Nonlinear Phenomena of the Buck-boost Converter Analyzed by Storage Energy

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Abstract–In this paper, the storage energy in dc system is used to investigate the nonlinear phenomena of the buck-boost converter under the peak current control mode. Bifurcation and chaos are observed based on the stroboscopic map, zero energy is obtained in stable period one and the storage energy with same value and opposite direction are obtained in period two, new phenomenon occurs in some parameters. Simulation of bifurcation and chaos figure of energy are given under different parameters. These results can provide a new guide line to design a stable system and provide a new control algorithm for power electronic system.

Keywords–Story energy, dynamic phenomena, Power converter.

I. INTRODUCTION

It is obvious that the power electronics system consists of many reactive components such as inductors and capacitors. The storage energy in this component has become a very interesting topic in power electronics. In 2003, the new concept of energy factor in dc converters was firstly put forward by Prof. Cheng in reference [1, 2]. In these papers, storage energy of inductor and capacitor is defined as the difference between maximum energy and minimum energy in one cycle. Energy factor is defined as the ratio of storage energy to output energy in one cycle. These new concepts provide a new measurement on the characteristics of dc converters. But those definitions of the new concepts have not been developed to analyze the circuit or guide the design of the power electronic system. Though some researchers has investigated and develop new control algorithm to analysis the stability of the system based on energy balance [3, 4], and proposed a new energy based switching scheme for controlling dc-dc converter circuit and analysis its stability by Pawan Gupta and Amit Patra based on the energy balance theory in 2005 [5, 6], however, no researchers investigate the stability of the energy which stored in capacitor and inductor, and how the energy evolves when the parameters change.

In this paper, the storage energy in dc system is used to investigate the instability of the buck-boost converter, and the bifurcation and chaos are observed based on the discrete map, zero energy is clearly obtained in stable period on and same value and opposite direction are obtained in period two, and new phenomenon occurs in some parameters. Simulation of bifurcation and chaos figure of energy are given under different parameters. These results can provide a new guide line to design a stable system and provide a new control algorithm for power electronic system

II. BUCK-BOOST CONVERTER

1. *Circuit and mathematic description*

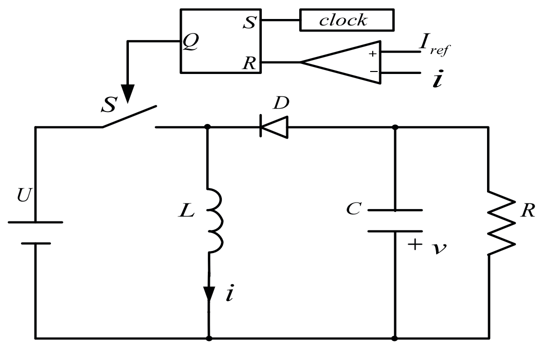

Fig. 1 buck-boost converter

Fig. 1 shows a typical buck-boost converter under peak current mode control. It is also assumed that the components in the circuit are ideal and no parasitic effects are considered. The equations can be summarized as follows when the buck-boost converter operates in discontinues mode.

(1) State 1: SW is on, D is off,

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1/(RC) \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} U/L \\ 0 \end{bmatrix} \quad (1)$$

(2) State 2: SW is off, D is on,

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/(RC) \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} \quad (2)$$

(3) State 3: SW is on, D is off,

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1/(RC) \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} \quad (3)$$

2. Piecewise discrete map derivation

In the following, T_s is the switching cycle, and the sample time is nT_s , then $i_n = i(nT_s)$, $v_n = v(nT_s)$, where i_n, v_n is the current through inductor and the voltage in the capacitor. Then the following equation (4), (5), (6) can represent equation (1), (2), (3) [7].

It is assumed that t_n is the time that the inductor current i reaches I_{ref} from the nT_s at state 1, t_d is the time that the inductor current reaches 0 from $(nT_s + t_n)$ in state 2.

If $t_n \ge T_s$, in $nT_s \sim (n+1)T_s$, the converter operates in state1, the equation (1) can be solved as follows

$$\begin{cases} i_{n+1} = i_n + \dfrac{UT_s}{L} \\ v_{n+1} = v_n e^{-T_s/RC} \end{cases} \quad (4)$$

if $(t_n + t_d) \ge T_s$, in $nT_s \sim (n+1)T_s$, the system operates in state 1 and state 2 , the inductor current and the capacitor voltage in $(n+1)T_s$ is as follows

$$\begin{cases} i_{n+1} = e^{\delta(T_s - t_n)}\left[a\cos\omega(T_s - t_n) + b\sin\omega(T_s - t_n)\right] \\ v_{n+1} = -Le^{\delta(T_s - t_n)}\left[(a\delta + b\omega)\cos\omega(T_s - t_n) + (b\delta - a\omega)\sin\omega(T_s - t_n)\right] \end{cases} \quad (5)$$

And if $(t_n + t_d) < T_s$, in $nT_s \sim (n+1)T_s$, the system operates in state 1, 2and comes into state 3.Then we can get solution of equation (3)

$$\begin{cases} i_{n+1} = 0 \\ v_{n+1} = v_2(t_d)e^{-(T_s - t_n - t_d)/RC} \end{cases} \quad (6)$$

Where, $t_n = \dfrac{L}{U}(I_{ref} - i_n)$,

$\delta = -1/2RC$ $\quad \omega = \sqrt{1/LC - 1/4R^2C^2}$ $\quad a = I_{ref}$

$b = -\dfrac{1}{\omega}\left(\dfrac{v_n e^{-t_n/RC}}{L} + \delta I_{ref}\right)$ $\quad t_d = \dfrac{1}{\omega}arctg\left(-\dfrac{a}{b}\right)$

$v_2(t_d) = -Le^{\delta t_d}\left[(a\delta + b\omega)\cos\omega t_d + (b\delta - a\omega)\sin\omega t_d\right]$.

