

# Sensorless Control Scheme for Continuously Estimating Rotor Position and Speed of Switched Reluctance Motor Drives Based on Two-Dimensional Least Squares

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*Abstract:* - This study presents a control scheme to continuously estimate the rotor position and speed of sensorless switched reluctance motor (SRM) drives. The estimation model is based on the two-dimensional (2D) least squares technique. For the proposed estimation scheme, only the rotor position to the optimal sensing phase being the active phase via measuring the current and the voltage is estimated. Furthermore, the position correction algorithm is proposed, which can be used to eliminate the fatal effect of the unexpected interface errors in the voltage or current measurement on the rotor position estimation. Because the presented scheme includes the estimation model based on the 2D least squares optimization technique, optimal sensing phase approach, and position correction algorithm, the high resolution of the rotor position estimation can be obtained. The simulation results under the current hysteresis and voltage single-pulse operations validated the proposed sensorless control scheme.

## 1 Introduction

As one of variable speed drives, SRM drives with commercial competition are found in many industrial applications, such as plotter drives, air-handler motor drives, washers, dryers, train air-conditioning drives, motor drives in electric vehicle, mining drives, fan, pump, screw rotary compressor drives, centrifuges, and aerospace applications. Controls of SRM drives need the information of the rotor position, to properly synchronize the phase excitations with the rotor position. Consequently shaft position sensors are usually employed to determine the rotor positions. However, these discrete position sensors not only add complexity and cost but also tend to reduce reliability of SRM drives. Hence, sensorless SRM drives become a challenging study area.

Some methods have been introduced to accomplish the rotor position estimation of sensorless SRM drives during the last fifteen years. According to the probing approaches, these sensorless position estimation methods are mainly classified into the three categories, as shown in Fig. 1

(a) The passive probing technique. It examines and analyzes the voltage and the current in the active phases of SRM drives and both signals are utilized to estimate rotor position. This technique is effective generally at standstill, low speed, and high speed. It includes the traditional methods being composed of the flux linkage-based approach [1] and the inductance-based approach [2], the artificial intelligent methods consisting of the artificial neural networks method [3], the fuzzy logic method [4], and the control theory methods, which are the sliding mode observer method [5], the binary observer method [6], the adaptive observer method [7], as well as the Luenberger non-linear observer method [8].

(b) The active probing technique. It injects the special signals (the voltage or the current) into the idle phases of SRM drives, the resulting signals (the current or the voltage) are then used to estimate the rotor position. In general, this technique is based on the inductance measurement and only applicable for standstill and very low speed. Some methods are developed, such as the amplitude modulation (AM) method [9], the power modulation (PM) method [10], the frequency modulation (FM) method [11], the square exciting method [12], and the resonant method [13].

(c) The hybrid probing technique. It acquires and analyzes the voltage and the current in the both active and idle phases, such as the recursive least squares method [14] and the mutual voltage method [15].

The authors developed a new sensorless control scheme to estimate the rotor position at standstill in [16]. This study is the further work of [16]. This study attempts to propose the new scheme to continuously and accurately estimate rotor position at running. The passive probing technique is used in this scheme. The works in this study are summarized as follows. 1) The new and accurate estimation model is presented, which is based on the 2D least squares optimization [17]. The bisection algorithm is employed to solve the presented estimation model fleetly. 2) The optimal sensing phase technique is directly introduced into the estimation algorithm, which can be helpful to obtaining the high resolution of the rotor position estimation. 3) The rotor position to the optimal sensing phase is only estimated. 4) The position correction algorithm is proposed based on the motion equation, to eliminate the fatal effect of the unexpected measurement interface on the rotor position estimation. 5) The simulations under the current hysteresis and voltage single-pulse controls validate the proposed estimation scheme.

## 2 Estimation Model

### 2.1 Analytically modeling flux linkage

In general, the rotor position estimation needs to know the flux linkage or inductance characteristics to the current and position. Hence, how to accurately describe these characteristics has a crucial effect on the accuracy of the rotor position estimation. For SRM drives, the flux linkage or the inductance are the nonlinear function of the current and the rotor position. It is clear that the accuracy of the resolution of the rotor position depends on the accuracy of the described functions if the current and the flux linkage/inductance are measured accurately.

