To appear in *Transportmetrica B: Transport Dynamics*
Vol. 00, No. 00, Month 20XX, 1–21

On the Impact of Obstructions on the Capacity of Nearby Signalized Intersections

Vikash V. Gayah[a], S. Ilgin Guler[a] and Weihua Gu[b*]

[a]*Department of Civil and Environmental Engineering, The Pennsylvania State Univeristy, University Park, PA, 16802*
[b]*Department of Electrical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong*

(*Received 00 Month 20XX; final version received 00 Month 20XX*)

Various obstructions exist that can impede maximum vehicle flow through signalized intersections. Examples include buses or freight vehicles dwelling at loading areas near the intersection, stalled vehicles, pre-signals that temporarily block car traffic to provide bus priority, on-street parking maneuvers and permanent road fixtures. If the effects of these obstructions are not recognized or accounted for, vehicle discharge capacities at these critical locations can be overestimated, leading to ineffective traffic management strategies. This paper examines the capacity of an isolated signalized intersection when a nearby roadway obstruction is present in either the upstream or downstream direction. To quantify the loss of capacity caused by an obstruction, the paper applies the variational theory of kinematic waves in a moving-time coordinate system, which simplifies the traditional variational theory by reducing the number of local path costs that must be considered. The result is a simple recipe that requires few calculations and can be used to gain insights into signal operations when obstructions are present. Capacity formulae for general cases are also developed from the recipe. The results, recipe and formulae can be used to guide policies on the location of obstructions that can be controlled, like bus stops, pre-signals or permanent road fixtures and to develop strategies to mitigate the effects of obstructions that can be identified in real-time. As an example, a simple adaptive signal control scheme is created using this methodology to more efficiently allocate green time between competing directions when an obstruction is present.

Keywords: Signalized Intersection Capacity; Obstructions; Variational Theory; Moving-Time Coordinate System

1. Introduction

Signalized intersections are often the most restrictive bottlenecks on urban streets. Thus, urban traffic management tends to focus on signalized intersection operations. For example, signals are coordinated to reduce the number of times vehicles need to stop (Roess et al. 2004; Girianna and Benekohal 2002), and green time is allocated dynamically to reduce vehicle delays (Miller 1963; Robertson and Bretherton 1974; Shelby 2004). Signalized intersections are also the focus of transit priority strategies (Christofa and Skabardonis 2011; Guler and Cassidy 2012), and play a vital role in how streets are organized across a network (Gayah and Daganzo 2012).

*Corresponding author. Email: weihua.gu@polyu.edu.hk

Other obstructions also exist that can impede traffic flow. Examples of these obstructions include: pre-signals for prioritizing bus movements (Wu and Hounsell 1998; Xuan et al. 2012; Guler and Menendez 2014); buses dwelling at curbside stops (Kim and Rilett 2005; Gu et al. 2014); freight delivery trucks (Yannis et al. 2006); on-street parking (Box 2004); bus bulbs (Fitzpatrick et al. 2002); and, the elimination of a travel lane. The presence of these additional obstructions near signalized intersections can exacerbate the existing signal bottleneck. If the impacts of these obstructions are not accounted for, vehicle discharge capacities at signalized intersections might be overestimated. This could result in the design and application of inadequate traffic management strategies.

Unfortunately, current understanding of intersection capacity fails to sufficiently account for the impact of these obstructions on signalized intersections. Previous analytical work has identified the locations where on-street parking maneuvers can negatively influence intersection operations and quantified the delays associated with these maneuvers (Cao et al. 2013; Ye and Chen 2011). However, these results cannot be used to estimate reductions in capacity that may occur. The Highway Capacity Manual (TRB 2010, henceforth HCM) furnishes formulae for estimating saturation flows at signalized intersections due to nearby on street parking maneuvers and dwelling buses. However, the HCM formulae assume that the impacts of these obstructions are independent of both their location and duration, which has been shown to be incorrect by Gu et al. (2013, 2014). The latter cited works model the impact of buses dwelling at near-side and far-side stops using kinematic wave theory, but they fail to furnish a general method for estimating the capacity reduction caused by obstructions of any type.

To our knowledge, an in-depth examination of the impacts of obstruction on intersection capacity currently does not exist. In light of this, the present work applies the well-known variational theory of kinematic waves (Daganzo 2005a,b; Daganzo and Menendez 2005) to determine the capacity of an isolated signalized intersection when an obstruction is present. Variational theory is ideal because it does not require enumeration of all traffic states that might arise. Previous studies have used this method to examine the capacity of a roadway with multiple traffic signals (Daganzo and Geroliminis 2008), bus lanes (Chiabaut et al. 2012), and merges (Leclercq et al. 2011), among other features. Additionally, Geroliminis and Boyaci (2012) examined the impacts of random bottlenecks created by dwelling buses on signalized arterials. However, analytical formulae were not furished due to the stochastic nature of the problem considered.

In this work, we apply variational theory in a moving-time coordinate system (Newell 1993), which simplifies analysis by reducing the number of local path costs that need to be considered to obtain a solution. The result is a simple recipe to calculate capacity which requires a few calculations. Using this recipe, we develop the general formulae for estimating vehicle discharge capacities at signals with obstructions located immediately upstream or downstream. The capacities provided here are a function of the intersection geometry, signal settings and obstruction features, and is independent of the demand conditions. This metric is well-suited to study overall operations at over-saturated signals, whereas other metrics are better suited to study undersaturated signals (e.g., delays due to obstructions at under-saturated signals are considered in Gu et al. (2014)).

The proposed methodology can be applied by both practitioners and researchers to easily and comprehensively identify and quantify any negative impacts obstructions may cause. In doing so, more effective traffic management strategies can be designed for and implemented at signalized intersections. The application of this methodology can also provide insights that might help to guide policies on how best to locate obstructions that can be controlled and design adaptive signal control schemes to reduce the negative impacts of obstructions.

The remainder of this paper is organized as follows. Section 2 describes the scenario and notation used in this analysis. Section 3 provides some background information on variational theory and discusses how it can be applied in a moving-time coordinate system. Section 4 derives the simple capacity recipe found by applying variational theory in a moving-time coordinate system to determine the capacity loss incurred by an obstruction. The recipe and the resulting capacity formulae are then used to gain insights about obstruction location and control, which is followed by a parametric analysis in Section 5. Finally, Section 6 provides some concluding remarks.

