Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2014, Article ID 536872, 12 pages
http://dx.doi.org/10.1155/2014/536872

Research Article

Exploring the Impact of Commuter's Residential Location Choice on the Design of a Rail Transit Line Based on Prospect Theory

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Received 7 October 2013; Accepted 18 June 2014; Published 4 August 2014

Academic Editor: Chaudry Masood Khalique

This paper explores the impact of prospect theory based commuter's residential location choice on the design problem of a rail transit line located in a monocentric city. A closed-form social welfare maximization model is proposed, with special consideration given to prospect theory based commuter's residential location choice over years. Commuters are assumed to make residential location choice by a trade-off between daily housing rent and generalized travel cost to minimize their prospect values. The solutions properties of the proposed model are explored and compared analytically. It is found that overestimation exists for the optimal solutions of rail line length, headway, and fare based on traditional utility theory, compared with the optimal solutions of the proposed prospect theory based model. A numerical example is given to illustrate the properties of the proposed model.

1. Introduction

Rail transit lines are being launched in many cities of China in recent years, due to the rapid development of economy and the dramatic growth of urban population. For instance, the Shanghai Municipal Government has commenced the project of extending railway line 11 about 5.76 km westwards with a total of four stations recently. In Hong Kong, a new metro line connecting Shatin new town to the central with a total length of 17 km and ten stations are also being built, which starts in 2011 and is expected to finish in 2019.

Rail transit lines can alleviate boring traffic congestion and make life more convenient as regards maneuverability for a specific set of people, namely, those living in the vicinity of the line and new stations to be constructed. Hence, commuters prefer living along the candidate rail transit lines so as to enjoy such advantage of rail service.

In many areas, especially in cities with high population densities like Shanghai and Hong Kong, commuters' behaviour of making residential location choice and rail travel mode choice simultaneously has been identified [1–3]. In other words, commuters' behaviour of simultaneous residential location choice and rail travel mode choice can directly affect the performance of the candidate rail transit line. The output results of the above commuter's simultaneous residential location choice and rail travel mode choice are the population densities in residential locations.

The discrete choice models were largely used to determine the residential location choice with generalized travel costs of various travel mode choices as the determinant factors [4–6]. The discrete choice models can help estimating population densities in residential locations, and explain the trade-offs commuters are faced with. Nevertheless, their use has been criticized in that most of the discrete choice models were proposed based on utility theory.

Although utility theory was applicable in many contexts, it may be inadequate in estimation of population densities over a long-term planning horizon. Before the reach of relative equilibrium of population densities, commuters undergo a relative long-term learning process of residential location choice and rail mode choice. This long-term learning process can be partially attributed to the existence of perception error and uncertainty on housing rent and generalized travel cost. Unfortunately, the long-term learning process cannot be captured by utility theory.

Table 1: Modes and reference points in some previous models.

Citation	Mode	Reference point
Katsikopoulos et al. [10]	Car	Travel time of the reference route
Senbil and Kitamura [11]	Commuters	Work start time and preferred arrival time
Avineri [7]	Private car	Average travel time
Jou et al. [12]	Commuters	Earliest acceptable arrival time and work start time
Xu et al. [9]	Travellers	Effective reversed time

Prospect theory can be seen as an extension of utility theory. Compared with utility theory, which are based on normative preference axioms, prospect theory describes lotteries choices by a two-step process: an initial phase of editing and a subsequent phase of evaluation [7, 8]. Because of the property of two-step process, prospect theory can be used to describe the above long-term learning process of commuters.

Reference point is a key parameter of prospect theory. However, there are no prefect models to predict the value of reference point in transportation models [9]. Some transportation models associated with prospect theory are summarized in Table 1. Katsikopoulos et al. [10] investigated car drivers' risk preference behaviour on route choice with travel time of the reference route as reference point. Senbil and Kitamura [11] explored commuters' departure time choice with work start time as reference point in decision frame 1 and preferred arrival time as reference point in decision frame 2. By contrast, Jou et al. [12] examined commuters' departure time with the reference point of earliest acceptable arrival time and work start time. Xu et al. [9] modelled drivers' route choice with effective reversed time as reference point.

Our goal is to explore the impacts of prospect theory based commuters' residential location choice on the design of a rail transit line. Commuters are assumed to choose residential locations along the candidate rail transit line by a trade-off between daily housing rent and generalized travel cost. To capture the above long-term learning process of residential location choice and rail mode choice simultaneously, two reference points are adopted: willingness-to-pay on daily housing rent and willingness-to-pay on generalized travel cost.

Commuters' residential location choice is affected by many design variables of a rail transit line, such as rail line length, rail station locations (spacing), headway, and fare. Specifically, rail line length is closely concerned with the coverage area of rail service; railway station locations (spacing) have a direct effect on the train operating speed, dwelling delays of trains, and in-vehicle time of commuters at stations; headway could be used to determine the waiting time of commuters at stations and fare is a component of generalized travel cost.

Normally, the above four design variables can be distinguished into two types: long-term and short-term decision variables. Long-term decision variables cannot be changed during operation stage, but short-term variables can be updated. For example, rail line length and rail station locations (spacings) should be determined during planning stage and are inflexible to change during operation stage, whereas fare and headway are still flexible to unevenly change in actual operation.

Commuters' generalized travel cost consists of fare and various travel costs, including access time cost, waiting time cost, and in-vehicle time cost. Specifically, access time cost is closely concerned with rail line length and rail station locations (spacing). Waiting time cost depends on headway. In-vehicle time cost is a function of distance between commuters' residential locations and central business district (CBD).

In this paper, all commuters are assumed to work in the CBD of a monocentric city, and thus homework is a compulsory trip of commuters each day. The long-term planning horizon of rail transit line design is divided into several equal periods. In each period, rail service can be improved. After the implementation of rail service in each period, commuters make residential location choice by trade-off between daily housing rent and generalized travel cost.

The reminder of this paper is organized as follows. In the next section, assumptions and notations are given. Section 3 presents model formation. Some model properties are examined. In Section 4, a numerical example is used to illustrate the insightful findings of the proposed models. Section 5 concludes this paper.

2. Assumptions and Notations

A transportation corridor of B km length is proposed, which extends from the CBD towards the boundary of the city, as shown in Figure 1. There is an ordered sequence of stations $\{1, 2, \ldots, N(\tau) + 1\}$. The symbol D_s represents the distance between station s and the CBD, $N(\tau)$ represents rail station number and D_1^τ is the rail line length in period τ. The considered designed variables include the combination of rail line length D_1^τ, station location D_s or spacing $(D_{s-1}-D_s)$, train headway $h(\tau)$, and fare f_s [2].

To facilitate the presentation of the essential ideas, without loss of generality, basic assumptions and notations are made in this paper, as follows.

2.1. Assumptions. (*A1*) Commuters are assumed to be homogeneous and they have the same preferred arrival time to the workplace located in the CBD and the same preferred daily housing rent for each residential location [13]. This assumption could be extended to multiclass commuters in further studies.

(*A2*) Commuters are assumed to board trains at the nearest rail station, and the trains stop at every station on the candidate rail transit line. This assumption has also been adopted by many previous works, such as those of Wirasinghe and Ghoneim [14] and Li et al. [2].

(*A3*) In-vehicle crowding cost in trains and moving costs for commuters from one place to another are not considered, since the proposed model is for long-term planning purpose of rail transit line. Other travel modes are not considered, because the main goal of this paper is to explore the prospect

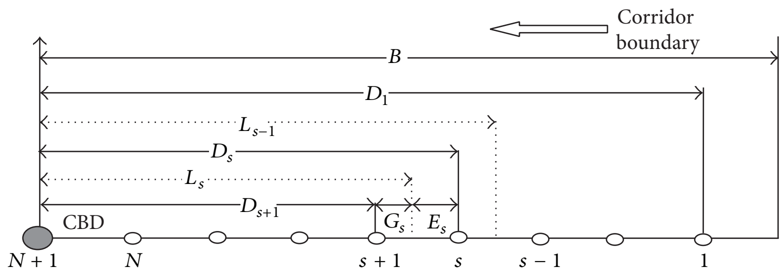

Figure 1: The rail transit line configuration in a monocentric city.

theory based commuters' residential location choice on the design of a rail transit line. The situation considered here may emerge in a monocentric city with highly compact city centre. Commuters in this monocentric city live in the dispersed surrounding suburban area [15].

