Lyapunov's Stability Theory-Based Model Reference Adaptive Control for Permanent Magnet Linear Motor Drives

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***Abstract*−The simplified mathematical model of the permanent magnet linear motor is developed in this paper. By applying Lyapunov's stability theory, the authors propose a model reference adaptive controller. Then the controller is evaluated by simulation. The results show that the MRAS is superior to the conventional PID controller in dynamic performance and steady precision. The MRAS is robust to the parameter variation and external disturbance.**

Index terms: *Permanent Magnet Linear Motors; Lyapunov's Stability Theory; Model Reference Adaptive Systems (MRAS)*

I. INTRODUCTION

With the development of digital circuits, computers, control theory and new material technology, linear motors are more and more employed in industry applications. Being compared with rotary motors plus transformation components to produce linear motions, linear motors have many advantages, such as quick response, high sensitivity, good tracking performance, etc, since they are usually connected in direct-drive systems. There is no backlash between the motor and the plant and therefore such non-linearity is eliminated. They can also simplify the devices, minimize the system inertia, lower the whole cost, and specially, they can be installed into a compact space. During the past decade, there are more and more investigations on linear motors applications.

In this paper, we will discuss a permanent magnet linear motor application in transportation system. Since the motor often carries from no load to full load, i.e., the mass of the moving part varies frequently, this will give rise to the variation of friction and other resistance forces. Thus a robust controller should be designed that is insensitive to the change of mass and resistance forces, and the system can run stably under any load situation. It is obvious that a conventional PID controller can not meet these specifications. Sometimes it even can not make the system stable. Some people may recommend us to use linear system theory. However, an accurate mathematical model is required to achieve high performance. In fact, the variation of mass and resistance forces may cause the model parameters changing in a big range. On the other hand, model reference adaptive control scheme can allow some uncertainties in a real system. The controller can force the system output follow up the reference model output, therefore MRAS has great practicability in control engineering applications.

This paper is organized as follows: First, the dynamic characteristic of the PM linear motor is analyzed. Then it is simplified and the state space model is developed according to the practical control scheme. In Section III, a model reference adaptive controller is designed by applying Lyapunov's stability theory. The validity of the proposed method is simulated in Section IV, where the dynamic performance of MRAS controller is compared with that of conventional PID regulator when the system is subject to external disturbance and parameter variation. Finally, some concluding remarks are drawn in Section V.

II. MATHEMATICAL MODEL OF THE SYSTEM

Some assumptions are made before we can develop the dynamic mathematical model of the PM linear motor.

1) The magnet saturation is ignored since it can be considered as parameter variation.

2) The induced EMF is sinusoidal. Actually the measured waveforms in experiment are almost sinewaves, so the motor is a PM synchronous motor.

3) Eddy current loss and magnetic hysteresis are regardless.

4) There is no excitation current dynamics.

5) There is no squirrel cage or short circuit ring in the mover.

In fact, the above assumptions are similar to real situation very much, so it is reasonable. According to the unification theory on motors, the dynamic characteristic of a PM linear motor can be described with Park's equations in d-q coordinate system, which moves synchronously with the mover, by applying some coordinates transformations.

$$v_q = Ri_q + p\lambda_q + \omega_s\lambda_d \qquad (1)$$

$$v_d = Ri_d + p\lambda_d - \omega_s\lambda_q \qquad (2)$$

$$\lambda_q = L_q i_q \qquad (3)$$

$$\lambda_d = L_d i_d + \lambda_{af} \qquad (4)$$

where

v_d, v_q stator voltages at direct-axis and quadrature-axis;

i_d, i_q stator currents at direct-axis and quadrature-axis;

R stator resistance;

p differential operator d/dt;

λ_d, λ_q stator linkages at direct-axis and quadrature-axis;

ω_s equivalent synchronous rotary angular velocity;

L_d, L_q stator inductances at direct-axis and quadrature-axis;

