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Equilibrium Pricing Sequence in a Co-Opetitive Supply Chain with the ODM as a Downstream Rival of Its OEM

Baozhuang Niu

Lingnan (University) College, Sun Yat-sen University, P.R.C, niubzh@mail.sysu.edu.cn

Yulan Wang*

Faculty of Business, the Hong Kong Polytechnic University, Hong Kong, yulan.wang@polyu.edu.hk

Pengfei Guo

Faculty of Business, the Hong Kong Polytechnic University, Hong Kong, pengfei.guo@polyu.edu.hk

We study three basic price competition games engaged in by an original equipment manufacturer (OEM) and its competitive original design manufacturer (ODM): a simultaneous pricing game, an OEM-pricing-early game, and an ODM-pricing-early game. The ODM provides contract manufacturing service to the OEM and competes with this OEM in the consumer market by selling self-branded products. We consider two market environments: the ODM market and the OEM market. For the ODM market, we show that a sequential pricing game arises as the outcome preferred by the OEM and its ODM. Moreover, the equilibrium that the OEM prices early risk-dominates the one that the ODM prices early. Nevertheless, for the OEM market, the simultaneous pricing game and the sequential pricing game can both arise and be sustained. We also demonstrate that it is in their mutual interest to be friends rather than foes.

Keywords: *Contract Manufacturing, Competitive ODM, Endogenous Pricing Game, Supply Chain Management*

1 Introduction

In the manufacturing industry, original equipment manufacturers (OEMs) often outsource their manufacturing and even some design functions to original design manufacturers (ODMs), the contract manufacturers (CMs) that design and manufacture the specified products for OEMs (McIvor and Humphreys 2004, Kaya 2011). For example, PalmOne worked with HTC, a Taiwan-based ODM, to design and manufacture its popular Treo 650 smartphones (Engardio and Einhorn 2005). Apple outsourced its product manufacturing and partially, its innovation to notebook ODMs including Quanta, Asus, and Flextronics (Mihailescu 2005; Engardio and Einhorn 2005).

However, outsourcing production to ODMs can be a double-edged sword for OEMs (Nellore and Soderquist 2000). On the one hand, by outsourcing production to ODMs, OEMs can minimize the risk of new product failure, shorten the introduction time of innovative products, speed up

*Corresponding Author.

product life cycles, reduce design and production costs, and expand product portfolios (Feng and Lu 2011). On the other hand, according to Ozkan and Wu (2009a), ODMs are getting more likely to launch their own branded products that are similar to the OEMs' and, thus, become their downstream competitors. For instance, in the consumer electronics industry, HTC, the ODM for Google, produces its self-branded HTC Desire smartphone and the Google Nexus One smartphone, which have similar specifications (Phones-review 2010).

The quality levels of the OEM products and self-branded ODM products can be very close, as they are produced by the same manufacturer using similar production technologies. In recent years, more and more OEMs are focusing on marketing activities (e.g., branding, advertising, and real-time customer interactions) while relying on their CMs to ensure the product quality (Kaya and Özer 2009). Compared to OEMs, ODMs often lack the marketing skills necessary to achieve success in the markets such as the United States, Japan and some European countries (Gao et al. 2003). The potential reasons are as follows: (1) ODMs are unfamiliar with the distributors in the countries that are likely to be the home markets of the OEMs, and, hence, face difficulty building efficient distribution channels and service networks; (2) they do not have sufficient knowledge about the consumer purchasing behaviors in those markets and have limited pricing skills; and (3) they invest little in branding and promoting their products. It is frequently reported that ODMs' products occupy much less market share in developed markets than in emerging markets. For example, Taiwan-based TPV Technology Limited (TPV) designs and produces computer monitors and LCD televisions not only for OEMs, such as Samsung, Philips, Sony, Vizio, and LG, but also for itself under its own brands, AOC and Envision. In 2009, AOC's market share in France and Germany was ranked 15th and 13th, respectively (AOC International 2010), but the company was officially ranked first in the Asia-Pacific region. AOC even achieved the largest market share (34.2%) in Philippines (Kant 2011). According to Ray Zhuo, the general manager in charge of AOC's operations in Asia-Pacific, Middle East Asia, and Central Asia, its success in these emerging markets was primarily due to its *local business network* advantage (D'Mello 2009). In another case, the ODM Asus produces and sells its self-branded Asus notebook. According to a report from RescueCom and IDConly, in 2010 Asus captured around 3% of the American computer market, whereas Asus's OEMs Apple and Toshiba account for 9.7% and 10.2%, respectively (Bart 2011). However, Asus performs very well in its close-to-home emerging markets. For example, Asus enjoyed a market share of 12.5% in China and has been fighting for second place, targeting a 15% market share (Yang and Lu 2011). Actually, due to its *close-to-home* marketing strategy, Asus has "penetrated into China commercial business aggressively" and launched a "ferocious attack to gain more mind share and market share" in the Asia-Pacific region (Parnell 2011). As a result, Asus has built a mature sales and service network in this region. These examples demonstrate that customers may prefer the self-branded products of competitive ODMs over those of OEMs in some local emerging markets due to their strong market position. However, the opposite may hold true in developed markets. To characterize the different market features, we refer to the market where the competitive ODM's (OEM's) product is preferred as the *ODM (OEM) market*, and we consider the price competition in both markets.

When an OEM contracts with a competitive ODM, the price competition is intriguing because the ODM is not only a downstream competitor of the OEM but also an upstream business partner. Will a firm's price be undercut by its competitor, and will the two firms engage in a price war? Indeed, in the traditional Bertrand competition game, the second-mover advantage exists because the firm that makes its pricing decision late can undercut the price of the firm that prices early so as to capture a larger market share (Gal-Or 1985, Eilon 1993). In such a traditional setting, a Stackelberg equilibrium cannot be sustained because firms always try to undercut the prices of other firms. Nevertheless, this intuition need not hold in our setting because the OEM and the ODM are not only competitors but also partners. Recall that the ODM's revenue comes from both the contract manufacturing business and selling self-branded products. Undercutting the OEM's price can attract more customers to the ODM's self-branded products, but such behavior also reduces the wholesale price for the ODM's contract manufacturing service and thus the related revenue. Knowing that, the ODM may not have enough incentive to undercut the OEM's price. Anticipating the ODM's response and without worrying about the price being undercut, the OEM may prefer to price early rather than late. Thus, the following questions arise: Could a sequential pricing game be sustained between the OEM and its competitive ODM? If it could, under which conditions?

To study this issue, we investigate an endogenous pricing game that was first introduced in Gal-Or (1985). There are two stages in this game. At the first stage, two players independently decide when to price their products, early or late. Their pricing timing choices are revealed at the beginning of the second stage: a *simultaneous game* is played if both players make the same pricing timing decisions; an *OEM-pricing-early game* is played if the OEM prefers pricing early while the ODM prefers pricing late, and an *ODM-pricing-early game* is played otherwise. Note that the first stage of this game is artificially constructed to enable examination of firms' incentives for choosing a particular pricing sequence (Amir and Stepanova 2006). The players' endogenized pricing timing choice will then be revealed by the subgame perfect Nash equilibrium (SPNE) of the endogenous timing game.

We show that the answer to the first question is positive: A Stackelberg equilibrium can indeed be sustained in the pricing game between an OEM and its ODM. Therefore, the conventional wisdom generated from the traditional Bertrand pricing game (i.e., two firms will frequently change their prices to compete for customers), does not hold in a situation where one firm's end-market competitor also serves as its upstream partner. We further find that the equilibrium of the endogenous pricing game is heavily affected by the market type.

In the ODM market, both the OEM-pricing-early sequential game and the ODM-pricing-early sequential game can be the equilibrium. In this market, neither party has a large incentive to undercut the other's price: The OEM has little incentive because customers prefer the ODM's products, and it is difficult to attract customers by undercutting the price; the ODM has even less incentive because it has revenues from the OEM via contract manufacturing. Consequently, a sequential pricing game can be sustained. Furthermore, we find that the player that prices late may even charge a higher price than the one that prices early. We also show that, between the two sequential games, the OEM-pricing-early game risk-dominates the ODM-pricing-early game. That is, the more uncertain the OEM and ODM are about each other's choice of pricing early or late, the more likely a sequential pricing game with the OEM pricing early becomes.

In the OEM market, either a sequential pricing game or a simultaneous pricing game can be the equilibrium. Their decisions on pricing early or late depend on the OEM's outside option (i.e., the non-competitive ODM's wholesale price) because this outside option determines the allocation of the profit margin between the OEM and its competitive ODM. If this outside option is favorable for the OEM (i.e., if the non-competitive ODM's wholesale price is low), the two parties will choose the sequential pricing game; otherwise, the competition will become fierce, and a simultaneous game will ensue. We also show that the OEM is more likely to prefer pricing early in the OEM market. This helps explain why the OEM usually announces price information earlier than the competitive ODM does in some developed countries/regions (i.e., where the OEM has a large market share).

Besides addressing the two aforementioned main questions, we consider two further questions: Does the competitive ODM have the incentive to reject performing the contract manufacturing service for the OEM? Should the OEM avoid doing business with the competitive ODM and, instead, outsource from non-competitive ODMs? We find that, if the wholesale price of the non-competitive ODMs is sufficiently low, then the OEM always stays in the market. Under this condition, engaging in both self-branded business and contract manufacturing is more beneficial to the competitive ODM than is doing only self-branded business. We also show that the OEM should outsource solely from its competitive ODM even if the competitive ODM charges the same wholesale price as that of the non-competitive ODMs. The underlying reason is that revenue from the upstream CM business tames the competitive ODM and mitigates the competition between the OEM and its competitive ODM in the consumer market. Or, to put it simply, a non-hungry ODM has less incentive to bite the hand that feeds it. In summary, as long as an attractive outside option exists for the OEM (i.e., when the non-competitive ODMs' wholesale price is sufficiently low), the OEM and the competitive ODM should choose to be friends rather than foes.

We organize the remainder of this paper as follows. The literature is reviewed in Section 2. Section 3 introduces the model settings. Sections 4 and 5 analyze the pricing sequence preference of the OEM and its competitive ODM in the ODM market and the OEM market, respectively. We discuss the impacts of market environments, wholesale price negotiation, the competitive ODM's limited capacity, and the outcomes with a general demand model in Section 6. Lastly, Section 7 provides the concluding remarks and suggestions for future research. We relegate the technical details and proofs to the appendix.

2 Related Studies

Studies on co-opetitive supply chains involving both competitive and cooperative parties are closely related. In economics, Spiegel (1993) investigates the manufacturer's market-entry incentives and shows that it is less likely for a manufacturer to become a competitor of its OEM if the latter outsources its production to the former. In operations management, Arya et al. (2007) study a scenario in which a manufacturer opens a direct channel to sell self-branded products and directly compete with the retailer under a Cournot competition setting. Ozkan and Wu (2009a, 2009b) adopt the product life cycle model to study the market-entry timing issue of the ODM and how it allocates capacity between the contracted production and the ODM's own production. Lim and Tan (2010) show that high brand equity can safeguard the OEM and deter the contract manufacturer from entering the same market. Chen et al. (2012) study the price masking behavior in a supply chain comprising an OEM and a contract manufacturer that purchases components for another small OEM. Sodhi and Tang (2013) study Chinese CMs and their tactics for dealing with Western OEMs from 2001 to 2011, and argue that a plus sum game can be achieved when the CM and the OEM cooperate and compete simultaneously.

In particular, Wang et al. (2013) investigate the quantity leadership preference of an OEM and its competitive ODM in a market where the OEM's products are preferred by consumers. We take a different approach by studying the endogenized pricing sequence decisions of an OEM and its competitive ODM, assuming either party's product can be preferred by the consumer. Because we study price competition, which corresponds to short-run competition (Davidson and Deneckere 1986), price-undercutting behavior plays a critical role in the competitors' strategic decisions. We fully explore the impact of this behavior and find that the equilibrium timing outcomes between the OEM and the competitive ODM are significantly different from those stemming from quantity competition. We also find that the market environments, the degree of intensity of downstream competition, the competitive ODM's profit resources and the price war among ODMs heavily influence the equilibrium timing outcomes, and that their roles are different from those found in Wang et al. (2013).

Studies on dual-channel management are also related. Tsay and Agrawal (2004) provide an extensive literature review of this research stream. In particular, Chiang et al. (2003) investigate a two-stage supply chain in which the supplier sells directly to the customers, and hence, becomes a competitor of its wholesale retailer. Yao and Liu (2005) study whether a manufacturer should open a direct channel and they consider two pricing schemes: simultaneous pricing and manufacturer-leader-retailer-follower pricing. Yao et al. (2009) compare three inventory strategies for a dual-channel supply chain: the centralized strategy, the decentralized strategy where the manufacturer (retailer) determines the direct (retail) channel quantity, and the outsourcing strategy where the direct channel inventory is managed by a logistics company. Caliskan-Demirag et al. (2011) explore the impact of both customer rebate and dealer rebate on the channel members' performance. Recently, Yoo and Lee (2011) consider different channel structures in which an Internet channel and/or a physical store channel can be added. They show that market conditions may determine whether retail prices will be reduced. Chen et al. (2012) examine the conditions under which the manufacturer and the retailer achieve Pareto-improvement when they operate in a dual-channel supply chain. Ryan et al. (2013) consider demand uncertainty and price competition in a dual channel supply chain and propose contracts for channel coordination.

Our work is also related to the literature on the endogenous timing game and its application. First proposed by Hamilton and Slutsky (1990), the endogenous timing game involves a two-stage framework for firms to endogenously determine their moving sequence: simultaneous or sequential. van Damme and Hurkens (1999) consider a duopoly quantity competition model and show that a low-cost firm will assume endogenized Stackelberg leadership. van Damme and Hurkens (2004) further consider a duopoly price competition model and show that it is more likely for the low-cost firm to be the Stackelberg price leader. Pastine and Pastine (2004) examine the impact of delay cost and reaction time on firms' endogenized pricing sequence preference. Amir and Stepanova (2006) apply the risk-dominance concept in a Bertrand duopoly setting and show that a low-unit-cost firm prefers pricing early. Wang et al. (2014) apply the endogenous timing game to a setting in which two firms make their respective choices to be either efficient (i.e., make quantity decisions before demand uncertainty is removed) or responsive (i.e., make quantity decisions after demand uncertainty is removed). They find that both *being efficient* and *being responsive* can sustain as the Nash equilibrium (NE) under conditions with respect to the degrees of demand uncertainty and/or competition intensity. For the other applications of the endogenous timing game in the field of operation management, see, e.g., Li et al. (2002) and Xie and Ai (2006). In contrast to existing studies, we apply the endogenous timing game to investigate the pricing sequence preferences of an OEM and its competitive ODM whose profit is generated from selling self-branded products and contract manufacturing for the OEM.

3 Model Setting

3.1 Notations and assumptions

We consider an OEM (labeled o) that outsources its entire production function to ODMs (labeled m). For simplicity, we assume that there exists a competitive ODM (representative of all competitive ODMs) that competes with the OEM in the consumer market as well as a non-competitive ODM (representative of all non-competitive ODMs) that engages only in contract manufacturing. The OEM allocates its production orders to the competitive ODM (in a proportion labeled as θ) and the non-competitive ODM (in a proportion labeled as $(1-\theta)$). Correspondingly, the competitive ODM and the non-competitive ODM will charge a unit wholesale price w and $\bar{w}$ for their services, respectively.