2. Scenario and Notation

Considered here is an isolated signalized intersection with fixed cycle length, C, and green ratio, g.[1] The signal alternates between fixed green periods of length gC and red periods of length $(1-g)C$. Traffic on the homogeneous road segments upstream and downstream of the signalized intersection is assumed to follow the kinematic wave theory of traffic flows (Lighthill and Whitham 1955; Richards 1956) with a triangular fundamental diagram; see top of Figure 1a, where Q_m denotes the capacity, v_f the free-flow speed, $-w$ ($w > 0$) the backward wave speed, and k_j the jam density. Furthermore, vehicles are assumed to be conserved at the intersection; i.e., vehicles do not turn into or out of the segment.

To simplify the capacity analysis, we adopt a moving-time coordinate system as described in Newell (1993). Using these moving-time coordinates, the time coordinate at any location is measured relative to the passing of a reference vehicle traveling throughout the segment at the free-flow speed v_f. In this representation, the fundamental diagram is modified and can be completely characterized by just two parameters: capacity, Q_m, and backward wave speed, $-w^{'} = -(1/v_f + 1/w)^{-1}$. This modified fundamental diagram is illustrated in the top of Figure 1b. Note that the "cost" functions shown in the bottom of Figures 1a and 1b are used when applying variational theory and will be described in the next section.

An obstruction may occur in the roadway segment either upstream or downstream of the intersection. The distance between the obstruction and the intersection is defined as d; see Figure 2. At this location, the obstruction creates a bottleneck on the roadway with a capacity of $Q_B < Q_m$. The capacity reduction at this location lasts the entire time that the obstruction is present, denoted S. In this analysis, we assume that only one obstruction occurs at a time and that obstructions occur infrequently enough that they do not interact; i.e., that each obstruction can be examined independently.

3. Determining capacity in a moving-time coordinate system

Variational theory is used to determine the impact of obstructions on the capacity of nearby signalized intersection. Section 3.1 provides some background on the variational theory method. Note that the description here is not comprehensive, but provided to give sufficient background on the techniques used in this paper. A complete description and derivation of variational theory can be found in Daganzo (2005a,b); Daganzo and Menendez (2005); Daganzo and Geroliminis (2008). Section 3.2 describes how variational theory can be modified and applied in a moving-time coordinate system, and the advantages of doing so.

[1]By isolated, we mean that signals upstream and downstream are sufficiently far away as to not interact with the intersection of interest. Although simple, this scneario can serve as a building block for more complicated situations in the future. Futhermore, recent studies have focused on a similar scenario to quantify queue lengths and delays due to obstructions (Gu et al. 2013, 2014).

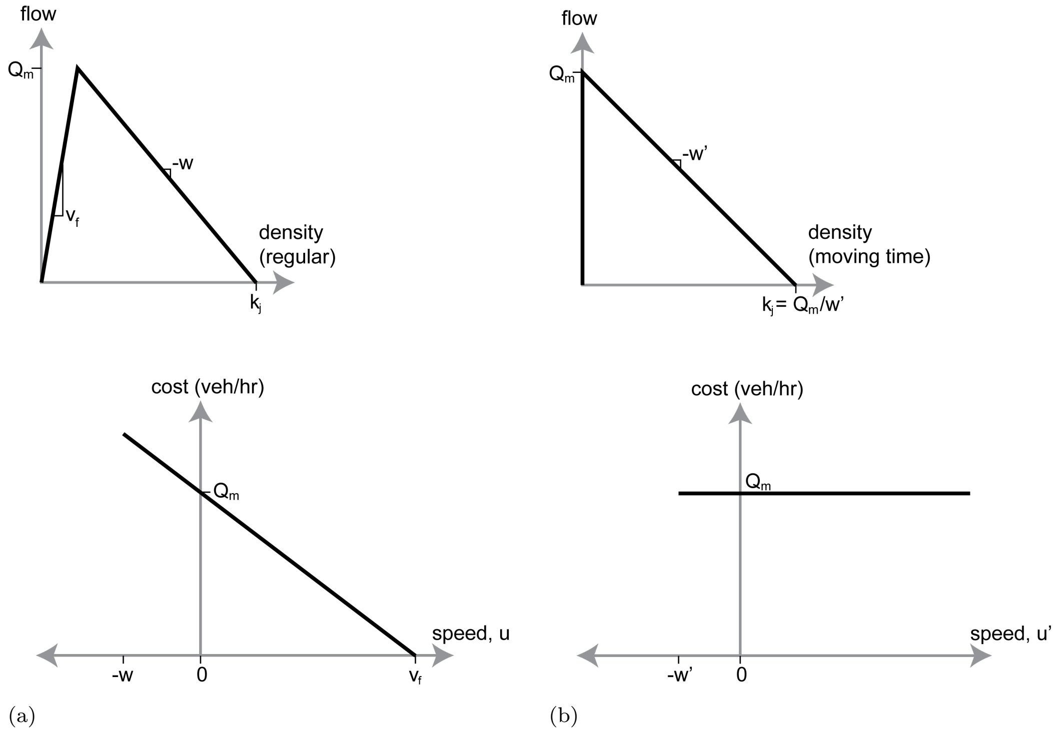


Figure 1.: Triangular fundamental diagram and associated cost function in a: (a) traditional coordinate system; and, (b) moving-time coordinate system.

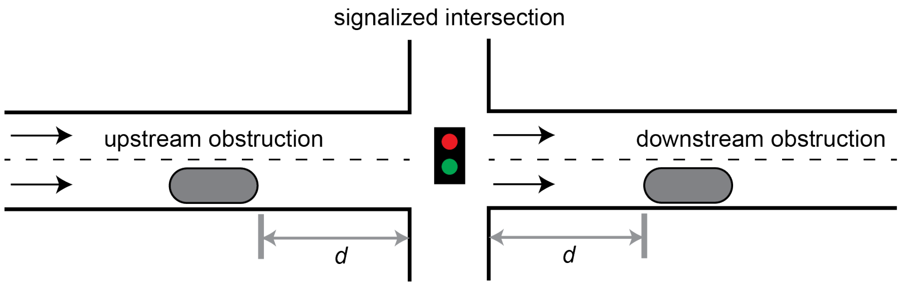


Figure 2.: Illustration of upstream and downstream obstructions

3.1. *Background on variational theory*

Variational theory allows the solution of the kinematic wave model to be reformulated as a shortest path problem. Doing so facilitates the direct calculation of a roadway's capacity without the need to determine the evolution of all traffic states. In the variational theory formulation, a "cost" function, $r(u)$, is defined which provides the local cost (in units of vehicle flow) of traveling along a path on the time-space plane with local speed u.[2] For a homogeneous roadway segment that obeys kinematic wave theory with a given fundamental diagram, $Q(k)$, $r(u) = \sup_k \{Q(k) - ku\}$.

