A Practical Dynamic Phasor Model of Static VAR Compensator

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Abstract – This paper presents a three phase dynamic phasor model of Static VAR Compensator which takes fundamental and fifth harmonic into consideration. The modelling approach is based on the time-varying Fourier coefficient series of the power system variables. By truncating the less important higher order terms and keeping the significant ones, the new dynamic phasor model can realistically simulate the dynamic behaviour of SVC with fast speed and high accuracy. Without any simplified assumption, the new model is applicable to fast numerical simulation of SVC. Computer simulation results show the accuracy and efficiency of the dynamic phasor model of SVC.

Keywords – Static VAR Compensator, Dynamic phasors, time-varying Fourier coefficient series, numerical simulation, power system.

INTRODUCTION

With the rapid development of power electronics technologies, more and more Flexible AC Transmission System (FACTS) devices have been widely applied in modern power systems for enhancing the controllability and power transfer capability of the AC system, and as a result, they play an important role in the power system operation and control. The static VAR compensator (SVC) is one of the most common FACTS devices used to improve the power system performance in steady-state and dynamic stability, voltage profile, and reactive power flow. For the stability assessment of large-scale power systems, it is necessary to model those FACTS device accurately and efficiently.

Because of the switching behaviour of thyristor controlled reactor (TCR), accurate modelling of SVC is non-trivial. The quasi-static approximation model commonly used in electro-mechanical transients (transient stability) simulation is not adequate enough to catch the dynamic behaviour of the switching. Even though in electro-magnetic transients simulation, the full device level time-domain model can provide detailed response of the devices, the computation burden of such model is very demanding and hence is impractical for daily usage in large-scale power system simulations.

Dynamic phasor models are developed from time-domain descriptions using the generalized averaging procedure [1-3]. Dynamic phasors can establish nonlinear time-invariant and large-signal model of FACTS devices such as TCSC [4], STATCOM [5] and UPFC [6]. The model of SVC in [7] only takes the fundamental phasor into consideration for sub-synchronous resonance (SSR) study.

In this paper, a dynamic phasor model of SVC is proposed and evaluated. This model truncates the less important higher order frequency components and keeps only the lower order significant ones, namely the fundamental and fifth harmonic. As a result, it can accurately catch the dynamic behaviour of SVC with fast simulation speed. This dynamic phasor model can either be used in dynamic phasor (DP) simulation of modern power systems or incorporated in the traditional transient stability (TS) simulation to form a TS-DP hybrid simulation for transient stability study of large power system with accurate modelling of FACTS devices.

The rest of this paper is organized as follows. In Section II, the basic concept of dynamic phasor modelling approach is introduced. Section III presents the dynamic phasor SVC model. Section IV provides the evaluation results of the proposed model using a simple test power system, and Section V concludes the paper.

BASIC CONCEPT OF DYNAMIC PHASORS

The approach of dynamic phasors is firstly called as the method of state-space averaging and based on the time-varying Fourier coefficients. A complex time domain waveform $x(\tau)$ can be represented on the interval $\tau \in (t-T, t]$ using a Fourier series of the form:

$$x(\tau) = \sum_{k=-\infty}^{\infty} X_k(t) e^{jk\omega_s \tau} \tag{1}$$

where $\omega_s = 2\pi / T$ and $X_k(t)$ are the complex time-varying Fourier coefficients, which called as dynamic phasors. The kth dynamic phasor at time t is determined by the following expression:

$$X_k(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)e^{-jk\omega_s\tau}d\tau = \langle x \rangle_k(t) \tag{2}$$

The dynamic phasor method is based on the idea of frequency decomposition, and focuses on the dynamics of the significant Fourier coefficient. There are two key and useful properties of the phasors:

1. k-phasor differential characteristic:

For the *k*th Fourier coefficient, the differential with time satisfies the following formula:

$$\frac{dX_k}{dt}(t) = \left\langle \frac{dx}{dt} \right\rangle_k (t) - jk\omega_s X_k(t) \tag{3}$$

