Dynamic Phasor Modelling of TCR based FACTS Devices for High Speed Power System Fast Transients Simulation

Zhijun E1, K. W. Chan2 and D. Z. Fang3

Abstract – This paper firstly proposes a generic dynamic phasor model of thyristor controlled reactor (TCR) at the device level. The modelling approach is based on the time-varying Fourier coefficient series of the power system variables. By truncating the less important higher order terms and keeping the significant ones, this dynamic phasor TCR model can realistically simulate the nonlinear characteristics of TCR with fast speed and high accuracy. The operation of anti-parallel-connected thyristor pair is simulated by a switching function. Based on this TCR dynamic phasor model, fast and efficient SVC and TCSC dynamic phasor models could then be implemented. While a simple power system was used for the device level evaluation, the 39-bus New England power system was built to fully verify the accuracy and efficiency of the propose dynamic phasor models by benchmarking with a commercial software named DCG-EMTP. The availability of fast and efficient dynamic phasor models for FACTS devices has provided a powerful basis for high speed power system fast transients simulation of large scale interconnected power systems.

Keywords - Dynamic phasors, thyristor-controlled reactor, thyristor-controlled series capacitor, static VAR compensator

I. INTRODUCTION

With the rapid development of power electronics technologies, more and more Flexible AC Transmission System (FACTS) devices have been widely adopted and applied in modern power systems for enhancing the controllability and power transfer capability of the AC system; and as a result, they play an important role in the power system operation and control. The static VAR compensators (SVCs) constitute the first generation of FACTS controller. It is shunt-connected static generator or absorber of reactive power in which the output is varied to maintain or control specific parameters of power system. Thyristor-controlled series capacitor (TCSC), on the other hand, is a series-controlled capacitive reactance that can provide continuous control of power on the AC line over a wide range. By now, SVC and TCSC are the most common FACTS devices used to improve the power system performance in steady-state and dynamic stability, voltage profile, and reactive power flow. For the stability assessment of large-scale power systems, it is necessary to model those FACTS device accurately and efficiently.

Because of the nonlinear switching behaviour of thyristor controlled reactor (TCR), accurate modelling of SVC or TCSC is non-trivial. The quasi-static approximation model commonly used in electromechanical transients (transient stability) simulation is not adequate to catch the dynamic behaviour of the switching. Even though in electromagnetic transient simulation, the full device level time-domain model can provide detailed response of the devices, the computation burden of such model is very demanding and hence is impractical for daily usage in large-scale power system simulations.

Dynamic phasor (DP) is developed from time-domain descriptions using the generalized averaging procedure [1-7], and can be used to establish nonlinear time-invariant and large-signal models of nonlinear devices. By now, HVDC system [8-9] and FACTS devices [10] such as SVC [11], TCSC [12-13], STATCOM [14] and UPFC [15-17] have been investigated and modelled using the dynamic phasor approach. However, the model of SVC in [11] and TCSC in [12] takes only the fundamental phasor into consideration for sub-synchronous resonance (SSR) study. In addition, simplified assumptions were taken in [13] to deduce the TCSC dynamic phasor model without the inclusion of the important self-governed control circuit.

In this paper, a generic dynamic phasor model of TCR is firstly proposed and evaluated. This model includes a switching function to simulate the operation of thyristor pair. By truncating the less important higher order frequency components and keeps only the lower order significant ones, the model of SVC and TCSC, which can accurately catch the dynamic behaviour of SVC and TCSC with fast simulation speed, are then implemented. Finally, the system level dynamic simulation of the 39-bus New England power system is implemented using the proposed dynamic phasor models. Balanced and unbalanced case studies on this 39-bus system show the validity of the proposed dynamic phasor models. The dynamic phasor models proposed in this paper can either be used in dynamic phasor (DP) simulation of modern power systems or incorporated in the traditional transient stability (TS) simulation to form a TS-DP hybrid simulation for transient stability study of large power system with accurate modelling of FACTS devices.

The rest of this paper is organized as follows. In Section II, the basic concept of dynamic phasor modelling approach is introduced. Section III presents the dynamic phasor models of TCR. Section IV provides the evaluation results of the proposed SVC and TCSC model on a simple power system. Section V shows the system level dynamic phasor simulation and Section VI concludes the paper.