λ_{af} permanent magnet linkage;

where d-, q-variables are obtained from a-, b-, c-variables via the following Park's transformation:

$$\begin{bmatrix} v_q \\ v_d \\ v_0 \end{bmatrix} = 2/3 \begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin(\theta) & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad (5)$$

where θ is the equivalent electrical angle, whereas a-, b-, c-variables can be obtained from d-, q-variables via the following inverse Park's transformation:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) & 1 \\ \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) & 1 \end{bmatrix} \begin{bmatrix} v_q \\ v_d \\ v_0 \end{bmatrix} \qquad (6)$$

The above transformations are also applicable to currents and magnet linkages. Thus the following equation on the motor's total input power in dq and abc coordinate systems holds.

$$\text{power} = v_a i_a + v_b i_b + v_c i_c = 3(v_d i_d + v_q i_q)/2 \qquad (7)$$

And the electromagnetic thrust and mechanical equations are described in eqns.(8) and (9), respectively.

$$F_e = 3P(\pi/\tau)(\lambda_d i_q - \lambda_q i_d)/2 = 3P(\pi/\tau)[\lambda_{af} i_q + (L_d - L_q) i_d i_q]/2 \qquad (8)$$

$$F_e = F_L + B_v v + Mpv \qquad (9)$$

where

F_e electromagnetical thrust;

P number of pole pair(s);

τ pole pitch;

F_L load friction;

v actual linear velocity;

B_v damping coefficient associated with velocity;

M mass of the moving part;

The constant electromagnetic energy principle is employed when the electromagnetic thrust expression is deduced, and linear motion is regarded as equivalent rotary motion, where

ω_r is the equivalent mechanical angular velocity. Then these relationships hold: $\omega_r = v\pi/\tau$ and $\omega_s = P\omega_r$.

In order to make the control objective more explicit, we can adapt the above dynamic equations to state-space format by choosing i_d, i_q and v as state variables.

$$pi_d = (v_d - Ri_d + P\pi L_q i_q v/\tau)/L_d \tag{10}$$

$$pi_q = (v_q - Ri_q - P\pi L_d i_d v/\tau - P\pi\lambda_{af} v/\tau)/L_q \tag{11}$$

$$pv = \{3P(\pi/\tau)[\lambda_{af} i_q + (L_d - L_q) i_d i_q]/2 - F_L - B_v v\}/M \tag{12}$$

We note there are product items of velocity v and currents i_q, i_d in eqns.(10) and (11), so the plant model is nonlinear. To simplify analysis and obtain maximum thrust-current ratio, we propose to apply the $i_d^* = 0$ control scheme, i.e., the direct axis current is forced to be zero, $i_d \equiv i_d^* = 0$. Then the motor model can be simplified as:

$$pi_q = (- Ri_q - K_t v + v_q)/L_q \tag{13}$$

$$pv = (1.5K_t i_q - B_v v - F_L)/M \tag{14}$$

where $K_t = P\pi\lambda_{af}/\tau$. The above equations can be converted into state equations format.

$$\frac{dx}{dt} = Ax + Bu \tag{15}$$

$$y = Cx \tag{16}$$

where x is state variable, $x = (i_q \quad v)^T$, u, control input, $u = v_q$, and y, measurement output, $y = v$. F_L is regarded as disturbance.

$$A = \begin{bmatrix} -R/L_q & -K_t/L_q \\ 1.5K_t/M & -B_v/M \end{bmatrix},$$

$B = (1/L_q \quad 0)^T$,

$C = (1 \quad 0)$.

Fig.1 shows the block diagram of simplified linear motor model. From fig.1 we know this is a second order linear system with single input and single output.

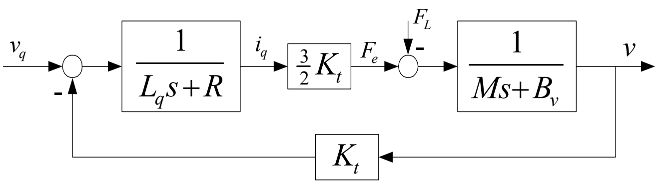

Fig. 1. Simplified block diagram of the PM Linear Motor

III. DESIGN OF ADAPTIVE CONTROLLER

In this section a model reference adaptive controller will be designed by using Lyapunov's stability theory, which can keep the motor dynamic performance consistent with the reference model and make the system insensitive to parameter variations and external disturbance, and the steady error goes to zero. The design steps are arranged as follows. First, a proper reference model is selected according to the performance index. Then the controller structure is determined and the error equation is deduced. Finally, a Lyapunov function is chosen and is used to develop parameter adaptation law, which can make the error approximate to zero.