(*A4*) The original population density in the monocentric city is specified as a linear function. The original population density at distance x from the CBD in period τ is defined as $g(x,\tau) = g_0(\tau)(1 - ex)$, $\forall x \in [0,B]$, where $g_0(\tau)$ represents the population density in the CBD of period τ and e represents the density gradient describing how rapidly the density falls as the distance increases. Here, $e > 0$ represents the fact that more commuters live at CBD area, while $e < 0$ represents the fact that more commuters live at suburban area. Smaller value of e means more decentralized city. Specifically, when e equals 0, this linear population density function is reduced to a uniform one. With this assumption, the total number of population $G(\tau)$ in period τ is given by $G(\tau) = \int_0^B g_0(\tau)(1 - ex)\,dx$ [2].

2.2. Notations. Consider the following:

$c(x,s,\tau)$: commuters' actual generalized travel cost for arriving at CBD from location x from station s by train in the τth period;

$r(x,\tau)$: commuters' actual housing rent at the location x in the τth period, $x \in \mathbf{X}$, and $\mathbf{X}$ is the choice set; that is, many types of houses exist at location x;

$\tilde{c}(x,s,\tau)$: commuters' perceived generalized travel cost for arriving at CBD from location x from station s by train in the τth period;

$\tilde{r}(x,\tau)$: commuters' perceived housing rent at the location x in the τth period;

$c_{\mathrm{WTP},x,\tau}$: commuters' reference point to decide whether the generalized travel cost is high or low at the location x in the τth period, which is called the commuters' willingness-to-pay (WTP) on travel cost;

$r_{\mathrm{WTP},x,\tau}$: commuters' reference point to decide whether the housing rent is high or low at the location x in the τth period, which is called the commuters' willingness-to-pay (WTP) on housing rent;

$\tilde{p}(x,\tau)$: the probability of obtaining low living cost in terms of the generalized travel cost and housing rent at location x in the τth period;

$\Delta rc(x,s,\tau)$: the deviation between perceived living cost and reference points in terms of the generalized travel cost and housing rent at location x in the τth period;

$\pi(\tilde{p}(x,\tau))$: the probability weighting function;

$V(\Delta rc(x,s,\tau))$: value function of living at location x and travelling to the CBD from station s by train in the τth period;

$\mathrm{PV}_{x,s,\tau}$: prospect value of living at location x and travelling to the CBD from station s by train in the τth period.

3. Model Formulation

The design of a rail transit line is considered over a planning horizon of $[0,T]$. This horizon is divided into M equal periods. The rail transit line is assumed to be implemented by an operator franchised by government. Social welfare maximization is the decision objective of rail transit line design. Commuters are assumed to make residential location choices as if they are prospect maximizers. This question can be formulated as a mathematically programming model with the objective of social welfare maximization, subjected to the constraints of prospect theory based residential location choice equilibrium condition.

3.1. Prospect Theory Based Residential Location Choice Equilibrium Condition. As stated above, prospect theory can be used to capture the learning process of commuters' location choice over rail design periods in the planning horizon. As in Avineri [7], Wardrop's [16] principle of user equilibrium could be extended to prospect theory based equilibrium, "Equilibrium under the condition that no commuter can decrease his/her choice prospect value by unilaterally switching his/her choice behaviour." Mathematically, this equilibrium condition can be expressed as

$$q(x,s,\tau) > 0 \Longrightarrow \mathrm{PV}_{x,s,\tau} = \min\{\mathrm{PV}_{x,s,\tau}\}, \quad \forall x \in [0,B], \quad s = 1,2,\ldots,N(\tau), \quad \tau \in [1,M], \tag{1}$$

where $PV_{x,s,\tau}$ represents prospect value of living at location x and travelling to the CBD from station s by train in the τth period and $q(x, s, \tau)$ is the peak-hour travel demand density at location x to the CBD by train through station s in period τ.

Prospect value $PV_{x,s,\tau}$ could be calculated by the following equations:

$$PV_{x,s,\tau} = \pi\left(\tilde{p}(x,\tau)\right) V\left(\Delta rc(x,s,\tau)\right), \quad (2)$$

$$\pi\left(\tilde{p}(x,\tau)\right) = \frac{\tilde{p}(x,\tau)^{\gamma}}{\left[\tilde{p}(x,\tau)^{\gamma} + \left(1-\tilde{p}(x,\tau)\right)^{1/\gamma}\right]}, \quad (3)$$

$$V\left(\Delta rc(x,s,\tau)\right) = \begin{cases} \Delta rc(x,s,\tau)^{\alpha'}, \Delta rc(x,s,\tau) \geq 0 \\ -\lambda\left(-\Delta rc(x,s,\tau)\right)^{\beta'}, \Delta rc(x,s,\tau) < 0, \end{cases} \quad (4)$$

$$\Delta rc(x,s,\tau) = \tilde{c}(x,s,\tau) + \tilde{r}(x,\tau) - c_{\mathrm{WTP},x,\tau} - r_{\mathrm{WTP},x,\tau}, \quad (5)$$

$$\tilde{c}(x,s,\tau) = c(x,s,\tau) + \xi, \quad (6)$$

$$\tilde{r}(x,\tau) = r(x,\tau) + \psi, \quad (7)$$

where ξ/ψ is a random disturbance term reflecting generalized travel cost/housing rent differences among residential locations.

The generalized travel cost, $c(x, s, \tau)$, consists of rail fare and various travel time cost, including access time cost, waiting time cost, and in-vehicle time cost. Specifically, it is defined as

$$c(x,s,\tau) = f_s(\tau) + \phi_u u_s(x) + \phi_s w_s(\tau) + \phi_t t_s(\tau), \quad (8)$$

where $f_s(\tau)$ represents rail fare from station s to the CBD in period τ, $u_s(x)$ represents commuters' average access time to station s from location x, $w_s(\tau)$ represents commuters' average waiting time at station s in period τ, $t_s(\tau)$ represents commuters' in-vehicle time to the CBD from station s, and ϕ_u, ϕ_w, ϕ_t represent commuters' value of time for access time, waiting time, and in-vehicle time, respectively.

The commuters' waiting time at station s in period τ, $w_s(\tau)$, can be given by

$$w_s(\tau) = \alpha h(\tau), \quad \forall s = 1, 2, \ldots, N(\tau), \quad (9)$$

where $h(\tau)$ represents the headway of railway service in period τ and α is a calibration parameter which depends on the distributions of train headway and commuter arrival.

The commuters' in-vehicle time from station s to the CBD in period τ, $t_s(\tau)$, can be calculated by

$$t_s(\tau) = T_{s1}(\tau) + T_{s2}(\tau), \quad \forall s = 1, 2, \ldots, N(\tau), \quad (10)$$

where

$$T_{s1}(\tau) = \frac{D_s}{V_t(\tau)}, \quad \forall s = 1, 2, \ldots, N(\tau),$$
$$T_{s2} = \beta_0 (N(\tau) + 1 - s), \quad \forall s = 1, 2, \ldots, N(\tau), \quad (11)$$

where $V_t(\tau)$ represents the average train cruise speed in period τ, D_s represents the station distance from station s to CBD defined as above, and β_0 represents the average train dwelling time at a station, which can be calibrated with observed data [17, 18].

To represent the demand-supply relationship of housing rental market, the following housing rent is assumed given by

$$r(x,\tau) = \alpha_s \left[1 + \frac{\beta_s P(x,\tau)}{(H(x,\tau) - P(x,\tau))}\right], \quad (12)$$

where $H(x, \tau)$ (in terms of housing unit) denotes potential housing supply density at location x in the τth period and α_s and β_s are positive scalar parameters that represent the fixed and demand-dependent components of the rent function around station s [19].

