Position Control of Induction Motor using Indirect Adaptive Fuzzy Sliding Mode Control

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Abstract—This paper presents the design of an indirect adaptive fuzzy sliding mode controller for a vector controlled induction motor position servo drive. The proposed adaptive controller takes advantage of sliding mode control (SMC) and proportional integral (PI) control. The chattering effect is attenuated and robust performance can be ensured. Moreover, the upper bound of the discontinuous control term is assumed to be unknown and adaptive algorithm is applied to estimate it. The stability analysis for the adaptive control scheme is provided. Simulation example is presented to verify the effectiveness of the proposed method.

Keywords—Sliding mode control, fuzzy logic control, induction motor.

I. INTRODUCTION

Servomotors are widely used in many industrial systems including drives for robot manipulators, electric cars, machine tool drives, etc. In such applications, the drive robustness for system parameter variation and load disturbance is very important. It is well known that the file-oriented methods [1-3] have been used in the design of induction motor (IM) drives for high performance applications. By using these control algorithms, the dynamic behavior of the induction motor is similar to a separately excited DC motor.

Sliding mode control (SMC) are widely used in the field of induction motor drive [4-6]. However, there exists a discontinuous control action, chattering phenomena will take place when implementing a sliding mode control. One common way to eliminate this drawback is to introduce a boundary layer neighboring the sliding surface [6], [7]. This method can lead to stable closed loop system with avoiding the chattering problem, but there exists a finite steady state error due to finite gain in steady state without switching control action.

The adaptive fuzzy controller incorporated with a sliding mode control [8]-[10] design to acquire stability and consistent performance is energetic in fuzzy control research. In order to improve the steady state performance, an indirect adaptive fuzzy logic controller that merges a proportional plus integral control and SMC is proposed. The control scheme can give good transient and robust performance and the bound of the external disturbances without needing to know the magnitude. Instead, an adaptive mechanism is developed such that the bound can be estimated. Moreover, the chattering phenomenon is alleviated with PI control in the design of sliding mode controller. It is proved that the closed-loop system is globally stable in the sense that all signals are bounded and the system output can track the desired input asymptotically with uncertainties and disturbances.

In this paper, an indirect fuzzy sliding mode control has been applied to a position servo that uses vector controlled induction motor. The basic concept of sliding mode control is presented in Section 2. Mathematical description of the induction motor is in Section 3. A brief description of fuzzy logic system is in Section 4. In section 5, the adaptive fuzzy sliding control is proposed. Simulation example for the proposed control concept is shown in Section 6. Finally, the paper is concluded in Section 7.

II. SLIDING MODE CONTROL BASIC CONCEPT

Consider a general class of single-input single-output nonlinear systems as follow form [7]:

\[ x'(t) = f(x) + g(x)u + d(t) \]

\[ y = x \]

where \( f \) and \( g \) are unknown nonlinear functions, \( x = [x_1, x_2, \ldots, x_n]^T \) \( y = [y_1, y_2, \ldots, y_m]^T \) \( u = [u_1, u_2, \ldots, u_m]^T \) \( d(t) \) is the unknown external disturbance. It is required the system \( (1) \) to be controllable, the input gain \( g(x) > 0 \) is necessary. The control problem is to obtain the state \( x \) for tracking a desired trajectory \( x_d \) in the presence of model uncertainties and external disturbance.

Define the tracking error

\[ e = x - x_d = [e_1, e_2, \ldots, e_n]^T \] (2)

A sliding surface in the space of the error state as

\[ s(e) = k_1 e_1 + k_2 e_2 + \cdots + k_n e_n^\alpha + e_\alpha = k^\alpha e \] (3)

where \( k = [k_1, k_2, \ldots, k_n]^T \) are the coefficients of the Hurwitzian polynomial \( h(\lambda) = \lambda^n + k_{n-1}\lambda^{n-1} + \cdots + k_1 \) such that all the roots are in the open left half-plane and \( \lambda \) is a Laplace operator. If the initial condition \( e(0) = 0 \), the tracking problem \( x = x_d \) can be considered as the state error vector remaining on the sliding surface \( s(e) = 0 \) for all \( t \geq 0 \). A sufficient condition to achieve this performance is to choose the control approach such that

\[ \frac{1}{2} \frac{d}{dt} (s^2(e)) \leq -\eta_s |s| \] (4)

where \( \eta_s \) is a small positive constant. The system \( (1) \) is controlled in such a way that the state always moves towards the sliding surface \( (3) \) and hits it. The sign of the
control value must change at the intersection between the state trajectory and sliding surface.

