Mixed Truck Delivery Systems with both Hub-and-Spoke and Direct Shipment

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Abstract

This paper studies a mixed truck delivery system that allows both hub-and-spoke and direct shipment delivery modes. A heuristic algorithm is developed to determine the mode of delivery for each demand and to perform vehicle routing in both modes of deliveries. Computational experiments are carried out on a large set of randomly generated problem instances to compare the mixed system with the pure hub-and-spoke system and the pure direct shipment system. The experiment results show that the mixed system can save around 10% total traveling distance on average as compared with either of the two pure systems.

Keywords: Truck delivery system, Hub-and-spoke, Direct shipment, Mixed system, Vehicle routing, Heuristic algorithm.

1. Introduction

The flow of physical goods from manufacturers to customers is a major focus of logistics systems. Moreover, product delivery at a reasonable cost has recently become a critical factor in the survival of emerging e-businesses (Lee and Whang, 2001). Logistics and operations researchers have done extensive research on the design and operations of local delivery systems in order to determine the most cost-effective methods of delivery. Because obtaining the optimal solutions to these problems is extremely difficult, it is essential to develop good heuristics for organizing and operating delivery systems.

The design or organization of a local truck delivery system has a critical impact on its performance. There are two common types of delivery system designs: the *direct shipment* system and the *hub-and-spoke* system. In a direct shipment system, each supplier operates independently with its own fleet delivering goods to customers. Each vehicle visits only one customer in a trip. This method should be utilized when the lead-time requirement is tight, the goods need to be isolated, or the shipment is large. If these criteria are not satisfied, then transportation costs can be reduced by having each delivery vehicle visit several customer locations, provided that the total quantity of goods to be delivered does not exceed the vehicle's capacity. This type of arrangement is called *direct shipment with milk runs*. Whenever a milk run is included, a decision on the routing of each vehicle needs to be made. When there are multiple suppliers in the delivery region, especially when customers have common suppliers, another type of delivery system can be utilized. In such a system, goods from all suppliers are collected and consolidated in a central facility, called the *hub*, and then redistributed to the customers. If each vehicle visits only one supplier or customer in a collection or redistribution trip, the system is called a *hub-and-spoke* system. When the delivery order sizes are relatively small, a vehicle can visit several stops in a collection trip or a redistribution trip. This delivery network is termed *hub-and-spoke with milk runs*. We will consider only the direct shipment with milk runs and hub-and-spoke with milk runs. For simplicity, we will simply refer to these systems as direct shipment and hub-and-spoke respectively in the following discussion.

The hub-and-spoke system takes advantage of the economies of scale in vehicle utilization. It can also improve customer service in terms of delivery frequency. When direct shipment is used, smaller suppliers need to wait until a sufficient amount of goods are ordered to maintain cost effectiveness in transportation. With the hub-and-spoke system, suppliers can provide a higher frequency of delivery (improved service quality) by combining the demands or orders of others. Intuitively, when the customers of each supplier are located very close to the supplier and the delivery quantity is large enough to justify the shipping of goods with full truckloads, the direct shipment system is better. Otherwise, the hub-and-spoke system is more appropriate.

In reality, suppliers and customers are located quite randomly and delivery quantities vary from order to order. The advantage of one of the systems over the other is neither obvious nor unchanged from day to day. In this situation, a *mixed delivery system* can be beneficial and better than either of the two pure delivery systems. Such a mixed system can be viewed as a hub-and-spoke system allowing some orders to be directly shipped whenever beneficial. Therefore, in the mixed system, different delivery modes may be used for different shipments depending on the quantity to be shipped and geographical locations of the supplier and the customer (see Figure 1).

There has been extensive research on the Vehicle Routing Problem (VRP), which is the main component of a direct shipment system with milk runs. It is a problem of determining routes through one or more depots and a set of customer locations to minimize the total distance traveled. A VRP can take various forms based on the constraints and requirements of the network and the shipment demands, such as vehicle capacity, the delivery time window, line-haul and back-haul demands, and multiple depots, etc. Bodin and Golden (1981) presented an overview of different types of VRPs. Our analysis is related to one of these VRPs, namely, the Capacitated Vehicle Routing Problem (CVRP), in which each vehicle has a given capacity. Various solution methods have been developed to solve the CVRP. Well-known solution techniques include savings heuristics (Clarke and Wright, 1964), the sweep heuristic (Gillett and Miller, 1974), λ-opt tour improvement methods (Lin, 1965), etc. For recent surveys on these solution techniques, see Laporte (1992), Fisher (1995), and Laporte *et al.* (2000).

In order to increase the efficiency of delivery systems, some researchers have studied the design and operation of hub-and-spoke systems, in which the hub location is a critical decision. For example, O'Kelly (1987), Campbell (1996), Abdinnour-Helm and Venkataramanan (1998), and Pirkul and Schilling (1998) solved the location-allocation problem that determines locations of hubs and the assignment of nodes to each hub. O'Kelly and Bryan (1998) considered the above problem with economies of scale taken into account, where the marginal cost decreases with flow volume. All these papers used air passenger flow data to illustrate their methods. But the models and algorithms in these studies are general and may be applied in air, truck or telecommunication networks. Recently, Sasaki *et al.* (1999) proposed solution algorithms to solve multi-hub location problems in the airline industry.