In order to determine travel demand density $q(x, s, \tau)$, we first define the potential travel demand density at location x in period τ, which is denoted by $P(x, \tau)$. Generally speaking, commuters' destinations are normally distributed along the rail line with more concentration close to the CBD of course. Denote η by the proportion of trips with CBD being the destinations in period τ and denote ρ by the ratio of peak-hour flow to the daily average flow, and then $\eta\rho g(x)$ represents the peak-hour potential travel demand density in terms of $(A4)$. We have

$$P(x,\tau) = \eta\rho g_0(\tau)(1 - ex)$$
$$= P_0(\tau)(1 - ex), \quad \forall x \in [0, B], \quad (13)$$

where $P_0(\tau)$ represents the peak-hour potential travel demand density in the CBD and $P_0(\tau) = \eta\rho g_0(\tau)$.

As stated above, travel demand density for rail service, $q(x, s, \tau)$, is closely concerned with several design variables, namely, rail line length, rail station or spacing, headway, and fare, in terms of the generalized travel cost. To represent such effect, a negative exponential elastic demand density function is used as follows:

$$q(x,s,\tau) = P(x,\tau)\exp\left(-\theta\tilde{c}(x,s,\tau)\right),$$
$$\forall x \in [0, B], \quad s = 1, 2, \ldots, N(\tau), \quad (14)$$

where θ ($\theta > 0$) represents the sensitivity parameter for the generalized travel cost and the perceived generalized travel cost $\tilde{c}(x, s, \tau)$ is given by (6) and (8).

3.2. Social Welfare of Candidate Rail Transit Line. Social welfare of the candidate rail transit line can be calculated by summation of operator's net profit and consumer surplus of commuters. Mathematically, the social welfare in the planning horizon $\tau \in [1, M]$, SW, can be expressed as

$$SW = \sum_{\tau=1}^{M}\left(P_\tau + CS_\tau\right), \quad (15)$$

where P_τ and CS_τ are the operator's net profit and consumer surplus of commuters in period τ, respectively.

The operator's net profit is closely concerned with revenue from fare and related construction and operation cost. Accordingly, P_τ could be calculated by

$$P_\tau = R_\tau - C_\tau, \quad (16)$$

where R_τ is the operator's revenue in period τ and C_τ is the related construction and operation cost in period τ.

The operator's revenue comes from fare. It could be calculated by summation of the number of commuters boarding at each station multiplied by the corresponding fare; that is,

$$R_\tau = \sum_{s=1}^{N} \frac{f_s(\tau) Q_{s,\tau}}{(1+i)^{\tau-1}}, \tag{17}$$

where $1/(1+i)^{\tau-1}$ is the discount factor in period τ, i is the interest rate, and $Q_{s,\tau}$ is the travel demand of station s in period τ.

In terms of (*A2*), the travel demand of each station comes from coverage area of this station; that is,

$$Q_{s,\tau} = \int_{L_s}^{L_{s-1}} q(x, s, \tau)\, dx, \quad \forall s = 1, 2, \ldots, N(\tau), \tag{18}$$

where $q(x, s, \tau)$ is peak-hour travel demand density of rail service at location x in period τ given by (14). L_s is commuter watershed line, which is located at the middle point of line segment $(s, s+1)$ and the distance of commuter watershed line L_s from the CBD is given by

$$L_s = \frac{(D_s + D_{s+1})}{2}, \quad \forall s = 1, 2, \ldots, N(\tau). \tag{19}$$

In particular, L_0 represents the maximum coverage location of rail service. Beyond this location, no one would patronize the rail service. Thus, L_0 holds

$$q(L_0^\tau, 1, \tau) = 0, \quad L_0^\tau \in [D_1^\tau, B], \tag{20}$$

where $q(L_0^\tau, 1, \tau)$ is travel demand density for station 1 at location L_0^τ in period τ. On the basis of (8)–(14), the maximum service coverage L_0 of the rail line can be given by

$$L_0 = \frac{1}{e}. \tag{21}$$

According to the (*A4*), (21) implies that railway service is available for all the residential people in the considered corridor [2]. Substituting (14) and (19) into (21), $Q_{s,\tau}$ can be rewritten as

$$\begin{aligned}
Q_{s,\tau} &= P_0(\tau) \int_{L_s}^{L_{s-1}} \exp\left(-\frac{\theta\phi_t}{V_t(\tau)} x + \lambda_s(\tau)\right) dx \\
&\quad - eP_0(\tau) \int_{L_s}^{L_{s-1}} x \exp\left(-\frac{\theta\phi_t}{V_t(\tau)} x + \lambda_s(\tau)\right) dx \\
&= -\frac{P_0(\tau) V_t(\tau)}{\theta\phi_t} \left[\exp\left(-\theta\tilde{c}(L_{s-1}, s, \tau)\right) \right. \\
&\quad \left. - \exp\left(-\theta\tilde{c}(L_s, s, \tau)\right)\right] + \frac{eP_0(\tau) V_t^2(\tau)}{\theta^2\phi_t^2} \\
&\quad \cdot \left[\left(\frac{\theta\phi_t}{V_t(\tau)} L_{s-1} + 1\right) \exp\left(-\theta\tilde{c}(L_{s-1}, s, \tau)\right) \right. \\
&\quad \left. - \left(\frac{\theta\phi_t}{V_t(\tau)} L_s + 1\right) \exp\left(-\theta\tilde{c}(L_s, s, \tau)\right)\right],
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\lambda_s(\tau) &= -\theta f_s(\tau) - \theta\phi_u u_s(x) - \theta\phi_s w_s(\tau) - \theta\phi_t\beta_0 \\
&\quad \times (N(\tau) + 1 - s) + \theta\phi_t \frac{u(x) V_w}{V_t(\tau)} - \theta\xi, \\
&\qquad \forall s = 1, 2, \ldots, N(\tau).
\end{aligned} \tag{23}$$

V_w is denoted as commuters' walking speed from location x to station s, and $u(x)V_w$ is the distance between location x and station s.

The discounted cost C_τ, which consists of three cost components, the train operations cost $C_{o,\tau}$, rail line cost $C_{L,\tau}$, and rail station cost $C_{s,\tau}$, could be expressed as

$$C_\tau = C_{o,\tau} + C_{L,\tau} + C_{s,\tau}. \tag{24}$$

The discounted train operating cost $C_{o,\tau}$ is given by

$$C_{o,\tau} = \frac{(\mu_o + \mu_1 F(\tau))}{(1+i)^{\tau-1}}, \tag{25}$$

where μ_o is the fixed operating cost, μ_1 is the operating cost per train in each period, and F is the fleet size (or the number of trains) on that line. $F(\tau)$ equals the vehicle round journey time $T_R(\tau)$ divided by the headway $H(\tau)$. Namely,

$$F(\tau) = \frac{T_R(\tau)}{h(\tau)}, \tag{26}$$

where the round journey time T_R is composed of the terminal time, line-haul travel time, and train dwelling delays at station [20], which could be expressed as

$$T_R = \varsigma T_o + 2(T_{11} + T_{12}), \tag{27}$$

where T_o is the constant terminal time on the circular line and ς is the number of terminal times on that line. T_{11} and T_{12} are, respectively, the total line-haul travel time and total dwelling delay for train's operations from station 1 to CBD, given by (11).

The discounted rail line cost $C_{L,\tau}$ is the sum of variable cost $\gamma_1 D_1$ (e.g., land acquisition cost, line construction cost) which is proportional to the rail line length D_1 and the fixed cost γ_0 (e.g., line overhead cost, maintenance cost, and labour cost), discounted to present value terms. Namely,

$$C_{L,\tau} = \gamma_1 D_1 (1+\chi)^{\tau-1} + \frac{\gamma_0}{(1+i)^{\tau-1}}, \tag{28}$$

where γ_1 is the fixed rail line cost per kilometre in each period. The term $1/(1+\chi)^{\tau-1}$ represents the inflation factor. It means that, for the same capacity enhancement, the fixed rail line cost increases χ each period.