II. BASIC CONCEPT OF DYNAMIC PHASORS

The approach of dynamic phasors is firstly known as the method of state-space averaging and is based on the time-varying Fourier coefficients. Generally, a complex time
domain waveform \( x(\tau) \) can be represented on the interval \( \tau \in (t-T,t] \) using a Fourier series of the form:

\[
x(\tau) = \Re\{\sum_{k=0} X_k(t)e^{j2\pi k\tau}\}
\]  
(1)

where \( \omega_k = 2\pi/T \) and \( X_k(t) \) are the complex time-varying Fourier coefficients called as dynamic phasors. The \( k \) th dynamic phasor at time \( t \) is determined by the following expression:

\[
X_k(t) = \frac{c}{T} \int_{t}^{t+T} x(\tau)e^{-j2\pi k\tau} d\tau = \langle x \rangle_k(t)
\]  
(2)

where \( c=1 \) if \( k=0 \) and \( c=2 \) if \( k > 0 \). The dynamic phasor method is based on the idea of frequency decomposition and focuses on the dynamics of the significant Fourier coefficient. The following are the two key and useful properties of the phasors:

1. \( k \)-phasor differential characteristic:

For the \( k \)th Fourier coefficient, the differential with time satisfies the following formula:

\[
\frac{dX_k}{dt}(t) = \frac{d}{dt}\langle x \rangle_k(t) - jk\omega X_k(t)
\]  
(3)

2. Product of dynamic phasors:

The \( k \)th phasor of a product of two time-domain waveform \( x(\tau) \) and \( y(\tau) \) can be obtained by the following operation:

\[
\langle xy \rangle_k = \sum_{l=0}^{\infty} \langle x \rangle_k \langle y \rangle_l
\]  
(4)

Also, the time domain waveform \( x(\tau) \) can be transformed back from its dynamic phasors by the following equation:

\[
x(\tau) = \Re\{X_k(t)e^{j2\pi k\tau}\} = X_k(t)e^{-j2\pi k\tau} + X_{-k-1}(t)e^{-j2\pi(k-1)\tau} + \ldots + X_{k-1}(t)e^{j2\pi(k-1)\tau} + X_k(t)e^{j2\pi k\tau}
\]  
(5)

Moreover, since \( x(\tau) \) is real,

\[
X_{-k} = X_k^*
\]

where the operator * means the conjugate of a complex number.

III. DYNAMIC PHASOR MODEL OF TCR

TCR is one of the most important building blocks of thyristor-based SVC and TCSC. Although it can be used alone, it is more often employed in conjunction with fixed or thyristor-switched capacitors to provide rapid and continuous control of reactive power over the entire selected lagging-to-leading range. As an example, the three-phase SVC shown in Fig. 1 consists of three delta-connected three-phase TCRs, shunt capacitors and filter circuits of odd harmonics.

A. Dynamic phasor model of TCR

Fig. 2 depicts a single phase TCR. If the two thyristor valves are fired symmetrically in the positive and negative half-cycles of supply voltage, and thus only odd-order harmonics would be produced. Harmonics analysis shows that seventh and higher order harmonics has less effect on TCR dynamic characteristics [18]. As a result, in this paper only the fundamental, third and fifth harmonics would be considered in the dynamic phasor model of TCR.

From the single phase TCR circuit as shown in Fig. 2, its time-domain model can be obtained as:

\[
\begin{align*}
\frac{dv}{dt} & = i - i_i \\
\frac{di}{dt} & = sv
\end{align*}
\]  
(6)

where \( v = u_i - u_x \) and \( s \) is the switching function. When one thyristor is full conducting, \( s = 1 \); and when both thyristors are shut, \( s = 0 \). With \( <x> \), rewritten as \( X_1 \), the dynamic phasor model of TCR can be obtained by the differential characteristic as the following formula:

\[
\begin{align*}
C \frac{dv_k}{dt} & = -jk\omega CV_k + I_k - I_i \\
L \frac{di}{dt} & = -jk\omega LI_k + <sv>_k
\end{align*}
\]  
(7)

where \( <sv>_k \) can be calculated by the product characteristic of dynamic phasors according to (8).

\[
\langle sv \rangle_k = \sum_{l=5,3,1,3,5} S_{kl} V_l
\]  
(8)