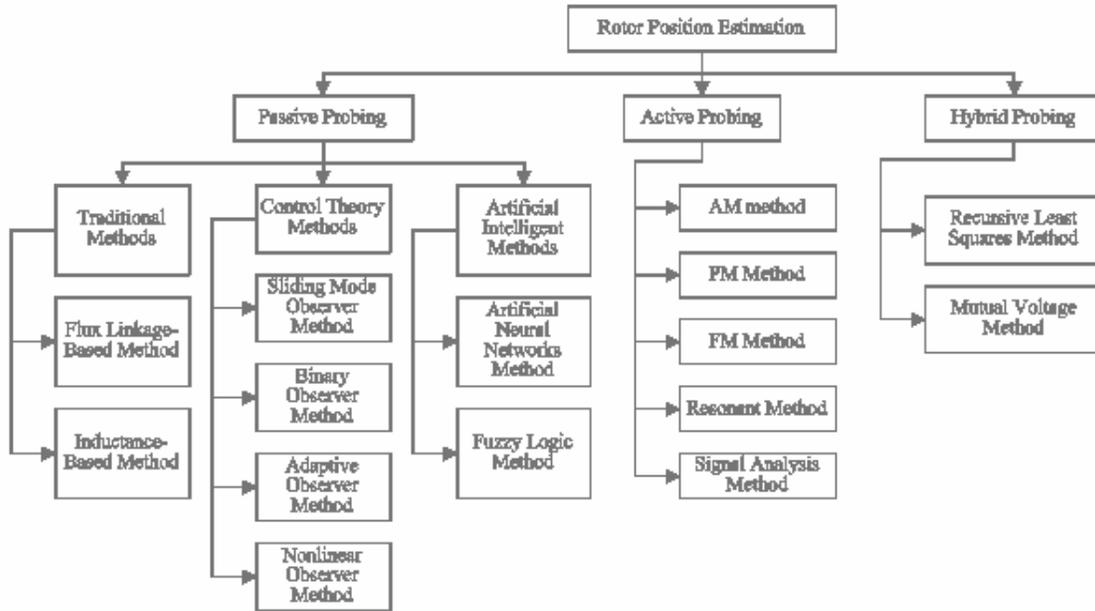


Fig. 1 Classification of rotor position estimation methods

Assuming that the  $N \times M$  flux linkage data  $\mathbf{y}_{kj}$  with respect to the rotor position  $\mathbf{q}_k$  and the current  $i_j$  are obtained through the measurements on an existing motor or through the numerical computations ( $k = 0, 1, \dots, N-1; j = 0, 1, \dots, M-1$ ), the analytical model proposed in this study is

$$\mathbf{y}(\mathbf{q}, i) = \sum_{k=0}^p \sum_{j=0}^q a_{kj} (\mathbf{q} - \bar{\mathbf{q}})^k (i - \bar{i})^j \quad (1)$$

$$\bar{\mathbf{q}} = \sum_{k=0}^{N-1} \mathbf{q}_k / N \quad (2)$$

$$\bar{i} = \sum_{j=0}^{M-1} i_j / M$$

where  $p$  and  $q$  are the integers and satisfy respectively  $N \geq p$  and  $M \geq q$ .

The coefficients  $a_{kj}$  in (1) are determined by using 2D least squares optimization. The detailed description is seen in [17].

## 2.2 Estimation model

The rotor position can be computed from (1), if the information of both the flux linkage and the current is obtained. In general, the phase voltage and the phase current of SRM drives can be acquired directly on line. Hence, the flux linkage has to be computed according to the measured voltage and current. Here, the flux linkage is computed by using the trapezoid integration method, which is described by (3).

$$\mathbf{y}(l+1) = \mathbf{y}(l) + \frac{1}{2} T_s [V(l+1) + V(l) - ri(l+1) - ri(l)] \quad (3)$$

$$\mathbf{y}(0) = 0$$

where  $\mathbf{y}(l+1)$  and  $\mathbf{y}(l)$  are the flux linkage values at sampling instants  $(l+1)$  and  $(l)$ ,  $V(l+1)$  and  $V(l)$  are the voltage values applied to the phase winding at sampling instants  $(l+1)$  and  $(l)$ ,  $i(l+1)$  and  $i(l)$  are the phase current values at sampling instants  $(l+1)$  and  $(l)$ ,  $r$  is the resistance of the phase winding,  $T_s$  is the sampling time, and  $l = 0, 1, 2, \dots$ .