Consider the control problem of nonlinear systems (1), if the nonlinear functions $f(x)$ and $g(x)$ are known. The SMC signal $u'$ guarantees the sliding condition of (4).

$$u' = \frac{1}{g(x)} \left[ \sum_{i=1}^{n} k_e e^{(i)} - f(x) + x_e^{(e)} - \eta_{\lambda} \operatorname{sgn}(s) \right]$$  \hspace{1cm} (5)

where

$$\operatorname{sgn}(s) = \begin{cases} 
1 & \text{for } s > 0 \\
0 & \text{for } s = 0 \\
-1 & \text{for } s < 0
\end{cases}$$  \hspace{1cm} (6)

Define the Lyapunov function $V_1$ as

$$V_1 = \frac{1}{2} s^2$$  \hspace{1cm} (7)

Differentiating (7) with respect to time, $V_1$ along the system trajectory as

$$\dot{V}_1 = s \cdot \dot{s} = s \cdot (k_e \dot{e} + k_2 \ddot{e} + \cdots + k_n \epsilon^{(n-1)} + x_e^{(e)} - x_e^{(e)})$$

$$= s \cdot \left( \sum_{i=1}^{n} k_e e^{(i)} + f(x) + g(x)u' + d(t) - x_e^{(e)} \right)$$  \hspace{1cm} (8)

$$\leq -\eta_{\lambda} |s|$$

Hence the SMC input $u'$ can be guaranteed the sliding condition of (4). However, the nonlinear functions $f$ and $g$ are unknown, it is difficult to apply the control law (5) for an unknown nonlinear system. Moreover, the switching-type control term will cause chattering problem. To solve these problems, the adaptive scheme using the fuzzy system and the PI control law is proposed in section V.

III. MATHEMATICAL DESCRIPTION OF THE INDUCTION MOTOR

The induction motor implemented in this paper is a three phase star-connected four-pole 600 $W$, 60Hz, 120V/olt/5Amp, type. The mechanical equation of induction servomotor drive can be written as [2].

$$J \ddot{\theta} + B \dot{\theta} + T_L = T_E$$  \hspace{1cm} (9)

where $J$ is the moment of inertia, $B$ is the damping coefficient, $T_E$ represents the electric torque and $T_L$ denotes the external load disturbance. By using the implementation of field-oriented control [2], the electric torque can be written as

$$T_E = K_r \dot{i}_e$$  \hspace{1cm} (10)

$$K_r = \frac{3N_p I_p^2}{2L_p}$$  \hspace{1cm} (11)

where $K_r$ is the electric torque constant, $i_e$ is the torque current command, $i_b$ is the flux current command, $N_p$ is the number of pole pairs, $L_p$ is the magnetizing inductance per phase and $I_p$ is the rotor inductance per phase. Then the description of the dynamic structure of the control induction motor can be represented in the following form.

$$\dot{\theta} = \frac{1}{J}[-B \dot{\theta} + K_r \dot{i}_e - T_L]$$  \hspace{1cm} (12)

Define $x_1 = \theta$ be the rotor angle of the induction motor and $x_2 = \dot{\theta}$ be the motor angular velocity. The dynamic equation of system (12) can be written as the form of (1) as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 r + c \end{bmatrix} d$$  \hspace{1cm} (13)

where

$$a = -B / J + b = K_r / J, \quad c = -1 / J, \quad d = T_L \quad \text{and} \quad u = \dot{i}_e$$ is the control command. The control objective is to design a control law so that the rotor position tracks the desired trajectory. Assume that the parameters of the induction motor system are unknown.

IV. FUZZY LOGIC SYSTEM

In this section, a brief description of fuzzy logic system is considered. The basic configuration of the fuzzy logic system [11] includes a fuzzy base, consists of a collection of fuzzy IF-THEN rules:

$$R^{i} : \text{IF } x_i = F_i^l \text{ and } \cdots \text{ and } x_n = F_n^l \text{ THEN } y = B^l$$  \hspace{1cm} (14)

Then $y$ is $B^l$.