Dynamic phasors are complex quantities. Each equation in (7) consists of two parts: the real and imaginary parts; and as a result, the dynamic model of single phase TCR would have more equations than the corresponding time domain model. Nevertheless, it could catch the dynamic behaviour of TCR with relatively larger integration step time. The complete TCR dynamic phasor model is as follow:

\[
\begin{align*}
C \frac{dv_k^r}{dt} - k\omega CV_k^r & = I_k^r - I_i^r \\
C \frac{dv_k^i}{dt} + k\omega CV_k^i & = I_k^i - I_i^i
\end{align*}
\]
\[
L \frac{dI_k^R}{dt} - k_\omega L I_k^I = \sum_{m \neq n - k} \left[ S_{mK}^R V_n^R - S_{mK}^I V_n^I \right] \\
+ \sum_{m \neq n - k} \left[ S_{mK}^R V_n^R + S_{mK}^I V_n^I \right] \\
+ \sum_{m \neq n - k} \left[ -S_{mK}^R V_n^R + S_{mK}^I V_n^I \right] \\
L \frac{dI_k^I}{dt} + k_\omega L I_k^R = \sum_{m \neq n - k} \left[ S_{mK}^I V_n^R - S_{mK}^R V_n^I \right] \\
+ \sum_{m \neq n - k} \left[ -S_{mK}^I V_n^R + S_{mK}^R V_n^I \right] \\
+ \sum_{m \neq n - k} \left[ S_{mK}^I V_n^R - S_{mK}^R V_n^I \right]
\]

As an illustration for (9), the fundamental phasor model is presented as follow:

\[
\langle \text{sv} \rangle = S_1 V_{1a} + S_1 V_{1b} + S_1 V_{1c} + S_1 V_{2a} + S_1 V_{2b} + S_1 V_{2c} = S_2 V_{1a} + S_2 V_{1b} + S_2 V_{1c} + S_2 V_{2a} + S_2 V_{2b} + S_2 V_{2c}
\]

\[
C \frac{dV_k^R}{dt} - \omega C V_k^I = I_k^R - I_k^I
\]

\[
C \frac{dV_k^I}{dt} + \omega C V_k^R = I_k^I - I_k^R
\]

\[
L \frac{dI_k^R}{dt} - \alpha_0 L I_k^I = S_k^R V_k^R + (S_k^R V_k^R + S_k^I V_k^R + S_k^I V_k^R)
\]

\[
+(S_k^R V_k^R + S_k^I V_k^R + S_k^I V_k^R + S_k^I V_k^R)
\]

\[
L \frac{dI_k^I}{dt} + \alpha_0 L I_k^R = S_k^I V_k^R + (-S_k^I V_k^R - S_k^I V_k^R - S_k^I V_k^R)
\]

\[
+(S_k^I V_k^R - S_k^I V_k^R - S_k^I V_k^R - S_k^I V_k^R)
\]

(10)

where the superscript \( R \) and \( I \) denote the real and imaginary parts of the defined quantities.

**B. Dynamic phasor model of switching function**

The nonlinear operation of a TCR is simulated by a switching function. The dynamic model of the switching function [13] is a key element of dynamic phasor model of a TCR and can be described as follow:

\[
S_R = \frac{1}{T} \int_{0}^{T} s(\tau) \cdot d\tau = \frac{\tau - \alpha}{\pi}
\]

\[
S_I = \frac{1}{T} \int_{0}^{T} s(\tau) \cdot e^{-j\omega \tau} \cdot d\tau = \frac{j}{m \pi} [e^{j\alpha \omega} - e^{-j\alpha \omega}]
\]

\[
= \frac{1}{m \pi} [\sin m(\alpha + \sigma) - \sin m\alpha] \\
+ \frac{j}{m \pi} [\cos m(\alpha + \sigma) - \cos m\alpha] \quad (m \neq 0)
\]

(11)

where, \( m \) is the subscript of the \( m \)th phasor of switching function, \( \alpha \) is the firing angle and \( \sigma \) is conduction angle of TCR which depend on the closed-loop control of the SVC or TCSC. For each simulation time step, \( \alpha \) and \( \tau \) are calculated from control loop.

Because of the angle difference between phases, the switch function models for phase B and C are different from phase A and can be expressed as:

\[
\begin{bmatrix}
S_{A,B} = e^{j\alpha(2\pi/3)} S_{A,B} \\
S_{C,B} = e^{j(2\pi/3)} S_{A,B}
\end{bmatrix}
\]

(12)

**C. Dynamic phasor model of controller circuit**

A practical model of the control system for the SVC and TCSC is essential for the proper representation of the system dynamic. A general SVC control system consists of a measurement system, a voltage regulator, a gate-pulse generator, a synchronizing system, and a supplementary control as shown in Fig. 3. The time-domain model of the controller circuit is shown in Fig. 4. In the dynamic model of SVC, the same voltage control unit is used. But the gate-pulse generator and synchronizing system is not included because only firing angle \( \alpha \) and conduction angle \( \tau \) are necessary for the dynamic phasor modelling work. The main difference is the calculation of \( V_{\text{acc}} \) is derived from the dynamic phasors of voltage instead of the instantaneous value. The same approach can also be applied in the modelling of the TCSC control system.