From (1) and (3), the proposed estimation model is given by (4).

$$f(\mathbf{q}) = \sum_{k=0}^{p-1} \sum_{j=0}^{q-1} a_{kj} (\mathbf{q} - \bar{\mathbf{q}})^k (i - \bar{i})^j - \mathbf{y} = 0 \quad (4)$$

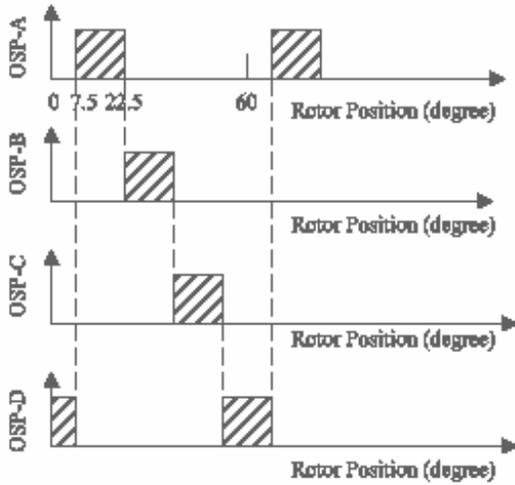
It can be observed that it is difficult to directly solve the rotor position from (4) if the flux linkage and the current are known. In this study, the bisection algorithm is utilized to solve the rotor position from (4).

## 3 Estimation Scheme

### 3.1 Optimal sensing phase at running

In this study, the four-phase SRM drive is selected as a case study. For a four-phase SRM drive, the phase is named as the optimal sensing phase if the rotor position to a phase is located within the range from 7.5 degree to 22.5 degree. Furthermore, the rotor position estimation to the optimal sensing phase has high resolution [16]. Supposing that a four-phase SRM drive runs under the motoring operation and the sequence of the conducting phase within a rotor period (60 degree for the four-phase SRM drive) is phase-A  $\rightarrow$  phase-B  $\rightarrow$  phase-C  $\rightarrow$  phase-D for a specified rotation direction, the sequence of the optimal sensing phase within a period is the optimal sensing phase-A  $\rightarrow$  the optimal sensing phase-B  $\rightarrow$  the optimal sensing phase-C  $\rightarrow$  the optimal sensing phase-D.

The schematic diagram of the optimal sensing phase distribution is shown in Fig. 2. It is evident that the domain of the optimal sensing phase is 15 degree for a four-phase SRM drive. There is an optimal sensing phase at any time because the rotor period is 60 degree for a four-phase SRM drive. Therefore each phase will have an optimized angle of one quarter of it.



Within a period, the relationship between the estimated rotor positions to all phases is shown in Table 1 for the four-phase SRM drive. It can be observed that the turn-on angles and the turn-off angles to all phases can be determined from the estimated positions to the optimal sensing phase.

Table 1 Relationship of the estimated rotor positions between the optimal sensing and other phases

Estimated position	Optimal sensing phase	
	Phase-A	Phase-B
$\theta_{eA}$	$\theta_{eA}$	$(\theta_{eB}+15)$
$\theta_{eB}$	$(\theta_{eA}-15)/(\theta_{eA}+45)$	$\theta_{eB}$
$\theta_{eC}$	$(\theta_{eA}+30)$	$(\theta_{eB}-15)/(\theta_{eB}+45)$
$\theta_{eD}$	$(\theta_{eA}+15)$	$(\theta_{eB}+30)$
Estimated position	Optima sensing phase	
	Phase-C	Phase-D
$\theta_{eA}$	$(\theta_{eC}+30)$	$(\theta_{eD}+45)/(\theta_{eD}-15)$
$\theta_{eB}$	$(\theta_{eC}+15)$	$(\theta_{eD}+30)$
$\theta_{eC}$	$\theta_{eC}$	$(\theta_{eD}+15)$
$\theta_{eD}$	$(\theta_{eC}-15)/(\theta_{eC}+45)$	$\theta_{eD}$