We assume that $\bar{w}$ is exogenously given. This is a reasonable assumption because non-competitive ODMs, such as Foxconn, Quanta, and Compal, are often involved in tense price wars, and their thin profit margin leaves little room for them to adjust prices (Schofield 2010). Nevertheless, we consider w, the competitive ODM's unit wholesale price, as a decision variable. Note that the competitive ODM has additional revenue from selling its self-branded products. This enables it to tolerate an even lower profit margin from contract manufacturing than the non-competitive ODMs. We also assume that the unit production costs for the products of the competitive ODM and the OEM are similar, and, thus, normalize them to zero. (The overall conclusions still hold if we consider positive production costs.) This assumption is reasonable because these two products are likely to be made on the *same* production line with identical design, production quality, and inspection requirements. For example, the Sony Ericsson XPERIA X1 handset and the HTC Touch were produced on the same manufacturing platform–HTC TouchFLO (Tofel 2008).

The OEM and the competitive ODM engage in a price competition in the downstream consumer market. Each firm produces a differentiated but substitutable product. The production quantities are jointly determined by their respective product prices, i.e., via the direct demand functions, and the reaction functions are both upward-sloping. In other words, the incumbent's demand increases as the competitor's price increases[†]. The demand for firm i's product, $i = o, m$ can be defined as follows:

$$q_i(p_i, p_j) = a - p_i + b_i p_j, \quad i, j = o, m; \quad i \neq j, \tag{1}$$

where a represents the market potential, p_i is its selling price, and q_i is its production quantity (effective demand)[‡]. Let $b_i \in (0, 1), i = o, m$ represent firm i's marketing power, which measures its efforts made in brand recognition, sales promotion, distribution network establishment as well as

[†]Similar settings have been widely adopted in the OM/marketing literature. See Bernstein and Federgruen (2004a) for a review of studies in this area.

[‡]We study the general case with $q_i(p_i, p_j) = a_i - \alpha_i p_i + b_i p_j, \quad i, j = o, m; \quad i \neq j$, in Section 6.4.

its familiarity with local consumers' purchasing behavior. b_i also measures the price competition intensity because the marketing powers of the competitors indicate the intensity of price competition in the downstream market. See Xiao and Qi (2008), Ha et al. (2011) and Shang et al. (2012) for a more in-depth discussion. Note that different b_is can result in different market share allocation. For example, if $p_o = p_m = p$ and $b_o > b_m$, then the demand for the OEM's products is $a-(1-b_o)p$, larger than that of the competitive ODM. Thus the OEM occupies a larger market share.

In this paper, to enhance our understanding of the consumer market, we consider two stylized market environments: the ODM market and the OEM market. In the former, consumers prefer the competitive ODM's product over that of the OEM; thus, we assume $b_o = b \leq 1$ and normalize b_m to one. Consequently, the profit functions of the OEM and the competitive ODM can be written as follows:

$$\Pi_o(p_o) = (a - p_o + bp_m)(p_o - \theta w - (1-\theta)\bar{w}), \tag{2}$$

$$\Pi_m(p_m) = (a - p_m + p_o)p_m + \theta w(a - p_o + bp_m). \tag{3}$$

Note that the competitive ODM's profit function can be divided into two parts. The first is the profit generated from the self-branded business, while the second is the profit from the contract manufacturing business. In contrast, in the OEM market, consumers usually consider the stability and reliability of brands more than other factors (Pinedo et al. 2008). Thus, they tend to prefer the OEM's product over that of the ODM. Therefore, we normalize $b_o = 1$ and assume $b_m = b \leq 1$. Thus, the profit functions of the OEM and the ODM are changed to

$$\Pi_o(p_o) = (a - p_o + p_m)(p_o - \theta w - (1-\theta)\bar{w}), \tag{4}$$

$$\Pi_m(p_m) = (a - p_m + bp_o)p_m + \theta w(a - p_o + p_m). \tag{5}$$

We then analyze the price competition between the OEM and the competitive ODM in the ODM market in Section 4 and that in the OEM market in Section 5, respectively.

3.2 Endogenous pricing game

When determining the selling prices of their end products, the OEM and the competitive ODM can decide either simultaneously or sequentially. In practice, although such decisions are rarely made at exactly the same time, they can be viewed as the outcomes of a simultaneous game if there is no communication among the players. Meanwhile, in a sequential game, two scenarios can arise: either the OEM or the competitive ODM decides its product selling price first. We denote the action of choosing the selling price *"early"* by E and that of choosing the selling price *"late"* by L. Let $\beta = (\beta_o, \beta_m)$ denote the two players' joint strategies. $\beta \in \{(E,E),(L,L),(E,L),(L,E)\}$. Note that, if both the OEM and the competitive ODM prefer making their pricing decisions early/late, then a simultaneous game will be played; otherwise, it is a sequential (Stackelberg) game with both parties assuming their respective roles. Denote the simultaneous basic game as S. Then Π_i^S is firm i's profit and p_o^S and p_m^S are the selling prices under S. Similarly, Π_i^E (Π_i^L) is firm i's profit when its retail price is determined early (late), and the corresponding product price is denoted as p_i^E (p_i^L), $i = o, m$.

We adopt the endogenous timing game to derive the endogenized pricing sequence. The two-stage game is illustrated by Figure 1. In stage 1, the players decide to price either early or late. In stage 2, a corresponding game is played: if they both make the same choice, then a simultaneous game is played; otherwise, a sequential game is played. By comparing the payoffs under different basic games, we can obtain the equilibrium outcomes of the endogenous pricing game, which reveals players' endogenized pricing sequence choice.

Note that, if multiple equilibria exist, the well-known criterion "*risk dominance*" can be used to select the globally optimal one (Harsanyi and Selten 1988, van Damme and Hurkens 2004, Amir and Stepanova 2006). Under the "risk dominance" criterion, players measure the risk from the viewpoint of the strategic uncertainty about their rivals' moves when they face multiple equilibria. The equilibrium that minimizes such a risk can allocate the total game surplus well and hence coordinate the players' expectations (van Damme and Hurkens 2004, Amir and Stepanova 2006). Extensive experimental studies by Van Huyck et al. (1990), Cooper et al. (1990), and Cabrales et al. (2000), show that the risk-dominant equilibrium is often predicted more successfully than the payoff-dominant one. This finding is also valid in evolutionary games (Kandori et al. 1993, Ellison 1993, Amir and Stepanova 2006). The definition of risk dominance can be illustrated by the following example. Suppose two equilibria (E, L) and (L, E) coexist. If the following is true

$$(\Pi_i^E - \Pi_i^S)(\Pi_j^L - \Pi_j^S) \geq (\Pi_i^L - \Pi_i^S)(\Pi_j^E - \Pi_j^S),$$

then it can be said that (E, L) risk-dominates (L, E). This concept has been widely adopted in the literature on endogenous timing game (see, e.g., Cabrales et al. 2000, van Damme and Hurkens 2004, and Amir and Stepanova 2006).

4 The ODM Market

In the ODM market, (2) and (3) are the profit functions of two players, respectively. We assume that the competitive ODM determines the contract manufacturing wholesale price first and after that the OEM decides how much to outsource from the competitive ODM, i.e., θ. A similar assumption has been adopted in the OM/marketing literature (see, e.g., Lariviere and Porteus 2001, and Wang et al. 2013). This assumption has been justified by industrial observations as well. For example, Asus, a Taiwan-based competitive ODM determines its contract manufacturing fee and reveals this information to its OEMs, which then make their own decisions on whether to outsource contract manufacturing to Asus(Lin 2005). Below, we first derive the equilibrium product

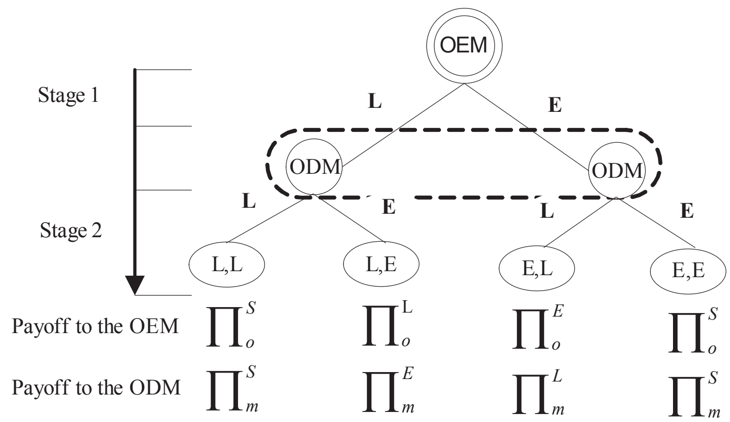

Figure 1: Illustration of the Endogenous Pricing Game

retail prices, optimal contract manufacturing wholesale price w, production outsourcing proportion θ and corresponding profits for both parties under each basic game by backward induction. We then apply the endogenous pricing game to investigate their pricing sequence decisions. In the interest of saving space, we include the detailed proofs in the online Appendix. Let superscript $*$ denote the optimal solutions.

4.1 Summary of the three basic games

The game outcomes are provided in Proposition 1. To ensure the co-existence of the OEM and the competitive ODM in the end market, $\bar{w} < a/(1-b)$ is required. Thus in the following analysis, we assume $\bar{w} < a/(1-b)$.

Proposition 1. *In the ODM market, the outcomes of three basic games are shown in Table 1.*

Table 1: Outcomes of three basic games in the ODM market

Simultaneous game	OEM-pricing-early game	ODM-pricing-early game
$w^{S*} = \bar{w}$, $\theta^* = 1$	$w^{L*} = \bar{w}$, $\theta^* = 1$	$w^{E*} = \bar{w}$, $\theta^* = 1$
$p_o^{S*} = \frac{(2+b)a+(2+b^2)\bar{w}}{4-b}$ $p_m^{S*} = \frac{3a+(1+2b)\bar{w}}{4-b}$	$p_o^{E*} = \frac{(2+b)a+(2-b+b^2)\bar{w}}{2(2-b)}$ $p_m^{L*} = \frac{(6-b)a+(2+3b-b^2)\bar{w}}{4(2-b)}$	$p_o^{L*} = \frac{(4+b)a+(4-b+b^2)\bar{w}}{4(2-b)}$ $p_m^{E*} = \frac{3a+(1+b)\bar{w}}{2(2-b)}$
$q_o^{S*} = \frac{(2+b)[a-(1-b)\bar{w}]}{4-b}$ $q_m^{S*} = \frac{3a+(1-2b+b^2)\bar{w}}{4-b}$	$q_o^{E*} = \frac{(2+b)[a-(1-b)\bar{w}]}{4}$ $q_m^{L*} = \frac{(6-b)a+(2-5b+3b^2)\bar{w}}{4(2-b)}$	$q_o^{L*} = \frac{(4+b)[a-(1-b)\bar{w}]}{4(2-b)}$ $q_m^{E*} = \frac{3a+(1-b)\bar{w}}{4}$
$\Pi_o^{S*} = \frac{(2+b)^2[a-(1-b)\bar{w}]^2}{(4-b)^2}$ $\Pi_m^{S*} = \frac{(2+b)[a-(1-b)\bar{w}]\bar{w}}{4-b}$ $+\frac{[3a+(1-2b+b^2)\bar{w}][3a+(1+2b)\bar{w}]}{(4-b)^2}$	$\Pi_o^{E*} = \frac{(2+b)^2[a-(1-b)\bar{w}]^2}{8(2-b)}$ $\Pi_m^{L*} = \frac{(2+b)[a-(1-b)\bar{w}]\bar{w}}{4}$ $+\frac{[(6-b)a+(2-5b+3b^2)\bar{w}][(6-b)a+(2+3b-b^2)\bar{w}]}{16(2-b)^2}$	$\Pi_o^{L*} = \frac{(4+b)^2[a-(1-b)\bar{w}]^2}{16(2-b)^2}$ $\Pi_m^{E*} = \frac{(4+b)[a-(1-b)\bar{w}]\bar{w}}{4(2-b)}$ $+\frac{[3a+(1-b)\bar{w}][3a+(1+b)\bar{w}]}{8(2-b)}$

Based on the forgoing outcomes, we offer the following proposition regarding the co-opetition between the OEM and the competitive ODM in the ODM market.

Proposition 2. *In the ODM market, if $\bar{w} \geq a/(1-b)$, then the OEM has to exit the market. Otherwise,*

(i) The competitive ODM tends to charge a wholesale price equal to that of the non-competitive ODM's; that is, $w^* = \bar{w}$.

(ii) The OEM sources solely from the competitive ODM; that is, $\theta^* = 1$.

(iii) The OEM's (competitive ODM's) profit is decreasing (increasing) in w^*.

Proposition 2 provides the following three insights. First, when outsourcing from the non-competitive ODM instead of the competitive one is costly such that $\bar{w} \geq a/(1-b)$, then the OEM could be expelled from the market. This is consistent with the industry observation that the OEM will choose to sell the product line/brand if the generated revenue is too low and the manufacturing cost is too high. For example, Philips sold its LCD TV business to TPV, a Chinese competitive ODM. Philips had been suffering from bad performance (i.e., small market share) in the Chinese market for an extended period of time (Xu 2010). Empirical work by Walker and Weber (1984) also supports our finding: high production costs may force OEMs to stop doing business. We note that the threshold value $a/(1-b)$ is increasing in a and b. Thus, the OEM is more likely to leave the ODM market when the market size a is small and/or it has weak marketing power b .

Second, when the OEM and the competitive ODM coexist in the market ($\bar{w} < a/(1-b)$), then the OEM tends to source *solely* from the competitive ODM at a low wholesale price. This finding can be supported by the story of Asus, a Taiwan-based competitive ODM. Asus got all the production orders of the 14-inch iBook from Apple, by quoting a manufacturing price *lower* than the non-competitive ODM, Quanta. At that time, the prices of Asus and Quanta were all very low due to the intense price war (Lin 2005, Wang et al. 2013). The reason why the OEM sources solely from the competitive ODM is elucidated by Wang et al. (2013). They explain that the contract manufacturing profit loss due to allocating the OEM's production order to the non-competitive ODMs will force the competitive ODM to strengthen its self-branded business, which may utimately hurt the OEM. Similarly, Spiegel (1993) cited the industrial examples of Mazda and Ford as well as Zenith and HP, explaining that horizontal subcontracting among rivals could effectively reduce the competitive ODMs' incentives to develop their own business. Boeing, for example, successfully controlled its competitive ODMs by signing manufacturing contracts with Lockheed, Mitsubishi Heavy Industries, Kawasaki Heavy Industries Ltd., and Fugi Heavy Industries (Chen 2011, Spiegel 1993). Readers, are cautioned, however, that this conclusion is based on pure profit analysis, without considering other issues such as safeguarding the supply chain against disruptions with dual outsourcing strategies; see Tomlin and Wang (2005) for a discussion of the incentives for adopting dual sourcing strategy.

Lastly, the OEM's profit is decreasing in w^* whereas that of the competitive ODM is increasing in w^*. A high wholesale price leads the OEM to reduce its product quantity, which utimately hurts the OEM's performance. However, for the competitive ODM, its profit comes from two resources: the self-branded business and the contract manufacturing service. Although a high wholesale price may lower the ODM's profit generated from contract manufacturing, the equilibrium production quantity and the selling price of its self-branded products are both increasing in w^*. The tradeoff between these two profit sources eventually motivates the competitive ODM to strengthen the self-branded business in the ODM market with a high w^*.

4.2 Endogenized pricing sequence

In this section, based on the results obtained through the three basic games, we compare the equilibrium prices and quantities.

Proposition 3. *In the ODM market, we obtain the following comparison results.*

(1) Under the simultaneous game, $p_o^{S*} \leq p_m^{S*}$ *and* $q_o^{S*} < q_m^{S*}$.

(2) Under the OEM-pricing-early game, $q_o^{E*} < q_m^{L*}$. *And* $p_m^{L*} \leq p_o^{E*}$ *if* $b \in [2/3, 1]$*; otherwise,* $p_m^{L*} > p_o^{E*}$.

(3) Under the ODM-pricing-early game, $p_o^{L*} < p_m^{E*}$. *And* $q_o^{L*} > q_m^{E*}$ *if* $b \in (1/2, 1]$ *and* $\bar{w} < (2b-1)a/3(1-b)$*; otherwise,* $q_o^{L*} \leq q_m^{E*}$.