The discounted rail station cost $C_{s,\tau}$ includes a fixed cost (e.g., station land acquisition cost and design and construction cost) and a variable cost (e.g., station overhead cost, operating cost, and maintenance cost), discounted to present value terms. Mathematically, $C_{s,\tau}$ can be expressed as

$$C_{s,\tau} = \kappa_0 (1+\chi)^{\tau-1} + \frac{\kappa_1 (N(\tau) + 1)}{(1+i)^{\tau-1}}, \tag{29}$$

where κ_0 is the fixed cost and κ_1 is the operating cost per station in each period.

Consumer surplus measures the difference between what consumers would be willing to pay for travel and what they actually pay. In order to obtain consumer surplus, the inverse demand function is calculated as follows:

$$[q(x,s,\tau)]^{-1}\left(q\left(x,s,\tau\right)\right)=c\left(x,s,\tau\right)+\xi=\frac{1}{\theta}\ln\frac{P\left(x,\tau\right)}{q\left(x,s,\tau\right)}, \tag{30}$$

with $\forall x\in[0,B]$, $s=1,2,\ldots,N(\tau)$. The consumer surplus at location x in period τ, denoted as $\mathrm{CS}(x,s,\tau)$, can be calculated by

$$\begin{aligned}\mathrm{CS}\left(x,s,\tau\right)&=\int_0^{q(x,s,\tau)}[q(x,s,\tau)]^{-1}\left(w\right)dw\\&\quad-q\left(x,s,\tau\right)\left(c\left(x,s,\tau\right)+\xi\right)=\frac{q\left(x,s,\tau\right)}{\theta}.\end{aligned} \tag{31}$$

The discounted consumer surplus in period τ, CS_τ, is then obtained by summing the consumer surplus along the candidate rail transit line, discounted to the present value. Namely,

$$\mathrm{CS}_\tau=\frac{\int_0^{L_0}\mathrm{CS}\left(w,s,\tau\right)dw}{(1+i)^{\tau-1}}. \tag{32}$$

3.3. Social Welfare Maximization Model. As stated above, the design goal of the rail transit line is social welfare maximization. Mathematically, this problem can be formulated as follows:

$$\begin{aligned}&\max\ \mathrm{SW}\left(\mathbf{D},h\left(\tau\right),f_s\left(\tau\right)\right)\\&=\sum_{\tau=1}^{M}\sum_{s=1}^{N(\tau)}\frac{\left(f_s\left(\tau\right)Q_{s,\tau}\right)}{(1+i)^{\tau-1}}\\&\quad-\sum_{\tau=1}^{M}\frac{\left[\mu_0+\left(\mu_1/h\left(\tau\right)\right)\left(\varsigma T_0+\left(2D_1^\tau/V_t\left(\tau\right)\right)+2\beta_0N\left(\tau\right)\right)\right]}{(1+i)^{\tau-1}}\\&\quad-\sum_{\tau=1}^{M}\left[\frac{\left(\gamma_0+\kappa_1\left(N\left(\tau\right)+1\right)\right)}{(1+i)^{\tau-1}}-(1+\chi)^{\tau-1}\right.\\&\qquad\left.\times\left(\gamma_1D_1^\tau+\kappa_0\right)+\frac{\int_0^{L_0^\tau}\left(q\left(w,s,\tau\right)/\theta\right)dw}{(1+i)^{\tau-1}}\right],\end{aligned} \tag{33}$$

where $\mathbf{D}$ represents the vector of station locations; namely, $\mathbf{D}=(D_{N(\tau)},\ldots,D_2,D_1^\tau)$. $Q_{s,\tau}$ can be determined by (22).

The optimal solutions for the rail line length, rail station location, headway, and fare can be obtained by setting the partial derivatives of objective function equation (33) with respect to these decision variables equal to zero and solving them simultaneously. The following proposition gives the optimal solutions. The proof is given in Appendix A.

Proposition 1. *With the given population density in a particular period, the optimal rail line length, rail station location, headway, and fare solutions with the objectives of social welfare maximization satisfy the systems of equations*

$$\begin{aligned}\frac{\partial SW\left(\cdot\right)}{\partial D_s}&=\sum_{\tau=1}^{M}\sum_{i=s-1}^{s+1}\frac{f_s\left(\tau\right)}{(1+i)^{\tau-1}}\frac{\partial Q_{i,\tau}}{\partial D_s}-\Delta_s\\&\quad\times\sum_{\tau=1}^{M}\left(\frac{1}{(1+i)^{\tau-1}}\frac{2\mu_1}{h\left(\tau\right)V_t\left(\tau\right)}+\left(1+\chi\right)^{\tau-1}\gamma_1\right)=0,\\h\left(\tau\right)&=\sqrt{\frac{\mu_1\left(\varsigma T_0+\left(2D_1^\tau/V_t\left(\tau\right)\right)+2\beta_0N\left(\tau\right)\right)}{\alpha\phi_s\left[\Delta_3-f_s\left(\tau\right)\sum_{i=1}^{N(\tau)}\left(\Delta_1^s-\Delta_2^s\right)\right]}},\\f_s\left(\tau\right)&=\frac{\sum_{\tau=1}^{M}\sum_{i=1}^{N(\tau)}Q_{i,\tau}-\sum_{\tau=1}^{M}\Delta_3}{\sum_{\tau=1}^{M}\sum_{i=1}^{N(\tau)}\left(\Delta_2^s-\Delta_1^s\right)},\end{aligned} \tag{34}$$

where $\Delta_s=1$ *if* $s=1$, *and 0 otherwise.* Δ_1^s, Δ_2^s, *and* Δ_3 *are given by*

$$\begin{aligned}\Delta_1^s&=\frac{P_0\left(\tau\right)V_t\left(\tau\right)}{\phi_t}\left[\exp\left(-\theta\widetilde{c}\left(L_{s-1},s,\tau\right)\right)\right.\\&\qquad\left.-\exp\left(-\theta\widetilde{c}\left(L_s,s,\tau\right)\right)\right],\\\Delta_2^s&=\frac{eP_0\left(\tau\right)V_t^2\left(\tau\right)}{\theta\phi_t^2}\left[\left(\frac{\theta\phi_t}{V_t\left(\tau\right)}L_{s-1}+1\right)\exp\left(-\theta\widetilde{c}\left(L_{s-1},s,\tau\right)\right)\right.\\&\qquad\left.-\left(\frac{\theta\phi_t}{V_t\left(\tau\right)}L_s+1\right)\exp\left(-\theta\widetilde{c}\left(L_s,s,\tau\right)\right)\right],\\\Delta_3&=P_0\left(\tau\right)\left[\exp\left(-\theta\widetilde{c}\left(L_0^\tau,N\left(\tau\right),\tau\right)\right)\right.\\&\qquad\times\frac{\theta\phi_t^2+e\theta\phi_t\phi_sV_t\left(\tau\right)+e\theta\phi_sV_t^2\left(\tau\right)}{\theta\phi_t^2}\\&\qquad\left.-\exp\left(-\theta\widetilde{c}\left(0,0,\tau\right)\right)\frac{\theta\phi_t^2+e\phi_sV_t^2\left(\tau\right)}{\theta\phi_t^2}\right],\end{aligned} \tag{35}$$

and $\partial Q_{i,\tau}/\partial D_s$ *are given by*

$$\begin{aligned}\frac{\partial Q_{s-1,\tau}}{\partial D_s}&=-\frac{P_0\left(\tau\right)}{2}\exp\left(-\theta\widetilde{c}\left(L_{s-1},s,\tau\right)\right)+\frac{eP_0\left(\tau\right)V_t\left(\tau\right)}{2\theta\phi_t}\\&\quad\times\left[\left(\frac{\theta\phi_t}{V_t\left(\tau\right)}L_{s-1}+\frac{1}{2}\right)\exp\left(-\theta\widetilde{c}\left(L_{s-1},s,\tau\right)\right)\right],\\&\qquad\forall s=2,\ldots,N\left(\tau\right)\end{aligned}$$