The mixed delivery system has received less attention than the two pure systems. Aykin (1995a) studied the location-routing problem. The problem was to find the hub locations and at the same time to determine the delivery mode for each demand. Aykin (1995b) proposed a simulated annealing procedure to solve the problem with an initial solution generated using a greedy interchange heuristic. The interchange was based on the "savings estimate" calculated for each hub–node pair if that node would be served by that hub. Hall (1987) developed similar models and used the EOQ concept to determine which delivery mode a demand should be assigned to with a predetermined hub location. However, all these models for mixed delivery systems assumed that each trip only involved one origin or one destination through the hub(s). These models were built for applications in air transportation networks. They considered only the assignment of demands to particular hub(s) without dealing with the issue of routing the vehicle in each trip. To the best of our knowledge, there has been no previous work on a mixed system with milk runs.

In this study, we propose a heuristic for scheduling vehicles in a mixed truck delivery system and evaluate through extensive computational experiments the traveling cost (distance) savings of the mixed system as compared with the traveling costs of the pure systems. For simplicity, we ignore the fixed cost of operating the hub. Furthermore, we assume that any variable cost of operating the hub is included in the transportation cost of entering and leaving the hub (see Section 2 for further discussion).

We assume that homogeneous vehicles are used. We further assume that the required delivery quantity from any supplier to any customer does not exceed the capacity of one vehicle. A customer order from a supplier cannot be split into two trips in the direct shipment system. Since demands are aggregated in the hub-and-spoke system, the total supply from one supplier or the total demand by one customer may be larger than a truckload. Therefore, it is inevitable for us to allow splitting of shipments into several vehicles.

This paper is organized as follows: Section 2 presents the model of our mixed delivery system. The heuristic procedure for scheduling vehicles in the delivery system is given in detail in Section 3. Section 4 reports the computational results and analyzes the relationship between the benefits of the mixed delivery system and the problem parameters. Finally, some concluding remarks are provided in Section 5.

2. Model Description

In our study, the delivery system is defined on an undirected network $G=(V,E)$. The vertex set is $V=\{u_0\}\cup V_s\cup V_c$, where u_0 is the given hub location, $V_s=\{u_1,u_2,\ldots,u_m\}$ is the set of suppliers, and $V_c=\{u_{m+1},u_{m+2},\ldots,u_{m+n}\}$ is the set of customers. Associated with network G is a shortest distance matrix with elements t_{ij} being the shortest distance from u_i to u_j, for $i,j=0,1,\ldots,m+n$. This shortest distance matrix is symmetric and satisfies the triangle inequality, that is, $t_{ij}=t_{ji}$ and $t_{ij}+t_{jk}\ge t_{ik}$ for any $i,j,k=0,1,\ldots,m+n$. We let

$$D=\{(u_i,u_j)\mid i=1,2,\ldots,m;\ j=m+1,m+2,\ldots,m+n\}$$

be the set of all supplier–customer pairs. Associated with each supplier–customer pair, $(u_i,u_j)\in D$, is a nonnegative demand parameter, q_{ij}, which indicates that a quantity of q_{ij} is required to be transported from supplier u_i to customer u_j. The goods are transported by homogeneous vehicles with capacity Q, and we assume that there is an infinite supply of vehicles. We further assume that $q_{ij}\le Q$ for all $(u_i,u_j)\in D$. The objective is to determine the vehicle routes, some of which will serve the customers directly from a supplier while others will be connected to the hub, to minimize the total travel distance of the vehicles. In our model, one of the decisions is to partition the set D into subsets D^d and D^h, where the demand in D^d will be satisfied via direct shipments (with milk runs) and the demand in D^h will be served by hub-and-spoke deliveries (with milk runs). For $i=1,2,\ldots,m$ and $j=m+1,m+2,\ldots,m+n$, let

$$q_{ij}^d=\begin{cases} q_{ij}, & \text{if } (u_i,u_j)\in D^d;\\ 0, & \text{otherwise;}\end{cases}$$

$$q_{ij}^h=\begin{cases} q_{ij}, & \text{if } (u_i,u_j)\in D^h;\\ 0, & \text{otherwise;}\end{cases}$$

$$\tilde{q}_i^h=\sum_{j=m+1}^{m+n} q_{ij}^h;$$

$$\tilde{q}_j^h=\sum_{i=1}^{m} q_{ij}^h.$$

Note that in our model, the "distance" between any two points can also be interpreted as the cost of traveling between those two points. Here, all fixed costs (e.g., the fixed cost of operating the hub) are ignored. Furthermore, if there is a variable cost, κ, for handling each unit of shipment at the hub, then our model can still be used for solving the mixed hub-and-spoke delivery problem by adding $\kappa/2$ to the "length" of every arc incident to the hub.

For any given set D^d, the optimal direct shipment is obtained as follows: For each supplier, u_i ($i = 1,2,\ldots,m$), we solve a CVRP with the depot located at u_i to satisfy all the demand, q_{ij}^d, incurred by each customer, u_j, that satisfies $(u_i, u_j) \in D^d$.

For any given set D^h, the optimal hub-and-spoke shipment is obtained as follows: To determine the optimal collection routes, we solve a CVRP with the depot located at u_0 to pick up the goods, $\tilde{q}_1^h, \tilde{q}_2^h, \ldots, \tilde{q}_m^h$, from suppliers, $u_1, u_2, \ldots, u_m$, respectively. To determine the optimal delivery routes, we solve a CVRP with the depot located at u_0 to satisfy the demand $\tilde{q}_{m+1}^h, \tilde{q}_{m+2}^h, \ldots, \tilde{q}_{m+n}^h$ of customers $u_{m+1}, u_{m+2}, \ldots, u_{m+n}$, respectively.