The fuzzy logic system performs a mapping from $U = U_1 \times \cdots \times U_n \subseteq R^n$ to $R$, where the input vector $x = [x_1, \cdots, x_n]^T \in R^n$ and the output variable $y \in R$ denote the linguistic variables associated with the inputs and output of the fuzzy logic system. $F_i^l$ and $B^l$ are labels of the input and output fuzzy sets respectively. Let $l = 1, 2, \cdots, n$ denotes the number of input for fuzzy logic system and $l = 1, 2, \cdots, m$ denotes the number of the fuzzy IF-THEN rules. By using the singleton fuzzification, product inference and center average defuzzification, the mathematical equation of the fuzzy system can be written as:

$$y(x) = \frac{\sum_{i=1}^{m} y^i (\prod_{l=1}^{n} \mu_{F_i^l}(x_l))}{\sum_{i=1}^{m} \prod_{l=1}^{n} \mu_{F_i^l}(x_l)}$$  \hspace{1cm} (15)

where $\mu_{F_i^l}(x_l)$ is the membership function of the linguistic variable $x_l$, and $y^i$ represents a crisp value. By introducing the concept of fuzzy basis function (FBF)[11], (15) can be rewritten as

$$y(x) = \theta^T \zeta(x)$$  \hspace{1cm} (16)

where $\theta = [y_1, \cdots, y_m]^T$ is a parameter vector and $\zeta(x) = [\zeta_1(x), \cdots, \zeta_m(x)]^T$ is a regressive vector with the regressor defined as

$$\zeta_i(x) = \frac{\prod_{l=1}^{n} \mu_{F_i^l}(x_l)}{\sum_{i=1}^{m} \prod_{l=1}^{n} \mu_{F_i^l}(x_l)}$$  \hspace{1cm} (17)

V. DESIGN OF ADAPTIVE FUZZY CONTROLLER

The result in (5) is possible only while $f(x)$ and $g(x)$ are
well known. To solve the problem of the ideal controller cannot be implemented, an indirect adaptive fuzzy controller is proposed. We replace the unknown functions $f(x)$ and $g(x)$ by the fuzzy logic system (16). Moreover, a PI control term is designed in order to avoid chattering problem by the switching-type control term. The input and output of the continuous time PI controller is in the form of:

$$u_p = \theta_p z + \theta z_2$$

(18)

where $z = s, z_2 = s, \theta$ and $\theta_p$ are PI control gains. (18) can be rewritten as

$$\hat{p}(s|\theta_p) = \theta_p \psi(z)$$

(19)

where $\theta_p = \theta_p \theta_p \in \mathbb{R}^2$ is an adjustable parameter vector and $\psi^T(z) = [z_1, z_2] \in \mathbb{R}^2$ is a regressive vector. We use fuzzy logic system to approximate the unknown function $f(x), g(x)$ and design an adaptive PI control term to attenuate chattering action problem in sliding mode control.

Hence, the resulting control law is as follows:

$$u = \frac{1}{\hat{g}(x|\theta_p)} \left[ -f(x|\theta_p) - \sum_{i=1}^{\infty} k_i e_i + x_i^{\text{ref}} - \hat{p}(s|\theta_p) \right]$$

(20)

where

$$\tilde{f}(x|\theta_p) = \theta_p \psi(x)$$

(21)

$$\hat{g}(x|\theta_p) = \theta_p \psi(x)$$

(22)

Define $\hat{p}(s|\theta_p) = D + \eta + \omega_{\text{max}}$, when $s \geq \Phi$ in the control process, when $s < \Phi$ the control is replaced by PI control term. $\Phi$ is the thickness of the boundary layer and $D$ is the upper bound of the external disturbance $d(t)$.

Theorem: Consider the control problem of the nonlinear system (1). If control law (20) is used $\tilde{f}, \hat{g}$ and $\hat{p}$ are given by (19), (21) and (22), the parameters vector $\theta, \theta_e, \theta_p$ and the estimated bound $\tilde{D}$ are adjusted by the adaptive law (23)-(26). The closed loop system signals will be bounded and the tracking error will converge to zero asymptotically.

$$\dot{\theta}_f = \gamma_s \zeta(x)$$

(23)

$$\dot{\theta}_e = \gamma_s \zeta(x)u$$

(24)

$$\dot{\theta}_p = \gamma_s \zeta(s)$$

(25)

$$\dot{\tilde{D}} = \gamma_s |s|$$

(26)

Proof: Define the optimal parameters of fuzzy systems

$$\theta_f^* = \arg \min_{\theta_f \in \Theta_f} \{ \sup_{x \in X} | \tilde{f}(x|\theta_f) - f(x) | \}$$

(27)

$$\theta_e^* = \arg \min_{\theta_e \in \Theta_e} \{ \sup_{x \in X} | \hat{g}(x|\theta_e) - g(x) | \}$$