$$\frac{\partial Q_{1,\tau}}{\partial D_1^{\tau}} = -\frac{P_0(\tau)}{2}\exp\left(-\theta\widetilde{c}\left(L_1,s,\tau\right)\right) + \frac{eP_0(\tau)V_t(\tau)\left(V_t(\tau)-\theta\phi_t\right)}{2\theta^2\phi_t^2}\exp\left(-\theta\widetilde{c}\left(L_1,s,\tau\right)\right)$$

$$\frac{\partial Q_{s,\tau}}{\partial D_s} = \frac{P_0(\tau)}{2}\left[\exp\left(-\theta\widetilde{c}\left(L_{s-1},s,\tau\right)\right) - \exp\left(-\theta\widetilde{c}\left(L_s,s,\tau\right)\right)\right] - \frac{eP_0(\tau)V_t(\tau)}{2\theta\phi_t} \times\left[\left(\frac{\theta\phi_t}{V_t(\tau)}L_{s-1}+\frac{1}{2}\right)\cdot\exp\left(-\theta\widetilde{c}\left(L_{s-1},s,\tau\right)\right) - \left(\frac{\theta\phi_t}{V_t(\tau)}L_s+\frac{1}{2}\right)\exp\left(-\theta\widetilde{c}\left(L_s,s,\tau\right)\right)\right], \quad \forall s=2,\ldots,N(\tau),$$

$$\frac{\partial Q_{s+1,\tau}}{\partial D_s} = \frac{P_0(\tau)}{2}\exp\left(-\theta\widetilde{c}\left(L_{s+1},s,\tau\right)\right) - \frac{eP_0(\tau)V_t(\tau)}{2\theta\phi_t} \cdot\left[\left(\frac{\theta\phi_t}{V_t(\tau)}L_s+\frac{1}{2}\right)\exp\left(-\theta\widetilde{c}\left(L_s,s,\tau\right)\right)\right], \quad \forall s=1,\ldots,N(\tau)-1. \tag{36}$$

Proposition 1 presents the partial derivatives of travel demand $Q_{i,\tau}$ with respect to railway line length D_1^{τ} and railway station location D_s. There is another alternative approach to determine these partial derivatives, implementing equilibrium sensitivity analysis of travel demand with respect to railway line length and railway station location. Details on sensitivity analysis approach could be seen in Friesz et al. [21] and Yan and Lam [22].

By contrast, the closed-form solutions, given by Proposition 1, can be used to examine the interrelationship between the optimal solutions of rail design variables directly. For instance, it could be seen that the optimal headway $h(\tau)$ will increase if the railway operating cost per train μ_1 increased. Li et al. [2] proposed a similar closed-form analysis with the objective of profit maximization based on utility theory and in a static situation.

To highlight the difference between the optimal solutions of the rail design variables based on prospect theory and traditional utility theory, the following proposition is given. The proof is given in Appendix B.

Proposition 2. *Overestimation exists for the optimal solutions of rail line length, headway, and fare based on traditional utility theory, compared with prospect theory.*

The most widely used solution algorithm for solving concave problem is the Frank-Wolfe searching algorithm. For solving the prospect theory based residential location equilibrium problem (1), this algorithm reduces to a sequence of shortest path computations and one-dimensional minimizations [23]. For the optimization of rail design variables, the heuristic algorithm proposed by Li et al. [2] is used here, which is directly based on the first-order optimality conditions of the social welfare with respect to the above rail design variables, as shown in Proposition 1.

4. Numerical Example

To facilitate the presentation of the essential ideas and contributions of this paper, an illustrative example is employed. Specifically, the difference between the optimal solutions of rail design variables based on traditional utility theory and prospect theory are compared.

The alignment of the rail transit line concerned is shown in Figure 1. The corridor length is fixed as 40 km. The time horizon is 3 years and M is 3. Without loss of generality, even station spacing is set as 1.0 km. Other parameters are given in the following Table 2.

From Table 3, it could be seen that the optimal solutions of rail line length, fare, and headway based on prospect theory were less than those based on utility theory. This result was in accord with Proposition 2. However, the social welfare based on prospect theory was larger than that based on utility theory; namely, $26669520 > 24278400$. These results can be attributed to the long-term learning behaviour of commuters' on residential location choice. This long-term learning behaviour reduced the investment of the rail transit line, but increased the social welfare.

5. Conclusions

This paper proposed closed-form models to explore the impacts of prospect theory based residential location choice on the design of a rail transit line in a monocentric city. Prospect theory was used to model the long-term learning behaviour of commuters' on residential location choice over a planning horizon. Trade-off exists between daily housing rent and generalized travel cost for commuters.

The analytical optimal solutions of rail design variables with social welfare maximization have been given. It is concluded that overestimation exists based on traditional utility theory, compared with prospect theory.

This study provides a new avenue for the design of a rail transit line. Further research is needed in the following directions.

(1) In this paper, a monocentric city is assumed, with only one CBD and several other residential locations. Thus, the commuters' mobility between different CBD(s) in larger cities cannot be explored. The city boundary is not explicitly considered. The proposed model can be extended into polycentric CBD model in a further study [24–26].

(2) All commuters were assumed to be homogenous in this study. However, previous studies have shown that income levels dominated the residential location choices [27, 28]. Therefore, the proposed model can be extended to incorporate the income levels

Table 2: Parameters.

Symbol	Definition	Value
i	Interest rate per period	0.03
χ	Inflation rate per period	0.01
f_0	Fixed component of distance-based fare (HK$)	2.5
ρ	Ratio of peak-hour flow to the daily average flow (%)	80%
η	Proportion of trips with the CBD as the destinations (%)	95%
e	Density gradient	0.04
θ	Sensitivity parameters in elastic demand function	0.066
ξ	Commuters' perceived random error for generalized travel cost	$N(0,3)$
$\phi_u/\phi_t/\phi_s$	Value of time for access time, waiting time, and in-vehicle, respectively, (HK$/hour)	80
α	Parameter for waiting cost	0.5
α'	Parameter for value function	0.88
β'	Parameter for value function	0.88
λ	Parameter for value function	2.25
$V_t(\tau)$	Average train operating speed (km/hour)	40
V_w	Average commuter walking speed from location x to station s (km/hour)	10
$u(x)$	Average distance between location x to station s (km)	0.5
β_0	Average train dwelling time at a station (hour)	0.03
μ_0	Fixed train operating cost per period (HK$/hour)	1350
μ_1	Variable train operating cost per train in each period (HK$/vehicle hour)	540
ς	Number of terminal times of train	2
T_0	Constant terminal time of train (hour)	0.2
γ	Parameter for weighting probability function	0.61/0.69
γ_0	Fixed component of rail line cost (HK$/hour)	750
γ_1	Variable component of rail line cost (HK$/vehicle hour)	300
κ_0	Fixed component of rail station cost (HK$/hour)	1250
κ_1	Variable component of rail station cost (HK$/ station hour)	500
α_s	Parameter representing fixed components of rent function around rail station s	80
β_s	Parameter representing demand-dependent components of rent function around rail station s	10
$H(x,\tau)$	Potential housing supply density at location x in period τ (housing unit)	40000
ψ	Random disturbance term of daily housing rent differences among houses	$N(0,8)$
$c_{\mathrm{WTP},x,\tau}$	Reference point for generalized travel cost (HK$/day)	80
$r_{\mathrm{WTP},x,\tau}$	Reference point for daily housing rent (HK$/day)	100
$\tilde{p}(x,\tau)$	Probability of obtaining low living cost at location x in period τ	90%

Table 3: Optimal solutions of rail design variables.