III. DEFINITION OF ENERGY STORAGE

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1. Storage energy for inductors

The variation of the stored energy for the inductor in one switching cycle in buck-boost converter of peak current mode is defined as S_L, which can be calculated by the following equation

$$S_L = \frac{1}{2}L(i_{n+1}{}^2 - i_n{}^2)$$

Where, i_{n+1}, i_n can be calculated by equation (4), (5), (6).

2. Storage energy for capacitor

The variation of the stored energy for the capacitor in one switching cycle in buck-boost converter of peak current mode is defined as S_C , which can be calculated by the following equation

$$S_C = \frac{1}{2}C(v_{n+1}{}^2 - v_n{}^2)$$

Where, v_{n+1}, v_n can be calculated by equation(4), (5), (6).

IV. SIMULATION

Different inductor value is used to investigate the characteristic of the storage energy of the system. The circuit parameters are given as follows: $T_s = 50\mu\text{S}$, $U = 35V$, $C = 4\mu\text{F}$, $R = 100\Omega$, $I_{ref} = 3\text{A}$. Dynamics behavior of bifurcation and chaos of storage energy are observed as Fig. 4 shown.

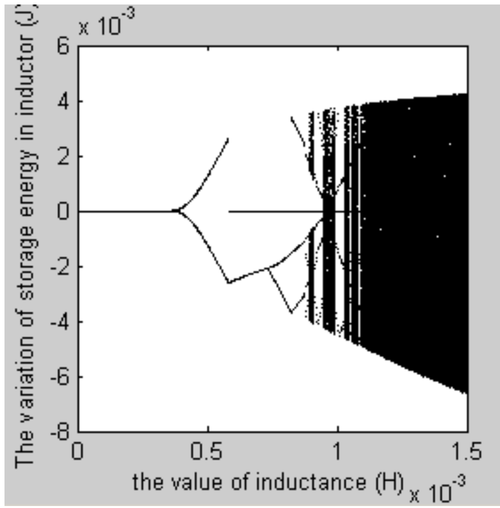

Fig. 4(a)

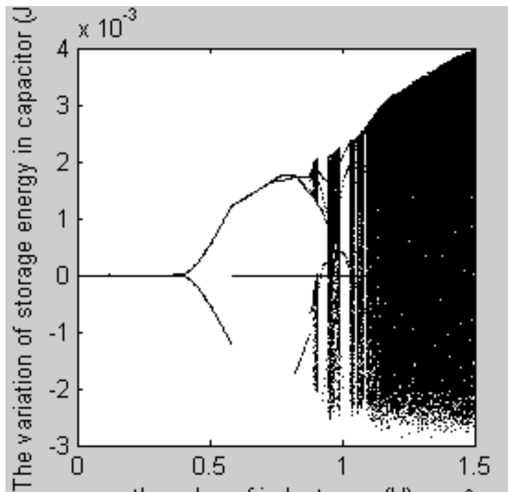

Fig. 4 (b)

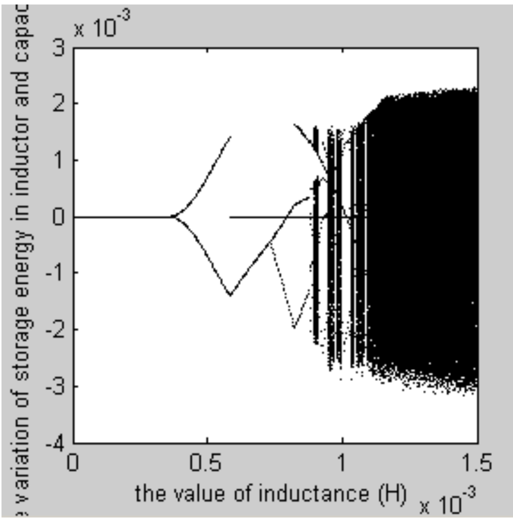

Fig. 4 (c)

Fig. 4(a), (b), (c) Bifurcation diagram of storage energy in inductor, capacitor and the sum of inductor and capacitor respectively (the value of inductance is the bifurcation parameter).

In the Fig. 4, abundant nonlinear dynamic phenomena have been observed that the orbit undergoes period bifurcation and period 3 which finally induce chaos. Fig. 4(a), (b), (c) shows bifurcation diagram of storage energy in inductor, capacitor and the sum of inductor and capacitor respectively, when the value of inductance changes. In Fig 4(a), the system operates on stable period 1 until the capacitance is greater than 0.37mH. The variation of the storage energy in inductor is zero when the system operates stable. Then the system smoothly goes to period 2 until the capacitance is greater than 0.58mH and then the system goes to period 2 which has a zero and a negative branch. After that, period 3 occurs which induce chaos finally. However, the system goes through period 4, 6 into chaos. The most important is that bifurcation and chaos period occurs and finally into chaos, which is similar with Fig. 4 (b) and (c).

V. Conclusion

The application of the storage energy in dc converter is investigated in this paper, bifurcation and chaos and some interesting phenomenon occurs in the bifurcation diagram when different parameter studied. Zero energy appears when the circuit operates in stable period 1, which will develop new control algorithm when design a stable power electronic system and a new angle to understand the dynamic behaviour of the circuit. Further research will be continuing on develop the new method of controlling the system and dynamic phenomenon of the system

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