### 3.2 Position correction algorithm

The accurate estimation of the rotor position is obtained from (4) by using the optimal sensing phase approach if the phase voltage and the current are measured accurately. However, if there would be sufficient large measurement errors in the voltage or the current, the yielded rotor position estimation deviates seriously from the actual rotor position, and even Equation (4) has no solution at all and consequently the rotor position estimation would fail. To avoid the emergence of the above case and to enhance the accuracy of the rotor position estimation, it is expected that the estimated rotor position can be corrected by using the adequate approach at real time. In this study, the approach named as the position correction algorithm is used, which is based on the motion equation of SRM drives. The position correction computation satisfies (5).

$$\mathbf{q}_e = \mathbf{q}'_e + \mathbf{w}_{er}T_s \quad (5)$$

where  $\mathbf{q}_e$  denotes the present position estimation output,  $\mathbf{q}'_e$  denotes the last position estimation output, and  $\mathbf{w}_{er}$  denotes the estimated rotor speed that will be discussed in the following section.

From (5), the developed position correction algorithm is depicted by Fig. 3.

In Fig. 3, the increment of the estimated rotor position represents the difference between the present position estimation and the last position estimation,  $ERR\_1$  denotes the upper limitation of the increment of the rotor position estimation, and  $ERR\_2$  denotes the lower limitation.  $ERR\_1$  and  $ERR\_2$  are not the constant values. These two limitations rely on both the estimated rotor speed and the sampling time if it is supposed that the angular acceleration during the sampling time is a constant.  $ERR\_1$  and  $ERR\_2$  satisfy (6) and (7), respectively.

$$ERR\_1 = k_1 \mathbf{w}_{er}T_s \quad (6)$$

$$ERR\_2 = k_2 \mathbf{w}_{er}T_s \quad (7)$$

where  $k_1$  and  $k_2$  are the constant values. In this study,  $k_1$  is selected as 1.9 and  $k_2$  is selected as 0.1.

### 3.3 Motor speed estimation

In this study, the motor speed estimation is performed every a 15 degree. To be specific, the speed estimation is just performed every when the turn-on angle to a phase is outputted. Consequently, the speed estimation is computed from (8).

$$\mathbf{w}_{er} = 15 / Dt \quad (8)$$

where  $Dt$  represents the time over a 15 degree.

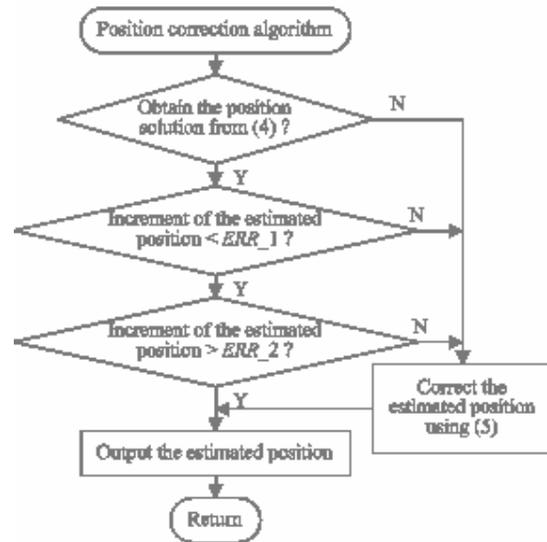


Fig. 3 Flowchart of the position correction algorithm

### 3.4 Estimation scheme

From the aforementioned analyses, the estimation scheme can be developed, which is illustrated by Fig. 4. Referring to Fig. 4, the developed estimation scheme is described as follows. Firstly, the optimal sensing phase is detected from the last estimated position. Next, the voltage and current for the optimal sensing phase are acquired. Then, integrating the acquired voltage and current with respect to the time, the flux linkages  $\mathbf{y}$  of the optimal sensing phase can be computed from (3). Hence, the rotor position  $\theta_o$  to the optimal sensing phase can be estimated from (4) by using the bisection solution algorithm. After that, the estimated position is corrected from the last estimated speed and the position correction algorithm shown in Fig. 3. Finally, the outputted position estimation is obtained and the motor speed is estimated.