(28)

$$\theta_p^* = \arg \min_{\theta_p \in \Theta_p} \{ \sup_{x \in X} | \hat{p}(s|\theta_p) - u_e | \}$$

(29)

where $u_e = 1/g(x) \cdot \eta_2 \cdot \text{sgn}(s)$, $\Omega_f$, $\Omega_e$ and $\Omega_p$ are constraint sets for $\theta_f, \theta_e$ and $\theta_p$, respectively. Define the minimum approximation error.

$$\omega = f(x) - \tilde{f}(x|\theta_f^*) + (g(x) - \hat{g}(x|\theta_e^*) \cdot u$$

(30)

Assumption:

$$\Omega_f = \{ \theta_f \in \mathbb{R}^2 \text{|| } \theta_f \leq M_f \}$$

$$\Omega_e = \{ \theta_e \in \mathbb{R}^2 \text{|| } 0 < e \leq \theta_e \leq M_e \}$$

$$\Omega_p = \{ \theta_p \in \mathbb{R}^2 \text{|| } \theta_p \leq M_p \}$$

where $M_f, e, M_e$ and $M_p$ are pre-specified parameters. And assume the fuzzy parameters $\theta_f, \theta_e$ and the PI control parameter $\theta_p$ never reach the boundaries.

From (8) and (20), we have

$$\dot{s} = \sum_{i=1}^{\infty} k_i e_i + x_i^{\text{ref}} - x_i^{\text{ref}}$$

$$= \sum_{i=1}^{\infty} k_i e_i + f(x) + g(x)u + d(t) - x_i^{\text{ref}}$$

$$= \sum_{i=1}^{\infty} k_i e_i - \tilde{f}(x|\theta_f^*) - (g(x) - \hat{g}(x|\theta_e^*) \cdot u - \hat{p}(s|\theta_p) + d(t) + \omega$$

$$= \tilde{f}(x|\theta_f^*) - \tilde{f}(x|\theta_f^*) + (\hat{g}(x|\theta_e^*) - \hat{g}(x|\theta_e^*) \cdot u$$

$$= \hat{p}(s|\theta_p) - \hat{p}(s|\theta_p) + \hat{p}(s|\theta_p) + d(t) + \omega$$

$$= \phi_f^*(x) + \phi_e^* \cdot \psi(x) - \hat{p}(s|\theta_p) + d(t) + \omega$$

$$= \phi_f^*(x) + \phi_e^* \cdot \psi(x) - \hat{p}(s|\theta_p) + d(t) + \omega$$

$$= \phi_f^*(x) + \phi_e^* \cdot \psi(x)$$

Define the Lyapunov candidate

$$V = \frac{1}{2} s^2 + \frac{1}{2} \phi_f^* \cdot \phi_f^* + \frac{1}{2} \phi_e^* \cdot \phi_e^* + \frac{1}{2} \phi_p^* \cdot \phi_p^* + \frac{1}{2} \tilde{D}$$

(32)

where $\tilde{D} = \tilde{D} - \tilde{D}$. The time derivative of $V$ along the error trajectory (31) is

$$\dot{V} = s \cdot s + \frac{1}{\gamma_1} \phi_f^* \cdot \phi_f^* + \frac{1}{\gamma_2} \phi_e^* \cdot \phi_e^* + \frac{1}{\gamma_3} \phi_p^* \cdot \phi_p^* + \frac{1}{\gamma_4} \tilde{D} \tilde{D}$$

$$= s \cdot (\phi_f^* \cdot \psi(x) + \phi_e^* \cdot \psi(x) \cdot u + \phi_p^* \cdot \psi(s) - \hat{p}(s|\theta_p^*) + \omega + d(t))$$

$$+ \frac{1}{\gamma_1} \phi_f^* \cdot \phi_f^* + \frac{1}{\gamma_2} \phi_e^* \cdot \phi_e^* + \frac{1}{\gamma_3} \phi_p^* \cdot \phi_p^* + \frac{1}{\gamma_4} \tilde{D} \tilde{D}$$

$$= s \cdot (\phi_f^* \cdot \psi(x) + \phi_e^* \cdot \psi(x) \cdot u + \phi_p^* \cdot \psi(s) + \phi_f^* \cdot \phi_f^* + \phi_e^* \cdot \phi_e^* + \phi_p^* \cdot \phi_p^* + \hat{D} \hat{D}$$

$$= \frac{1}{\gamma_1} \phi_f^* \cdot \phi_f^* + \frac{1}{\gamma_2} \phi_e^* \cdot \phi_e^* + \frac{1}{\gamma_3} \phi_p^* \cdot \phi_p^* + \hat{D} \hat{D}$$