Optimal solutions in year 3	Based on prospect theory	Based on utility theory
Rail line length (km)	23.50	25.00
Fare (HK$/km)	11.51	12.41
Headway (hour)	0.12	0.14
Total travel demand (persons/day)	12,230	14,401
Social welfare[1] (HK$)	**26,669,520**	**24,278,400**

[1]The parameter 360 (days) was used to convert daily social welfare to yearly social welfare.

for determining the residential location choices and population density.

(3) Only rail travel mode is considered in this paper. To investigate the effects of commuters' travel mode choice behaviour on the design of a rail transit line; more travel modes should be taken into account, for instance, autobus or park-and-ride modes [29–31].

Appendices

A. Proof of Proposition 1

To obtain the optimal solutions of rail line length and rail station locations, the partial derivatives of the objective function $\mathrm{SW}(\cdot)$ with respect to D_s are set to zero; namely,

$$\frac{\partial \mathrm{SW}(\cdot)}{\partial D_s} = \sum_{\tau=1}^{M} \frac{f_s(\tau)}{(1+i)^{\tau-1}} \frac{\partial \sum_{i=1}^{N} Q_{i,\tau}}{\partial D_s} - \Delta_s$$

$$\times \sum_{\tau=1}^{M} \left[\left(\frac{1}{(1+i)^{\tau-1}} \frac{2\mu_1}{h(\tau) V_t(\tau)} + (1+\chi)^{\tau-1} \gamma_1 \right) \right.$$

$$+\frac{1}{(1+i)^{\tau-1}}\frac{\partial\int_0^{L_0^\tau}(q(w,s,\tau)/\theta)\,dw}{\partial D_s}\Bigg]=0,$$

$$\forall s=1,2,\ldots,N(\tau),\qquad\text{(A.1)}$$

where $\Delta_s = 1$ if $s = 1$, and 0 otherwise. In terms of (18)–(23), $Q_{s,\tau}$ is a function of $D_s, \lambda_s, \lambda_{s-1}$, which are functions of D_{s-1}, D_s and D_{s+1}; namely,

$$Q_{s,\tau}=Q_{s,\tau}(D_{s-1},D_s,D_{s+1}),\quad \forall s=1,2,\ldots,N(\tau).\qquad\text{(A.2)}$$

Thus, the following equation holds:

$$\frac{\partial Q_{i,\tau}}{\partial D_s}=0,\quad \forall s\neq s-1,s,s+1.\qquad\text{(A.3)}$$

The derivative of $\int_0^{L_0^\tau}(q(w,s,\tau)/\theta)dw$ with respect to D_s is calculated as follows:

$$\frac{\partial\int_0^{L_0^\tau}(q(w,s,\tau)/\theta)\,dw}{\partial D_s}=\frac{\partial\int_0^{D_s}(q(w,s,\tau)/\theta)\,dw}{\partial D_s}+\frac{\partial\int_{D_s}^{L_0^\tau}(q(w,s,\tau)/\theta)\,dw}{\partial D_s}.\qquad\text{(A.4)}$$

Since

$$\frac{\partial\int_0^{D_s}(q(w,s,\tau)/\theta)\,dw}{\partial D_s}=\frac{q(D_s,s,\tau)}{\theta},\quad\forall s=1,2,\ldots,N(\tau),$$

$$\frac{\partial\int_{D_1^\tau}^{L_0^\tau}(q(w,s,\tau)/\theta)\,dw}{\partial D_1^\tau}=\frac{1}{\theta}\left[q(L_0,1,\tau)\frac{\partial L_0^\tau}{\partial D_1^\tau}-q(D_1^\tau,1,\tau)\right]=-\frac{q(D_1^\tau,1,\tau)}{\theta},$$

$$\frac{\partial\int_{D_s}^{L_0^\tau}(q(w,s,\tau)/\theta)\,dw}{\partial D_s}=-\frac{q(D_s,s,\tau)}{\theta},\quad\forall s=2,3,\ldots,N(\tau),\qquad\text{(A.5)}$$

we have

$$\frac{\partial\int_0^{L_0^\tau}(q(w,s,\tau)/\theta)\,dw}{\partial D_s}=0,\quad\forall s=1,2,\ldots,N(\tau).\qquad\text{(A.6)}$$

Substituting (A.3) and (A.6) into (A.1), one immediately obtains

$$\frac{f_s(\tau)}{(1+i)^{\tau-1}}\frac{\sum_{i=s-1}^{s+1}\partial Q_{i,\tau}}{\partial D_s}-\Delta_s\left(\frac{1}{(1+i)^{\tau-1}}\frac{2\mu_1}{h(\tau)V_t(\tau)}+(1+\chi)^{\tau-1}\gamma_1\right)=0,\quad\forall s=1,\ldots,N(\tau).\qquad\text{(A.7)}$$

The partial derivative of the objective function SW(·) with respect to headway $h(\tau)$ is

$$\frac{\partial\mathrm{SW}(\cdot)}{\partial h(\tau)}=\sum_{\tau=1}^{M}\frac{f_s(\tau)}{(1+i)^{\tau-1}}\frac{\partial\sum_{i=1}^{N}Q_{i,\tau}}{\partial h(\tau)}+\sum_{\tau=1}^{M}\left[\frac{(\mu_1/h^2(\tau))(\varsigma T_0+(2D_1^\tau/V_t(\tau))+2\beta_0 N(\tau))}{(1+i)^{\tau-1}}+\frac{1}{(1+i)^{\tau-1}}\frac{\partial\int_0^{L_0^\tau}(q(w,s,\tau)/\theta)\,dw}{\partial h(\tau)}\right]=0,$$

$$\forall s=1,2,\ldots,N(\tau).\qquad\text{(A.8)}$$

From (13), $Q_{s,\tau}$ is a function of $\tilde{c}(L_{s-1},s,\tau)$ and $\tilde{c}(L_s,s,\tau)$ and thus a function of $h(\tau)$. In terms of (5) and (15), L_s ($s = 0,\ldots,N(\tau)$) is independent of headway $h(\tau)$. With the given population density at the CBD in period τ, $P_0(\tau)$, we have

$$\begin{aligned}\frac{\partial Q_{s,\tau}}{\partial h(\tau)}=&\frac{\alpha P_0(\tau)\phi_s V_t(\tau)}{\phi_t}\times[\exp(-\theta\tilde{c}(L_{s-1},s,\tau))-\exp(-\theta\tilde{c}(L_s,s,\tau))]\\&-\frac{\alpha e P_0(\tau)\phi_s V_t^2(\tau)}{\theta\phi_t^2}\cdot\left[\left(\frac{\theta\phi_t}{V_t(\tau)}L_{s-1}+1\right)\exp(-\theta\tilde{c}(L_{s-1},s,\tau))-\left(\frac{\theta\phi_t}{V_t(\tau)}L_s+1\right)\exp(-\theta\tilde{c}(L_s,s,\tau))\right].\end{aligned}\qquad\text{(A.9)}$$

The derivative of $\int_0^{L_0^\tau}(q(w,s,\tau)/\theta)dw$ with respect to $h(\tau)$ is

$$\begin{aligned}\frac{\partial\int_0^{L_0^\tau}(q(w,s,\tau)/\theta)\,dw}{\partial h(\tau)}&=\int_0^{L_0^\tau}-\alpha\phi_s P_0(\tau)(1-ew)\exp(-\theta\tilde{c}(w,s,\tau))\,dw\\&=-\alpha\phi_s P_0(\tau)\exp(-\theta\tilde{c}(w,s,\tau))\times\left[1+\frac{e\phi_s V_t^2(\tau)}{\theta\phi_t^2}\left(\frac{\theta\phi_t}{V_t(\tau)}w+1\right)\right]\Bigg|_0^{L_0^\tau}\\&=-\alpha\phi_s P_0(\tau)\Big[\exp(-\theta\tilde{c}(L_0^\tau,N(\tau),\tau))\end{aligned}$$

$$\times \frac{\phi_t^2 + e\phi_t\phi_s V_t(\tau) + e\phi_s V_t^2(\tau)}{\theta\phi_t^2} - \exp(-\theta\widetilde{c}(0,0,\tau)) \frac{\theta\phi_t^2 + e\phi_s V_t^2(\tau)}{\theta\phi_t^2}\Bigg]. \quad \text{(A.10)}$$