Since the plant model has the format as eqn.(15), we assume the reference model as follows.

$$\frac{dx_m}{dt} = A_m x_m + B_m u_c \tag{17}$$

Then select a control law as eqn.(18).

$$u = K_1 u_c - K_2 x \tag{18}$$

Thus the model reference adaptive system is shown in fig.2. Now the state equation of the closed loop system has been changed to the following equation.

$$\frac{dx}{dt} = (A - BK_2)x + BK_1 u_c$$

$$= A_c(\kappa)x + B_c(\kappa)u_c \tag{19}$$

where the parameters in matrices K_1 and K_2 can be selected in any way, there can also exist some constraints between them. We suppose the closed loop system can be described with eqn.(19), where matrices A_c and B_c depend on the parameter κ, and κ is a certain combination of K_1 and K_2. If eqn.(19) is equivalent to eqn.(17) at any time, then the original system can follow the reference model completely. A sufficient condition is there exist a parameter κ^0 that makes eqn.(20) hold.

$$\begin{aligned} A_c(\kappa^0) &= A_m \\ B_c(\kappa^0) &= B_m \end{aligned} \tag{20}$$

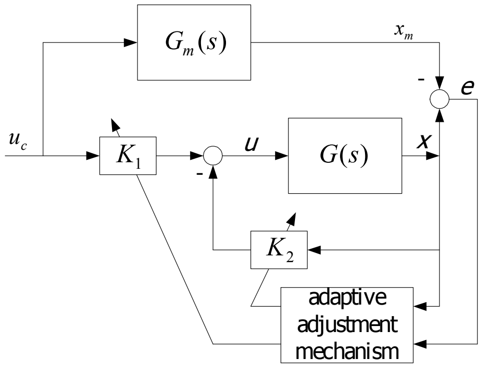

Fig. 2. Block diagram of the MRAS

Here we introduce error e, which is defined in eqn.(21).

$$e = x - x_m \tag{21}$$

By subtracting eqn.(17) from eqn.(15), we get

$$\frac{de}{dt} = \frac{dx}{dt} - \frac{dx_m}{dt} = Ax + Bu - A_m x_m - B_m u_c \tag{22}$$

Adding and subtracting a term $A_m x$ at right-hand of eqn.(22), we get

$$\frac{de}{dt} = A_m e + (A - A_m - BK_2)x + (BK_1 - B_m)u_c$$
$$= A_m e + (A_c(\kappa) - A_m)x + (B_c(\kappa) - B_m)u_c$$
$$= A_m e + \Psi(\kappa - \kappa^0) \tag{23}$$

The last equality of above equation is derived when extract model tracking condition is met. To deduce the parameter tuning law, we introduce a function $V(e, \kappa)$.

$$V(e,\kappa) = \frac{1}{2}(\gamma e^T Pe + (\kappa - \kappa^0)^T(\kappa - \kappa^0)) \tag{24}$$

where P is a positive definite matrix. $V(e, \kappa)$ is obviously a positive definite function. If its first order derivative to time is not positive definite, then V is a Lyapunov function. Now we solve the derivative of V to time t.

$$\frac{dV}{dt} = -\frac{\gamma}{2}e^T Qe + \gamma(\kappa - \kappa^0)\Psi^T Pe + (\kappa - \kappa^0)^T \frac{d\kappa}{dt}$$
$$= -\frac{\gamma}{2}e^T Qe + (\kappa - \kappa^0)^T(\frac{d\kappa}{dt} + \gamma\Psi^T Pe) \tag{25}$$

where Q is a positive definite matrix, which meets the following equation.

$$A_m^T P + PA_m = -Q \tag{26}$$

According to Lyapunov's stability theory, as long as A_m is stable, there always exist such positive definite matrices P and Q.