(33)
where \( \phi_f = -\theta_f \), \( \phi_g = -\theta_g \), \( \phi_p = -\theta_p \) and \( \dot{D} = -\dot{D} \). Substitute (23)-(26) into (33), then we have

\[
\dot{V} = s(d(t) + \omega - (\dot{D} + \eta_h) \text{sgn}(s)) - (\dot{D} - \dot{D}) |s| \leq s(\omega - \eta_h) |s| \leq 0
\]

(34)

\( \omega \) is the minimum approximation error, (34) is the best we can obtain. Therefore, all signals in the system are bounded. Obviously, if \( \epsilon(0) \) is bounded, then \( \epsilon(t) \) is also bounded for all \( t \). Since the reference signal \( x_d \) is bounded, then the system states \( x \) is bounded as well. To complete the proof and establish asymptotic convergence of the tracking error, we need proving that \( s \to 0 \) as \( t \to \infty \).

Assume that \( |s| \leq \eta_s \), then equation (34) can be rewritten as

\[
\dot{V} \leq s |\omega| - |s| \eta_s \leq \eta_s |s| - |s| \eta_s
\]

(35)

Integrating both sides of (35), we have

\[
\int_0^t |s| \, dt \leq \frac{1}{\eta_s} \int_0^t (|V(0)| - |\dot{V}(0)| + \eta_s \int_0^t |s| \, dt) \, dt
\]

(36)

then \( s \in L_\infty \). Form (31), we know that \( s \) is bounded and every term in (33) is bounded. Hence, \( s, \dot{s} \in L_\infty \), use of Barbalat’s lemma, \( s(t) \to 0 \) as \( t \to \infty \), the system is stable and the error will asymptotically converge to zero.

VI. SIMULATION EXAMPLE

In this section, we apply our proposed adaptive fuzzy controller for induction motor position servo drive. The parameters of induction motor are [2]:

\[
J = 4.78 \times 10^{-3} \text{Nm} / \text{s}^2
\]

\[
B = 5.34 \times 10^{-3} \text{Nms} / \text{rad}
\]

\[
K_T = 0.4851 \text{Nm} / \text{A}
\]

\[
T_I = 0.5 \text{Nm}
\]

The example is to let the rotor angle to track a sin-wave trajectory \( x_d = \theta_d = \pi \sin(t) \). We applied the external load disturbance \( T_I \) at \( t = 10 \text{sec} \). Choose the sliding surface as \( s = k_2e + \dot{e} \), \( k_1 = 4 \). The initial values of parameters \( \theta_p \) are set by \( \theta_{p_1} = 10 \) and \( \theta_{p_2} = 3.5 \). The membership functions for state \( x_i \), \( i = 1, 2 \) are selected as:

\[
\mu_{\eta_1}(x) = \exp\left(-((x_i + \pi/6) / (\pi/24))^2\right)
\]

\[
\mu_{\eta_2}(x) = \exp\left(-((x_i + \pi/12) / (\pi/24))^2\right)
\]

\[
\mu_{\eta_3}(x) = \exp\left(-((x_i, \pi/24))^2\right)
\]

\[
\mu_{\eta_4}(x) = \exp\left(-((x_i - \pi/6) / (\pi/24))^2\right)
\]

then there are 25 rules to approximate the system functions. The initial consequent parameters of fuzzy rules are chosen randomly in the interval [0.5, 2]. Let the learning rate \( \gamma_1 = 4, \gamma_2 = 1, \gamma_3 = 8 \) and \( \gamma_4 = 10 \). Choose the initial condition \( x = [-\pi/4, 0, 0]^T \) and step size 0.01s. Fig. 1 shows the simulation results. It can be seen that the tracking performance is good even presence of disturbance.

In this paper, the indirect fuzzy sliding mode control and the robust control using the adaptive control strategy are proposed. The proposed adaptive fuzzy sliding mode control has been applied to position control of induction servomotor. Based on the Lyapunov synthesis approach, the PI control parameters can be tuned on-line and the chattering effect is attenuated and robust performance can be ensured. Moreover, the upper bound of the discontinuous control term is assumed to be unknown and adaptive algorithm is applied to estimate it. The simulation results show the effectiveness of the proposed method in induction servomotor.

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