Hence, our problem is to find a partition $\{D^d, D^h\}$ of D such that (i) the demand q_{ij}^d is satisfied by direct shipment for every supplier–customer pair, $(u_i, u_j) \in D^d$, where the delivery arrangements are determined by solving a CVRP for each supplier, and (ii) the demand q_{ij}^h is to be satisfied by hub-and-spoke deliveries for every supplier–customer pair, $(u_i, u_j) \in D^h$, where the delivery arrangements are determined by solving two CVRPs, one for the collection of goods from the suppliers and one for the delivery of goods to the customers (see Figure 1). We will call this model the mixed hub-and-spoke and direct shipment delivery problem, or simply the *mixed delivery problem*.

Two other problems related to our mixed delivery problem can be described as follows: (i) Instead of determining the optimal partition $\{D^d, D^h\}$ of D, suppose that $D^d = D$ and $D^h = \emptyset$ are given. Then, our decision is to determine the optimal direct shipment arrangements to satisfy all the customers' demands. We will refer to this problem as the *pure direct shipment problem*. (ii) Instead of determining the optimal partition $\{D^d, D^h\}$ of D, suppose that $D^h = D$ and $D^d = \emptyset$ are given. Then, our decision is to determine the optimal hub-and-spoke deliveries to satisfy all the customers' demands. We will refer to this problem as the *pure hub-and-spoke problem*. Clearly, for any given set of data, the optimal objective function value of the mixed delivery problem must be no larger than that of either pure problem. In Section 4, we will study the benefits of the mixed delivery model as compared with the two pure delivery systems.

Note that both the mixed delivery problem and the two pure delivery problems involve solving the CVRP as a subproblem. However, the CVRP belongs to the class of *NP*-hard problems, which indicates that the existence of an efficient algorithm to solve the problem optimally is unlikely (see, for example, Christofides, 1985). Hence, we solve our CVRP subproblems using the well-known Clarke–Wright savings heuristic (Clarke and Wright, 1964). In solving each CVRP, those vertices with zero quantity will not be considered, i.e., they need not be visited by any vehicle.

3. Solution Procedure

In this section, we present a heuristic algorithm to find a near-optimal solution to the mixed delivery system. In this heuristic, we first obtain the solution to the pure direct shipment problem and the solution to the pure hub-and-spoke delivery problem. The better of them is then taken as the initial solution of our *improvement procedure* that searches for improvements in the solution. Thus, the solution generated by the heuristic will be guaranteed to be no worse than the solutions of the pure delivery systems obtained by the Clarke–Wright heuristic. The heuristic can be described as follows.

Heuristic H:

Step 1. Solve the pure direct shipment problem. This is done as follows: For $i = 1,2,\ldots,m$, solve a CVRP using the Clarke–Wright heuristic with the depot located at u_i to serve the n customers with demands $q^d_{i,m+1}, q^d_{i,m+2},\ldots, q^d_{i,m+n}$. Let the solution value (i.e., the total travel distance) be Z^d.

Step 2. Solve the pure hub-and-spoke delivery problem. This is done as follows: Solve a CVRP using the Clarke–Wright heuristic with the depot located at u_0 to collect the goods from the m suppliers with supplies $\tilde{q}^h_1, \tilde{q}^h_2,\ldots,\tilde{q}^h_m$. Next, solve a CVRP using the Clarke–Wright heuristic with the depot located at u_0 to serve the n customers with demands $\tilde{q}^h_{m+1}, \tilde{q}^h_{m+2},\ldots,\tilde{q}^h_{m+n}$. Let the solution value (i.e., the total travel distance of all collection and distribution trips) be Z^h.

Step 3. If $Z^d \le Z^h$, then let the pure direct shipment solution be the current solution, put the direct shipment delivery in "sending mode", put the hub-and-spoke delivery in "receiving mode", and let $D^d = D$, $D^h = \varnothing$. Otherwise, let the pure hub-and-spoke delivery solution be the current solution, put the hub-and-spoke delivery in sending mode, put the direct shipment delivery in receiving mode, and let $D^h = D$, $D^d = \varnothing$. Let $Z = \min\{Z^d, Z^h\}$ be the value (i.e., the total travel distance) of the current solution. Set $Z^m \leftarrow Z$, which is the value of the best solution obtained so far.

Step 4. Consider the current solution.

Case (i): If the direct shipment delivery is in sending mode, then for every $(u_i, u_j) \in D^d$, compute v_{ij}^d, which is an estimate of the improvement in the solution value if the supplier–customer pair (u_i, u_j) is transferred from D^d to D^h. Transfer all those pairs with positive v_{ij}^d from direct shipment delivery to hub-and-spoke delivery, i.e., set

$$D^d \leftarrow D^d \setminus \{(u_i, u_j) \mid v_{ij}^d > 0\} \text{ and } D^h \leftarrow D^h \cup \{(u_i, u_j) \mid v_{ij}^d > 0\},$$

Case (ii): If the hub-and-spoke delivery is in sending mode, then for every $(u_i, u_j) \in D^h$, compute v_{ij}^h, which is an estimate of the improvement in the solution value if the supplier–customer pair (u_i, u_j) is transferred from D^h to D^d. Transfer all those pairs with positive v_{ij}^h from hub-and-spoke delivery to direct shipment delivery, i.e., set

$$D^h \leftarrow D^h \setminus \{(u_i, u_j) \mid v_{ij}^h > 0\} \text{ and } D^d \leftarrow D^d \cup \{(u_i, u_j) \mid v_{ij}^h > 0\}.$$

(The estimates v_{ij}^d and v_{ij}^h will be discussed in detail later.)

Step 5. Solve the mixed delivery problem with demand partition $\{D^d, D^h\}$. This includes solving a subproblem with direct shipment for the supplier–customer pairs in D^d and a subproblem with hub-and-spoke delivery for the supplier–customer pairs in D^h. Let Z' be the solution value of the mixed system.