This cost function represents the maximum rate that vehicles on the road can overtake a moving observer traveling at constant speed u in the absence of any bottlenecks. The presence of a bottleneck that restricts vehicle movement along the roadway provides a 'shortcut' on the time-space plane with a lower path cost. Examples of these shortcuts include the red period of a traffic signal that completely eliminates flow, or an obstruction

[2]Note that this cost function actually provides a cost rate. However, we use the term cost function to be consistent with the literature.

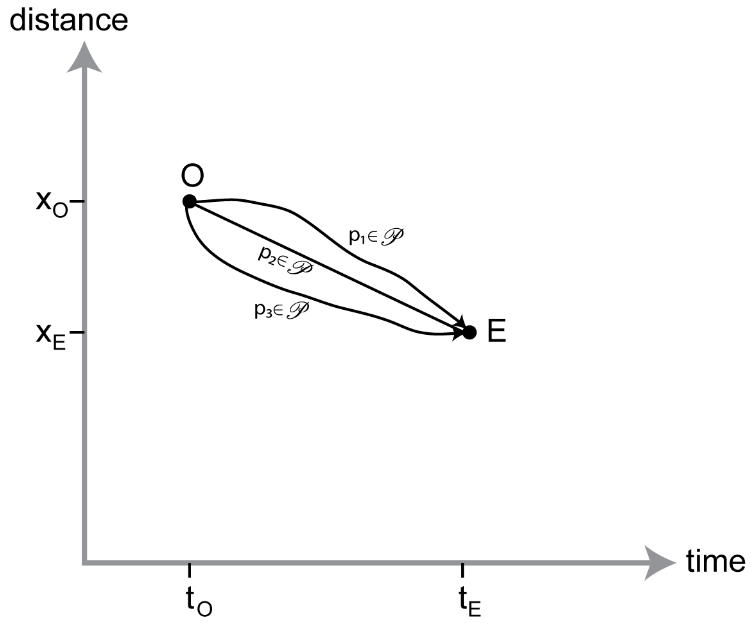


Figure 3.: Time-space diagram depicting subset of paths used to connect two points.

along the roadway that impedes traffic. The cost of traveling along such a shortcut at a given location (with speed $u = 0$) is given by the capacity of the bottleneck it represents.

Paths are only considered "valid" if they are continuous, piece-wise differentiable curves on the time-space plane that have local speeds contained within the range $u \in [\frac{dQ(k_j)}{dk}, \frac{dQ(0)}{dk}]$ (Daganzo 2005a,b). The restriction of continuous paths ensure that observers do not disappear and reappear on the time-space plane, while the path speed is restricted to the speed of kinematic waves that might arise in an LWR solution. The total cost of traveling along a valid path Z_i with a local speed of $z(t)$ at any time t is:

$$\Delta Z_i = \int_{t_0}^{t_1} r(z(t))dt, \tag{1}$$

where t_0 and t_1 are the times the path begins and ends, respectively, and $r(z(t))$ includes the effect of any shortcuts present. The value ΔZ_i represents an upper bound on the number of vehicles that can overtake a moving observer that travels along the path Z_i. This upper bound can be achieved only when there is no downstream restriction that limits the overtaking vehicular flow.

Consider now any two points on the time-space plane, O and E, that can be connected by at least one valid path; see Figure 3. Define the coordinates of these points on the time-space plane as $O(t_O, x_O)$ and $E(t_E, x_E)$, respectively, where $t_O < t_E$, and let $\mathscr{P}$ be the set of all valid paths that connect these two points. The path with the least cost that connects O and E provides a tight upper bound on the maximum number of vehicles that can overtake any observer traveling between these two points. It follows that the maximum rate that vehicles can overtake such an observer, i.e., the capacity for vehicular flow between those two points, is:

$$q_c(O, E) = \inf_{p \in \mathscr{P}} \frac{\Delta Z_p}{(t_E - t_O)}, \tag{2}$$

which turns out to be a shortest path problem. The capacity of the segment at location x, $q_c(x)$, is defined as the maximum rate at which vehicles are able to pass an observer with $x_O = x_E = x$ over a period of time; i.e., $q_c(x) = q_c(O, E)$ for $x_O = x_E = x$.

When the fundamental diagram is triangular, the shortest path problem can be simplified even further. In this case, the cost function $r(u)$ is linear; see Figure 1a. Furthermore, Daganzo (2005b) shows that the shortest path is piece-wise linear and made up only of segments with local speeds $u = -w$ and $u = v_f$, as well as $u = 0$ along any fixed shortcuts that might exist. Other examples of using a triangular fundamental diagram when applying the variational theory can also be found in the literature (Daganzo and Geroliminis 2008; Leclercq and Geroliminis 2013).

3.2. *Application to moving-time*

In this paper we apply the variational theory methodology in a moving-time coordinate system as defined in Newell (1993) to facilitate an analytical solution for the capacity of a signalized intersection when an obstruction is present. As will be shown, doing so reduces the number of local path costs and simplifies the analysis.

Denote the modified triangular fundamental diagram shown in Figure 1b for such a coordinate system as $Q'(k')$, where k' represents the density measured in these moving-time coordinates. The cost of paths traveling at speed u' in the moving-time coordinate system, $r'(u')$, is:

$$r'(u') = \sup_{k'}\{Q'(k') - k'u'\}. \tag{3}$$

Here the set of valid local speeds increases when compared to the traditional coordinate system: it ranges from $u' = -w'$ to $u' = \infty$. However, the cost function becomes much simplier in this case. From (3) we find that $r'(\infty) = 0$ and $r'(u') = Q_m, \forall\ u' < \infty$; see Figure 1b. This makes physical sense. Recall that these costs represent the maximum rates that traffic can overtake a moving observer traveling at speed u' and consider Figure 4. The lighter lines in the figure represent vehicle trajectories traveling at the maximum flow, Q_m. The vehicle trajectories all have a slope of ∞, which is analogous to the free flow speed, v_f in a traditional time-space diagram without moving-time coordinates. The darker black lines represent observer trajectories traveling at different speeds. Note that the observers pass vehicles at the same rate over time in the moving-time coordinate system, except when traveling at the free-flow speed (parallel to the vehicle trajectories). This verifies that the path costs predicted from (3) are consistent with the physical observations in the moving-time plane.