Proposition 3 shows that, in the simultaneous game, the competitive ODM charges a higher retail price and sells more than the OEM does. The main intuition behind this is the ODM's market dominance. Proposition 3 also shows that the player that prices late can charge a higher price than that of the player who prices early. This result is significantly different from that of the traditional Bertrand game, in which the player who prices late tends to undercut the earlier price (Gal-Or 1985). For instance, in the OEM-pricing-early game, as the OEM's sales quantity is low ($q_o^{E*} < q_m^{L*}$) and its marketing power is also low, the competitive ODM can charge a higher price than the OEM can; see Figure 2 for an illustration. Because of the complex competitor-partner relationship between the OEM and its competitive ODM, price undercutting is not always the best response for a player: the ODM has little incentive to do so, as undercutting the price will indirectly reduce the gain from the contract manufacturing business; the OEM has little incentive to do so, as it may encounter tough bargaining with the ODM on the wholesale price for the contract manufacturing service.

Next, we derive the endogenized pricing sequences of the OEM and its competitive ODM assuming that they coexist in the market, and obtain the following proposition.

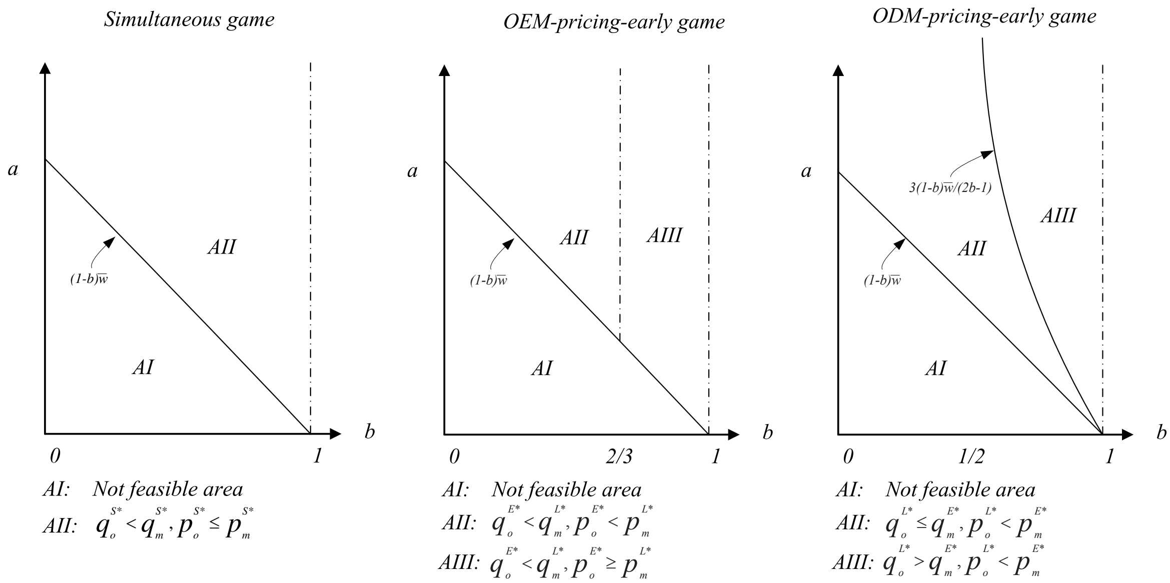

Figure 2: Pricing and Quantity Equilibria in the ODM Market

Proposition 4. *Comparing the three basic games yields the following*

(1) Neither (L, L) nor (E, E) is the equilibrium.

(2) Both (E, L) and (L, E) are SPNE. There also exists a mixed-strategy equilibrium whereby the OEM and the ODM randomize over E and L.

It is generally believed that a simultaneous pricing game represents a fiercer competition than a sequential pricing game, which can be regarded as a form of collusion between players. Yano and Komatsubara (2006) refer to sequential pricing as "implicit collusion". Proposition 4 clearly demonstrates that the partnership between the OEM and its competitive ODM mitigates the competition between them in the consumer market and allows them to choose a more collusive game, a sequential pricing game, in the ODM market.

Next we compare these two NE, (E, L) and (L, E), from the perspectives of risk dominance, a concept for refining NE proposed by Harsanyi and Selten (1988). With risk dominance, the idea is to *"pick the equilibrium that has the largest basin of attraction in the initial beliefs players ascribe to each other's behavior... In other words, it minimizes the risk of a coordination failure due to strategic uncertainty"* (Amir and Stepanova 2006). As we have explained in Section 3.2, the equilibrium selected by risk dominance better coordinates the players' expectations and hence, is more frequently observed in experimental studies. Therefore, the globally optimal equilibrium will be sufficiently stable.

Proposition 5. *In the ODM market, the NE (E, L) strictly risk dominates the NE (L, E).*

Proposition 5 shows that the more uncertainty the OEM and the competitive ODM have about each other's actions, the more likely they are to choose the NE (E, L). That is, the OEM-pricing-early game is less risky and more likely to appear in the ODM market.

4.3 Conditions for the ODM to retain the contract manufacturing business

Here we examine whether the competitive ODM should stop the contract manufacturing and concentrate solely on its self-branded business. We aim to investigate whether this motion would ultimately be beneficial for the ODM. When the OEM allocates all of its the production orders to the non-competitive ODMs, the corresponding profit functions can be written as follows:

$$\Pi_o(p_o) = (a - p_o + bp_m)(p_o - \bar{w}), \tag{6}$$

$$\Pi_m(p_m) = (a - p_m + p_o)p_m. \tag{7}$$

By assuming that the OEM shifts all the production orders to the non-competitive ODMs, we resolve the three basic games. The game results are included in Proposition 6.

Proposition 6. *(1) For the simultaneous game, if $\bar{w} < (2+b)a/(2-b)$, then*

(i) $p_o^{SN} = \frac{(2+b)a+2\bar{w}}{4-b}$; $p_m^{SN} = \frac{3a+\bar{w}}{4-b}$.

(ii) $\Pi_o^{SN} = \frac{[(2+b)a-(2-b)\bar{w}]^2}{(4-b)^2}$; $\Pi_m^{SN} = \frac{[3a+\bar{w}]^2}{(4-b)^2}$.

(2) For the OEM-pricing-early game, if $\bar{w} < (2+b)a/(2-b)$, then

(i) $p_o^{EN} = \frac{(2+b)a+(2-b)\bar{w}}{2(2-b)}$; $p_m^{LN} = \frac{(6-b)a+(2-b)\bar{w}}{4(2-b)}$.

(ii) $\Pi_o^{EN} = \frac{[(2+b)a-(2-b)\bar{w}]^2}{8(2-b)}$; $\Pi_m^{LN} = \frac{[(6-b)a+(2-b)\bar{w}]^2}{16(2-b)^2}$

(3) For the ODM-pricing-early game, if $\bar{w} < (4+b)a/(4-3b)$, then

(i) $p_o^{LN} = \frac{(4+b)a+(4-b)\bar{w}}{4(2-b)}$; $p_m^{EN} = \frac{3a+\bar{w}}{2(2-b)}$.

(ii) $\Pi_o^{LN} = \frac{[(4+b)a-(4-3b)\bar{w}]^2}{16(2-b)^2}$; $\Pi_m^{EN} = \frac{[3a+\bar{w}]^2}{8(2-b)}$.

Thus, we obtain the forgoing proposition in which the superscript N represents "No contract manufacturing service from the competitive ODM". Note that the assumption $\bar{w} < (2+b)a/(2-b)$ (under both the simultaneous game and the OEM-pricing-early game) and $\bar{w} < (4+b)a/(4-3b)$ (under the ODM-pricing-early game) are needed to guarantee the positive profit margins of the OEM. We fix the pricing sequence, compare the competitive ODM's profits with and without the contract manufacturing service, and obtain the following finding.

Proposition 7. *In the ODM market, the competitive ODM prefers being a co-opetitor to being a pure competitor.*

Proposition 7 shows that, although its self-branded product is preferred by the customer, retaining the contract manufacturing business and earning revenue from both the product sales and the manufacturing service is the optimal choice for the competitive ODM. This result may explain why competitive ODMs try very hard to secure OEM's contract manufacturing orders. For example, Asus's chairman Johnny Shih told the shareholders that, although the branding business seems profitable, maintaining manufacturing strength is also Asus's core competitiveness (Taipei Times 2004). Consequently, competitive ODMs, such as Asus, Micro Star International (MSI), Daphne and Chang'an Automobile have chosen to generate profits from dual sources (Shih et al. 2009, Cheng 2010, Sodhi and Tang 2013).

5 The OEM Market

In the OEM market, consumers purchase based on the stability/reliability of the brands (Pinedo et al. 2008). (4) and (5) are the two players' respective profit functions, where $b_o = 1$ and $b_m = b \leq 1$.

5.1 Results for three basic games and the endogenized pricing sequence

To derive the endogenized pricing sequence choices of the OEM and the competitive ODM in the OEM market, we first derive the optimal selling prices and the wholesale prices under the three basic games. The analysis is similar to that used for the ODM market. The details are provided in Proposition 8.

Proposition 8. *Under three basic games, define* $w^S = \frac{3(4+b+b^2)}{2(1-b)(8+b)}a$, $w^L = \frac{24-12b+5b^2-b^3}{(1-b)(32-16b-b^2+b^3)}a$, *and* $w^E = \frac{6+b+b^2}{(1-b)(7+b)}a$, *then the outcomes are shown in Table 2.*

Based on the outcomes, the following proposition arises.

Proposition 9. *In the OEM market, the OEM sources solely from the competitive ODM. However, the equilibrium wholesale price in the OEM market can be lower than that in the ODM market; that is,* $w^* \leq \bar{w}$.

As with the ODM market, in the OEM market, the OEM sources solely from the competitive ODM. Chen and Chen (2011) show a similar finding. However, dampened by the OEM's strong marketing power in the OEM market, the competitive ODM tends to charge a lower wholesale price for its manufacturing service in exchange for a larger order quantity.

Next, by comparing the equilibrium product retail prices (provided in Appendix C) we find the following.

Proposition 10. *In the OEM market:*

(1) Under the simultaneous game, $p_m^{S*} < p_o^{S*}$ *and* $q_m^{S*} \leq q_o^{S*}$.

Table 2: Outcomes of three basic games in the OEM market

Simultaneous game ($\bar{w} \geq w^S$)	Simultaneous game ($\bar{w} < w^S$)
$w^{S*} = w^S$, $\theta^* = 1$	$w^{S*} = \bar{w}$, $\theta^* = 1$
$p_o^{S*} = \frac{3(7-b)}{2(1-b)(8+b)}a$ $p_m^{S*} = \frac{(2+b)(7-b)}{2(1-b)(8+b)}a$	$p_o^{S*} = \frac{3(a+\bar{w})}{4-b}$ $p_m^{S*} = \frac{(2+b)(a+\bar{w})}{4-b}$
$q_o^{S*} = \frac{3(3+b)}{2(8+b)}a$ $q_m^{S*} = \frac{1+2b}{8+b}a$	$q_o^{S*} = \frac{3a-(1-b)\bar{w}}{4-b}$ $q_m^{S*} = \frac{(2+b)a-2(1-b)\bar{w}}{4-b}$
$\Pi_o^{S*} = \frac{9(3+b)^2}{4(8+b)^2}a^2$ $\Pi_m^{S*} = \frac{17+14b+5b^2}{4(1-b)(8+b)}a^2$	$\Pi_o^{S*} = \frac{[3a-(1-b)\bar{w}]^2}{(4-b)^2}$ $\Pi_m^{S*} = \frac{(2+b)(a+\bar{w})[(2+b)a-2(1-b)\bar{w}]}{(4-b)^2} + \frac{[3a-(1-b)\bar{w}]\bar{w}}{4-b}$
OEM-pricing-early game ($\bar{w} \geq w^L$)	OEM-pricing-early game ($\bar{w} < w^L$)
$w^{L*} = w^L$, $\theta^* = 1$	$w^{L*} = \bar{w}$, $\theta^* = 1$
$p_o^{E*} = \frac{42-30b+3b^2+b^3}{(1-b)(32-16b-b^2+b^3)}a$ $p_m^{L*} = \frac{28-9b-5b^2+2b^3}{(1-b)(32-16b-b^2+b^3)}a$	$p_o^{E*} = \frac{3a+(3-b)\bar{w}}{2(2-b)}$ $p_m^{L*} = \frac{(4+b)a+(4+b-b^2)\bar{w}}{4(2-b)}$
$q_o^{E*} = \frac{18-9b-2b^2+b^3}{32-16b-b^2+b^3}a$ $q_m^{L*} = \frac{4+7b-3b^2}{32-16b-b^2+b^3}a$	$q_o^{E*} = \frac{3a-(1-b)\bar{w}}{4}$ $q_m^{L*} = \frac{(4+b)a-(1-b)(4-b)\bar{w}}{4(2-b)}$
$\Pi_o^{E*} = \frac{2(18-9b-2b^2+b^3)^2}{(2-b)(32-16b-b^2+b^3)^2}a^2$ $\Pi_m^{L*} = \frac{17-b^3}{(1-b)(32-16b-b^2+b^3)}a^2$	$\Pi_o^{E*} = \frac{[3a-(1-b)\bar{w}]^2}{8(2-b)}$ $\Pi_m^{L*} = \frac{[(4+b)a-(1-b)(4-b)\bar{w}][(4+b)a+(4+b-b^2)\bar{w}]}{16(2-b)^2} + \frac{[3a-(1-b)\bar{w}]\bar{w}}{4}$
ODM-pricing-early game ($\bar{w} \geq w^E$)	ODM-pricing-early game ($\bar{w} < w^E$)
$w^{E*} = w^E$, $\theta^* = 1$	$w^{E*} = \bar{w}$, $\theta^* = 1$
$p_o^{L*} = \frac{9-b}{(1-b)(7+b)}a$ $p_m^{E*} = \frac{5+3b}{(1-b)(7+b)}a$	$p_o^{L*} = \frac{(6-b)a+(5-b)\bar{w}}{4(2-b)}$ $p_m^{E*} = \frac{(2+b)a+(1+b)\bar{w}}{2(2-b)}$
$q_o^{L*} = \frac{3+b}{7+b}a$ $q_m^{E*} = \frac{2(1+b)}{7+b}a$	$q_o^{L*} = \frac{(6-b)a-3(1-b)\bar{w}}{4(2-b)}$ $q_m^{E*} = \frac{(2+b)a-(1-b)\bar{w}}{4}$
$\Pi_o^{L*} = \frac{(3+b)^2}{(7+b)^2}a^2$ $\Pi_m^{E*} = \frac{4+3b+b^2}{(1-b)(7+b)}a^2$	$\Pi_o^{L*} = \frac{[(6-b)a-3(1-b)\bar{w}]^2}{16(2-b)^2}$ $\Pi_m^{E*} = \frac{[(2+b)a-(1-b)\bar{w}][(2+b)a+(1+b)\bar{w}]}{8(2-b)} + \frac{[(6-b)a-3(1-b)\bar{w}]\bar{w}}{4(2-b)}$