Step 6. If $Z' < Z$ (i.e., the new solution is better than the previous one), then let the new solution be the current solution and set $Z \leftarrow Z'$, otherwise, interchange the sending and receiving modes of the direct shipment delivery and the hub-and-spoke delivery. Now, Z equals the value of the current solution. If $Z < Z^m$, then the new solution becomes the best solution obtained so far, and we set $Z^m \leftarrow Z$. If the best solution has not been improved for N consecutive iterations or the current solution has not been improved for N' consecutive iterations, then stop; otherwise, go to Step 4.

Note that Steps 1–3 of Heuristic H determine an initial solution, while Steps 4–6 form a solution improvement procedure. The solution values of the mixed delivery problem, the pure direct shipment problem, and the pure hub-and-spoke problem generated by the algorithm are Z^m, Z^d, and Z^h, respectively.

In Case (i) of Step 4, the quantity v_{ij}^d is an estimate of the improvement in the solution value if demand q_{ij} is transferred from direct shipment to hub-and-spoke delivery, i.e., if the supplier–customer pair (u_i, u_j) is moved from D^d to D^h. Similarly, in Case (ii) of Step 4, v_{ij}^h is an estimate of the improvement in the solution value if demand q_{ij} is transferred from hub-and-spoke delivery to direct shipment.

Since v_{ij}^d or v_{ij}^h are updated at every iteration, it is possible that some demands transferred from one delivery mode to the other are transferred back later. In the following, we discuss how the estimates v_{ij}^d and v_{ij}^h are obtained. Since they are updated frequently, the computational time spent on determining them significantly affects the efficiency of Heuristic H. Therefore, we suggest a simple formula for determining these estimates. We let

$$v_{ij}^d = p_{ij}^d - c_{ij}^h, \quad (1)$$

where p_{ij}^d is an estimate of the savings if demand q_{ij} is removed from the current direct shipment requirements and c_{ij}^h is an estimate of the cost increase if demand q_{ij} is added to the predicted hub-and-spoke delivery requirements. However, without resolving the modified CVRP for the hub-and-spoke mode in the mixed problem, it is not easy to predict to which route q_{ij} should be added. To avoid resolving the CVRP, we let c_{ij}^h be the cost savings of taking out q_{ij} from the pure hub-and-spoke problem. This estimate can be calculated from the result of Step 2 and used whenever needed in the iterations.

Similarly, we let

$$v_{ij}^h = p_{ij}^h - c_{ij}^d, \quad (2)$$

where p_{ij}^h is an estimate of the savings if demand q_{ij} is removed from the current hub-and-spoke delivery requirements and c_{ij}^d is an estimate of the cost increase if demand q_{ij} is added to the predicted direct shipment requirements. Again, c_{ij}^d here is set to the cost savings of taking out q_{ij} from the pure direct shipment problem. It can be calculated from the result of Step 1.

To show the calculation of p_{ij}^d and p_{ij}^h, we define the following notation for the current solution:

n_i = number of direct shipment routes from supplier u_i ($i = 1,2,\ldots,m$);

τ_i^r = length of the r-th direct shipment route from supplier u_i ($i = 1,2,\ldots,m$; $r = 1,2,\ldots,n_i$);

s_{ij}^r = savings in travel distance if we remove customer u_j and its demand from the r-th direct shipment route of supplier u_i ($i = 1,2,\ldots,m$; $j = m+1, m+2,\ldots,m+n$; $r = 1,2,\ldots,n_i$);

n^{col} = number of collection routes from the hub;

n^{del} = number of delivery routes from the hub;

$\xi^{col,r}$ = quantity handled by the r-th collection route in the hub-and-spoke system ($r = 1,2,\ldots,n^{col}$);

$\xi^{del,r}$ = quantity handled by the r-th delivery route in the hub-and-spoke system ($r = 1,2,\ldots,n^{del}$);

$\tau^{col,r}$ = length of the r-th collection route ($r = 1,2,\ldots,n^{col}$);

$\tau^{del,r}$ = length of the r-th delivery route ($r = 1,2,\ldots,n^{del}$);

R_i^{col} = set of collection routes containing the goods of supplier u_i ($i = 1,2,\ldots,m$);

R_j^{del} = set of delivery routes containing the goods of customer u_j ($j = m+1, m+2,\ldots,m+n$).

We first consider the cost savings when the demand, q_{ij}, is removed from the current direct shipments. Let $\rho(i)$ denote the direct shipment route that handles the customer demand, q_{ij}. Let

$$D^{d}_{\rho(i)} = \{\, (u_i, u_j) \,|\, \text{demand } q_{ij} \text{ is handled by route } \rho(i) \,\}.$$

Then, we set

$$p^{d}_{ij} = \frac{s^{\rho(i)}_{ij} \cdot \tau^{\rho(i)}_{i}}{\sum_{k \text{ s.t.} (u_i, u_k) \in D^{d}_{\rho(i)}} s^{\rho(i)}_{ik}}. \qquad (3)$$

To understand the rationale behind this formula, consider the removal of demand q_{ij} from the route. When this demand is removed, the length of the route decreases by $s^{\rho(i)}_{ij}$. However, when two demands q_{ij}, q_{ik} are removed from the same route, the decrease in route length is not necessarily equal to $s^{\rho(i)}_{ij} + s^{\rho(i)}_{ik}$ if customers u_j and u_k are adjacent to each other (see Figure 2). In the extreme case, if all demands in the route are removed, then the reduction in route length should equal $\tau^{\rho(i)}_{i}$ instead of $\sum_{k \text{ s.t.} (u_i, u_k) \in D^{d}_{\rho(i)}} s^{\rho(i)}_{ik}$. Hence, for the removal of each customer, u_j, from the route, we approximate the cost savings by an amount proportional to $s^{\rho(i)}_{ij}$, with proportionality constant $\tau^{\rho(i)}_{i} \Big/ \sum_{k \text{ s.t.} (u_i, u_k) \in D^{d}_{\rho(i)}} s^{\rho(i)}_{ik}$.