**D. Dynamic phasor model of filter circuit**

The filter circuit is also necessary for proper modelling of SVC or TCSC. In short, it is an RLC circuit which consists of one RL circuit and one capacitor circuit shown as Fig. 5. The dynamic phasor model of RL circuit and capacitor circuit is deduced as follows [19]:

\[
\begin{bmatrix}
\frac{1}{L} \frac{dL_i}{dt} = \frac{1}{C} \frac{dC_i}{dt} = \frac{1}{R} \frac{dV_i}{dt} - \frac{V_i}{R} \\
\frac{1}{C} \frac{dC_i}{dt} + \frac{1}{L} \frac{dL_i}{dt} = \frac{1}{R} \frac{dV_i}{dt}
\end{bmatrix}
\]

(19)

Fig. 3: A general scheme diagram of SVC control system

Fig. 4: Logic block diagram of SVC control circuit

Fig. 5: Single-phase filter circuit
1. DP model of RL circuit

For RL circuit, time-domain model is:

\[ v(t) = L \frac{di(t)}{dt} + R i(t) \]  \hspace{1cm} (13)

and the dynamic phasors are obtained by

\[ L \frac{dl_k}{dt} = V_k - j \omega L I_k - R I_k \hspace{1cm} k = 1, 3, 5 \]  \hspace{1cm} (14)

Then the complete dynamic phasor model becomes

\[ \begin{cases} \frac{dl_k^R}{dt} = (V_k^R - R I_k^R) + j \omega L I_k^R \\ \frac{dl_k^I}{dt} = (V_k^I - R I_k^I) - j \omega L I_k^I \end{cases} \]  \hspace{1cm} k = 1, 3, 5 \]  \hspace{1cm} (15)

2. DP model of capacitor circuit

For capacitor circuit, we have the time-domain model:

\[ i(t) = C \frac{dv(t)}{dt} \]  \hspace{1cm} (16)

and the dynamic phasors are obtained by

\[ C \frac{dv_k}{dt} = I_k - j \omega C V_k \hspace{1cm} k = 1, 3, 5 \]  \hspace{1cm} (17)

Then the complete dynamic phasor model becomes

\[ \begin{cases} C \frac{dv_k^R}{dt} = I_k^R + j \omega C V_k^R \\ C \frac{dv_k^I}{dt} = I_k^I - j \omega C V_k^I \end{cases} \]  \hspace{1cm} k = 1, 3, 5 \]  \hspace{1cm} (18)

IV. EVALUATION OF TCR MODEL

The TCR dynamic phasor model presented in this paper is evaluated via the modeling of the SVC and TCSC. For the evaluation of the dynamic phasor model, full DCG EMTP simulations for the entire test systems are carried out to produce the benchmark results for each case.

A. SVC DP model evaluation

In SVC, three single phase TCRs is connected in delta to prevent the third harmonics from percolating into the transmission lines. As a result, in the SVC DP model, the third harmonics is not taken into consideration, and the term of third harmonics in the equations of other harmonics is also eliminated.
Fig. 8 and 9 show the comparison of instantaneous voltages between DCG EMTP and dynamic phasors. The instantaneous voltages of dynamic phasors are reversely transformed using equation (5). It is obviously that the dynamic phasor model has practically the same results as DCG EMTP software.

2. Efficiency evaluation

Various dynamic phasor models of SVC with different combinations of harmonics terms have also been built using the same modelling method. The run times for dynamic phasor simulations with different dynamic phasor models incorporated into the test system in Fig.5 are listed in Table 2. In Table 2, DP, denotes the model only including fundamental phasor, DP,,,denotes the model including fundamental and fifth phasors, and DP, denotes the model including fundamental, fifth and seventh phasors. Comparing with full time-domain model, DP, is fastest but least accurate model. DP, is most accurate but most time demanding among the three dynamic phasor models. Overall, DP, is the best compromise in term of accuracy and simulation efficiency.