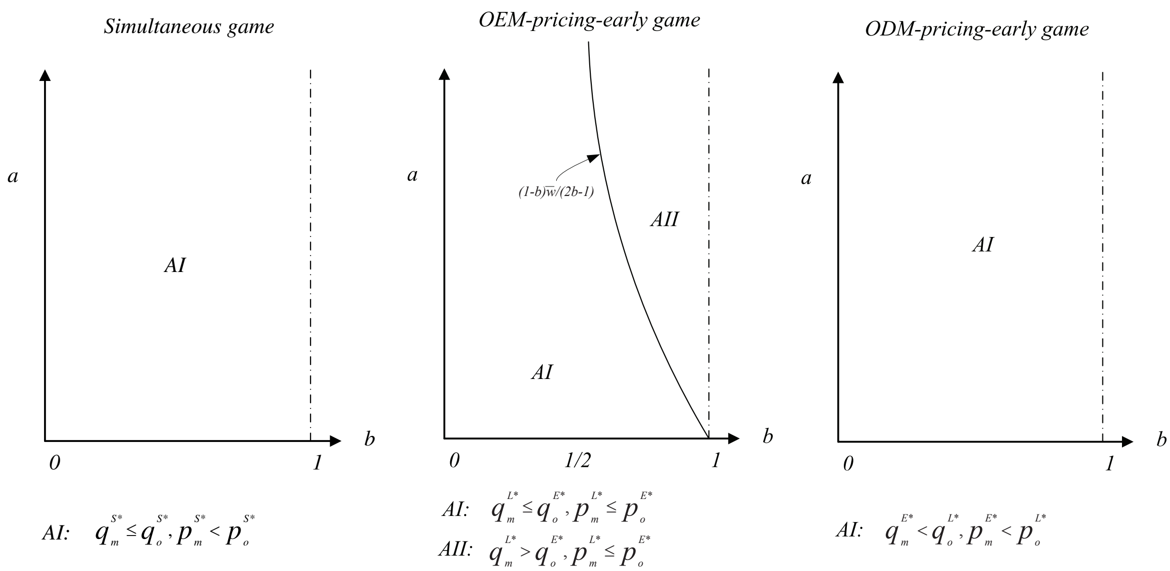

Figure 3: Pricing and Quantity Equilibrium in the OEM Market

(2) Under the OEM-pricing-early game, $p_m^{L*} \le p_o^{E*}$*. And* $q_o^{E*} < q_m^{L*}$ *if* $b \in (1/2, 1]$ *and* $\bar{w} < \frac{2b-1}{1-b}a$*; otherwise,* $q_o^{E*} \ge q_m^{L*}$*.*

(3) Under the ODM-pricing-early game, $p_m^{E*} \le p_o^{L*}$ *and* $q_m^{E*} \le q_o^{L*}$*.*

Proposition 10 shows that in the OEM market, no matter which basic game is played, the OEM's equilibrium product retail price is higher than that of the competitive ODM. Interestingly, in the consumer market, when the ODM makes its pricing decision early, the OEM can obtain a larger market share. The reason is as follows. In the OEM market, the ODM's product is less preferred by the customer, so the ODM mainly relies on the contract manufacturing business to generate profits. When the ODM moves first, q_o^{L*} can be viewed as the ODM's residual demand (Gal-Or 1985). Because the OEM's product is more preferred, it naturally follows that $q_m^{E*} \le q_o^{L*}$. Then the OEM enjoys a second-mover advantage and extracts a larger market share. This also benefits the ODM by increasing its contract manufacturing order quantity, and yields a win-win situation for both the OEM and the ODM. Under the OEM-pricing-early game, the ODM may achieve a larger market share by selling cheaply when its marketing power is relatively high ($b > 1/2$) and the market size is sufficiently large ($a \ge \frac{1-b}{2b-1}\bar{w}$). See Figure 3 for an illustration.

Next, we derive the endogenized pricing sequence choices of the OEM and its competitive ODM in the OEM market. To characterize the pricing sequence equilibrium, let

$$w_E = \frac{b(32 - 5b^2)}{(1-b)(64 - 16b - 8b^2 + b^3)}a; \quad w_L = \frac{b(2+b)}{(1-b)(4+b)}a.$$

It can be verified that $w_L \le w_E$. Thus, we obtain the following proposition.

Proposition 11. *In the OEM market, E dominates the other strategies when* $\bar{w} > w_E$*; (E, L) is the unique equilibrium when* $\bar{w} \in [w_L, w_E]$*; and (E, L) and (L, E) are the two pure Nash equilibria when* $\bar{w} < w_L$*.*

Proposition 11 shows that, in an OEM market, all the basic games can arise as a result of the endogenous pricing sequence choice selection. This result is different from that found for the ODM market where (L,L) and (E,E) can never be the NE, and the simultaneous game will not be played (Proposition 4). Here, because the outside option cost $\bar{w}$ is high, the competitive ODM can charge relatively high wholesale price, and the profit margin for the OEM is relatively low. Recall that an important advantage of pricing late is the ability to undercut the earlier price. Nevertheless, in this situation, neither would like to make its pricing decisions late: the ODM has little incentive because its main revenue comes from the contract manufacturing business, while the OEM has few incentive because its profit margin is very small, and hence reducing the price is not beneficial. Consequently, we observe that (E, E) appears. As $\bar{w}$ is reduced, the OEM can obtain a larger profit margin, and the ODM obtains less revenue from the contract business, making the ODM more willing to price late to undercut the OEM's price. We therefore observe (E, L) in this range. Because $\bar{w}$ is further reduced, the OEM also has a motivation to price late as the OEM has a sufficiently large profit margin and can therefore decrease its price. As a result, we obtain two equilibria. In summary, in the OEM market, the OEM's outside option has a vital impact on the pricing sequence game that the OEM and the ODM will play: As this outside option becomes less favorable to the OEM, the competition between the OEM and the ODM becomes fiercer, and a sequential pricing game becomes less likely.

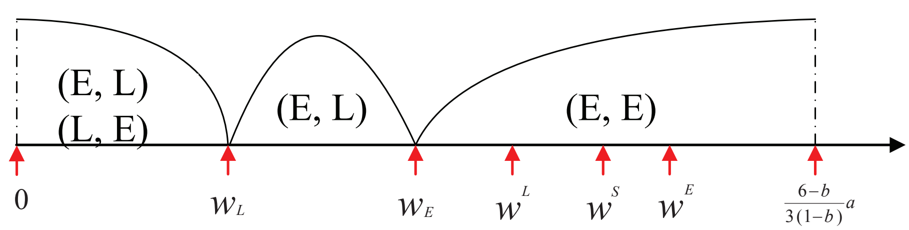

Figure 4: Impact of $\bar{w}$ on the Price Timing Equilibrium

5.2 Conditions for the ODM to retain the contract manufacturing business

As in Section 4.3, we study whether the competitive ODM should keep its contract manufacturing business in the OEM market. If the competitive ODM terminates the contract manufacturing service, then the profit functions of the ODM and the OEM are as follows:

$$\Pi_o(p_o) = (a - p_o + p_m)(p_o - \bar{w}), \quad (8)$$

$$\Pi_m(p_m) = (a - p_m + bp_o)p_m. \quad (9)$$

We first solve the three basic games by assuming that the competitive ODM does not conduct the contract manufacturing service. The following results can be derived analogously to those in the ODM market. Note that, to guarantee the positive profit margin of the OEM, the following constraints are required: $\bar{w} \leq \frac{3}{2-b}a$ (for the simultaneous game and the OEM-pricing-early game) and $\bar{w} \leq \frac{6-b}{4-3b}a$ (for the ODM-pricing-early game).

Proposition 12. *In the OEM market, assume that the competitive ODM now is a pure OEM. For the simultaneous game, if $\bar{w} < 3a/(2-b)$, then*

(1) $p_o^{SN} = \frac{3a+2\bar{w}}{4-b}$; $p_m^{SN} = \frac{(2+b)a+b\bar{w}}{4-b}$.

(2) $\Pi_o^{SN} = \frac{[3a-(2-b)\bar{w}]^2}{(4-b)^2}$; $\Pi_m^{SN} = \frac{[(2+b)a+b\bar{w}]^2}{(4-b)^2}$.

For the OEM-pricing-early game, if $\bar{w} < 3a/(2-b)$, then

(1) $p_o^{EN} = \frac{3a+(2-b)\bar{w}}{2(2-b)}$; $p_m^{LN} = \frac{(4+b)a+b(2-b)\bar{w}}{4(2-b)}$.

(2) $\Pi_o^{EN} = \frac{[3a-(2-b)\bar{w}]^2}{8(2-b)}$; $\Pi_m^{LN} = \frac{[(4+b)a+b(2-b)\bar{w}]^2}{16(2-b)^2}$

For the ODM-pricing-early game, if $\bar{w} < (6-b)a/(4-3b)$, then

(1) $p_o^{LN} = \frac{(6-b)a+(4-b)\bar{w}}{4(2-b)}$; $p_m^{EN} = \frac{(2+b)a+b\bar{w}}{2(2-b)}$.

(2) $\Pi_o^{LN} = \frac{[(6-b)a-(4-3b)\bar{w}]^2}{16(2-b)^2}$; $\Pi_m^{EN} = \frac{[(2+b)a+b\bar{w}]^2}{8(2-b)}$.

Comparing the results yields the following proposition.

Proposition 13. *In the OEM market, the competitive ODM prefers being a co-opetitor to being a pure competitor.*

Thus, in the OEM market, a mixed structure in which the competitive ODM engages in both a contract manufacturing business and a self-branded business makes the competitive ODM better off, a result that also holds in the ODM market. Thus, regardless of whether the ODM has strong or weak marketing power, it should continue its contract manufacturing business with the OEM.

6 Discussions

6.1 A comparison of equilibrium outcomes in two markets

From Sections 4 and 5, it is clear that the equilibria of the endogenous timing game are significantly different. That is, in the ODM market, a sequential game sustains as the equilibrium, while in the OEM market, both sequential games and simultaneous games can arise and be sustained. This indicates that the downstream market competition is tenser in the OEM market. Having said that, due to the OEM's large marketing power, we find that the competitive ODM's optimal wholesale price in the OEM market is lower than that in the ODM market, which benefits the OEM by reducing the manufacturing cost. See Proposition 9 and the proofs therein for further details. Considering this, we further compare the outcomes summarized in Propositions 1 and 8 and find that the OEM's retail prices and production quantities are all higher in the OEM market; see Proposition 14. This implies that *the OEM will be strictly better off in a market environment where it has a larger marketing power*. From the customers' perspective, a close look at the equilibrium outcomes (retail prices and supply values) in two markets show that the customers have to pay more for their preferred products, however, they also benefit from a higher product availability rate, which can be viewed as an index of service quality (Chen et al. 2008).

Proposition 14. *The OEM tends to set a higher retail price and provide more products when operating in the OEM market rather than in the ODM market.*

We then conduct extensive numerical studies to see whether it is possible for the ODM and OEM to align their incentives to prefer the OEM market. Our numerical studies show that the OEM market can indeed be by preferred by both parties. As Figure 5 illustrates, the competitive ODM's performance in the OEM market is better when $\bar{w}$ is sufficiently large. The main reason is that the ODM generates considerable profits from its contract manufacturing business. Its production order quantity in the OEM market is significantly larger than that in the ODM market. In addition, its retail price increases along with the OEM's, because, in price competition, the players' retail prices are strategic complements (Amir and Stepanova 2006). Considering these two factors, it is possible to observe that the ODM also prefers the OEM market given a large $\bar{w}$.

6.2 The impact of wholesale price negotiation

As Wang et al. (2013) illustrate, the timing game is generally not tractable when the manufacturing wholesale price is negotiated via a generalized Nash bargaining (GNB) process, rather than determined solely by the competitive ODM. However, we can still provide some qualitative analysis regarding the impact of wholesale price negotiation. Clearly, if we take the OEM's bargaining power into consideration, then the competitive ODM's wholesale price will be lowered. This indicates a

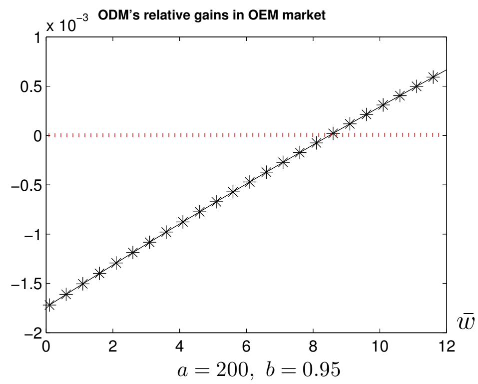

Figure 5: The Competitive ODM's Gains in the OEM Market

larger profit margin for the OEM, which gives the OEM more flexibility to undercut the competitive ODM's retail price, if the OEM acts as the price follower. On the other hand, the competitive ODM may tend to be the price leader when it cannot fully control the manufacturing wholesale price. There are two underlying forces. First, although w is lowered, the production orders from the OEM can be increased, if the OEM acts as the price follower: the OEM's price-undercutting behavior lowers the retail price but enlarges its customer demand, which also benefits the competitive ODM. Second, the competitive ODM's lower profit margin reduces its incentives to undercut the OEM's retail price as the price follower, which may even lower its profit margin. Therefore, we predict that, *considering wholesale price negotiation, the OEM (competitive ODM) has more incentives to act as the price follower (leader).* The forgoing analysis is consistent with that of van Damme and Hurkens (2004), where the more cost-efficient player assumes the price leadership (In our context, the OEM's production cost is w while the competitive ODM's is zero, so the ODM is more cost-efficient.).

6.3 When the competitive ODM is capacitated

When the ODM produces and sells its self-branded products, it is natural to question how it allocates the production capacity between the OEM's contract manufacturing order and its own production demand. We have shown that it is in the best interest of the OEM to source from the competitive ODM when the charged manufacturing wholesale price w is lower than the non-competitive ODMs' $\bar{w}$. Therefore, if the ODM has a capacity constraint K, as long as $w \leq \bar{w}$, the ODM can always increase w to reduce the OEM's order quantity and thus keep the total production demand (q_o+q_m) below K. However, when $w = \bar{w}$, a lower bound quantity is reached. If the OEM's order quantity is large, then the competitive ODM cannot further increase w and hence has to face a production capacity allocation problem.

According to Cachon and Lariviere (1999), there are three main rules for solving this problem: linear, proportional and uniform. They show that, even when a non-competitive linear demand model is assumed, the capacity allocation problem is very complicated. In our context, considering the co-opetitive relationship between the OEM and the competitive ODM, the capacity allocation problem would be exceedingly difficult to solve. Therefore, we leave this as one important future research direction. We predict that our previous finding $\theta^* = 1$ may not necessarily hold when the competitive ODM is capacitated.

6.4 General demand model

Lastly, we study a general demand model in which $q_i(p_i, p_j) = a_i - \alpha_i p_i + b_i p_j$, for $i, j = o, m;\ i \neq j$. Similar models have been widely studied in OM/marketing literature (see, e.g., Ryan et al. 2013). The parameter a_i stands for product i's market potential, α_i stands for product i's own-price sensitivity of demand, and b_i stands for the degree of intensity of market competition. As such, the market heterogeneity can be captured by three forgoing parameters. The ODM market and the OEM market represent two extreme scenarios of this general demand model. We are interested in whether our previous findings under the two extreme cases still hold under this general demand setting. To make the model tractable, we employ an assumption termed "*diagonal dominance*", which is "*highly intuitive and satisfied in most industries*" (Bernstein and Federgruen 2003, Bernstein and Federgruen 2004a, Bernstein and Federgruen 2004b). In our context, this assumption implies that $\alpha_i > b_i$ for $i, j = o, m,\ i \neq j$. Hence, $\alpha_o \alpha_m > b_o b_m$.

Based on the equilibrium outcomes of the three basic games under this general setting, we have the following proposition. (Readers can find the details of the equilibrium outcomes of the three basic games in the proof of Proposition 15; see Appendix.)

Proposition 15. *Under the general demand model, the OEM tends to set* $\theta^* = 1$ *in the three basic games.*

Therefore, the OEM will source solely from the competitive ODM regardless of the demand form. This implies that the results under the two extreme demand scenarios are quite robust. We also examine the competitive ODM's incentives to drop the contract manufacturing business under the general setting, and find the following:

Proposition 16. *Under the general demand model, the competitive ODM prefers being a co-opetitor to being a pure competitor.*

Regarding the endogenous timing game, unfortunately, there are no analytical results. We then conduct extensive numerical studies with respect to the non-competitive ODMs' wholesale price $\bar{w}$, and two critical ratios: a_o/a_m and b_o/b_m. The former ratio stands for the relative market potential difference while the latter stands for the relative marketing power difference.