If we choose the parameter tuning law as follows,

$$\frac{d\kappa}{dt} = -\gamma\Psi^T Pe \tag{27}$$

then we get

$$\frac{dV}{dt} = -\frac{\gamma}{2}e^T Qe \tag{28}$$

i.e., the derivative of Lyapunov function V to time t is half negative definite. According to Lyapunov's stability theorem, now the output error between real system and reference model will approximate to zero, and the whole system will be asymptotically stable. Therefore, eqn.(27) is the Lyapunov's stability theory-based parameter tuning law for the model reference adaptive system.

IV. SIMULATION RESULTS

The parameters of the PM linear motor are listed in table 1. In this example, we select the following reference model after several trial-and-errors.

$$G_m(s) = \frac{100}{s^2 + 16s + 100}$$

It has fast response and little overshoot. The matrices K_1 and K_2 in the control law now degrade to two scalars. They are given by the following expressions to complete parameter update.

Table 1 Parameters of the PM linear motor

Phase resistance R (Ω)	8.6
Quadrature-axis synchronous inductance L_q (mH)	6.0
Permanent magnet linkage λ_{af} (V.s)	0.35
Pole pitch τ (m)	0.031
Number of pole pair(s) P	1
Mass of the moving part without load M (kg)	1.635
Viscous damping coefficient B_v (N.s/m)	0.1
Total friction coefficient μ	.

$$\frac{dK_1}{dt} = -10000u_c e$$

$$\frac{dK_2}{dt} = 10000ye$$

This paper has compared the performance of adaptive controller with that of conventional PID regulator. The three parameters of PID regulator are chosen as $K_p = 2$, $K_i = 220$, $K_d = 2.5$. The simulation results are shown in figs.3 to 8, respectively.

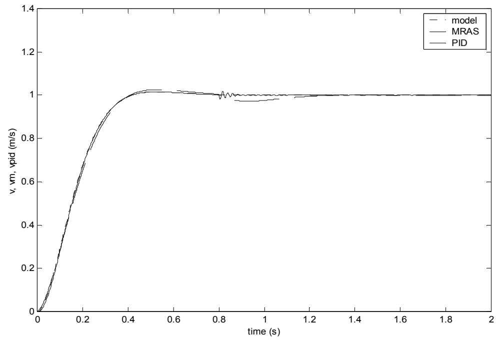

Fig.3. Step response waveforms of the model, MRAS and PID tuning system

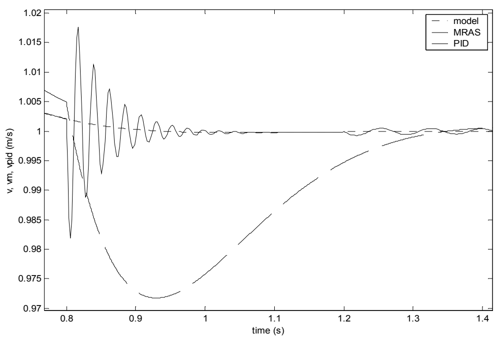

Fig.4. Zoom of the response waveforms when disturbance and parameter variation occur