2. Product of dynamic phasors:

The *k*th phasor of a product of two time-domain waveform $x(\tau)$ and $y(\tau)$ can be obtained by the following operation:

$$\langle xy \rangle_k = \sum_{i=-\infty}^{\infty} \langle x \rangle_{k-i} \langle y \rangle_i \tag{4}$$

Also, the time domain waveform $x(\tau)$ can be transformed back from its dynamic phasors by the following equation:

$$\begin{aligned} x(\tau) &= \mathrm{Re}(X_k(t)e^{jk\omega_0\tau}) \\ &= X_{-k}(t)e^{-jk\omega_0\tau} + X_{-(k-1)}(t)e^{-j(k-1)\omega_0\tau} + \cdots \\ &\quad + X_{k-1}(t)e^{j(k-1)\omega_0\tau} + X_k(t)e^{jk\omega_0\tau} \end{aligned} \tag{5}$$

Moreover, since $x(\tau)$ is real,

$$X_{-k} = X_k^*$$

where the operator * means the conjugate of a complex number.

THREE PHASE DYNAMIC PHASOR SVC MODELLING

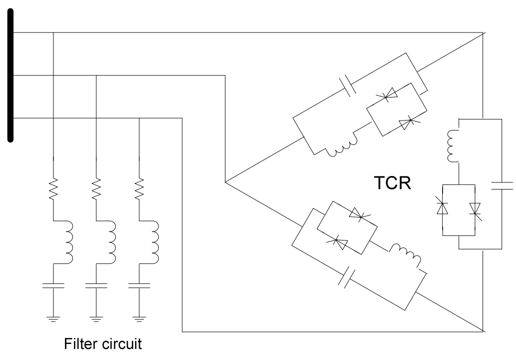

Fig. 1. Three phase SVC circuit

The basic three phase static VAR compensator consists of three delta-connected single phase SVCs and filter circuits of fifth harmonic. The dynamic phasor model of each component is described in the following sections.

DYNAMIC PHASOR MODEL OF SVC

A single phase SVC is shown in Fig. 2. If the two thyristor valves are fired symmetrically in the positive and negative half-cycles of supply voltage, only odd-order harmonics would be produced. In addition, the delta connection of the three single phase SVCs could prevent the triplen harmonics from percolating into the transmission lines. However, since the 5th harmonics cannot be cancelled out in the lines, it has to be taken into consideration in the dynamic phasor model of SVC for realistic modelling of its dynamics.

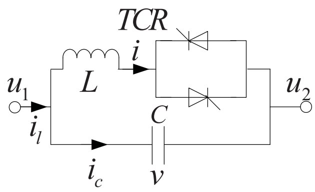

Fig. 2. Single phase circuit of SVC

From the simple SVC circuit, its time-domain model can be obtained as:

$$\begin{cases} C\dfrac{dv}{dt} = i_l - i \\ L\dfrac{di}{dt} = sv \end{cases} \tag{6}$$

where $v = u_1 - u_2$, s is the switching function. $s = 1$ when one thyristor is full conducting, and $s = 0$ when both thyristors are shut. With $< x >_k$ rewritten as X_k, the dynamic phasor model of SVC is obtained by the differential characteristic as the following formula:

$$\begin{cases} \dfrac{dV_k}{dt} = -jk\omega_s V_k + \dfrac{I_{lk} - I_k}{C} \\ \dfrac{dI_k}{dt} = -jk\omega_s I_k + \dfrac{< sv >_k}{L} \end{cases} \tag{7}$$

where $\langle sv \rangle_k$ can be calculated with

$$\langle sv \rangle_k = \sum_{l=-5,-1,1,5} S_{k-l} V_l \tag{8}$$

Since dynamic phasors are complex quantities, each equation in (7) consists of two parts: the real and imaginary parts, and the dynamic model of single phase SVC is an 8^{th}-order model including the fundamental and fifth harmonic phasors. Though the dynamic phasor model has more equations than the time domain model, it could catch the dynamic behaviour of SVC with relatively larger integration step time. The whole model of SVC can be derived as follow:

For the fundamental phasor:

$$
\begin{aligned}
<sv>_1 &= S_6 V_{-5} + S_2 V_{-1} + S_0 V_1 + S_{-4} V_5 \\
&= S_6 V_5^* + S_2 V_1^* + S_0 V_1 + S_4^* V_5
\end{aligned}
$$

$$
\begin{aligned}
\frac{dV_1^R}{dt} &= \omega_s V_1^I + \frac{1}{C}(I_{l1}^R - I_1^R) \\
\frac{dV_1^I}{dt} &= -\omega_s V_1^R + \frac{1}{C}(I_{l1}^I - I_1^I) \\
\frac{dI_1^R}{dt} &= \omega_s I_1^I + \frac{1}{L}(S_6^R V_5^R + S_6^I V_5^I) \\
&\quad + \frac{1}{L}(S_2^R V_1^R + S_2^I V_1^I + S_0 V_1^R) + \frac{1}{L}(S_4^R V_5^R + S_4^I V_5^I) \\
\frac{dI_1^I}{dt} &= -\omega_s I_1^R + \frac{1}{L}(S_6^I V_5^R - S_6^R V_5^I) \\
&\quad + \frac{1}{L}(S_2^I V_1^R - S_2^R V_1^I + S_0 V_1^I) + \frac{1}{L}(S_4^R V_5^I - S_4^I V_5^R)
\end{aligned}
$$

For 5th phasor:

$$
\begin{aligned}
<sv>_5 &= S_{10} V_{-5} + S_6 V_{-1} + S_4 V_1 + S_0 V_5 \\
&= S_{10} V_5^* + S_6 V_1^* + S_4 V_1 + S_0 V_5
\end{aligned}
$$

$$
\begin{aligned}
\frac{dV_5^R}{dt} &= 5\omega_s V_5^I + \frac{1}{C}(I_{l5}^R - I_5^R) \\
\frac{dV_5^I}{dt} &= -5\omega_s V_5^R + \frac{1}{C}(I_{l5}^I - I_5^I) \\
\frac{dI_5^R}{dt} &= 5\omega_s I_5^I + \frac{1}{L}(S_{10}^R V_5^R + S_{10}^I V_5^I) \\
&\quad + \frac{1}{L}(S_6^R V_1^R + S_6^I V_1^I + S_0 V_5^R) + \frac{1}{L}(S_4^R V_1^R - S_4^I V_1^I) \\
\frac{dI_5^I}{dt} &= -5\omega_s I_5^R + \frac{1}{L}(S_{10}^I V_5^R - S_{10}^R V_5^I) \\
&\quad + \frac{1}{L}(S_6^I V_1^R - S_6^R V_1^I + S_0 V_5^I) + \frac{1}{L}(S_4^R V_1^I + S_4^I V_1^R)
\end{aligned}
\tag{9}
$$

where the superscript R and I denote the real and imaginary parts of the defined quantities.

DYNAMIC PHASOR MODEL OF SWITCHING FUNCTION

The dynamic model of the switching function [8] is calculated by

$$S_0 = \frac{1}{T}\int_{t-T}^{t} s(\tau)\cdot d\tau = \frac{\tau-\alpha}{\pi} \tag{10}$$

and

$$
\begin{aligned}
S_k &= \frac{1}{T}\int_{t-T}^{t} s(\tau)\cdot e^{-jm\omega_s\tau} d\tau = \frac{j}{m\pi}[e^{-jm\tau} - e^{-jm\alpha}] \\
&= \frac{1}{m\pi}(\sin m\tau - \sin m\alpha) + \frac{j}{m\pi}(\cos m\tau - \cos m\alpha)
\end{aligned}
\quad (k \neq 0) \tag{11}
$$

The delay angle α and conduction angle τ are variables depended on the closed-loop control of the SVC. For each simulation time step, α and τ are calculated from control loop.