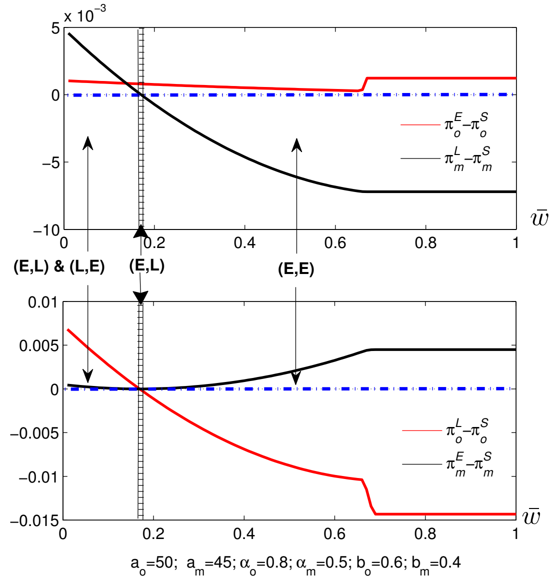

Figure 6: Impact of $\bar{w}$ under General Demand Model

Consistent with our previous findings, there exist two threshold values for $\bar{w}$, by which the OEM and the competitive ODM select their roles of price leadership/followership. That is, when $\bar{w}$ is small, both (E, L) and (L, E) are the equilibria; when $\bar{w}$ is moderate, (E, L) is the unique equilibrium; and when $\bar{w}$ is large, (E, E) is the dominant strategy. See Figure 6 for an illustration of the findings.

The impacts of a_o/a_m and b_o/b_m are also consistent with the previous findings. In the feasible area, when a_o/a_m and b_o/b_m are both small, the results are similar to those in the ODM market. Thus, we observe that (E, L) and (L, E) coexist. When a_o/a_m and b_o/b_m are both large, it reduces to the OEM market, where (E, E), (E, L) and (L, E) possibly arise as the equilibria. Regarding b_o/b_m, we find that there exist two threshold values, between which (E, L) is the unique equilibrium. When b_o/b_m is large, (E, E) dominates. When b_o/b_m is small, (E, L) and (L, E) coexist. Regarding a_o/a_m, we find that it is less likely for (E, E) to arise as the equilibrium when the OEM's product has a market potential sufficiently larger than that of the competitive ODM. Note that a simultaneous game is played when (E, E) is adopted. And (E, E) arises when b_o/b_m is large, that is, when the OEM's product is more competitive over that of the ODM. Thus, a simultaneous game is more likely to appear when the products are highly differentiated. The intuition behind this is that the ODM prefers to move first to induce the OEM to order more while the OEM prefers to move first to take the market advantage because waiting is not beneficial (see the similar explanations for Proposition 11). However, a sequential game will appear if the market potential for the OEM's product is large. Note that when a_o is large while a_m is small, the ODM's product has less market potential. Thus, the ODM mainly relies on its contract manufacturing business to generate profits.

Therefore, the ODM tends to cooperate with the OEM by avoiding the direct competition and thus playing a sequential game (Wang et al. 2013).

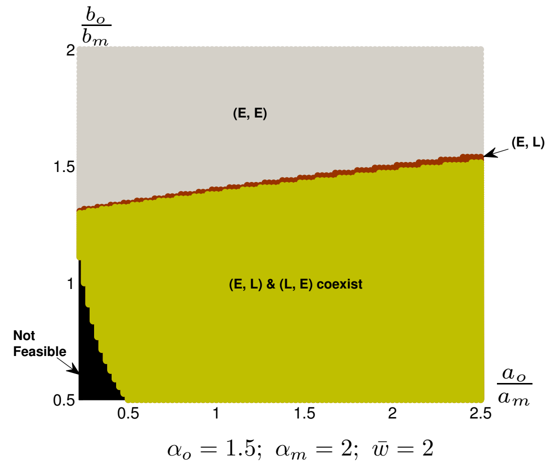

Figure 7: Impact of Relative Market Potential and Substitutability under General Demand Model

7 Concluding Remarks

This paper considers price competition between an OEM and its competitive ODM, that provides contract manufacturing service for the OEM and also produces self-branded products to compete with the OEM in the consumer market. The ODM's complex partner-competitor role leads to different pricing game conclusions than those found in a traditional oligopoly pricing game.

We first demonstrate that this partner-competitor relationship between the OEM and its ODM is actually stable and neither party wants to deviate from it, as long as the OEM has a favorable outside option (i.e., the non-competitive ODMs provide a sufficiently low wholesale price). Specifically, it is better for the OEM to source solely from the competitive ODM; it is also in the best interest of the ODM to maintain the contract manufacturing service and sell self-branded products.

Our second main conclusion shows that the partnership between the OEM and its competitive ODM mitigates the competition between them in the consumer market. The player that prices late has less incentive to undercut the earlier price. Moreover, the player that prices late may actually charge a higher product retail price than the one who prices early. In the ODM market, the two parties endogenously choose a more collusive game, that is, a sequential pricing game, instead of a simultaneous pricing game. Furthermore, the sequential pricing game with the OEM pricing early risk dominates the sequential game with the ODM pricing early. In the OEM market, their decisions on choosing pricing sequence depends on the OEM's outside option: they will choose the sequential pricing game if the OEM has a very favorable outside option (i.e., if the non-competitive

ODM's wholesale price is low). However, if the OEM's outside option is less favorable (i.e., the non-competitive ODM's wholesale price is high), the competition will become fierce, and a simultaneous game will ensue.

We also discuss the differences between the two market environments, the impacts of wholesale price negotiation, the competitive ODM's limited capacity, and the results of a general demand model. We find that our main conclusions are robust. Our study helps to answer the question of whether the OEM should treat its competitive ODM as a friend or a foe. Our conclusions strongly depend on the OEM's outside option (i.e., the wholesale price provided by other non-competitive ODMs). As this outside option becomes more favorable for the OEM, the OEM is more likely to collaborate with its competitive ODM. If we consider such an outside option as an equilibrium price arising from the competition between ODMs, then it would be interesting for future researchers to study how the number of ODMs in the market affects the price and the competition game between the OEM and its competitive ODM.

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Appendix

Proof of Proposition 1: We provide only the proofs for the simultaneous game. Due to the space limitations, we omit the detailed proofs for both the OEM-pricing-early game and the ODM-pricing-early game (the proofs are similar and straightforward.).

Simultaneous game

Under the simultaneous game, the competitive ODM determines its unit manufacturing price w first, and the OEM then determines its proportion of the production order allocated to the competitive ODM, i.e., θ. After that, the competitive ODM and the OEM decide their retail prices, respectively.

We maximize profit functions (2) and (3) simultaneously. As

$$\begin{aligned}\frac{\partial \Pi_o}{\partial p_o} &= -(p_o - \theta w - (1-\theta)\bar{w}) + a - p_o + bp_m = -2p_o + \theta w + (1-\theta)\bar{w} + a + bp_m, \\ \frac{\partial \Pi_m}{\partial p_m} &= -p_m + a - p_m + p_o + \theta bw = -2p_m + a + p_o + \theta bw.\end{aligned}$$

Thus, the best response functions are

$$p_o^*(p_m) = \frac{\theta w + (1-\theta)\bar{w} + a + bp_m}{2}; \quad p_m^*(p_o) = \frac{a + p_o + \theta bw}{2}.$$

Solving them simultaneously yields

$$p_o(w,\theta) = \frac{(2+b)a + 2\theta w + 2(1-\theta)\bar{w} + \theta b^2 w}{4-b}; \quad p_m(w,\theta) = \frac{3a + (1+2b)\theta w + (1-\theta)\bar{w}}{4-b},$$

and the corresponding sales quantities are

$$q_o(w,\theta) \quad = \quad \frac{(2+b)a - (2-b-b^2)\theta w - (2-b)(1-\theta)\bar{w}}{4-b}; q_m(w,\theta) = \frac{3a + (1-b)^2\theta w + (1-\theta)\bar{w}}{4-b}.$$

We then substitute them into the OEM's profit function and have

$$\Pi_o(w,\theta) = q_o^2(w,\theta) = \frac{[(2+b)a + ((2-b)(\bar{w}-w) + b^2 w)\theta - (2-b)\bar{w}]^2}{(4-b)^2}.$$

Thus,

$$\frac{\partial \Pi_o(w,\theta)}{\partial \theta} = 2q_o(w,\theta)\frac{\partial q_o(w,\theta)}{\partial \theta} = \frac{2[(2-b)(\bar{w}-w) + b^2 w]q_o(w,\theta)}{4-b}.$$

$(2-b)(\bar{w}-w) + b^2 w > 0$ as long as $w \le \bar{w}$. In Proposition 7 of §4.3, we will show that it is in the best interest of the competing ODM to both conduct contract manufacturing business and sell its self-branded products. That is, the competing ODM will offer $w \le \bar{w}$. As a result, Π_o is increasing in θ. The OEM tends to set $\theta^* = 1$. Given this, we rewrite the competitive ODM's profit function and have

$$\Pi_m(w) = \frac{[3a + (1-2b+b^2)w][3a + (1+2b)w]}{(4-b)^2} + \frac{(2+b)[a-(1-b)w]w}{4-b}.$$

It can be verified that $\Pi_m(w)$ is concave in w, and

$$\frac{\partial \Pi_m(w)}{\partial w} = \frac{2(7+b+b^2)[a-(1-b)w]}{(4-b)^2}.$$

Letting $\frac{\partial \Pi_m(w)}{\partial w} = 0$, then $w = a/(1-b)$. If $\bar{w} \geq a/(1-b)$, then $w^{S*} = a/(1-b)$, and hence $\Pi_o^S = 0$. Otherwise, $w^{S*} = \bar{w}$. As $\theta^* = 1$, the equilibrium retail prices, sales quantities, and profits of the OEM and the competitive ODM are

$$\begin{aligned}
p_o^{S*} &= \frac{(2+b)a+(2+b^2)\bar{w}}{4-b}; \quad p_m^{S*} = \frac{3a+(1+2b)\bar{w}}{4-b}, \\
q_o^{S*} &= \frac{(2+b)[a-(1-b)\bar{w}]}{4-b}; \quad q_m^{S*} = \frac{3a+(1-2b+b^2)\bar{w}}{4-b}, \\
\Pi_o^{S*} &= (a-p_o^{S*}+bp_m^{S*})(p_o^{S*}-\bar{w}) = \frac{(2+b)^2[a-(1-b)\bar{w}]^2}{(4-b)^2}, \\
\Pi_m^{S*} &= \frac{[3a+(1-2b+b^2)\bar{w}][3a+(1+2b)\bar{w}]}{(4-b)^2} + \frac{(2+b)[a-(1-b)\bar{w}]\bar{w}}{4-b}.
\end{aligned}$$

It is straightforward to show that Π_o^{S*} is decreasing in $\bar{w}$. And we can show that $\frac{\partial \Pi_m^S}{\partial \bar{w}} = \frac{2(7+b+b^2)[a-(1-b)\bar{w}]}{(4-b)^2} > 0$, so Π_m^{S*} increases in $\bar{w}$.

Proof of Proposition 2: The results are immediate based on Proposition 1, thus we omit the details here.

Proof of Proposition 3: We can show that

$$\begin{aligned}
p_o^{S*} - p_m^{S*} &= \frac{(2+b)a+(2+b^2)\bar{w}}{4-b} - \frac{3a+(1+2b)\bar{w}}{4-b} = \frac{-a+(1-b)^2\bar{w}}{4-b} < \frac{-a+(1-b)a}{4-b} \leq 0; \\
p_o^{L*} - p_m^{E*} &= \frac{(4+b)a+(4-b+b^2)\bar{w}}{4(2-b)} - \frac{3a+(1+b)\bar{w}}{2(2-b)} = -\frac{a-(1-b)\bar{w}}{4} \leq 0; \\
q_o^{S*} - q_m^{S*} &= \frac{(2+b)[a-(1-b)\bar{w}]}{4-b} - \frac{3a+(1-2b+b^2)\bar{w}}{4-b} = \frac{-a-3(1-b)\bar{w}}{4-b} < 0; \\
q_o^{E*} - q_m^{L*} &= \frac{(2+b)[a-(1-b)\bar{w}]}{4} - \frac{(6-b)a+(2-5b+3b^2)\bar{w}}{4(2-b)} \\
&= \frac{-(2-b+b^2)a-(1-b)(6-3b-b^2)\bar{w}}{4(2-b)} < 0; \\
p_o^{E*} - p_m^{L*} &= \frac{(2+b)a+(2-b+b^2)\bar{w}}{2(2-b)} - \frac{(6-b)a+(2+3b-b^2)\bar{w}}{4(2-b)} = \frac{(3b-2)[a-(1-b)\bar{w}]}{4(2-b)}; \\
q_o^{L*} - q_m^{E*} &= \frac{(4+b)[a-(1-b)\bar{w}]}{4(2-b)} - \frac{3a+(1-b)\bar{w}}{4} = \frac{-(1-2b)a-3(1-b)\bar{w}}{2(2-b)}.
\end{aligned}$$

So $p_o^{E*} - p_m^{L*}$ is positive if $b \geq 2/3$, and $q_o^{L*} - q_m^{E*}$ is positive if $b > 1/2$ and $\bar{w} < (2b-1)a/3(1-b)$.

Proof of Proposition 4: When $\bar{w} < a/(1-b)$, all three basic games exist. We then compare the payoffs of the OEM and the competitive ODM to derive the equilibrium of the endogenous timing game. First, we show that

$$\begin{aligned}\Pi_o^E - \Pi_o^S &= \frac{b^2[(2+b)a-(2-b-b^2)\bar{w}]^2}{8(2-b)(4-b)^2} \geq 0;\\ \Pi_m^L - \Pi_m^S &= \frac{b(2+b)(48-22b+b^2)[a-(1-b)\bar{w}]^2}{16(2-b)^2(4-b)^2} \geq 0.\end{aligned}$$

Therefore, the equilibrium (E, L) is an NE. Next, we can show that (L, E) is an NE as

$$\begin{aligned}\Pi_m^E - \Pi_m^S &= \frac{9b^2[a-(1-b)\bar{w}]^2}{8(2-b)(4-b)^2} \geq 0.\\ \Pi_o^L - \Pi_o^S &= \frac{3b^2(32-5b^2)[a-(1-b)\bar{w}]^2}{16(2-b)^2(4-b)^2} \geq 0.\end{aligned}$$

Proof of Proposition 5: Based on the proof of Proposition 4, we have

$$\begin{aligned}(\Pi_o^E - \Pi_o^S)(\Pi_m^L - \Pi_m^S) &= \frac{(48-22b+b^2)b^3(2+b)^3[a-(1-b)\bar{w}]^4}{128(2-b)^3(4-b)^4}.\\ (\Pi_o^L - \Pi_o^S)(\Pi_m^E - \Pi_m^S) &= \frac{27(32-5b^2)b^4[a-(1-b)\bar{w}]^4}{128(2-b)^3(4-b)^4}.\end{aligned}$$

According to Harsanyi and Selten (1988), the equilibrium (E, L) risk dominates (L, E) if

$$\left(\Pi_o^E - \Pi_o^S\right)\left(\Pi_m^L - \Pi_m^S\right) \geq \left(\Pi_o^L - \Pi_o^S\right)\left(\Pi_m^E - \Pi_m^S\right).$$

Note that

$$\frac{(\Pi_o^E - \Pi_o^S)(\Pi_m^L - \Pi_m^S)}{(\Pi_o^L - \Pi_o^S)(\Pi_m^E - \Pi_m^S)} = \frac{(2+b)^3(48-22b+b^2)}{27b(32-5b^2)},$$

and $(2+b)^3(48-22b+b^2) - 27b(32-5b^2) = (1-b)(4-b)^2(24+7b-b^2) \geq 0$. So (E, L) risk dominates (L, E).