Combining (A.10) and (A.13), one immediately obtains

$$h(\tau) = \sqrt{\frac{\mu_1 \left(\varsigma T_0 + \left(2D_1^\tau / V_t(\tau)\right) + 2\beta_0 N(\tau)\right)}{\alpha\phi_s \left[\Delta_3 - \overline{f}\sum_{i=1}^{N(\tau)} \left(\Delta_1^s - \Delta_2^s\right)\right]}}, \quad \text{(A.11)}$$

where

$$\Delta_1^s = \frac{P_0(\tau) V_t(\tau)}{\phi_t} \times \left[\exp(-\theta\widetilde{c}(L_{s-1}, s, \tau)) - \exp(-\theta\widetilde{c}(L_s, s, \tau))\right],$$

$$\Delta_2^s = \frac{eP_0(\tau) V_t^2(\tau)}{\theta\phi_t^2} \times \left[\left(\frac{\theta\phi_t}{V_t(\tau)} L_{s-1} + 1\right)\exp(-\theta\widetilde{c}(L_{s-1}, s, \tau)) - \left(\frac{\theta\phi_t}{V_t(\tau)} L_s + 1\right)\exp(-\theta\widetilde{c}(L_s, s, \tau))\right],$$

$$\Delta_3 = P_0(\tau)\Bigg[\exp(-\theta\widetilde{c}(L_0^\tau, N(\tau), \tau)) \times \frac{\phi_t^2 + e\phi_t\phi_s V_t(\tau) + e\phi_s V_t^2(\tau)}{\phi_t^2} - \exp(-\theta\widetilde{c}(0,0,\tau)) \frac{\theta\phi_t^2 + e\phi_s V_t^2(\tau)}{\theta\phi_t^2}\Bigg]. \quad \text{(A.12)}$$

The partial derivative of the objective function SW(·) with respect to flat fare $f_s(\tau)$ is

$$\frac{\partial \mathrm{SW}(\cdot)}{\partial f_s(\tau)} = \sum_{\tau=1}^{M} \frac{1}{(1+i)^{\tau-1}} \times \left(\sum_{i=1}^{N(\tau)} Q_{i,\tau} + f_s(\tau) \frac{\partial \sum_{i=1}^{N(\tau)} Q_{i,\tau}}{\partial f_s(\tau)} + \frac{\partial \int_0^{L_0^\tau} (q(w,s,\tau)/\theta)\, dw}{\partial f_s(\tau)}\right) = 0, \quad \text{(A.13)}$$

where

$$\frac{\partial Q_{s,\tau}}{\partial f_s(\tau)} = \frac{P_0(\tau) V_t(\tau)}{\phi_t}\left[\exp(-\theta\widetilde{c}(L_{s-1}, s, \tau)) - \exp(-\theta\widetilde{c}(L_s, s, \tau))\right] - \frac{eP_0(\tau) V_t^2(\tau)}{\theta\phi_t^2} \cdot \left[\left(\frac{\theta\phi_t}{V_t(\tau)} L_{s-1} + 1\right)\exp(-\theta\widetilde{c}(L_{s-1}, s, \tau)) - \left(\frac{\theta\phi_t}{V_t(\tau)} L_s + 1\right)\exp(-\theta\widetilde{c}(L_s, s, \tau))\right],$$

$$\frac{\partial \int_0^{L_0^\tau} (q(w,s,\tau)/\theta)\, dw}{\partial f_s(\tau)} = \int_0^{L_0^\tau} -P_0(\tau)(1 - ew)\exp(-\theta\widetilde{c}(w,s,\tau))\, dw$$
$$= -P_0(\tau)\exp(-\theta\widetilde{c}(w,s,\tau)) \times \left[1 + \frac{e\phi_s V_t^2(\tau)}{\theta\phi_t^2}\left(\frac{\theta\phi_t}{V_t(\tau)} w + 1\right)\right]\Bigg|_0^{L_0^\tau}$$
$$= -P_0(\tau)\Bigg[\exp(-\theta\widetilde{c}(L_0^\tau, N(\tau), \tau)) \times \frac{\phi_t^2 + e\phi_t\phi_s V_t(\tau) + e\phi_s V_t^2(\tau)}{\phi_t^2} - \exp(-\theta\widetilde{c}(0,0,\tau)) \frac{\theta\phi_t^2 + e\phi_s V_t^2(\tau)}{\theta\phi_t^2}\Bigg]. \quad \text{(A.14)}$$

Thus,

$$f_s(\tau) = \frac{\sum_{\tau=1}^{M}\sum_{i=1}^{N(\tau)} Q_{i,\tau} - \sum_{\tau=1}^{M}\Delta_3}{\sum_{\tau=1}^{M}\sum_{i=1}^{N(\tau)}\left(\Delta_2^s - \Delta_1^s\right)}, \quad \text{(A.15)}$$

where Δ_1^s, Δ_2^s, and Δ_3 are the same as in (A.11). In view of the above system of equations, which consist of (A.7), (A.11), and (A.15), the optimal rail line length, rail station location (or spacing), headway, and fare can be calculated.

B. Proof of Proposition 2

In terms of Proposition 1, the rail length D_1^τ and rail station location (or spacing) D_s can be determined by

$$\frac{f_s(\tau) P_0(\tau)}{2}\left[\exp(-\theta\widetilde{c}(L_2, s, \tau)) - \exp(-\theta\widetilde{c}(L_1, s, \tau))\right] + f_s(\tau)\left(\frac{eP_0(\tau) V_t(\tau)\left(V_t(\tau) - \theta\phi_t\right)}{2\theta^2\phi_t^2} - \frac{eP_0(\tau) V_t(\tau)}{2\theta\phi_t}\right) \cdot \left(\frac{\theta\phi_t}{V_t(\tau)} L_1 + \frac{1}{2}\right)\exp(-\theta\widetilde{c}(L_1, s, \tau))$$

$$-\left(\frac{1}{(1+i)^{\tau-1}}\frac{2\mu_1}{h(\tau)V_t(\tau)}+(1+\chi)^{\tau-1}\gamma_1\right)=0, \tag{B.1}$$

$$\exp\left(-\theta\widetilde{c}\left(L_{s+1},s,\tau\right)\right)-\exp\left(-\theta\widetilde{c}\left(L_s,s,\tau\right)\right)=0, \quad \forall s=2,\ldots,N(\tau), \tag{B.2}$$

combined with (19).

The condition of traditional stochastic user equilibrium based on utility theory can be expressed as $\widetilde{c}(x,s,\tau) = \min\{\widetilde{c}(x,s,\tau)\}$, for locations with $q(x,s,\tau) > 0$. Submitting this condition into (19) and (B.1), we could have the optimal solution of rail line length for traditional stochastic user equilibrium based on utility theory, $(D_1^\tau)^*$, shown as follows:

$$\begin{aligned}
&(D_1^\tau)^* \\
&= -\frac{V_t(\tau)}{\theta\phi_t} \\
&\quad\times\ln\left(\left(\frac{1}{(1+i)^{\tau-1}}\frac{2\mu_1}{h(\tau)V_t(\tau)}+(1+\chi)^{\tau-1}\gamma_1\right)\right. \\
&\quad\times\left(\overline{f}\left(\frac{eP_0(\tau)V_t(\tau)(V_t(\tau)-\theta\phi_t)}{2\theta^2\phi_t^2}-\frac{eP_0(\tau)V_t(\tau)}{2\theta\phi_t}\right)\right. \\
&\quad\left.\left.\times\frac{\theta\phi_t}{V_t(\tau)}L_1+\frac{1}{2}\right)^{-1}\right) \\
&\quad-\frac{V_t(\tau)}{\phi_t}\left(f_s(\tau)+\phi_u u_s(L_1)+\phi_s\alpha h(\tau)+\phi_t\beta_0 N(\tau)+\xi\right), \\
&0<\left(\frac{1}{(1+i)^{\tau-1}}\frac{2\mu_1}{h(\tau)V_t(\tau)}+(1+\chi)^{\tau-1}\gamma_1\right) \\
&\quad\times\left(f_s(\tau)\left(\frac{eP_0(\tau)V_t(\tau)(V_t(\tau)-\theta\phi_t)}{2\theta^2\phi_t^2}-\frac{eP_0(\tau)V_t(\tau)}{2\theta\phi_t}\right)\right. \\
&\quad\left.\times\left(\frac{\theta\phi_t}{V_t(\tau)}L_1+\frac{1}{2}\right)\right)^{-1}<1.
\end{aligned} \tag{B.3}$$