For phase B and C, the models of switch function are different with phase A because of the angle difference between phases. The switch function models of phase B and C can be calculated by the following equations.

$$
\begin{cases}
S_{k,B} = e^{k*(-2\pi/3)} S_{k,A} \\
S_{k,C} = e^{k*(2\pi/3)} S_{k,A}
\end{cases}
\tag{12}
$$

DYNAMIC PHASORS MODEL OF FILTER CUICIRTS

The filter circuit is a RLC circuit which consists of one RL circuit and one capacitor circuit shown as Fig.3.

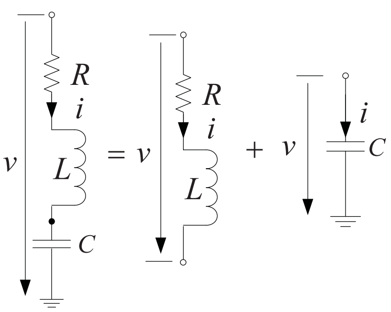

Fig. 3. Single-phase filter circuit

The dynamic phasor model of RL circuit and capacitor circuit is deduced as follow:

1. DP model of RL circuit

For RL circuit, we have the time-domain model:

$$v(t) = L\frac{di(t)}{dt} + Ri(t) \tag{13}$$

and the dynamic phasors are

$$L\frac{dI_k}{dt} = V_k - jk\omega_s L I_k - RI_k \tag{14}$$

$$
\begin{cases}
\dfrac{dI_k^R}{dt} = \dfrac{1}{L}(V_k^R - RI_k^R) + k\omega_s I_k^I \\
\dfrac{dI_k^I}{dt} = \dfrac{1}{L}(V_k^I - RI_k^I) - k\omega_s I_k^R
\end{cases}
\quad k = 1, 5 \tag{15}
$$

2. DP model of capacitor circuit

For capacitor circuit, we have the time-domain model:

$$i(t) = C\frac{dv(t)}{dt} \tag{16}$$

and the dynamic phasors are

$$\frac{dV_k}{dt} = \frac{1}{C}I_k - jk\omega_s V_k \tag{17}$$

$$\begin{cases} \dfrac{dV_k^R}{dt} = \dfrac{1}{C}I_k^R + k\omega_s V_k^I \\ \dfrac{dV_k^I}{dt} = \dfrac{1}{C}I_k^I - k\omega_s V_k^R \end{cases} \quad k = 1,5 \tag{18}$$

As a result, the three phase dynamic phasors model of SVC with the 5th harmonic and filter circuit considered is a 60th-order model with 72 variables. For the evaluation of those variables, 24 more equations derived from the KCL in the SVC bus are required.

MODEL EVALUATION

A simple test system as shown in Fig. 4 is created to evaluate the performance of the dynamic phasor SVC model. The three single-phase SVC are delta connection. The SVC circuit is powered by a constant current source. The impedance of the current source is 159.39+j48.86 Ω. The capacity of the SVC is ±100MVar. The reference voltage of the test system is 230KV.

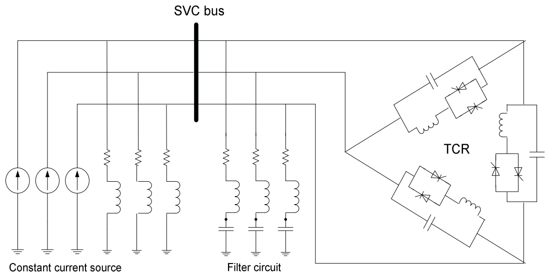

Fig. 4. the test system including SVC

FOUR PHASES OF EVALUATION

The evaluation includes four phases to cover all the TCR operating regions ranged from full conduction to close shown as Table I. In total, the simulation lasts for two seconds.