<table>
<thead>
<tr>
<th>Model type</th>
<th>DP</th>
<th>DP,</th>
<th>DP,</th>
<th>EMTP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time consumed(s)</td>
<td>17</td>
<td>24</td>
<td>36</td>
<td>28</td>
</tr>
</tbody>
</table>

B. TCSC DP model evaluation

Different from the SVC DP model, TCSC DP model cannot ignore the effect of triplen harmonics because of the TCRs connection. So the third harmonics phasors have to be taken into consideration in TCSC dynamic phasor model. A simple test system similar to the previous one is used in the evaluation of TCSC DP model. The constant current source with same impedance is used as the power source in the TCSC circuit. The RMS of the current source is 1kA. The parameters of limped line are R=5.87Ω, L=134.20mH. The parameters of the TCSC are L=6.85mH, C=175μF. The evaluation includes three phases which lasts for 1.5 seconds in total. The simulation starts with firing angle α = 60° at t=0, then α = 75° at t=0.5s, and the simulation end with α = 85° at t=1.5s. The emphasis of the comparison is placed on the waveforms of capacitor voltage. The results of comparison are shown in Fig.10-12.

Fig. 11: Capacitor dynamic phasor voltage

Fig. 12: Instantaneous capacitor voltage – DCG EMTP

Fig. 13: Instantaneous capacitor voltage – dynamic phasor

2. Efficiency evaluation

Several dynamic phasor models of TCSC with different harmonics considered are also implemented. Table 3 lists the simulation run times for the simple test system with different types of dynamic phasor models incorporated. In Table 3, DP, denotes the model only including fundamental phasor, DP, denotes the model including fundamental and third phasors, and DP, denotes the model including fundamental, third and fifth phasors. Comparing with the dynamic phasor models of SVC, the dynamic phasor models of TCSC have the similar conclusion that DP, is the best compromise in term of accuracy and simulation efficiency.

<table>
<thead>
<tr>
<th>Model type</th>
<th>DP</th>
<th>DP,</th>
<th>DP,</th>
<th>EMTP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time consumed(s)</td>
<td>18</td>
<td>25</td>
<td>38</td>
<td>32</td>
</tr>
</tbody>
</table>
V. DYNAMIC PHASOR SIMULATION AT SYSTEM LEVEL

Apart from evaluating the performance of the dynamic phasors at the device level modeling, the overall system dynamic performance is also considered. For the system level assessment, the 39-bus New England test system shown in Fig.14 is adopted. Again, the results obtained from the DP simulation are compared against with the benchmark results generated from the DCG EMTP.

In the EMTP simulation, generators and transmission lines are represented by two-axis generator models and distribution parameter line models, respectively. All loads are modeled as constant impedance. In the DP simulation, the dynamic phasor models of generators are derived from voltage flux-linkage equations directly and transmission lines are derived from distributed parameters model using the averaging approach. The step time of EMTP model is 50\(\mu\)s whereas the one for DP is 0.2ms. Two cases are performed and simulation comparison between DP and EMTP model is presented in the following sections.

A. Balanced fault case

In case 1, three phase to ground fault happens on bus 28 at 0.04s. The voltage of bus 26 is the focus of comparison. The results show in Fig.15.

From Fig.15-16, the waveforms of bus voltage using different models are generally matched each other while the ones from the DP simulation is relative smoother with less high frequency transients. This is as expected since the time step adopted by the DP simulation is relatively larger and hence the simulation resolution (bandwidth) is lower while the simulation speed is higher. As indicated from Fig.15-16, DP simulation is indeed able to capture the dynamic characteristic of the system during the fault on and post-fault stage for both symmetry and asymmetry fault conditions.

B. Unbalanced fault case

In case 2, phase A to ground fault happens on bus 28 at 0.05s and then cleared at 0.07s. The voltage of bus 28 is the focus of comparison. The results show as Fig.16.

VI. CONCLUSION

A practical dynamic phasor model of TCR is proposed in this paper. The dynamic phasor modelling approach is developed from the time-domain descriptions using time-varying Fourier coefficients. This new model is then used in development of the SVC and TCSC dynamic phasor model which includes the effects of dominant harmonics and hence is more accurate than the conventional transient stability counterpart models. Without the simplifications as taken in transient stability simulation, dynamic phasor simulation is capable of catching the fast dynamics characteristics as the EMTP simulation while its simulation speed is much faster then the EMTP. The
newly developed models have been fully evaluated on a simple test power system with constant current source and the 39-bus New England test system for device and system level assessment, respectively. By benchmarking with the DCG EMTP software, it is clearly shown that the dynamic phasor models are accurate and efficient, and could be practically applied to large scale power system for accurate system dynamic modelling with extensive use of fast acting power electronics devices such as SVC, TCSC and HVDC, etc.

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**REFERENCES**


**BIOGRAPHIES**

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