Clearly, application of variational theory in the moving-time coordinate system simplifies the shortest path problem because now *any* movement in the positive time direction is associated with the same local cost, Q_m, except for movements along fixed obstructions (e.g., red phases at traffic signals and other bottlenecks). The cost of moving along these obstructions remains unchanged and is equal to Q_B. This approach facilitates the derivation of simple analytical formulae to solve the shortest path problem.

Once the shortest path problem has been solved, the capacity in the moving-time coordinate system can then be determined by substituting the parameters in (1) and (2) with their moving-time counterparts. This will be equal to the capacity in the traditional coordinate system since the two representations are physically equivalent.

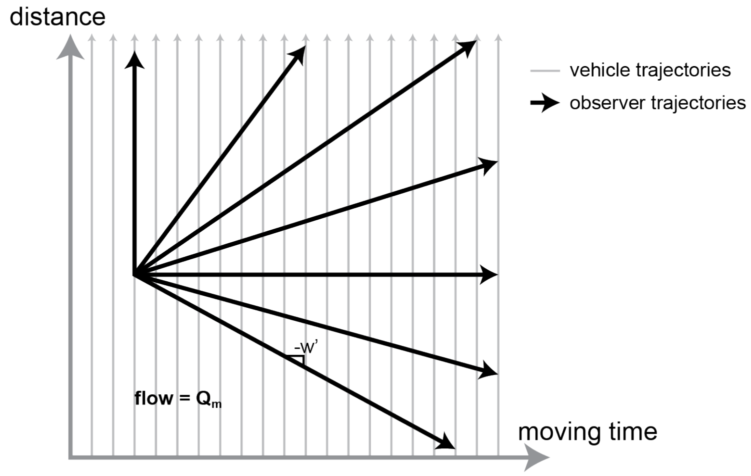


Figure 4.: Time-space diagram showing maximum passing rates in a moving-time coordinate system.

4. Capacity at Signalized Intersections

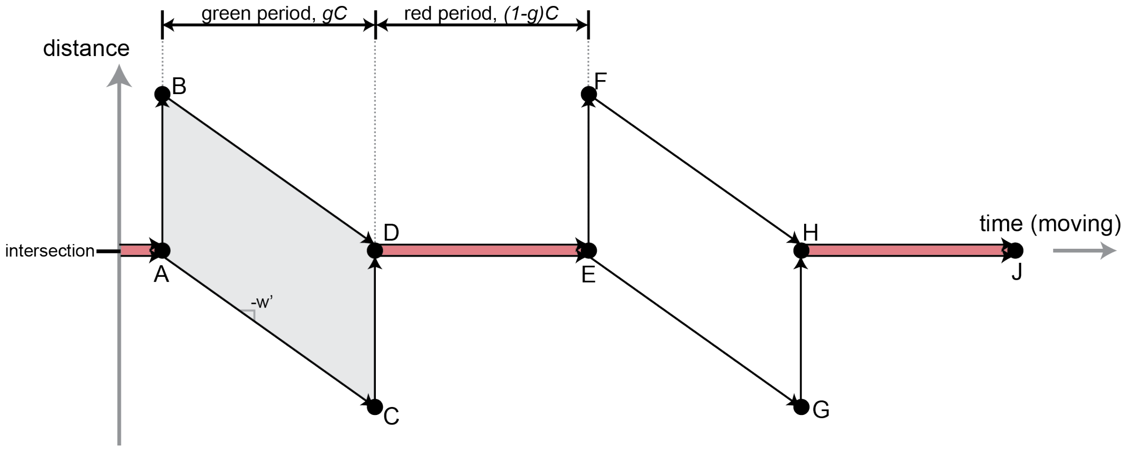
We now apply variational theory in the moving-time coordinate system to determine the capacity of a signalized intersection with a nearby obstruction. Section 4.1 uses this method to calculate the capacity in the simple case in which no obstruction is present. Though the outcome is trivial, the analysis provides a nice illustration of the methodology, and identifies time-space regions where obstructions might be damaging. Section 4.2 develops a simple recipe to determine the capacity of the intersection when an obstruction is present.

4.1. *Without obstructions*

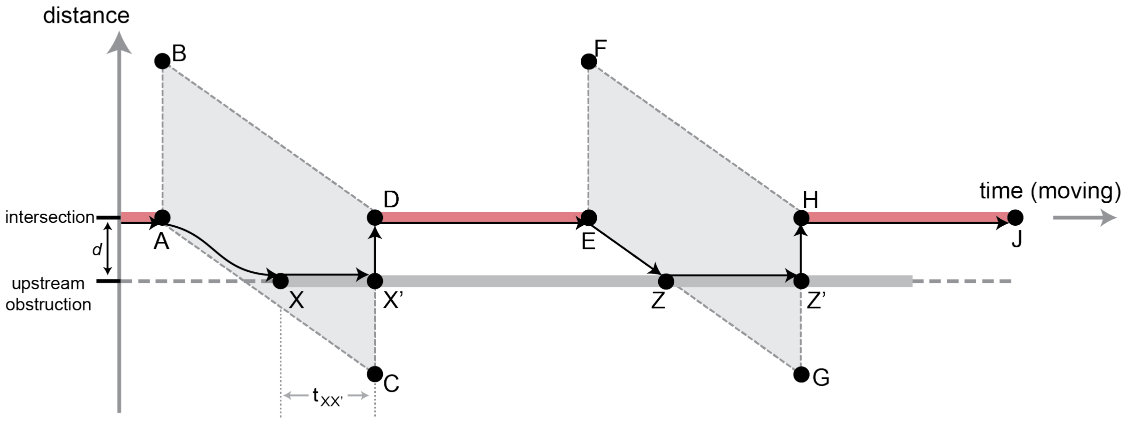
Figure 5a presents the time-space diagram for a signalized intersection with no obstruction in the moving-time coordinate system. The capacity of the signal can be determined by calculating q_c at the signal location.[3] Since the signal is cyclical, it follows that we only need to determine the least cost path during a single cycle (e.g., between points A and E) and then repeat this pattern over subsequent cycles. Clearly, the least cost path should include the signal's red period, as this duration of $(1-g)C$ has no cost associated with it. (Recall that a red period provides a shortcut where the cost is given by its capacity, 0.) It then remains to find the shortest path between points A and D. As discussed in Section 3.2, the cost of traveling between these two points depends only on the time distance separating these points (gC), as all valid paths would have the same local cost per unit time, Q_m. Therefore, any of the infinitely many valid paths between A and D would have the same total cost of $Q_m gC$, and the shortest path between A and E would have the same cost.