Next, we consider the cost reduction when the demand, q_{ij}, is removed from the current hub-and-spoke system. Note that when the customer demand, q_{ij}, is removed from a collection route in the hub-and-spoke system, the length of that route remains unchanged unless q_{ij} is the only demand from supplier u_i to be collected on that route. Thus, we need a fair assessment on the attractiveness of removing demand q_{ij} from the route. Note that $\sum_{r \in R^{col}_i} \tau^{col,r}$ is the total length of collection routes carrying products of supplier u_i in the hub-and-spoke system, and $\sum_{r \in R^{col}_i} \xi^{col,r}$ is the total demand handled by those routes. In other words, if the demand $\sum_{r \in R^{col}_i} \xi^{col,r}$ is removed from the hub-and-spoke system, the reduction in the total length of collection routes would be $\sum_{r \in R^{col}_i} \tau^{col,r}$. Hence, we estimate the reduction in the total length of collection routes due to the removal of demand q_{ij} as $q_{ij} \sum_{r \in R^{col}_i} \tau^{col,r} \Big/ \sum_{r \in R^{col}_i} \xi^{col,r}$. Similarly, we estimate the reduction in the total length of delivery routes due to the removal of q_{ij} as $q_{ij} \sum_{r \in R^{del}_j} \tau^{del,r} \Big/ \sum_{r \in R^{del}_j} \xi^{del,r}$. Therefore, we set

$$p_{ij}^{h} = q_{ij} \cdot \left(\frac{\sum_{r \in R_i^{col}} \tau^{col,r}}{\sum_{r \in R_i^{col}} \xi^{col,r}} + \frac{\sum_{r \in R_j^{del}} \tau^{del,r}}{\sum_{r \in R_j^{del}} \xi^{del,r}} \right). \tag{4}$$

Finally, we consider the cost of inserting demand q_{ij} into the existing direct shipment requirements and the savings from removing this demand from the existing hub-and-spoke delivery requirements. According to earlier discussions, we set the estimates of these quantities to

$$c_{ij}^{d} = \text{“ } p_{ij}^{d} \text{ in the pure direct shipment problem”} \tag{5}$$

and

$$c_{ij}^{h} = \text{“ } p_{ij}^{h} \text{ in the pure hub-and-spoke problem”.} \tag{6}$$

Corresponding to the notations used for the current mixed solution, notations for the two pure delivery problems are defined as $\hat{n}_i$, $\hat{\tau}_i^r$, $\hat{s}_{ij}^r$, $\hat{n}^{col}$, $\hat{n}^{del}$, $\hat{\xi}^{col,r}$, $\hat{\xi}^{del,r}$ $\hat{\tau}^{col,r}$ $\hat{\tau}^{del,r}$ $\hat{R}_i^{col}$, $\hat{R}_j^{del}$, and $\hat{D}_{\rho(i)}^{d}$, respectively.

From equations (1), (3), (4), and (6), we have

$$v_{ij}^{d} = \frac{s_{ij}^{\rho(i)} \cdot \tau_i^{\rho(i)}}{\sum_{k \text{ s.t. } (u_i, u_k) \in D_{\rho(i)}^{d}} s_{ik}^{\rho(i)}} - q_{ij} \left(\frac{\sum_{r \in \hat{R}_i^{col}} \hat{\tau}^{col,r}}{\sum_{r \in \hat{R}_i^{col}} \hat{\xi}^{col,r}} + \frac{\sum_{r \in \hat{R}_j^{del}} \hat{\tau}^{del,r}}{\sum_{r \in \hat{R}_j^{del}} \hat{\xi}^{del,r}} \right).$$

From equations (2), (3), (4), and (5), we have

$$v_{ij}^{h} = q_{ij} \cdot \left(\frac{\sum_{r \in R_i^{col}} \tau^{col,r}}{\sum_{r \in R_i^{col}} \xi^{col,r}} + \frac{\sum_{r \in R_j^{del}} \tau^{del,r}}{\sum_{r \in R_j^{del}} \xi^{del,r}} \right) - \frac{\hat{s}_{ij}^{\rho(i)} \cdot \hat{\tau}_i^{\rho(i)}}{\sum_{k \text{ s.t. } (u_i, u_k) \in \hat{D}_{\rho(i)}^{d}} \hat{s}_{ik}^{\rho(i)}}.$$

4. Computational Experiments

In this section, we assess via computational experiments the savings on the total travel distance of vehicles due to the use of the mixed delivery system, in comparison with the use of a pure delivery system. In the implementation of Heuristic H, the termination condition is set as $N = 7$ and $N' = 5$. In general, the termination condition should set to balance the solution quality and the computational effort. The above setting is based on test run results, which indicate that more iterations will hardly make further improvement. The experiments are carried out on a variety of different problem settings. Results are analyzed in terms of the relative improvements and the number of cases improved. Additional experiments are also carried out on the impact of including milk runs in the mixed system.