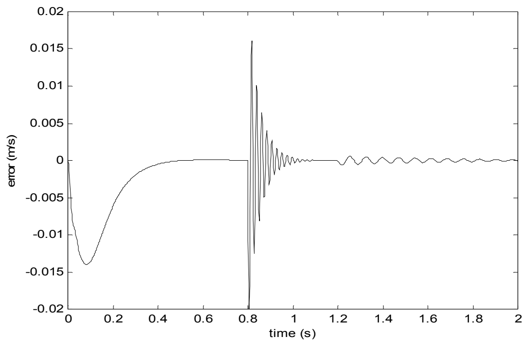

Fig.5. The waveform of output error between the MRAS and the model

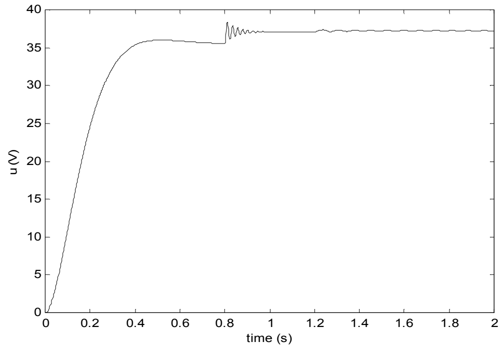

Fig.6. The control input of the MRAS

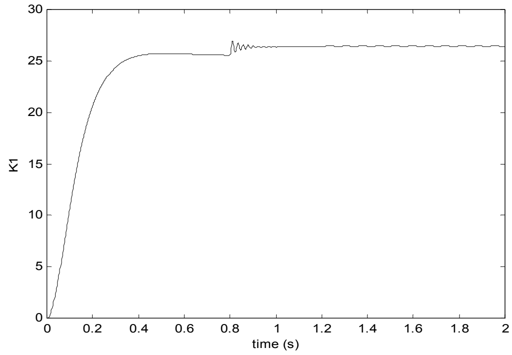

Fig.7. Tuning of the parameter K_1 of the adaptive control law

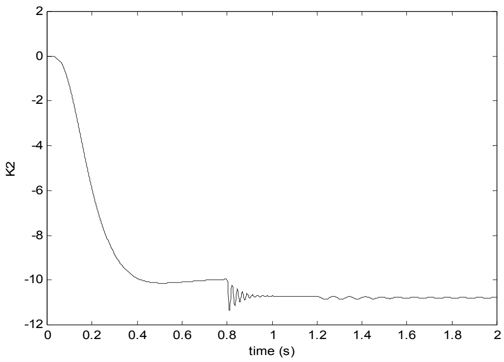

Fig.8. Tuning of the parameter K_2 of the adaptive control law

In simulation, the system ability of rejecting external disturbance and parameter variation is studied. First, when the system is under zero initial condition, the motor can follow the reference model perfectly starting from rest to steady state. Then, when the time is 0.8s, the load F_L suddenly changes from 0 to 10N. At this time the model reference adaptive system suffers slight oscillation, but it can be stable very soon. Finally, at the time of 1.2s, the mass of moving part increases to 10 times of its original mass, but the effect of this parameter variation on the system is so subtle that it can be ignored. However, if we use a PID regulator as a controller for this controlled plant, its disturbance rejecting ability is apparently worse, and the settle time becomes very long, it can not meet the request of fast following at all. Fig.3 shows the step response output waveforms of reference model, MRAS and PID regulator system. In this figure, the dynamic performance of MRAS is much better than that of PID regulator system owing to its shorter settle time and less overshoot. Fig.4 is the zoom of response curve when the load disturbance is exerted and then the parameter variation occurs. The MRAS has little deflection from the steady state with small magnitude of oscillation, but it is stabilized very soon. As a comparative controller, the PID regulator system has great deviate from steady state and its settle time is very long. The output error between MRAS and reference model during the whole dynamic process is illustrated in fig.5. From this figure we can see the error occurs mainly at the startup stage and the settling stage while external disturbance is exerted. The error goes to zero when the system is at steady state. Fig.6 presents the input signal of MRAS, i.e., the motor quadrature-axis voltage. Figs.7 and 9 give the updating process of parameters K_1 and K_2 of the adaptive control law, respectively. The control law can automatically adjust itself when external disturbance is exerted to the system and parameter variation occurs.

V. Concluding remarks

In this paper we have developed the simplified mathematical model of the PM linear motor. Then a model reference adaptive controller has been designed based on the Lyapunov's stability theory, and a comparative study on conventional PID regulator has been completed. Simulation results show that the Lyapunov's stability theory-based model reference adaptive system is robust and stable, which has better dynamic performance and stronger disturbance rejecting ability than PID regulator system. The adaptive control law is independent of plant parameters and easy to implement. Therefore the proposed method is effective.

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Biographies

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