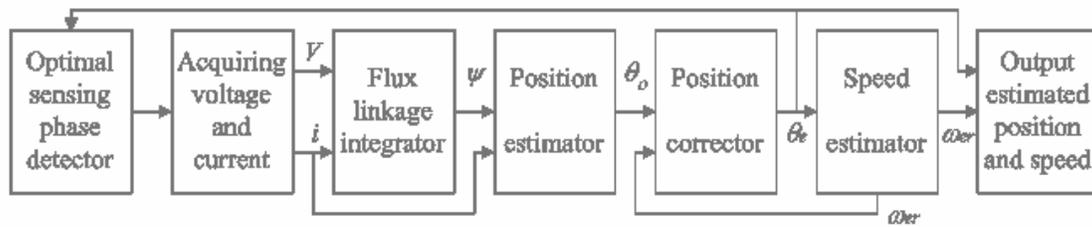


Fig. 4 Block diagram of the proposed scheme to estimate the rotor position and speed

## 4 Validation

To validate the proposed scheme to continuously estimate the rotor position and speed at running, a prototype of the four-phase, 8/6 poles SRM drive is simulated. The phase resistance is  $0.687 \Omega$ . The sampling frequency is 50 kHz. The simulations for the sensorless SRM drive are performed under the two operations, which are the low speed one under the current hysteresis control and the high speed one under the voltage single-pulse control.

### 4.1 Current hysteresis operation

SRM drives run generally under the current hysteresis control (constant torque operation) at low speed. Fig. 5 shows the simulation results when the sensorless SRM drive is running at 300 rpm. The maximum absolute position estimation error is equal to 0.684 degree and the maximum relative position estimation error is 1.16%. The maximum speed estimation error is 10.211 rpm, the maximum relative speed estimation error is 3.4%, and the estimated average speed is 300.243 rpm. It can be seen that the estimated results agree well with the actual results. Therefore, these simulation results under the current hysteresis control validate that the proposed scheme is effective and the rotor position and speed estimations are accurate.

### 4.2 Voltage single-pulse operation

The high speed operation (constant power operation) is implemented under the voltage single-pulse control. Fig. 6 depicts the simulation results under the voltage single-pulse operation at 1200 rpm. The maximum absolute position estimation error is 0.694 degree and the maximum relative position estimation error is 1.34%. The maximum absolute speed estimation error is  $-6.555$  rpm, the maximum relative speed estimation error is  $-0.55\%$ , and the estimated average speed is 1200.28 rpm. Hence, the simulation results under the single-pulse operation also demonstrate that the estimated positions and speeds using the proposed scheme agree well with the actual ones, respectively.

## 5 Conclusion

This study presents a sensorless control scheme to continuously estimate the rotor position and speed for SRM drives. The developed scheme including the estimation model based on the 2D least squares and bisection algorithm, optimal sensing phase approach, and position correction algorithm, can provide the high resolution of the rotor position and speed estimations.

The proposed estimation scheme only needs to acquire the voltage and the current for the optimal sensing phase being the active phase. The developed position correction algorithm can avoid the fatal effect of the unexpected interface from the voltage and current measurement on the rotor position estimation. The simulation results under the

current hysteresis and voltage single-pulse controls demonstrated that the estimated rotor position and speed are agreement with the actual ones, respectively. Thus, the proposed sensorless control scheme is effective.