4.1 Problem instance generation

All the problem instances are defined within a square of unit length, which may be considered as a scaled version of practical delivery regions. For each instance, the $m+n$ supplier and customer locations are uniformly distributed in the square area. Euclidean distance is taken as the travel distance between any two of these locations. There is a delivery order for each customer from each of the suppliers. The delivery demand of customer j from supplier i, q_{ij}, is randomly generated from Uniform$[a,b]$, where a and b are prespecified numbers. The hub location is determined by solving the "gravity problem" that minimizes $\sum_{i=1}^{m+n} \tilde{q}_i t_{0i}$ (see Francis and White, 1974, p. 170). The vehicle capacity, Q, is set to 10 units, and the number of suppliers, m, is set to 5.

To represent a wide range of situations, the number of customers and the parameters of the demand distribution are set to vary at several levels. The number of customers, n, is set to 10, 15, 20, and 25. The values of a and b, which determine the mean and coefficient of variation of the demand distribution, are set according to Table 1. We only use those combinations of a and b that satisfy the condition of $b \le 10$, since, by our assumption, all demands should not be larger than the vehicle capacity. There are 72 such combinations. Taking into account the four different values of n, there are altogether 288 different settings of problem parameters. For each of these settings, 40 problem instances are randomly generated. Therefore, a total of 11,520 problem instances are generated and used in the experiments.

<insert Table 1 about here>

4.2 Experiment results

For each of the above problem instances, we apply Heuristic H to obtain Z^d, Z^h, and Z^m. The relative savings from adopting the mixed system as compared with the pure systems, $(\min\{Z^d, Z^h\} - Z^m) / \min\{Z^d, Z^h\}$, can then be calculated. For any group of the problem instances, we can analyze the savings using the following two performance measures:

- The average relative savings;
- The proportion of instances with positive savings.

For all the problem instances used in the experiments, the overall average relative savings is 4.1% with a maximum of 24.1%; the proportion of instances with positive savings is 69.4%. This indicates that the mixed delivery system operated using our heuristic creates savings in most cases with different parameter settings and the savings are substantial. This comparison is made with the better of the two pure delivery systems. Note that since the pure systems can be considered as special cases of the mixed system, the performance of the mixed system is at least the same as the better of the two pure systems. When compared with either of the pure systems, the savings is even more significant. The overall average relative savings and the proportion of instances with positive savings are 9.9% and 73.4%, respectively, when compared with the pure direct shipment system. The figures are 11.5% and 84.1%, respectively, when compared with the pure hub-and-spoke system.

If the mixed system cannot be implemented, e.g., if the hub cost is too high, there is no information system support, or suppliers are unwilling to cooperate, then we need to identify the best alternative. To compare the two pure systems based on the experimental results, we define the relative change in total travel distance of the pure hub-and-spoke system with the pure direct shipment system as a reference: $(Z^h - Z^d)/Z^d$. Again, for any group of problem instances, the performance measures below can be calculated and evaluated.

- The average relative change in total travel distance (*T*);
- The proportion of instances when pure direct shipment is better than pure hub-and-spoke delivery (*B*).

The overall average *T* is only 1.3%. The overall *B* is around 61%. This means that the overall difference between the performances, in terms of the total distance traveled, of the two pure systems is minimal though pure direct shipment performs slightly better. This result also indicates that the problem instances used in this study do not particularly favor either pure delivery system.

4.3 Further discussion and analysis

To analyze the impact of the problem parameters on the savings of the mixed system, we use a regression model to relate the *average relative savings* to the following factors: the number of customers (n), the demand mean ($mean$), and the coefficient of variance of the demand (cv), and their interactions, $n \times mean$, $n \times cv$, and $mean \times cv$. Each parameter combination and its corresponding average relative savings are taken as a data entry for the regression model. The sample size is 288. The resulting coefficient of determination for the model R^2=39.1%. Hypothesis testing is conducted to verify whether the coefficient of each factor in the regression model is significantly different from zero. The estimated coefficients of the factors in the model and their P values in the hypothesis testing are summarized in Table 2. From the P values, we can see that all factors except $n \times mean$ and $mean \times cv$ are insignificant. The most significant factor is $mean \times cv$. This indicates that the distribution of the demand quantities significantly affects the relative savings of the mixed system.

Note that milk runs are included in our study. The proposed heuristic algorithm not only decides the mode of delivery for each demand, but also gives vehicle routing results for the deliveries. If milk runs are not allowed, then the problem will become only to decide the delivery mode of each demand and therefore will be much easier to solve. Such a problem can be formulated as the following integer programming model:

$$\text{Minimize} \quad \sum_{i=1}^{m} \sum_{j=m+1}^{m+n} t_{ij} x_{ij} + \sum_{i=1}^{m} t_{i0} y_i + \sum_{j=m+1}^{m+n} t_{0j} z_j$$

$$\text{subject to} \quad \sum_{j=m+1}^{m+n} q_{ij}(1 - x_{ij}) \le Q y_i, \qquad i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} q_{ij}(1 - x_{ij}) \le Q z_j, \qquad j = m+1, m+2, \ldots, m+n$$

$x_{ij} \in \{0,1\}$, y_i and z_j are integers,

where variable x_{ij} indicates in which mode demand q_{ij} should be delivered (equal to 1 if by direct shipment; 0 otherwise), y_i is the number of trips needed between supplier i and the hub, and z_j is the number of trips needed between the hub and customer j.

To sense how milk runs affect the total traveling distance, we take various problem instances (n = 10, 20; $mean$ = 3, 6; cv = 0, $cv_{\max}$, where $cv_{\max}$ is the highest value of cv shown in Table 1) and optimize the mixed system without milk runs using this model. The total distance traveled in the system is on average 17.75% longer than that in the system with milk runs. This explains why most practical delivery systems allow milk runs.