Proof of Proposition 6: Following Fudenberg and Tirole's setting (2000), we can easily derive the results. Due to space limitations, we omit the details.

Proof of Proposition 7: Based on the forgoing outcomes derived from three basic games, we conduct the comparisons as follows. We can show that

$$\begin{aligned}\Pi_m^S - \Pi_m^{SN} &= \frac{(8+2b+2b^2)a\bar{w} + (b^3+6b-8)\bar{w}^2}{(4-b)^2}\\ &\geq \frac{(8+2b+2b^2)\frac{2-b}{2+b}\bar{w}^2 + (b^3+6b-8)\bar{w}^2}{(4-b)^2} = \frac{8b^2+b^4}{(2+b)(4-b)^2}\bar{w}^2 \geq 0.\end{aligned}$$

Note that the first inequality is due to the assumption $\bar{w} < (2+b)a/(2-b)$. Similarly,

$$\begin{aligned}\Pi_m^E - \Pi_m^{EN} &= \frac{(8+2b)a\bar{w} - (8-6b-b^2)\bar{w}^2}{8(2-b)}\\ &\geq \frac{(8+2b)\frac{4-3b}{4+b}\bar{w}^2 - (8-6b-b^2)\bar{w}^2}{8(2-b)} = \frac{b^2\bar{w}^2}{8(2-b)} \geq 0.\end{aligned}$$

$$\begin{aligned}\Pi_m^L - \Pi_m^{LN} &= \frac{(32-16b+4b^2+2b^3)a\bar{w} - (32-48b+20b^2-2b^3-b^4)\bar{w}^2}{16(2-b)^2} \\ &\geq \frac{(32-16b+4b^2+2b^3)\frac{2-b}{2+b}\bar{w}^2 - (32-48b+20b^2-2b^3-b^4)\bar{w}^2}{16(2-b)^2} \\ &= \frac{b^2(32-16b+2b^2+b^3)\bar{w}^2}{16(2+b)(2-b)^2} \geq 0.\end{aligned}$$

Proof of Proposition 8: The proof is similar to that of Proposition 1. They are straightforward and tedious. We omit the details here.

Proof of Proposition 9: Based on Propositions 8, Proposition 9 can be easily obtained.

Proof of Proposition 10: We first compare the wholesale prices. For the simultaneous game, if $w^{S*} = w^S$, then

$$p_o^{S*} - p_m^{S*} = \frac{3(7-b)}{2(1-b)(8-b)}a - \frac{(2+b)(7-b)}{2(1-b)(8-b)}a = \frac{(7-b)}{2(8-b)}a > 0;$$

otherwise,

$$p_o^{S*} - p_m^{S*} = \frac{3(a+\bar{w})}{4-b} - \frac{(2+b)(a+\bar{w})}{4-b} = \frac{(1-b)(a+\bar{w})}{4-b} \geq 0.$$

For the OEM-pricing-early game, if $w^{L*} = w^L$, then

$$p_o^{E*} - p_m^{L*} = \frac{42-30b+3b^2+b^3}{(1-b)(32-16b-b^2+b^3)}a - \frac{28-9b-5b^2+2b^3}{(1-b)(32-16b-b^2+b^3)}a = \frac{14-7b+b^2}{32-16b-b^2+b^3}a > 0;$$

otherwise,

$$p_o^{E*} - p_m^{L*} = \frac{3a+(3-b)\bar{w}}{2(2-b)} - \frac{(4+b)a+(4+b-b^2)\bar{w}}{4(2-b)} = \frac{a+(1-b)\bar{w}}{4} > 0.$$

For the ODM-pricing-early game, if $w^{E*} = w^E$, then

$$p_o^{L*} - p_m^{E*} = \frac{9-b}{(1-b)(7+b)}a - \frac{5+3b}{(1-b)(7+b)}a = \frac{4}{7+b}a > 0;$$

otherwise,

$$\begin{aligned}p_o^{L*} - p_m^{E*} &= \frac{(6-b)a+(5-b)\bar{w}}{4(2-b)} - \frac{(2+b)a+(1+b)\bar{w}}{2(2-b)} = \frac{(2-3b)a+3(1-b)\bar{w}}{4(2-b)} \\ &> \frac{\frac{(2-3b)(1-b)(7-b)}{6+b+b^2}\bar{w}+3(1-b)\bar{w}}{4(2-b)} = \frac{2(1-b)(16-13b+3b^2)}{4(2-b)(6+b+b^2)} > 0,\end{aligned}$$

where the first inequality is due to $\bar{w} < w^E$.

Analogously, we compare the sales quantities and the results can be derived accordingly.

Proof of Proposition 11: Below we first assume that the wholesale price is exogenously given and denote it as w, based on which we then derive the corresponding pricing sequence equilibrium. Then we compare the endogenized wholesale prices with w to derive the endogenized pricing sequence equilibrium.

First, if the wholesale price is exogenously given as w, then we re-investigate the three basic games while assuming $\theta^* = 1$.

In an OEM market,

(1) under the simultaneous game,

(i) $p_o^S = \frac{3(a+w)}{4-b}$; $p_m^S = \frac{(2+b)(a+w)}{4-b}$.

(ii) $q_o^S = \frac{3a-(1-b)w}{4-b}$; $q_m^S = \frac{(2+b)a-2(1-b)w}{4-b}$.

(iii) $\Pi_o^S = \frac{[3a-(1-b)w]^2}{(4-b)^2}$; $\Pi_m^S = \frac{(2+b)(a+w)[(2+b)a-2(1-b)w]}{(4-b)^2} + \frac{[3a-(1-b)w]w}{4-b}$.

(2) under the OEM-pricing-early game,

(i) $p_o^E = \frac{3a+(3-b)w}{2(2-b)}$; $p_m^L = \frac{(4+b)a+(4+b-b^2)w}{4(2-b)}$.

(ii) $q_o^E = \frac{3a-(1-b)w}{4}$; $q_m^L = \frac{(4+b)a-(1-b)(4-b)w}{4(2-b)}$.

(iii) $\Pi_o^E = \frac{[3a-(1-b)w]^2}{8(2-b)}$; $\Pi_m^L = \frac{[(4+b)a-(1-b)(4-b)w][(4+b)a+(4+b-b^2)w]}{16(2-b)^2} + \frac{[3a-(1-b)w]w}{4}$.

(3) under the ODM-pricing-early game,

(i) $p_o^L = \frac{(6-b)a+(5-b)w}{4(2-b)}$; $p_m^E = \frac{(2+b)a+(1+b)w}{2(2-b)}$.

(ii) $q_o^L = \frac{(6-b)a-3(1-b)w]}{4(2-b)}$; $q_m^E = \frac{(2+b)a-(1-b)w}{4}$.

(iii) $\Pi_o^L = \frac{[(6-b)a-3(1-b)w]^2}{16(2-b)^2}$; $\Pi_m^E = \frac{[(2+b)a-(1-b)w][(2+b)a+(1+b)w]}{8(2-b)} + \frac{[(6-b)a-3(1-b)w]w}{4(2-b)}$.

To ensure that the OEM obtains the positive profit margin, it is required that $w \le \{\frac{3}{1-b}a, \frac{(6-b)}{3(1-b)}a\}$. We can verify that $\frac{3}{1-b}a > \frac{(6-b)}{3(1-b)}a$. So let's assume that $w \le \frac{(6-b)}{3(1-b)}a$. Then,

$$\begin{aligned}
\Pi_o^E - \Pi_o^S &= \frac{[3a-(1-b)w]^2}{8(2-b)} - \frac{[3a-(1-b)w]^2}{(4-b)^2} = \frac{b^2[3a-(1-b)w]^2}{8(2-b)(4-b)^2} \ge 0, \\
\Pi_m^E - \Pi_m^S &= \frac{[b(2+b)a-(1-b)(4+b)w]^2}{8(2-b)(4-b)^2} \ge 0.
\end{aligned}$$

Hence $\Pi_o^E \ge \Pi_o^S$ and $\Pi_m^E \ge \Pi_m^S$. Next,

$$\Pi_m^L - \Pi_m^S = \frac{b[3a-(1-b)w][b(32-5b^2)a-(1-b)(64-16b-8b^2+b^3)w]}{16(2-b)^2(4-b)^2}.$$

Thus, the sign of $\Pi_m^L - \Pi_m^S$ depends on that of $b(32-5b^2)a-(1-b)(64-16b-8b^2+b^3)w$. Define

$$w_E = \frac{b(32-5b^2)}{(1-b)(64-16b-8b^2+b^3)}a,$$

Then $\Pi_m^L \ge \Pi_m^S$ if $w \le w_E$. And correspondingly, (E, L) is a Nash equilibrium if $w \le w_E$.

Similarly, we can show that the sign of $\Pi_o^L - \Pi_o^S$ is the same as that of $b(2+b)a-(1-b)(4+b)w$. Define

$$w_L = \frac{b(2+b)}{(1-b)(4+b)}a,$$

then $\Pi_o^L \geq \Pi_o^S$ if $w \leq w_L$. And correspondingly, (L, E) is a Nash equilibrium for $w \leq w_L$.

And we can show that

$$w_E - w_L = \frac{(4-b)(b+3)ab^3}{(4+b)(1-b)(64+b^3-18b-8b^2)} > 0.$$

Then we have the following conclusion: Assume that the exogenous wholesale price is w, where $w \leq \frac{(6-b)}{3(1-b)}a$. Comparing the three basic games in an asymmetric price competition leads to in the following findings.

(1) E dominates the other strategies for $w > w_L$.

(2) (E, L) is the unique pure NE for $w \in [w_L, w_E]$. (E, L) and (L, E) are the two NEs for $w < w_F$.

(3) Lis impossible to be a dominant strategy as $\Pi_o^E > \Pi_o^S$ and $\Pi_m^E > \Pi_m^S$.

To facilitate the next proofs, we need a lemma described as follows, whose proofs are straightforward and we omit the details.

Lemma 1. *The optimal wholesale prices have the following relationship:* $w^{L*} \leq w^{S*} \leq w^{E*}$.

From Lemma 1 we have $w_L \leq w_E \leq w^L \leq w^S \leq w^E$, so $w^{L*} \leq w^{S*} \leq w^{E*}$. Moreover, $w^L \leq w^{L*} \leq w^{S*} \leq w^{E*}$ if $\bar{w} \geq w^L$, and $w^{L*} = w^{S*} = w^{E*} = \bar{w} \leq w^L$ if $\bar{w} < w^L$. Besides,

$$\frac{(6-b)}{3(1-b)}a - w^E = \frac{(6-b)}{3(1-b)}a - \frac{6+b+b^2}{(1-b)(7+b)}a = \frac{4(b+3)(2-b)}{3(1-b)(7+b)}a \geq 0.$$

Thus, Proposition 11 then follows from the above analysis results.

Proof of Proposition 12: Following Fudenberg and Tirole's setting (2000), we can easily derive the results. Due to space limitations, we omit the details here.

Proof of Proposition 13: The game results are provided in Proposition 12. Note that in the OEM market, under each basic game, the competitive ODM's profit is maximized at $w^{j*} = w^j, j = S, L, E$. Thus, $\Pi_m^j|_{w=w^j} \geq \Pi_m^j|_{w=\bar{w}}, j = S, L, E$. We can show that

$$\Pi_m^S|_{w=\bar{w}} - \Pi_m^{SN} = \frac{(12-b+b^2)a\bar{w} - (8-7b)\bar{w}^2}{(4-b)^2} \geq \frac{(12-b+b^2)\frac{2-b}{3}\bar{w}^2 - (8-7b)\bar{w}^2}{(4-b)^2} \geq 0,$$

where the first inequality is due to $\bar{w} < 3a/(2-b)$. Similarly, we can show

$$\begin{aligned}
\Pi_m^L|_{w=\bar{w}} - \Pi_m^{LN} &= \frac{(48-40b+14b^2)a\bar{w} - (32-48b+19b^2-2b^3)\bar{w}^2}{16(2-b)^2} \\
&\geq \frac{(48-40b+14b^2)\frac{2-b}{3}\bar{w}^2 - (32-48b+19b^2-2b^3)\bar{w}^2}{16(2-b)^2} \\
&= \frac{b(16+11b-8b^2)}{48(2-b)^2}\bar{w}^2 \geq 0; \\
\Pi_m^E|_{w=\bar{w}} - \Pi_m^{EN} &= \frac{2(6-b)a\bar{w} - (7-6b)\bar{w}^2}{8(2-b)} \\
&\geq \frac{2(6-b)\frac{4-3b}{6-b}\bar{w}^2 - (7-6b)\bar{w}^2}{8(2-b)} = \frac{1}{8(2-b)}\bar{w}^2 \geq 0.
\end{aligned}$$

Thus, $\Pi_m^{j*} \geq \Pi_m^j|_{w=\bar{w}} \geq \Pi_m^{jN}$. Therefore, we prove Proposition 13.

Proof of Proposition 14 In the ODM market, the global optimal equilibrium is (E, L), so we take the outcomes $p_o = \frac{(2+b)a+(2-b+b^2)\bar{w}}{2(2-b)}$ and $q_o = \frac{(2+b)[a-(1-b)\bar{w}]}{4}$ in this case as the benchmark. In the OEM market, it is possible for (E, E), (E, L) and (L, E) to sustain as the equilibrium, so we examine the retail prices and the production quantities one by one.

(E, E):

When $\bar{w} < w^S = \frac{3(4+b+b^2)}{2(1-b)(8+b)}a$, we have $w^{S*} = \bar{w}$, then $p_o^{S*} = \frac{3(a+\bar{w})}{4-b}$; $q_o^{S*} = \frac{3a-(1-b)\bar{w}}{4-b}$. Comparing the retail prices we find that $\frac{3(a+\bar{w})}{4-b} - \frac{(2+b)a+(2-b+b^2)\bar{w}}{2(2-b)} = \frac{(4-8b+b^2)a+(4-b^2+b^3)\bar{w}}{2(2-b)(4-b)} > 0$, so the retail price in the OEM market is higher. We then compare the quantities and find that $\frac{3a-(1-b)\bar{w}}{4-b} - \frac{(2+b)[a-(1-b)\bar{w}]}{4} = \frac{(4-2b+b^2)a+(1-b)(4+2b-b^2)\bar{w}}{4(4-b)} > 0$.

When $\bar{w} \geq w^S$, we have $w^{S*} = w^S$, then $p_o^{S*} = \frac{3(7-b)}{2(1-b)(8+b)}a$; $q_o^{S*} = \frac{3(3+b)}{2(8+b)}a$. Comparing the retail prices we find that $\frac{3(7-b)}{2(1-b)(8+b)}a > \frac{(2+b)a+(2-b+b^2)\bar{w}}{2(2-b)}$ requires $\bar{w} < \frac{26-21b+12b^2+b^3}{(1-b)(8+b)(2-b+b^2)}a$, which is larger than $a/(1-b)$, so the retail price in the OEM market is higher. We then compare the quantities and find that $\frac{3(3+b)}{2(8+b)}a > \frac{(2+b)[a-(1-b)\bar{w}]}{4}$ requires $\bar{w} > \frac{-4+8b+2b^2}{2(8+b)(2+b)(1-b)}a$. Since $\frac{-4+8b+2b^2}{2(8+b)(2+b)(1-b)}a < w^S$, we prove that the quantity in the OEM market is larger.

(E, L):

When $\bar{w} < w^L = \frac{24-12b+5b^2-b^3}{(1-b)(32-16b-b^2+b^3)}a$, we have $w^{L*} = \bar{w}$, then $p_o^{E*} = \frac{3a+(3-b)\bar{w}}{2(2-b)}$; $q_o^{E*} = \frac{3a-(1-b)\bar{w}}{4}$. Comparing the retail prices we find that $\frac{3a+(3-b)\bar{w}}{2(2-b)} - \frac{(2+b)a+(2-b+b^2)\bar{w}}{2(2-b)} = \frac{(1-b)(a+(1+b)\bar{w})}{2(2-b)} > 0$. We then compare the quantities and find that $\frac{3a+(3-b)\bar{w}}{2(2-b)} - \frac{(2+b)[a-(1-b)\bar{w}]}{4} = \frac{(1-b)(a+(1+b)\bar{w})}{4} > 0$.