For the evaluation of the dynamic phasors model, DCG EMTP simulation of the entire test system is carried out to produce the benchmark results. The emphasis of the comparison is placed on the waveforms of phase voltage of the SVC bus. The results of comparison are shown in Figs. 5-7.

TABLE I OPERATING REGIONS OF SVC

Phase	1	2	3	4
t (s)	0~0.5	0.5~1.0	1.0~1.5	1.5~2.0
M (A)	950	860	760	650
α (°)	90	100	150	180
Region	Full inducting	inductive	capacitive	close

Fig. 5 shows the phase A dynamic phasor voltage of the SVC bus with the instantaneous results obtained from the DCG-EMTP superimposed. It is clearly shown that the dynamic voltage phasors closely trace the envelop of the instantaneous voltage over the whole simulation period. This shows that the model including fundamental and fifth phasors is accurate enough to catch the dynamic response of SVC.

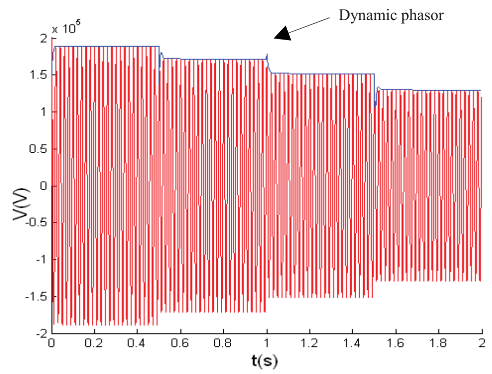

Fig. 5. SVC bus dynamic phasor voltage

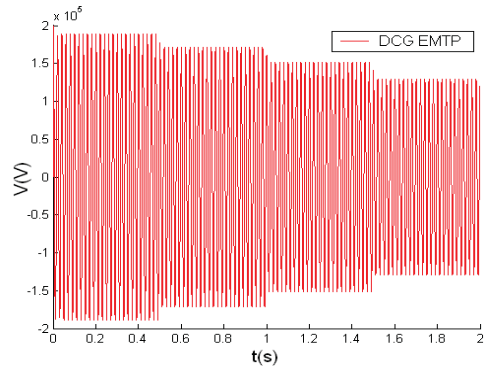

Fig. 6. Instantaneous SVC bus voltage – DCG EMTP

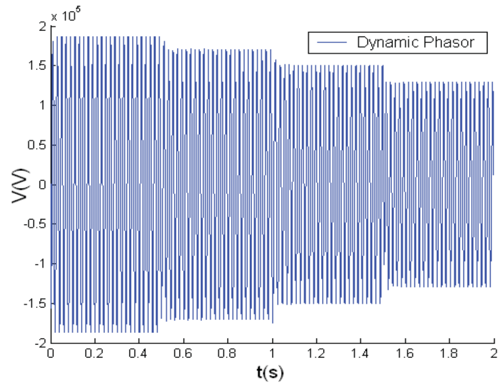

Fig. 7. Instantaneous SVC bus voltage – dynamic phasor

Fig. 6 and 7 show the comparison of instantaneous voltages between DCG EMTP and dynamic phasors. The instantaneous voltages of dynamic phasors are reversely transformed using equation (5). It is obviously that the dynamic phasor model has practically the same results as DCG EMTP software.

CONCLUSION

A practical three phase dynamic phasor model of static VAR compensator (SVC) is proposed in this paper. The dynamic phasor modelling approach is developed from the time-domain descriptions using time-varying Fourier coefficients. The new model includes fundamental and fifth harmonics to improve the model accuracy. Without the simplifications as taken in traditional transient stability simulation, the new model is capable of catching the fast dynamics characteristics of the SVC while it is still applicable to fast numerical simulation. The new model is evaluated on a simple test power system which consists of a SVC with constant current source. By comparing the simulation results with benchmarks obtained from the DCG EMTP software, it is clearly shown that the dynamic phasor model has high efficiency and accuracy. The modelling method can also be applied in the dynamic simulation of other FACTS devices.

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BIOGRAPHIES

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