When milk runs are not allowed, the distance of the pure systems will also increase. The results can also be obtained using the above model by fixing all the x values to 1 for the pure direct shipment system and to 0 for the pure hub-and-spoke system. If all three systems do not include milk runs, our test on the selected problem instances shows that the mixed system can save 13.28% of traveling distance compared with the better pure system, and save 24.76% and 24.22% compared with the pure direct shipment system and pure hub-and-spoke system, respectively. All these savings are higher than in the situation that includes milk runs.

5. Conclusions

In this paper, we studied a mixed truck delivery system that allows both direct shipment and hub-and-spoke deliveries. A heuristic was developed to determine the mode of delivery for each demand and to perform vehicle routing in both modes of delivery. Computational experiments were carried out to compare the mixed delivery system with the pure delivery systems. From the computational results, we can conclude that the mixed system is more effective than both pure systems. The delivery plan produced using the heuristic for the mixed system saves about 4% total distance on average compared with the best of the pure systems. The savings is about 10% on average if compared with any one of the pure systems. Analysis was also done on the impacts of problem parameters on the relative savings of the mixed system. The results showed that the demand distribution affects the relative performance of the mixed system most significantly. The effect of including milk runs in the systems is also discussed. Note that in our computational study, the fixed cost of operating the hub was ignored. Thus, to evaluate the overall cost of the systems, one should compare the transportation costs of the systems obtained from this study with the costs of operating the hub in order to make a fair comparison. In fact, the traveling cost savings of adopting the mixed system can be viewed as an upper bound on the hub cost in order to make the system more cost effective than pure direct shipment.

This study is limited to the problem with one hub and homogeneous vehicles. However, it can serve as a basis for further research in a number of directions. The heuristic proposed here is based on local search. A direct extension is to try other types of methods such as genetic algorithm, tabu search, etc., to search for better solutions. The problem studied here may be extended to more complex situations, for instance, to consider multiple hubs, heterogeneous vehicles, or delivery time window constraints. In particular, the problem with multiple hubs involves many new features and is more difficult to solve. Examples of new decisions in the multi-hub system include which hub to use for a demand and how to arrange the transportation among the hubs. The hub-and-spoke system in such a problem is more complicated.

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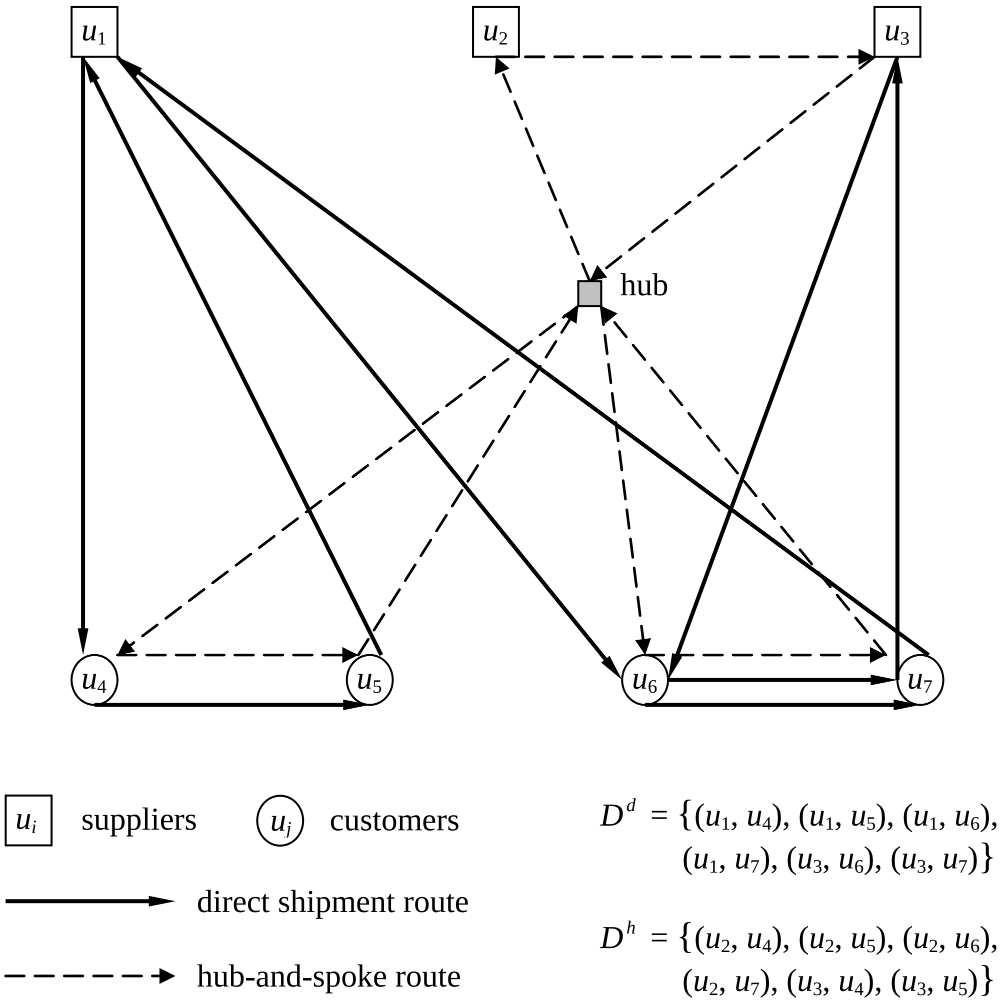

Figure 1. A mixed delivery system with both direct shipment and hub-and-spoke deliveries