Figure 5a illustrates the two valid shortest paths between points A and E that travel the furthest distance away from the signal during the green period: paths $A-B-D-E$ and $A-C-D-E$. These two paths define a boundary within which all other valid shortest paths between points A and E must exist. Let us refer to the tessellation of these two specific paths over all cycles as "signal paths", and denote their costs $\Delta Z_S = nQ_m gC$, where n is the number of cycles considered in the analysis period. Furthermore, let us refer

[3]Note that we will drop the use of $'$ to denote parameters measured in the moving-time coordinate system for notational simplicity, since the remainder of the paper uses a moving-time coordinate system.



(a)



(b)

Figure 5.: Time-space diagram showing: (a) signal paths and critical region when no obstruction is present; and, (b) modified signal path when an obstruction is present.

to the region on the time-space plane enclosed by the signal paths (shaded in Figure 5a) as the “critical region”. Note that this critical region extends a maximum distance of $w'gC$ away from the signal. As will be shown next, the presence of an obstruction within the critical region always provides a lower cost path between A and E, and reduces the capacity of the intersection.

4.2. *With obstructions*

Obstructions create shortcuts on the time-space plane with a reduced cost equal to the capacity of the obstruction, Q_B, that could potentially be used to provide a shorter path over one or multiple signal cycles[4]. Only two types of these valid paths exist that can make use of the nearby obstruction shortcut, as will be described next. Both of these paths must be considered and the path which provides the lowest total cost determines the actual impact of the obstruction on the capacity of the intersection. In the following analysis, we denote t_a and $t_e = t_a + S$ as the start and the end time of the obstruction, respectively. For illustration, the origin of the time axis is set at the start of an arbitrary green period.

[4]We assume here that this reduced capacity Q_B incorporates any capacity lost due to vehicles merging at the location of the obstruction.

4.2.1. Modified signal path

The first path is a modification of the signal path, termed the 'modified signal path'. If an obstruction provides a shortcut within the critical region, then a lower cost path will *always* exist that makes use of the shortcut for the entirety of the time it exists within that region. This is easy to see with an example. Consider the time-space diagram shown in Figure 5b with an upstream obstruction denoted by the thick gray horizontal line. Let us examine the impact of this obstruction during the first cycle; i.e., the impact on the cost of the path connecting points A and E. If this obstruction was not present, the path $A - X - X' - D$ would have the same cost as any of the other valid paths connecting points A and D within the shaded region. (And, of course, the least cost path connecting points D and E would be zero.) However, when the obstruction is present the cost of path $A - X - X' - D$ reduces to $Q_m(gC - t_{XX'}) + Q_B t_{XX'} = Q_m gC - (Q_m - Q_B)t_{XX'}$ due to the impact of the shortcut, where $t_{XX'}$ denotes the time period from X to X'. The same logic holds if the obstruction shortcut bisects one of the critical regions (as illustrated during the second cycle), or if the obstruction ends or is fully contained within the critical region (not shown).

Denote the path that travels along obstruction shortcuts only within the critical region but otherwise follows the nearest signal path as the "modified signal path". An example is illustrated over two cycles in Figure 5b as path $A-X-X'-D-E-Z-Z'-H-J$. Let us also define the total cost of a modified signal path as $\Delta Z_{S'}$; note that $\Delta Z_{S'} \leq \Delta Z_S$ by definition. The values of ΔZ_S and $\Delta Z_{S'}$ could depend upon the length of the analysis period of interest, but their difference, denoted $\Delta N_{S'} = \Delta Z_S - \Delta Z_{S'}$, is independent of the chosen analysis period as long as that period include all the cycles that are affected by the obstruction. In light of the above, for any given obstruction the $\Delta N_{S'}$ is given by the following formula:

$$\Delta N_{S'} = (Q_m - Q_B)C \begin{cases} 0 & d \geq w'gC, \\ \max(g - \max(\frac{d}{w'C}, \{\frac{t_a}{C}\}), 0) + \\ \max(\min(\{\frac{t_e}{C}\}, g) - \frac{d}{w'C}, 0) + \\ (\lfloor \frac{t_e}{C} \rfloor - \lceil \frac{t_a}{C} \rceil) * (g - \frac{d}{w'C}) & d < w'gC \text{ (upstream)} \\ \max\left(g - \frac{d}{w'C} - \left\{\frac{t_a}{C}\right\}, 0\right) + \\ \min\left(\left\{\frac{t_e}{C}\right\} - g + \frac{d}{w'C}, 0\right) + \\ (\lfloor \frac{t_e}{C} \rfloor - \lceil \frac{t_a}{C} \rceil + 1)(g - \frac{d}{w'C}) & d < w'gC \text{ (downstream)} \end{cases} \tag{4}$$

where $\lceil x \rceil$ and $\lfloor x \rfloor$ denote the smallest integer greater than x and the largest integer smaller than x, respectively, and $\{x\} = x - \lfloor x \rfloor$ denotes the fractional part of x. $\Delta N_{S'}$ represents the reduction in total cost (i.e., vehicle throughput) due to the presence of the obstruction in the critical region. Qualitatively, it is equal to the product of the amount of time the obstruction spends in the critical region and the reduction of vehicular movement due to it, $Q_m - Q_B$.

4.2.2. Obstruction path

The modified signal path considers only the influence of the obstruction when it is within the critical region; i.e., only uses the obstruction as a shortcut when it is within the critical region. However, the obstruction shortcut may also be used along its entire length to provide a shorter path. This leads to the second path that must be considered, which uses almost the entire shortcut length. Consider the following path that we call an "obstruction path": 1) start from the beginning of a green period; 2) travel to the location of the obstruction at maximum speed w' (∞) for an upstream (downstream) obstruction; 3) travel horizontally along the obstruction location until the obstruction is completed (using the entire length of the obstruction as a shortcut); 4) travel back to the signal location at maximum speed ∞ (w') for an upstream (downstream) obstruction; and, 5) end at the beginning of the next green period. An example, $A-Y-Y'-I-J$, is depicted in Figure 6a for the same obstruction shown in Figure 5b.