Since it is impossible for (E, L) to sustain as the equilibrium when $\bar{w} \geq w^L$ (Proposition 11), we need not consider the case in which $w^{L*} = w^L$.

(L, E):

When $\bar{w} < w^E = \frac{6+b+b^2}{(1-b)(7+b)}a$, we have $w^{E*} = \bar{w}$, then $p_o^{L*} = \frac{(6-b)a+(5-b)\bar{w}}{4(2-b)}$; $q_o^{L*} = \frac{(6-b)a-3(1-b)\bar{w}}{4(2-b)}$. Comparing the retail prices we find that $\frac{(6-b)a+(5-b)\bar{w}}{4(2-b)} - \frac{(2+b)a+(2-b+b^2)\bar{w}}{2(2-b)} = \frac{2(2-b)a+(3-b^2)\bar{w}}{2(2-b)} > 0$. We then compare the quantities and find that $\frac{(6-b)a-3(1-b)\bar{w}}{4(2-b)} - \frac{(2+b)[a-(1-b)\bar{w}]}{4} = \frac{(2-b+b^2)a+(1-b)(1-b^2)\bar{w}}{4(2-b)} > 0$.

Similarly, we need not consider the case in which $w^{E*} = w^E$ because it is impossible for (L, E) to sustain as the equilibrium when $\bar{w} \geq w^E$ (Proposition 11).

Proof of Proposition 15 Below we first summarize the equilibrium outcomes under the general demand model for all the three games. (The derivations are straightforward. Due to the space limitations, we omit the details.)

Simultaneous game:

It can be shown that $\Pi_o(w, \theta)$ is increasing in θ when $w \leq \bar{w}$ and $\alpha_o\alpha_m \geq b_o b_m$. Thus, $\theta^* = 1$. Define

$$w^S = \frac{b_o^2 b_m^2 a_o + \alpha_o b_m(2\alpha_m(-b_o + b_m)a_o + b_o^2 a_m) + 4\alpha_o^2\alpha_m(2\alpha_m a_o + (b_o + b_m)a_m)}{2\alpha_o(8\alpha_o^2\alpha_m^2 - b_o^3 b_m - \alpha_o\alpha_m b_m(6b_o + b_m))}. \tag{10}$$

If $\bar{w} > w^S$, then $w^{S*} = w^S$. The equilibrium retail prices and profits of the OEM and the competitive ODM are

$$p_o^{S*} = \frac{-b_o^3 b_m a_o + \alpha_o b_o(2\alpha_m(b_o - 2b_m)a_o + b_o^2 a_m) + 2\alpha_o^2\alpha_m(6\alpha_m a_o + (3b_o + b_m)a_m)}{2\alpha_o(8\alpha_o^2\alpha_m^2 - b_o^3 b_m - \alpha_o\alpha_m b_m(6b_o + b_m))};$$
$$p_m^{S*} = \frac{-b_o b_m^2 a_o + 8\alpha_o^2\alpha_m a_m + \alpha_o(\alpha_m(4b_o a_o + 6b_m a_o) + b_o(2b_o - b_m)a_m)}{2(8\alpha_o^2\alpha_m^2 - b_o^3 b_m - \alpha_o\alpha_m b_m(6b_o + b_m))};$$
$$\Pi_o^{S*} = \frac{(2\alpha_o\alpha_m + b_o^2)^2(b_m(b_o + b_m)a_o + \alpha_o(-2\alpha_m a_o + (-b_o + b_m)a_m))^2}{4b_o(-8\alpha_o^2\alpha_m^2 + b_o^3 b_m + \alpha_o\alpha_m b_m(6b_o + b_m))^2};$$
$$\Pi_m^{S*} = \frac{b_o^2 b_m^2 a_o^2 + 8\alpha_o^3\alpha_m a_m^2 + 2\alpha_o b_m a_o(2\alpha_m b_m a_o + b_o^2 a_m) + \alpha_o^2(4\alpha_m^2 a_o^2 + 4\alpha_m(b_o + 3b_m)a_o a_m + b_o^2 a_m^2)}{4\alpha_o(8\alpha_o^2\alpha_m^2 - b_o^3 b_m - \alpha_o\alpha_m b_m(6b_o + b_m))}.$$

If $\bar{w} \leq w^S$, then $w^{S*} = \bar{w}$. We have

$$p_o^{S*} = \frac{b_o(a_m + b_o\bar{w}) + 2\alpha_m(a_o + \alpha_o\bar{w})}{4\alpha_o\alpha_m - b_o b_m};$$
$$p_m^{S*} = \frac{2\alpha_o(a_m + b_o\bar{w}) + b_m(a_o + \alpha_o\bar{w})}{4\alpha_o\alpha_m - b_o b_m};$$
$$\Pi_o^{S*} = \frac{(\alpha_o(2\alpha_m(a_o - \alpha_o barw) + b_o(a_m + (b_o + b_m)\bar{w}))^2}{(-4\alpha_o\alpha_m + b_o b_m)^2};$$
$$\Pi_m^{S*} = \frac{\alpha_m b_m^2 a_o^2 + 4\alpha_o\alpha_m b_m a_o a_m + 4\alpha_o^2\alpha_m a_m^2}{(-4\alpha_o\alpha_m + b_o b_m)^2} + \frac{-\alpha_0(8\alpha_o^2\alpha_m^2 - 6\alpha_o\alpha_m b_o b_m - b_o^3 b_m - \alpha_o\alpha_m b_m^2)\bar{w}^2}{(-4\alpha_o\alpha_m + b_o b_m)^2}$$
$$+ \frac{(8\alpha_o^2\alpha_m^2 a_o - 2\alpha_o\alpha_m b_o b_m a_o + 2\alpha_o\alpha_m b_m^2 a_o + b_o^2 b_m^2 a_o + 4\alpha_o^2\alpha_m b_o a_m + 4\alpha_o^2\alpha_m b_m a_m + \alpha_o b_o^2 b_m a_m)\bar{w}}{(-4\alpha_o\alpha_m + b_o b_m)^2}.$$

OEM-pricing-early game:

Again, it can be shown that $\Pi_o(w, \theta)$ is increasing in θ when $w < \bar{w}$ and $\alpha_o\alpha_m \geq b_o b_m$. Thus,

$\theta^* = 1$. Define

$$w^L = \frac{8\alpha_o^2\alpha_m^2(2\alpha_m a_o + (b_o + b_m)a_m) + b_o b_m^2(\alpha_m(6b_o a_o - 2b_m a_o) + b_o(b_o + b_m)a_m)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)}$$
$$+ \frac{-2\alpha_o\alpha_m b_m(\alpha_m(8b_o a_o - 2b_m a_o) + b_o(2b_o + 3b_m)a_m)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)}.$$

If $\bar{w} > w^L$, then $w^{L*} = w^L$. The equilibrium retail prices and profits of the OEM and the competitive ODM are

$$p_o^{E*} = \frac{(b_o^2 b_m(-2\alpha_m(b_o - 3b_m)a_o + b_m(b_o + b_m)a_m) + 4\alpha_o^2\alpha_m^2(6\alpha_m a_o + (3b_o + b_m)a_m)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)}$$
$$+ \frac{2\alpha_o\alpha_m b_o(2\alpha_m(b_o - 6b_m)a_o + (b_o^2 - 4b_o b_m - 2b_m^2)a_m)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)};$$
$$p_m^{L*} = \frac{2b_o^2 b_m^2(b_o + b_m)a_o + 16\alpha_o^3\alpha_m^2 a_m + 2\alpha_o^2\alpha_m(\alpha_m(4b_o a_o + 6b_m a_o) + b_o(2b_o - 7b_m)a_m)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)}$$
$$+ \frac{-\alpha_o b_o b_m(2\alpha_m(3b_o + 5b_m)a_o + b_o(b_o - 3b_m)a_m)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)};$$

$$\Pi_o^{E*} = \frac{2\alpha_m(2\alpha_o\alpha_m + b_o(b_o - b_m))^2(2\alpha_o\alpha_m - b_o b_m)(b_m(b_o + b_m)a_o + \alpha_o(-2\alpha_m a_o + (-b_o + b_m)a_m))^2}{(-32\alpha_o^3\alpha_m^3 + b_o^2 b_m^2(b_o + b_m)^2 - 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) + 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m))^2};$$
$$\Pi_m^{L*} = \frac{(8\alpha_o^3\alpha_m^2 a_m^2 + b_o b_m^2 a_o(2\alpha_m(b_o - b_m)a_o + b_o(b_o + b_m)a_m)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)}$$
$$+ \frac{\alpha_o^2\alpha_m(4\alpha_m^2 a_o^2 + 4\alpha_m(b_o + 3b_m)a_o a_m + b_o(b_o - 6b_m)a_m^2)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)}$$
$$+ \frac{\alpha_o b_m(4\alpha_m^2(-b_o + b_m)a_o^2 - 2\alpha_m b_o(b_o + 4b_m)a_o a_m + b_o^2 b_m a_m^2)}{32\alpha_o^3\alpha_m^3 - b_o^2 b_m^2(b_o + b_m)^2 + 4\alpha_o\alpha_m b_o b_m^2(5b_o + b_m) - 4\alpha_o^2\alpha_m^2 b_m(12b_o + b_m)}.$$

If $\bar{w} \leq w^L$, then $w^{L*} = \bar{w}$. We have

$$p_o^{E*} = \frac{2\alpha_m(a_o + \alpha_o\bar{w}) + b_o(a_m + b_o\bar{w} - b_m\bar{w})}{4\alpha_o\alpha_m - 2b_o b_m};$$
$$p_m^{L*} = \frac{-b_o b_m(a_m + b_o\bar{w} + b_m\bar{w}) + 2\alpha_m(2\alpha_o(a_m + b_o\bar{w}) + b_m(a_o + \alpha_o\bar{w}))}{4\alpha_m(2\alpha_o\alpha_m - b_o b_m)};$$
$$\Pi_o^{E*} = \frac{(2\alpha_m(a_o - \alpha_o\bar{w}) + b_o(a_m + (b_o + b_m)\bar{w}))^2}{8\alpha_m(2\alpha_o\alpha_m - b_o b_m)};$$
$$\Pi_m^{L*} = \frac{4\alpha_m^2 b_m^2 a_o^2 + 16\alpha_o\alpha_m^2 b_m a_o a_m - 4\alpha_m b_o b_m^2 a_o a_m + 16\alpha_o^2\alpha_m^2 a_m^2 - 8\alpha_o\alpha_m b_o b_m a_m^2 + b_o^2 b_m^2 a_m^2}{16\alpha_m(-2\alpha_o\alpha_m + b_o b_m)^2}$$
$$+ \frac{(32\alpha_o^2\alpha_m^3 a_o - 32\alpha_o\alpha_m^2 b_o b_m a_o + 8\alpha_o\alpha_m^2 b_m^2 a_o + 12\alpha_m b_o^2 b_m^2 a_o - 4\alpha_m b_o b_m^3 a_o)\bar{w}}{16\alpha_m(-2\alpha_o\alpha_m + b_o b_m)^2}$$
$$+ \frac{(16\alpha_o^2\alpha_m^2 b_o a_m + 16\alpha_o^2\alpha_m^2 b_m a_m - 8\alpha_o\alpha_m b_o^2 b_m a_m - 12\alpha_o\alpha_m b_o b_m^2 a_m + 2b_o^3 b_m^2 a_m + 2b_o^2 b_m^3 a_m)\bar{w}}{16\alpha_m(-2\alpha_o\alpha_m + b_o b_m)^2}$$
$$+ \frac{(-32\alpha_o^3\alpha_m^3 + 48\alpha_o^2\alpha_m^2 b_o b_m + 4\alpha_o^2\alpha_m^2 b_m^2 - 20\alpha_o\alpha_m b_o^2 b_m^2 + b_o^4 b_m^2 - 4\alpha_o\alpha_m b_o b_m^3 + 2b_o^3 b_m^3 + b_o^2 b_m^4)\bar{w}^2}{16\alpha_m(-2\alpha_o\alpha_m + b_o b_m)^2}.$$

ODM-pricing-early game

Similarly, under our assumption $w \le \bar{w}$ and $\alpha_o\alpha_m \ge b_o b_m$, $\Pi_o(w,\theta)$ is increasing in θ. Thus, $\theta^* = 1$. Define

$$w^E = \frac{b_m(-b_o+b_m)a_o + 2\alpha_o(2\alpha_m a_o + (b_o+b_m)a_m)}{\alpha_o(8\alpha_o\alpha_m - b_o^2 - 6b_o b_m - b_m^2)}.$$

If $\bar{w} \ge w^E$, then $w^{E*} = w^E$. We have

$$p_o^{L*} = \frac{(-2b_o b_m a_o + \alpha_o(6\alpha_m a_o + (3b_o+b_m)a_m)}{\alpha_o(8\alpha_o\alpha_m - b_o^2 - 6b_o b_m - b_m^2))}; \quad p_m^{E*} = \frac{(b_o a_o + 3b_m a_o + 4\alpha_o a_m}{8\alpha_o\alpha_m - b_o^2 - 6b_o b_m - b_m^2)};$$

$$\Pi_o^{L*} = \frac{(b_m(b_o+b_m)a_o + \alpha_o(-2\alpha_m a_o + (-b_o+b_m)a_m))^2}{\alpha_o(-8\alpha_o\alpha_m + b_o^2 + 6b_o b_m + b_m^2)^2};$$

$$\Pi_m^{E*} = \frac{(b_m^2 a_o^2 + 2\alpha_o^2 a_m^2 + \alpha_o a_o(\alpha_m a_o + (b_o+3b_m)a_m)}{\alpha_o(8\alpha_o\alpha_m - b_o^2 - 6b_o b_m - b_m^2))}.$$

If $\bar{w} \le w^E$, then $w^{E*} = \bar{w}$. We have

$$p_o^{L*} = \frac{a_o + \alpha_o\bar{w}}{2\alpha_o} + \frac{b_o(b_m a_o + 2\alpha_o a_m + \alpha_o b_o\bar{w} + \alpha_o b_m\bar{w})}{4\alpha_o(2\alpha_o\alpha_m - b_o b_m)}; \quad p_m^{E*} = \frac{b_m a_o + 2\alpha_o a_m + \alpha_o b_o\bar{w} + \alpha_o b_m\bar{w}}{2(2\alpha_o\alpha_m - b_o b_m)};$$

$$\Pi_o^{L*} = \frac{(b_o b_m a_o + 4\alpha_o^2\alpha_m\bar{w} - \alpha_o(4\alpha_m a_o + b_o(2a_m + b_o\bar{w} + 3b_m\bar{w})))^2}{16\alpha_o(-2\alpha_o\alpha_m + b_o b_m)^2};$$

$$\Pi_m^{E*} = \frac{b_m^2 a_o^2 + 4\alpha_o b_m a_o a_m + 4\alpha_o^2 a_m^2}{8\alpha_o(2\alpha_o\alpha_m - b_o b_m)} + \frac{(-8\alpha_o^3\alpha_m + \alpha_o^2 b_o^2 + 6\alpha_o^2 b_o b_m + \alpha_o^2 b_m^2)\bar{w}^2}{8\alpha_o(2\alpha_o\alpha_m - b_o b_m)}$$
$$+ \frac{(8\alpha_o^2\alpha_m a_o - 2\alpha_o b_o b_m a_o + 2\alpha_o b_m^2 a_o + 4\alpha_o^2 b_o a_m + 4\alpha_o^2 b_m a_m)\bar{w}}{8\alpha_o(2\alpha_o\alpha_m - b_o b_m)}.$$

Proof of Proposition 16

Below we first list out the profits of the OEM and the competitive ODM when they are pure competitors, that is, the competitive ODM does not conduct the contract manufacturing business for the OEM. (We omit the detailed derivations due to the space limitations.)