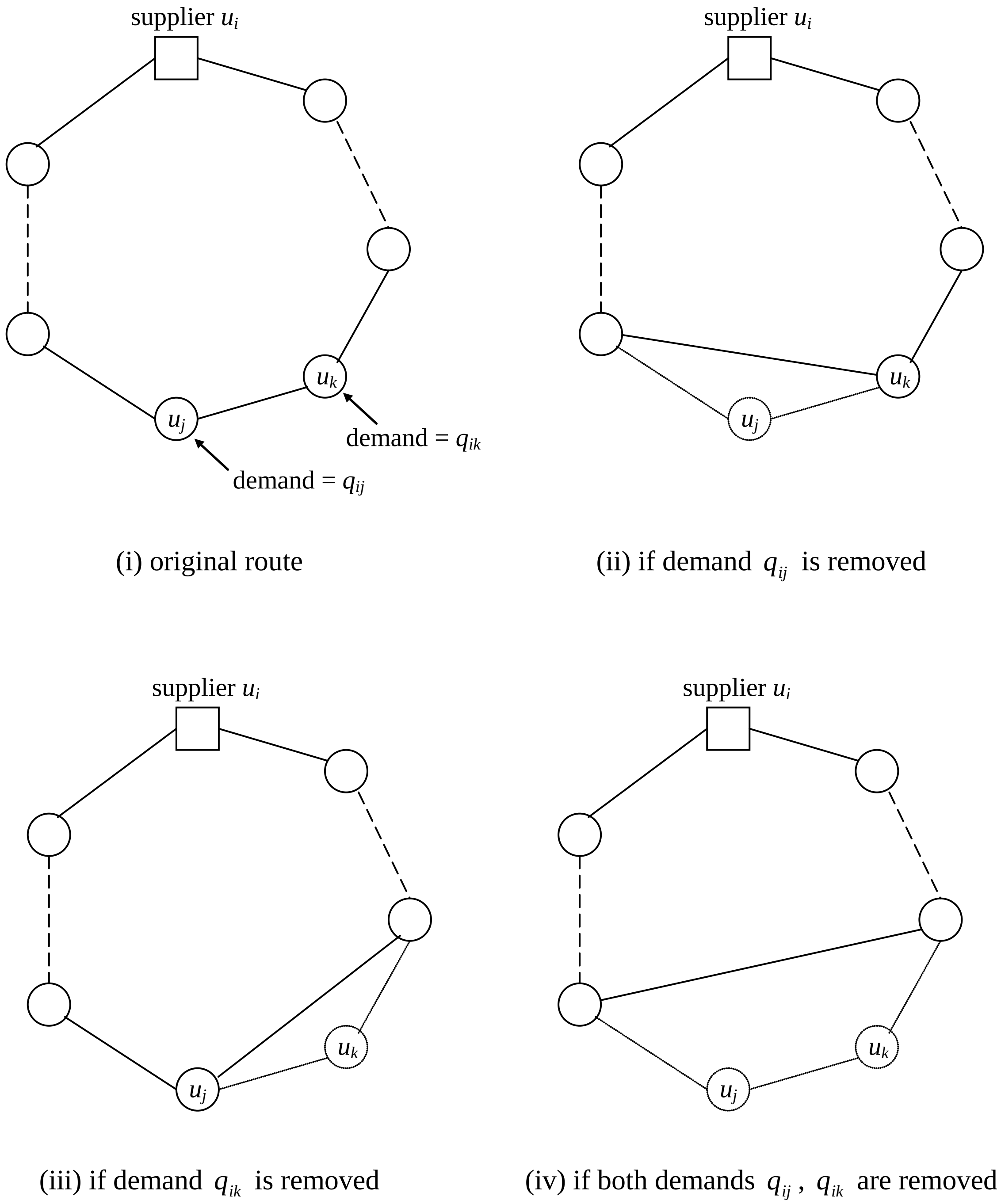

Figure 2. The situation when demands q_{ij} and q_{ik} are removed from the route

Table 1. Combinations of a and b used in the computational experiments

Mean		Coefficient of Variation										
		0	0.058	0.115	0.173	0.231	0.289	0.346	0.404	0.462	0.520	0.577
1	b	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	a	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
2	b	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
	a	2.0	1.8	1.6	1.4	1.2	1.0	0.8	0.6	0.4	0.2	0.0
3	b	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
	a	3.0	2.7	2.4	2.1	1.8	1.5	1.2	0.9	0.6	0.3	0.0
4	b	4.0	4.4	4.8	5.2	5.6	6.0	6.4	6.8	7.2	7.6	8.0
	a	4.0	3.6	3.2	2.8	2.4	2.0	1.6	1.2	0.8	0.4	0.0
5	b	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
	a	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0
6	b	6.0	6.6	7.2	7.8	8.4	9.0	9.6	-	-	-	-
	a	6.0	5.4	4.8	4.2	3.6	3.0	2.4	-	-	-	-
7	b	7.0	7.7	8.4	9.1	9.8	-	-	-	-	-	-
	a	7.0	6.3	5.6	4.9	4.2	-	-	-	-	-	-
8	b	8.0	8.8	9.6	-	-	-	-	-	-	-	-
	a	8.0	7.2	6.4	-	-	-	-	-	-	-	-
9	b	9.0	9.9	-	-	-	-	-	-	-	-	-
	a	9.0	8.1	-	-	-	-	-	-	-	-	-

Table 2. Coefficients of the factors in the regression model and their *P* values

Factor	Coefficient	*P* value
constant	0.03501	0.020
n	-0.0013336	0.094
mean	0.001085	0.668
cv	0.00393	0.904
n × *mean*	0.0003065	0.021
n×*cv*	-0.002142	0.171
mean × *cv*	0.014708	0.001