In many cases, interactions between the signal and the obstruction may provide additional shortcuts that reduce the cost of this obstruction path. We now discuss two rules to modify the obstruction path at its start and end, respectively, to reduce its cost.

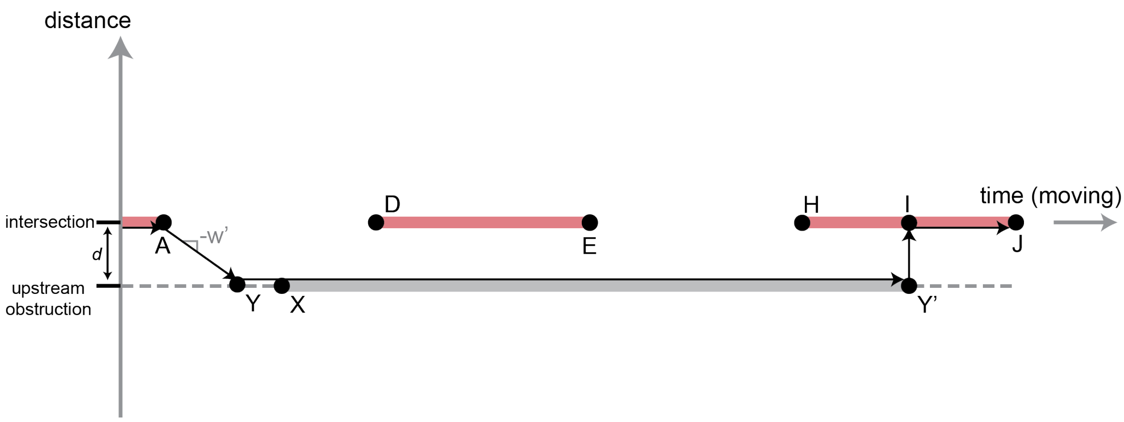
Rule 1 (obstruction start): This rule applies if the start of an obstruction lies within the shaded region shown in Figure 6b defined as: $t_a \in [(n-1)C + gC + \frac{d}{w'}, nC + \frac{d}{w'}]$ for some integer n for an upstream obstruction; and, $t_a \in [(n-1)C + gC, nC]$ for a downstream obstruction. For upstream obstructions, if t_a falls within this range, then a shorter path exists that stays at the signal location until time nC, then travels to the obstruction at speed w' and continues along the obstruction. Similarly, for downstream obstructions a shorter path exists that stays at the signal location until time nC, then travels to the obstruction at speed ∞ and continues along the obstruction.

To see why this is true, consider the upstream obstruction shown in Figure 6b. Let us calculate the total costs between points O and E using the original obstruction path and the modified path using Rule 1 for comparison. Note that after point E both the original and modified paths would be identical. The original obstruction path $O-A-C-E$ has a total cost of $Q_m(t_{OA} + t_{AB} + t_{BC}) + Q_B(t_{CD} + t_{DE})$. The modified path defined by Rule 1, $O-B-D-E$, has a total cost of $Q_m(t_{OA} + t_{AB}) + Q_m t_{DE}$. Since $t_{DE} \leq t_{BC}$, the modified path provides a lower total cost than the original path. Additionally, path $O-B-D-E$ would have the least cost of all paths connecting points O and E, since any path must be at least between the beginning and end of the green period (with total cost $(t_{OA}+t_{AB})Q_M$) and from the signal location to the obstruction location (with total cost $Q_m\frac{d}{w'}$. A similar comparison can be made to show that this rule also applies for downstream obstructions (not shown).

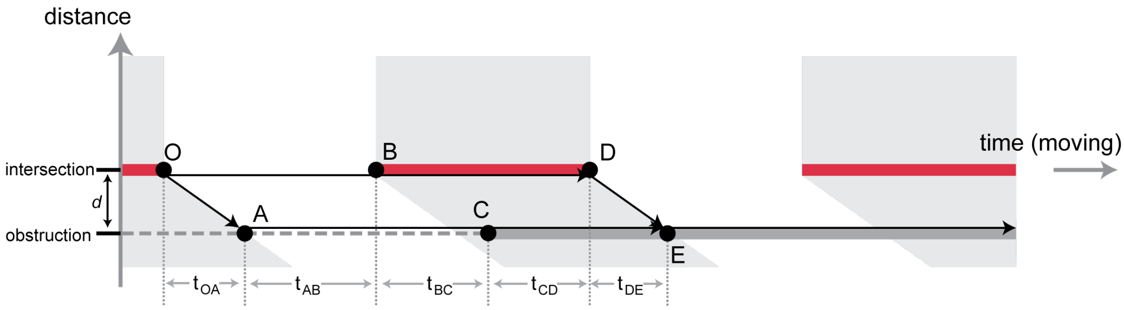
Rule 2 (obstruction end): This rule applies if the end of the obstruction lies within the shaded areas shown in Figure 6c defined as: $t_e \in [(n-1)C+gC, nC]$ for some integer n for an upstream obstruction; and, $t_e \in [(n-1)C+gC+\frac{d}{w'}, nC+\frac{d}{w'}]$ for a downstream obstruction. For upstream obstructions, a shorter path exists that leaves the obstruction location at $(n-1)C+gC$, travels to the signal location at speed ∞, and remains there until the end of the present red period. Similarly, for downstream obstructions, a shorter path exists that leaves the obstruction location at $(n-1)C+gC-\frac{d}{w'}$, travels to the signal location at w', and stays at the signal until the end of the present red period.

The logic behind the rule is the same as before. Consider the costs of the obstruction path and modified path applying Rule 2 that connect points X and Z shown in Figure 6c. The obstruction path, $X-Y-Z$, has total cost $Q_B t_{XY}$ while the modified path $X-W-Z$ has total cost 0. As expected, no shorter path can exist that connects these points. A similar comparison can also be performed for downstream obstructions to verify that the rule applies in this case as well.

