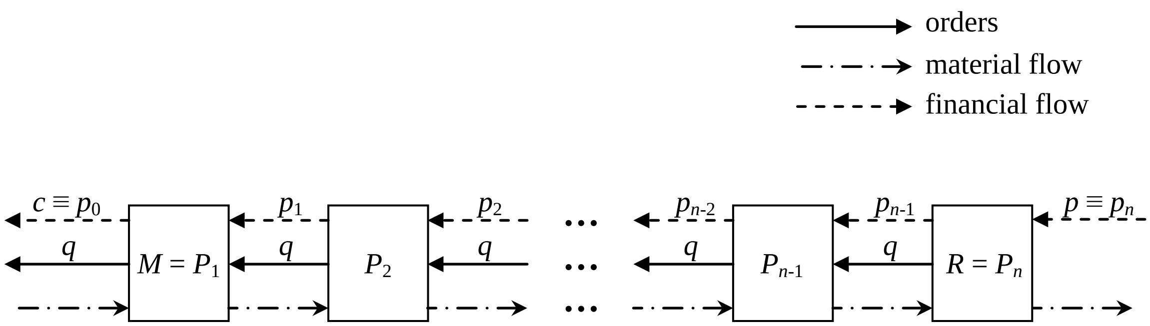

Figure 1. The *n*-stage supply chain

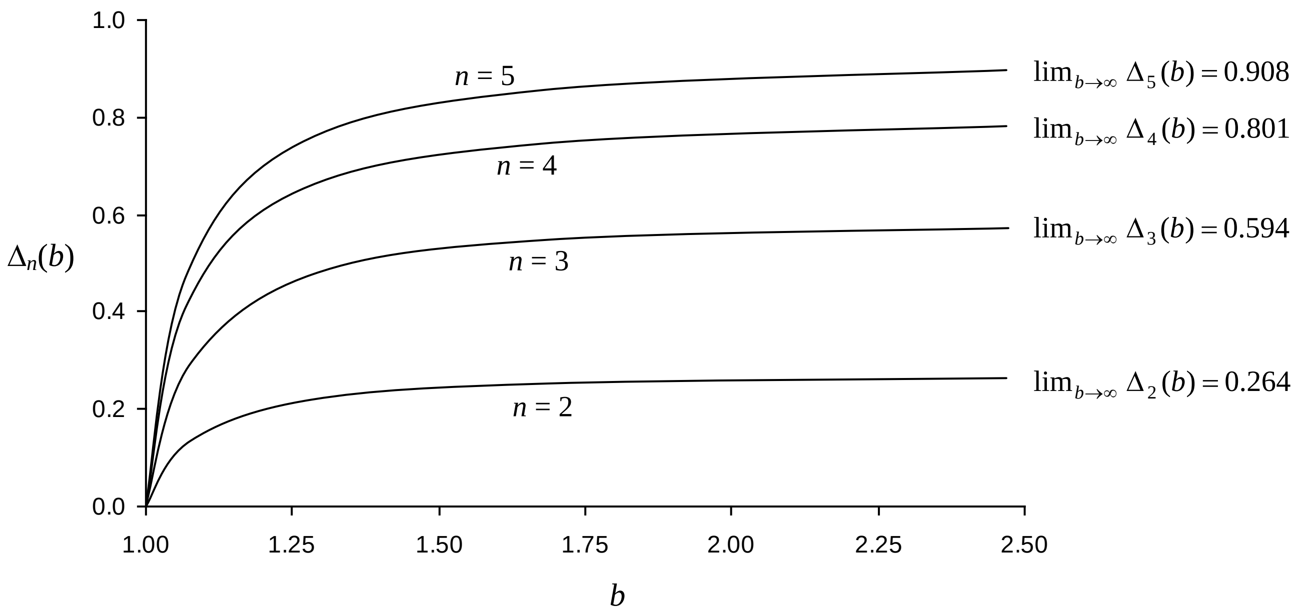

Figure 2. Percentage loss of channel profit versus price-elasticity index

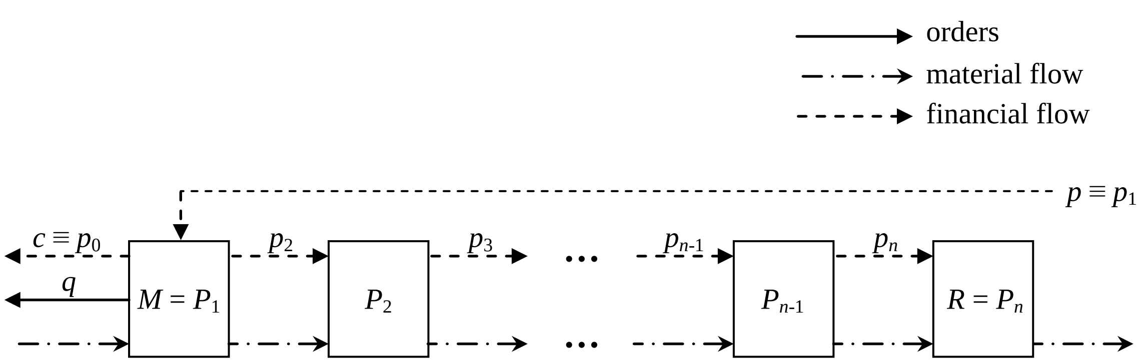

Figure 3. The *n*-stage supply chain with consignment contracts