## Acknowledgement

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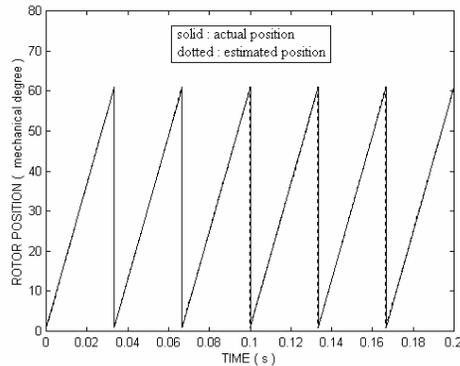
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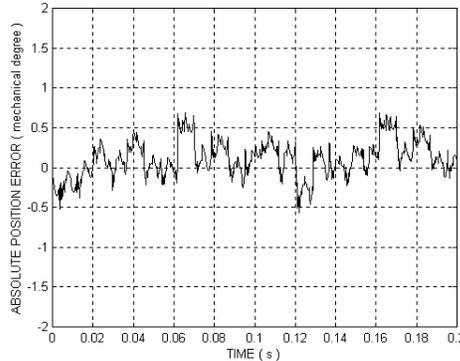
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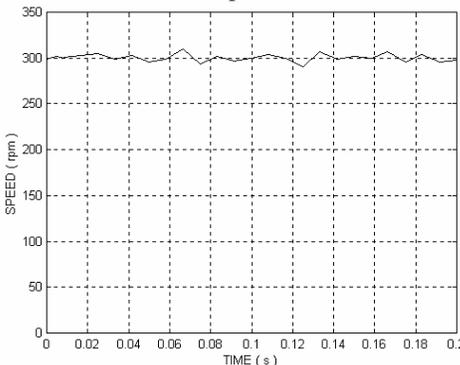
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(a) Comparison between the estimated and actual rotor positions

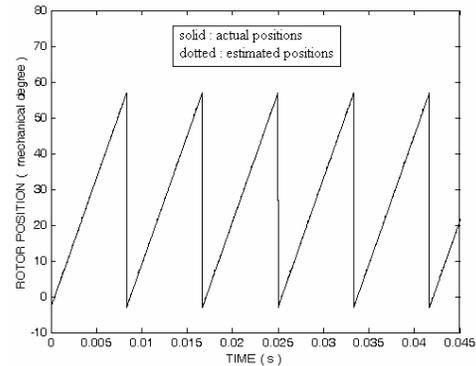


(b) Distribution of the absolute errors between the estimated and actual rotor positions

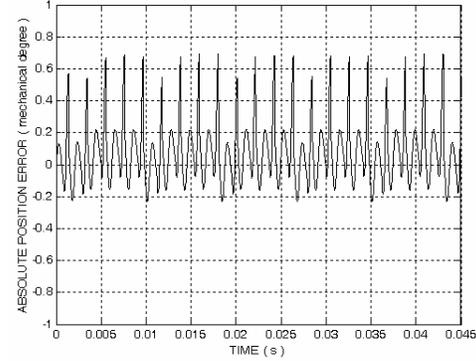


(c) Estimated speed

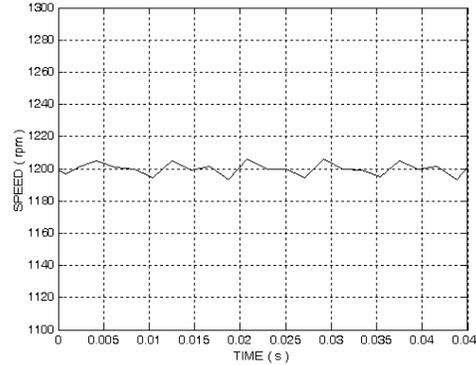
Fig. 5 Simulation results under the current hysteresis operation (expected speed = 300 rpm, turn-on angle =  $1^\circ$ , turn-off angle =  $23^\circ$ )



(a) Comparison between the estimated and actual rotor positions



(b) Distribution of the absolute errors between the estimated and actual rotor positions



(c) Estimated speed

Fig. 6 Simulation results under the voltage single-pulse operation (expected speed = 1200 rpm, turn-on angle =  $-3^\circ$  degree, and turn-off angle =  $18^\circ$ )