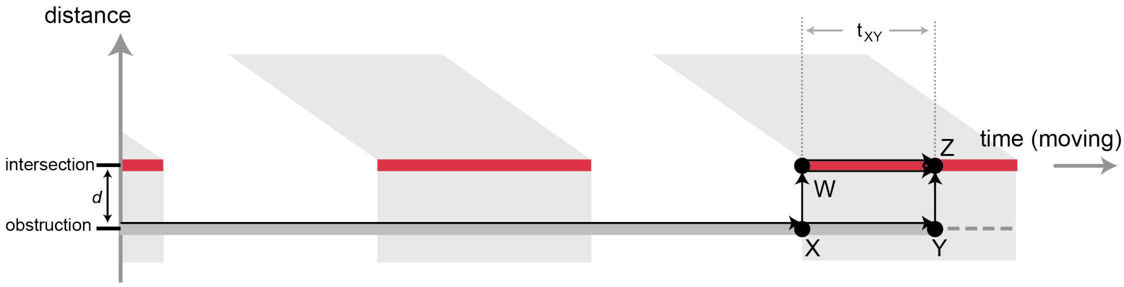
The applicability of the two rules depend entirely on the location of the obstruction, its start time and duration, and the signal timing settings at the intersection. Each may act on any real-world obstruction considered based on these parameters. Note that it is also possible for both rules to be applied simultaneously, as well. We denote the final obstruction path after applying the two rules the "modified obstruction path" and its total cost as $\Delta Z_{o'}$. The difference between ΔZ_S and $\Delta Z_{o'}$, denoted $\Delta N_{o'}$, can be



(a)



(b)



(c)

Figure 6.: Time-space diagram showing: (a) obstruction path; (b) modified obstruction path due to Rule 1; and, (c) modified obstruction path due to Rule 2.

expressed as:

$$\Delta N_{o'} = \begin{cases} Q_m C\left(g\left(\left\lceil \frac{t_e}{C}\right\rceil - \left\lfloor \frac{t_a}{C} - \frac{d}{w'C}\right\rfloor\right) - \min\left(\left\{\frac{t_a}{C} - \frac{d}{w'C}\right\}, g\right)\right) - \\ Q_m C\left(\frac{d}{w'C} + \max\left(g - \left\{\frac{t_e}{C}\right\}, 0\right)\right) - \\ Q_B * \max(0, C * \min\left(\frac{t_e}{C}, g + \left\lfloor \frac{t_e}{C}\right\rfloor\right) - t_a - \\ I\left(\left\{\frac{t_a}{C} - \frac{d}{w'C}\right\} > g\right) * C\left(1 - \left\{\frac{t_a}{C} - \frac{d}{w'C}\right\}\right)) & \text{upstream obstruction} \\ \\ Q_m C\left(g\left(\left\lceil \frac{t_e}{C} + \frac{d}{w'C}\right\rceil - \left\lfloor \frac{t_a}{C}\right\rfloor\right) - \min\left(\left\{\frac{t_a}{C}\right\}, g\right)\right) - \\ Q_m C\left(\frac{d}{w'C} + \max\left(g - \left\{\frac{t_e}{C} + \frac{d}{w'C}\right\}, 0\right)\right) - \\ Q_B * \max(0, \min\left(\left\lfloor \frac{t_e}{C} + \frac{d}{w'C}\right\rfloor C + gC - \frac{d}{w'}, t_e\right) - t_a - \\ I\left(\left\{\frac{t_a}{C}\right\} > g\right) * C\left(1 - \left\{\frac{t_a}{C}\right\}\right)) & \text{downstream obstruction} \end{cases} \tag{5}$$

where $I(\bullet)$ is an indicator function which takes the value 1 if the argument is true and 0 otherwise. This term represents the reduction in cost (i.e., vehicle throughput) due to the entirety of the obstruction's presence. Note that when the rules are applied, it might be possible that no portion of the obstruction path exists along the shortcut provided by the obstruction. This would occur when the obstruction is entirely contained in the union of the shaded regions shown in Figures 6b and 6c. In this case, the modified obstruction path can be entirely ignored because its cost will always be greater than that of the modified signal path.

4.2.3. Capacity calculation

The capacity of the signal when an obstruction is present can be determined using only the modified signal and modified obstruction paths.[5] This capacity, q_c, equals:

$$q_c = \frac{\Delta Z_s - \max\{\Delta N_{s'}, \Delta N_{o'}\}}{nC} \tag{6}$$

where n is the number of cycles considered in the analysis period; n should be no less than the number of cycles affected by the obstruction, i.e., $\left\lceil \frac{t_e}{C}\right\rceil - \left\lceil \frac{t_a}{C} - \frac{d}{w'C}\right\rceil$ for an upstream obstruction and $\left\lceil \frac{t_e}{C} + \frac{d}{w'C}\right\rceil - \left\lceil \frac{t_a}{C}\right\rceil$ for a downstream one.

The binding path also unveils the mechanism that limits capacity the most. If the modified obstruction path has the lower cost, the obstruction is the most restrictive bottleneck and queues will form at the obstruction location while it is present (excluding the portions that are trimmed by Rules 1 and 2). If the modified signal path has the lower cost, the signal is the bottleneck and the obstruction merely starves vehicle discharge flows during portions of the signal green periods.

5. Insights and Analysis

Section 5.1 applies the simple recipe from the previous section to develop insights on the placement and timing of obstructions that can be controlled. Section 5.2 presents numerical results of these models to examine how various properties of the obstruction impact capacity. Finally, Section 5.3 shows how this recipe can be used to develop a

[5]The reader can confirm that the obstruction cannot be used to create any shorter paths than these two.

simple adaptive signal control strategy to reduce the impact of some obstructions and demonstrates the potential capacity saving provided.

5.1. *Insights on timing and location of obstructions*

The first insight pertains to the choice of the obstruction's location between either upstream and downstream of the signal. Debates regarding this choice have long existed; e.g., researchers have argued where best to locate a bus stop relative to its nearby intersection (Terry and Thomas 1971; Fitzpatrick et al. 1997). Though a comprehensive comparison is yet to be conducted, the conclusion drawn from this methodology is simple if the only concern is the impact on the vehicle discharge capacity at the intersection: if d is given and t_a is uniformly distributed over time, an upstream obstruction yields the same expected capacity as a downstream one. This can be easily proved using our methodology presented above. Next, we will further scrutinize the effects of d and t_a for two special types of obstructions: short-lasting obstructions and permanent obstructions.