Simultaneous game: The profits of the OEM and the ODM are

$$\Pi_o^{SN} = \frac{\alpha_o(2\alpha_m(a_o - \alpha_o\bar{w}) + b_o(a_m + b_m\bar{w}))^2}{(-4\alpha_o\alpha_m + b_o b_m)^2}; \Pi_m^{SN} = \frac{\alpha_m(2\alpha_o a_m + b_m(a_o + \alpha_o\bar{w}))^2}{(-4\alpha_o\alpha_m + b_o b_m)^2}.$$

Note that we need to guarantee $\bar{w} < \frac{2\alpha_m a_o + b_o a_m}{2\alpha_o\alpha_m - b_o b_m}$ such that the OEM is selling a positive amount of goods in the market.

OEM-pricing-early game: The profits of the OEM and the ODM are

$$\Pi_o^{EN} = \frac{(2\alpha_m(a_o - \alpha_o\bar{w}) + b_o(a_m + b_m\bar{w}))^2}{8\alpha_m(2\alpha_o\alpha_m - b_o b_m)}; \Pi_m^{LN} = \frac{(b_o b_m(a_m + b_m\bar{w}) - 2\alpha_m(2\alpha_o a_m + b_m(a_o + \alpha_o\bar{w})))^2}{16\alpha_m(-2\alpha_o\alpha_m + b_o b_m)^2}.$$

Note that we need to guarantee $\bar{w} < \frac{2\alpha_m a_o + b_o a_m}{2\alpha_o\alpha_m - b_o b_m}$ such that the OEM is selling a positive amount of goods in the market.

ODM-pricing-early game: The profits of the OEM and the ODM are

$$\Pi_o^{LN} = \frac{(b_o b_m a_o + 4\alpha_o^2\alpha_m\bar{w} - \alpha_o(4\alpha_m a_o + 2b_o a_m + 3b_o b_m\bar{w}))^2}{16\alpha_o(-2\alpha_o\alpha_m + b_o b_m)^2}; \Pi_m^{EN} = \frac{(2\alpha_o a_m + b_m(a_o + \alpha_o\bar{w}))^2}{8\alpha_o(2\alpha_o\alpha_m - b_o b_m)}.$$

Again, we need to guarantee $\bar{w} < \frac{4\alpha_o\alpha_m a_o - b_o b_m a_o + 2\alpha_o b_o a_m}{4\alpha_o^2\alpha_m - 3\alpha_o b_o b_m}$ such that the OEM is selling a positive amount of goods in the market.

Now we are ready to compare. First, we compare Π_m^{S*} and Π_m^{SN} under the general demand setting. Recall that in calculating Π_m^{S*}, $\bar{w}$ may or may not be binding. If $\bar{w}$ is binding, $\Pi_m^{S*} = \Pi_m^S(\bar{w})$. If $\bar{w}$ is not binding, then $\Pi_m^{S*} > \Pi_m^S(\bar{w})$. Hence, if we could show $\Pi_m^S(\bar{w}) > \Pi_m^{SN}$, then $\Pi_m^{S*} > \Pi_m^{SN}$ will automatically apply. We can show that

$$\Pi_m^S(\bar{w}) - \Pi_m^{SN} = \frac{(8\alpha_o^2\alpha_m^2 a_o - 2\alpha_o\alpha_m b_o b_m a_o + b_o^2 b_m^2 a_o + 4\alpha_o^2\alpha_m b_o a_m + \alpha_o b_o^2 b_m a_m)}{(-4\alpha_o\alpha_m + b_o b_m)^2}\bar{w} + \frac{(-8\alpha_o^3\alpha_m^2 + 6\alpha_o^2\alpha_m b_o b_m + \alpha_o b_o^3 b_m)}{(-4\alpha_o\alpha_m + b_o b_m)^2}\bar{w}^2.$$

If $(-8\alpha_o^3\alpha_m^2 + 6\alpha_o^2\alpha_m b_o b_m + \alpha_o b_o^3 b_m)$ is larger than 0, the quadratic function is convex. It can be proven that $(8\alpha_o^2\alpha_m^2 a_o - 2\alpha_o\alpha_m b_o b_m a_o + b_o^2 b_m^2 a_o + 4\alpha_o^2\alpha_m b_o a_m + \alpha_o b_o^2 b_m a_m) > 0$ under our assumption $\alpha_o\alpha_m > b_o b_m$. Therefore, the symmetry axis of this quadratic function is smaller than 0. The root at $\bar{w} = 0$ is then the larger root. This indicates that $\Pi_m^S(\bar{w}) - \Pi_m^{SN}$ remains positive for all $\bar{w} > 0$.

If $(-8\alpha_o^3\alpha_m^2 + 6\alpha_o^2\alpha_m b_o b_m + \alpha_o b_o^3 b_m)$ is smaller than 0, the quadratic function is concave. Its symmetry axis is larger than 0 and the smaller root is $\bar{w} = 0$. Hence we need to prove that under the constraint $\bar{w} < \frac{2\alpha_m a_o + b_o a_m}{2\alpha_o\alpha_m - b_o b_m}$, $\Pi_m^S(\bar{w}) - \Pi_m^{SN}$ remains positive.

The larger root of this quadratic function is

$$\bar{w}' = \frac{8\alpha_o^2\alpha_m^2 a_o - 2\alpha_o\alpha_m b_o b_m a_o + b_o^2 b_m^2 a_o + 4\alpha_o^2\alpha_m b_o a_m + \alpha_o b_o^2 b_m a_m}{8\alpha_o^3\alpha_m^2 - 6\alpha_o^2\alpha_m b_o b_m - \alpha_o b_o^3 b_m}.$$

Now we compare the value of $\bar{w}'$ and the constraint.

$$\bar{w}' - \frac{2\alpha_m a_o + b_o a_m}{2\alpha_o\alpha_m - b_o b_m} = \frac{b_o^2 b_m(-b_o b_m^2 + 2\alpha_o\alpha_m(b_o + 2b_m))a_o + b_o^2(4\alpha_o^2\alpha_m + \alpha_o b_o(b_o - b_m))b_m a_m}{\alpha_o(2\alpha_o\alpha_m - b_o b_m)(8\alpha_o^2\alpha_m^2 - 6\alpha_o\alpha_m b_o b_m - b_o^3 b_m)}.$$

Under the condition $(-8\alpha_o^3\alpha_m^2 + 6\alpha_o^2\alpha_m b_o b_m + \alpha_o b_o^3 b_m) > 0$ and the assumption $\alpha_o\alpha_m > b_o b_m$, we can easily show that this value is larger than 0. Hence, the larger root $\bar{w}'$ is greater than the constraint of $\bar{w}$. As a result, for all values such that $\bar{w} \in \left(0, \frac{2\alpha_m a_o + b_o a_m}{2\alpha_o\alpha_m - b_o b_m}\right]$, the concave quadratic function $\Pi_m^S(\bar{w}) - \Pi_m^{SN}$ remains positive. Combining these two conditions, we show that $\Pi_m^S(\bar{w}) > \Pi_m^{SN}$, which implies $\Pi_m^{S*} > \Pi_m^{SN}$. In other words, the competitive ODM earns more when it acts as a co-opetitor of the OEM.

Next, we compare $\Pi_m^L(\bar{w})$ with Π_m^{LN}. We have

$$\Pi_m^L(\bar{w}) - \Pi_m^{LN} = \frac{(-32\alpha_o^3\alpha_m^3 + 48\alpha_o^2\alpha_m^2 b_o b_m - 20\alpha_o\alpha_m b_o^2 b_m^2 + b_o^4 b_m^2 + 2b_o^3 b_m^3)}{16\alpha_m(-2\alpha_o\alpha_m + b_o b_m)^2}\bar{w}^2 + \frac{(32\alpha_o^2\alpha_m^3 a_o - 32\alpha_o\alpha_m^2 b_o b_m a_o + 12\alpha_m b_o^2 b_m^2 a_o + 16\alpha_o^2\alpha_m^2 b_o a_m - 8\alpha_o\alpha_m b_o^2 b_m a_m + 2b_o^3 b_m^2 a_m)}{16\alpha_m(-2\alpha_o\alpha_m + b_o b_m)^2}\bar{w}.$$

It is easy to show that $(32\alpha_o^2\alpha_m^3a_o - 32\alpha_o\alpha_m^2b_ob_ma_o + 12\alpha_mb_o^2b_m^2a_o + 16\alpha_o^2\alpha_m^2b_oa_m - 8\alpha_o\alpha_mb_o^2b_ma_m + 2b_o^3b_m^2a_m) > 0$ under the assumption $\alpha_o\alpha_m > b_ob_m$.

If $(-32\alpha_o^3\alpha_m^3 + 48\alpha_o^2\alpha_m^2b_ob_m - 20\alpha_o\alpha_mb_o^2b_m^2 + b_o^4b_m^2 + 2b_o^3b_m^3)$ is larger than 0, then the quadratic function $\Pi_m^L(\bar{w}) - \Pi_m^{LN}$ is convex with a negative symmetry axis and a larger root at $\bar{w} = 0$. Consequently, $\Pi_m^L(\bar{w}) > \Pi_m^{LN}$.

If $(-32\alpha_o^3\alpha_m^3 + 48\alpha_o^2\alpha_m^2b_ob_m - 20\alpha_o\alpha_mb_o^2b_m^2 + b_o^4b_m^2 + 2b_o^3b_m^3)$ is smaller than 0, the quadratic function $\Pi_m^L(\bar{w}) - \Pi_m^{LN}$ is concave with a positive symmetry axis and a smaller root at $\bar{w} = 0$. Its larger root is given as

$$\bar{w}' = \frac{32\alpha_o^2\alpha_m^3a_o - 32\alpha_o\alpha_m^2b_ob_ma_o + 12\alpha_mb_o^2b_m^2a_o + 16\alpha_o^2\alpha_m^2b_oa_m - 8\alpha_o\alpha_mb_o^2b_ma_m + 2b_o^3b_m^2a_m}{32\alpha_o^3\alpha_m^3 - 48\alpha_o^2\alpha_m^2b_ob_m + 20\alpha_o\alpha_mb_o^2b_m^2 - b_o^4b_m^2 - 2b_o^3b_m^3}.$$

We then compare $\bar{w}'$ with the constraint value of $\bar{w}$.

$$\bar{w}' - \frac{2\alpha_ma_o + b_oa_m}{2\alpha_o\alpha_m - b_ob_m} = \frac{(16\alpha_o\alpha_m^2b_m + 2\alpha_mb_o(b_o - 4b_m)b_m)a_o + (16\alpha_o^2\alpha_m^2 - 8\alpha_o\alpha_mb_ob_m + b_o^3b_m)a_m}{(2\alpha_o\alpha_m - b_ob_m)(32\alpha_o^3\alpha_m^3 - 48\alpha_o^2\alpha_m^2b_ob_m + 20\alpha_o\alpha_mb_o^2b_m^2 - b_o^3b_m^2(b_o + 2b_m))}.$$

It could be easily proven that the forgoing is positive under the condition $(-32\alpha_o^3\alpha_m^3 + 48\alpha_o^2\alpha_m^2b_ob_m - 20\alpha_o\alpha_mb_o^2b_m^2 + b_o^4b_m^2 + 2b_o^3b_m^3) < 0$ and the assumption $\alpha_o\alpha_m > b_ob_m$. In other words, the larger root is greater than the constraint value of $\bar{w}$. Hence for all values such that $\bar{w} \in \left(0, \frac{2\alpha_ma_o + b_oa_m}{2\alpha_o\alpha_m - b_ob_m}\right]$, the concave quadratic function $\Pi_m^L(\bar{w}) - \Pi_m^{LN}$ remains positive. Combining these two conditions, we prove $\Pi_m^L(\bar{w}) > \Pi_m^{LN}$, which implies $\Pi_m^{L*} > \Pi_m^{LN}$. The competitive ODM prefers being a co-opetitor of the OEM.

Last, we compare $\Pi_m^E(\bar{w})$ and Π_m^{EN}. We have

$$\Pi_m^E(\bar{w}) - \Pi_m^{EN} = \frac{(8\alpha_o\alpha_ma_o - 2b_ob_ma_o + 4\alpha_ob_oa_m)}{16\alpha_o\alpha_m - 8b_ob_m}\bar{w} + \frac{-8\alpha_o^2\alpha_m + \alpha_ob_o^2 + 6\alpha_ob_ob_m}{16\alpha_o\alpha_m - 8b_ob_m}\bar{w}^2.$$

We could prove $\frac{(8\alpha_o\alpha_ma_o - 2b_ob_ma_o + 4\alpha_ob_oa_m)}{16\alpha_o\alpha_m - 8b_ob_m} > 0$ directly from our assumption $\alpha_o\alpha_m > b_ob_m$.

If $\frac{-8\alpha_o^2\alpha_m + \alpha_ob_o^2 + 6\alpha_ob_ob_m}{16\alpha_o\alpha_m - 8b_ob_m}$ is positive, which is equivalently assuming $(-8\alpha_o^2\alpha_m + \alpha_ob_o^2 + 6\alpha_ob_ob_m) > 0$ based on our assumption $\alpha_o\alpha_m > b_ob_m$, we can conclude that: the quadratic function $\Pi_m^E(\bar{w}) - \Pi_m^{EN}$ is convex, with a negative symmetry axis and a larger root at $\bar{w} = 0$. Hence for all $\bar{w} > 0$, $\Pi_m^E(\bar{w}) - \Pi_m^{EN}$ remains positive.

If $(-8\alpha_o^2\alpha_m + \alpha_ob_o^2 + 6\alpha_ob_ob_m)$ is smaller than 0, $\Pi_m^E(\bar{w}) - \Pi_m^{EN}$ is then concave, with a positive asymmetry axis and a smaller root at $\bar{w} = 0$. The larger root is given as

$$\bar{w}' = \frac{(8\alpha_o\alpha_ma_o - 2b_ob_ma_o + 4\alpha_ob_oa_m)}{8\alpha_o^2\alpha_m - \alpha_ob_o^2 - 6\alpha_ob_ob_m}.$$

We then compare $\bar{w}'$ with the constraint value of $\bar{w}$.

$$\bar{w}' - \frac{4\alpha_o\alpha_ma_o - b_ob_ma_o + 2\alpha_ob_oa_m}{4\alpha_o^2\alpha_m - 3\alpha_ob_ob_m} = \frac{b_o^2(4\alpha_o\alpha_ma_o - b_ob_ma_o + 2\alpha_ob_oa_m)}{\alpha_o(4\alpha_o\alpha_m - 3b_ob_m)(8\alpha_o\alpha_m - b_o(b_o + 6b_m))}.$$

We could easily show that this value is positive under the assumption $\alpha_o\alpha_m > b_ob_m$ and the condition $(-8\alpha_o^2\alpha_m + \alpha_ob_o^2 + 6\alpha_ob_ob_m) < 0$. The larger root is greater than the constraint value of

$\bar{w}$. Hence, for all values such that $\bar{w} \in \left(0, \frac{4\alpha_o\alpha_m a_o - b_o b_m a_o + 2\alpha_o b_o a_m}{4\alpha_o^2\alpha_m - 3\alpha_o b_o b_m}\right]$, $\Pi_m^E(\bar{w}) - \Pi_m^{EN}$ remains positive. Combining these two conditions, we could conclude that $\Pi_m^E(\bar{w}) > \Pi_m^{EN}$ and hence $\Pi_m^{E*} > \Pi_m^{EN}$. The competitive ODM prefers being a co-opetitor of the OEM.