In contrast with the optimal solution of rail line length for the proposed prospect theory based residential location choice equilibrium, $(D_1^\tau)^+$, we could have

$$(D_1^\tau)^* < (D_1^\tau)^+, \tag{B.4}$$

if and only if $(f_s(\tau)P_0(\tau)/2)[\exp(-\theta\widetilde{c}(L_2,s,\tau))-\exp(-\theta\widetilde{c}(L_1,s,\tau))] > 0$.

Under the proposed prospect theory based residential location choice equilibrium, travel cost will be higher for commuters living far away from the CBD; thus,

$$\widetilde{c}\left(L_2,s,\tau\right)-\widetilde{c}\left(L_1,s,\tau\right)<0 \tag{B.5}$$

exists. Since $\theta > 0$ and $f_s(\tau)P_0(\tau)/2 > 0$, we have

$$\frac{f_s(\tau)P_0(\tau)}{2}\left[\exp\left(-\theta\widetilde{c}\left(L_2,s,\tau\right)\right)-\exp\left(-\theta\widetilde{c}\left(L_1,s,\tau\right)\right)\right]>0. \tag{B.6}$$

Therefore, compared to the proposed prospect theory based residential location choice equilibrium, overestimation exists for the optimal solution of rail line length with the traditional stochastic user equilibrium based on utility theory.

Similarly, under the traditional stochastic user equilibrium based on utility theory, we have

$$\begin{aligned}
\Delta_1^s &= \frac{P_0(\tau)V_t(\tau)}{\phi_t} \\
&\quad\times\left[\exp\left(-\theta\widetilde{c}\left(L_{s-1},s,\tau\right)\right)-\exp\left(-\theta\widetilde{c}\left(L_s,s,\tau\right)\right)\right]=0, \\
\Delta_2^s &= \frac{eP_0(\tau)V_t^2(\tau)}{\theta\phi_t^2}\left[\left(\frac{\theta\phi_t}{V_t(\tau)}L_{s-1}+1\right)\exp\left(-\theta\widetilde{c}\left(L_{s-1},s,\tau\right)\right)\right. \\
&\quad\left.-\left(\frac{\theta\phi_t}{V_t(\tau)}L_s+1\right)\exp\left(-\theta\widetilde{c}\left(L_s,s,\tau\right)\right)\right] \\
&= \frac{eP_0(\tau)V_t^2(\tau)}{\theta\phi_t^2}\frac{\theta\phi_t}{V_t(\tau)}\left(L_{s-1}-L_s\right) \\
&\quad\times\exp\left(-\theta\min\left\{\widetilde{c}(x,s,\tau)\right\}\right).
\end{aligned} \tag{B.7}$$

In terms of Proposition 1, we have $(h(\tau))^* < (h(\tau))^+$ and $(f_s(\tau))^* < (f_s(\tau))^+$. In conclusion, overestimation exists for the optimal solutions of headway and fare, comparing the traditional stochastic user equilibrium based on utility theory with the proposed prospect theory based residential location choice equilibrium.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work described in this paper was jointly supported by a Grant from the Research Grant Council of the Hong Kong Special Administrative Region (Project no. PolyU 5215/09E) and a Postgraduate Studentship from the Research Committee of the Hong Kong Polytechnic University. The authors would like to thank the anonymous referees for their valuable comments.

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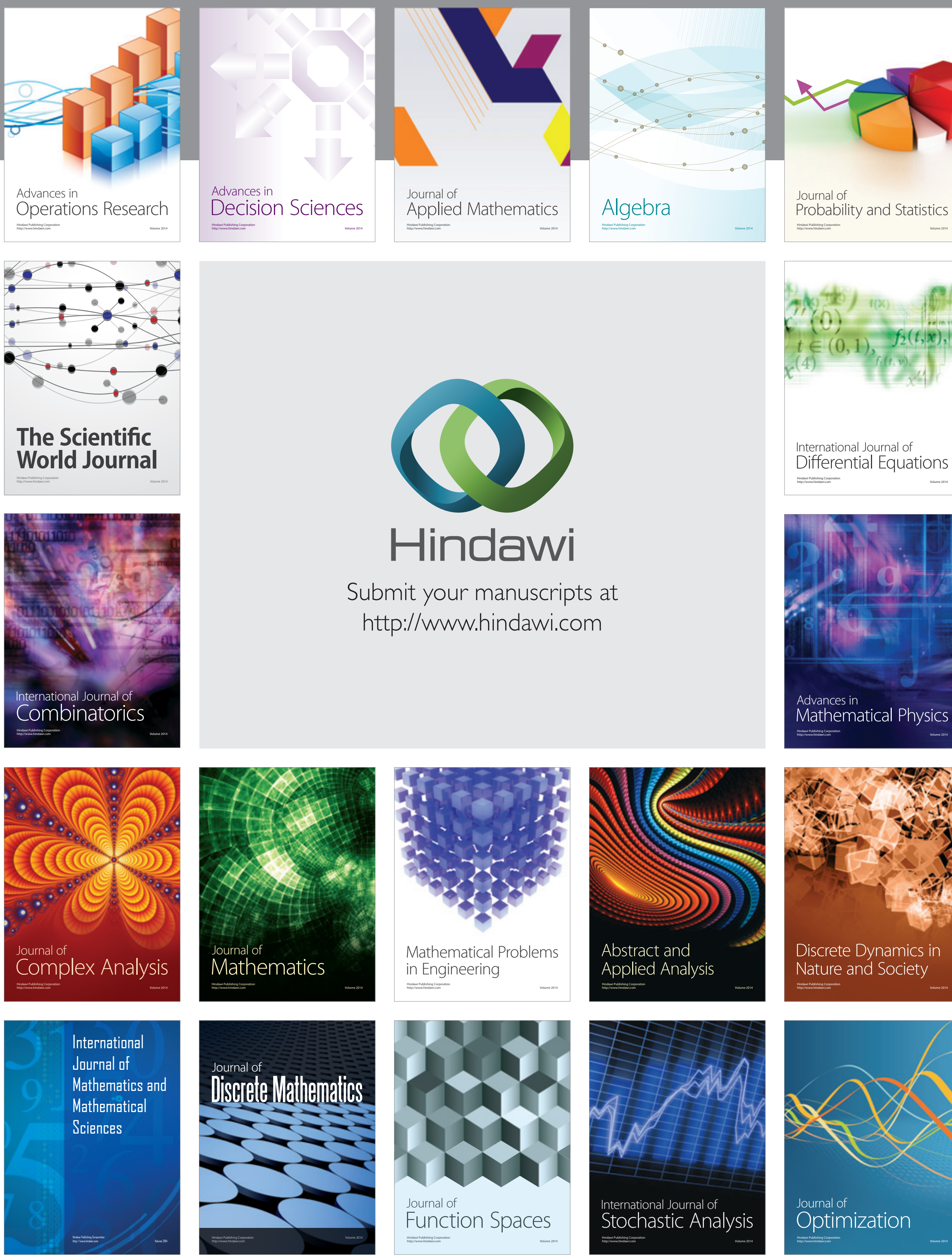
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