Let us first consider a short-lasting obstruction with $S < (1-g)C$ to determine insights on its location and potential control measures. Examples of obstructions of this type may include a pre-signal upstream of the main signal that temporarily stops car movement to allow buses to change lanes, a bus dwelling at a stop with a low passenger demand or vehicles egressing from driveways near the intersection. In this case, the modified signal path would always provide the least cost as the traffic signal is always more restrictive; i.e., $\Delta Z_{s'} < \Delta Z_{o'}$. Remember that obstructions only reduce the cost of the signal path if they occur within the critical region (see again Figures 5a and 5b), and this region only extends a maximum distance $w'gC$ away from the intersection. Therefore, short obstructions will never impact capacity if they are located at $d > w'gC$. If proximity to the intersection is also desired, the optimal obstruction location to avoid capacity reduction is $d^* = w'gC$.

If further proximity to the intersection is desired, then capacity reductions may occur. However, these reductions in capacity can be avoided if the obstruction can somehow be controlled to never occur within the critical region. Knowing the boundaries of this region, policies could be implemented to move the obstruction start time to periods when the obstruction will not affect capacity. (For upstream obstructions this start time is $nC+gC$, while for downstream obstructions it is $nC+gC-\frac{d}{w'}$, for some integer n.) For example, driveway egress could be controlled with additional signals on the driveways and parking maneuvers could be prohibited dynamically during these periods. Additionally, pre-signals could be controlled to never activate during these critical times or buses could be held upstream and only allowed to arrive to stops near the intersection outside of the critical region as in Gu et al. (2013). Of course, these latter strategies would impose some delays onto buses in exchange for increased car capacity. If obstructions cannot be controlled in this manner, then the methodology could be used to determine the closest location of the obstruction such that some maximum capacity reduction threshold is never exceeded.

Now, let us apply the methodology to identify the optimal location of a permanent obstruction with $S = \infty$. Examples of obstructions of this type include the elimination of a travel lane or installation of a bus bulb that impedes car traffic. If $d > w^{'}gC$, the modified signal and modified obstruction paths never intersect and can be treated independently. The cost of each path during any cycle is $\Delta Z_{s'} = Q_m gC$ and $\Delta Z_{o'} = Q_B C$, both of which are independent of d. In this case, the interaction of the obstruction with the signal does not affect capacity, and the maximum throughput is governed by the more restrictive of the two bottlenecks. On the other hand, if $d < w^{'}gC$, the modified signal and modified obstruction paths intersect once per cycle, at the end of every green period; see

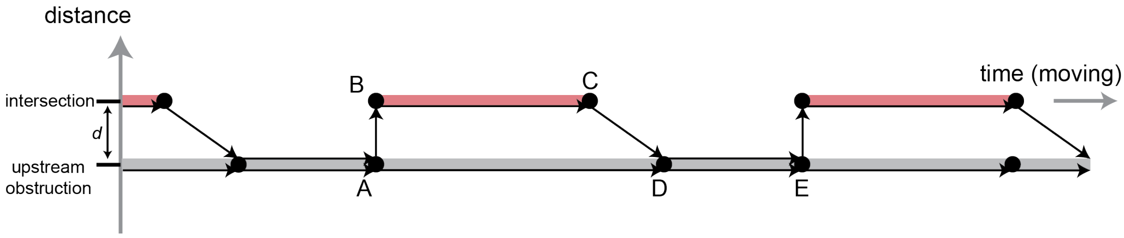


Figure 7.: Modified signal path and obstruction path if the obstruction is permanent.

Figure 7. The costs of the two paths during any cycle are $\Delta Z_{s'} = Q_B gC + (Q_m - Q_B)\frac{d}{w'}$ and $\Delta Z_{o'} = Q_B C$. Notice that $\Delta Z_{s'}$ increases with d but $\Delta Z_{o'}$ is independent of d. Thus, when d is small, the capacity of the intersection can be increased by moving the obstruction away from the intersection until the modified signal path has the same cost as the modified obstruction path; i.e., location $d^* = \frac{Q_B}{Q_m - Q_B}(1-g)Cw'$. For $d > d^*$, moving the obstruction further away does not increase capacity because at these locations the obstruction serves as the more restrictive bottleneck.

5.2. *Numerical Analysis*

We now use the previous methodology to quantify the impacts of obstructions on the capacity of signalized intersections. Dimensional analysis reveals that only five dimensionless parameters are needed to completely characterize a given obstruction: $\frac{Q_B}{Q_m}$, $\frac{t_a}{C}$, $\frac{S}{C}$, $\frac{d}{w'C}$ and g. The first describes the restrictiveness of the obstruction relative to the roadway segment. The second and third describe the start time and length of the obstruction, respectively. The fourth describes the location of the obstruction, and the fifth is just the fraction of green time available in the direction of interest. The capacity loss due to the obstruction can also be expressed dimensionlessly as the equivalent number of cycles worth of vehicle discharge lost due to the obstruction's presence. This term is equal to $\frac{max\{\Delta N_{s'}, \Delta N_{o'}\}}{Q_m gC}$.

Figure 8 presents the expected dimensionless capacity reduction for upstream obstructions located various distances away from the signal and lasting for various durations. For these calculations, we assume that the obstruction start time is uniformly distributed over the length of a cycle.

The capacity reductions can be quite significant: up to 6 cycles worth of vehicle throughput lost due to the obstruction for the parameters shown in Figure 8. As expected, longer lasting obstructions and obstructions closer to the signal provide a larger reduction in capacity. Figure 8 shows that the range of locations that cause a capacity reduction is a function of both $\frac{Q_B}{Q_m}$ and g. As the obstruction gets more restrictive (lower $\frac{Q_B}{Q_m}$), the range of locations at which it can affect signal capacity increases significantly, and as the signal becomes more restrictive (lower g), the range of locations decreases. The expected capacity reduction is also non-linear with respect to the distance of the obstruction from the signal, d. When d is small, increasing d can significantly reduce the capacity loss incurred by the obstruction (especially when S/C is large). However, this benefit almost vanishes when d is large. This highlights that significant capacity benefits can be achieved by moving nearby obstructions just a short distance away from signalized intersections.

These results are robust even when the actual, dimensionless duration of the obstruction is random and drawn from a distribution with mean $\frac{S}{C}$. Numerical simulations confirm that the expected capacity reductions presented in Figure 8 do not change significantly even if the variation in duration is as much as 50% of the mean value $\frac{S}{C}$.

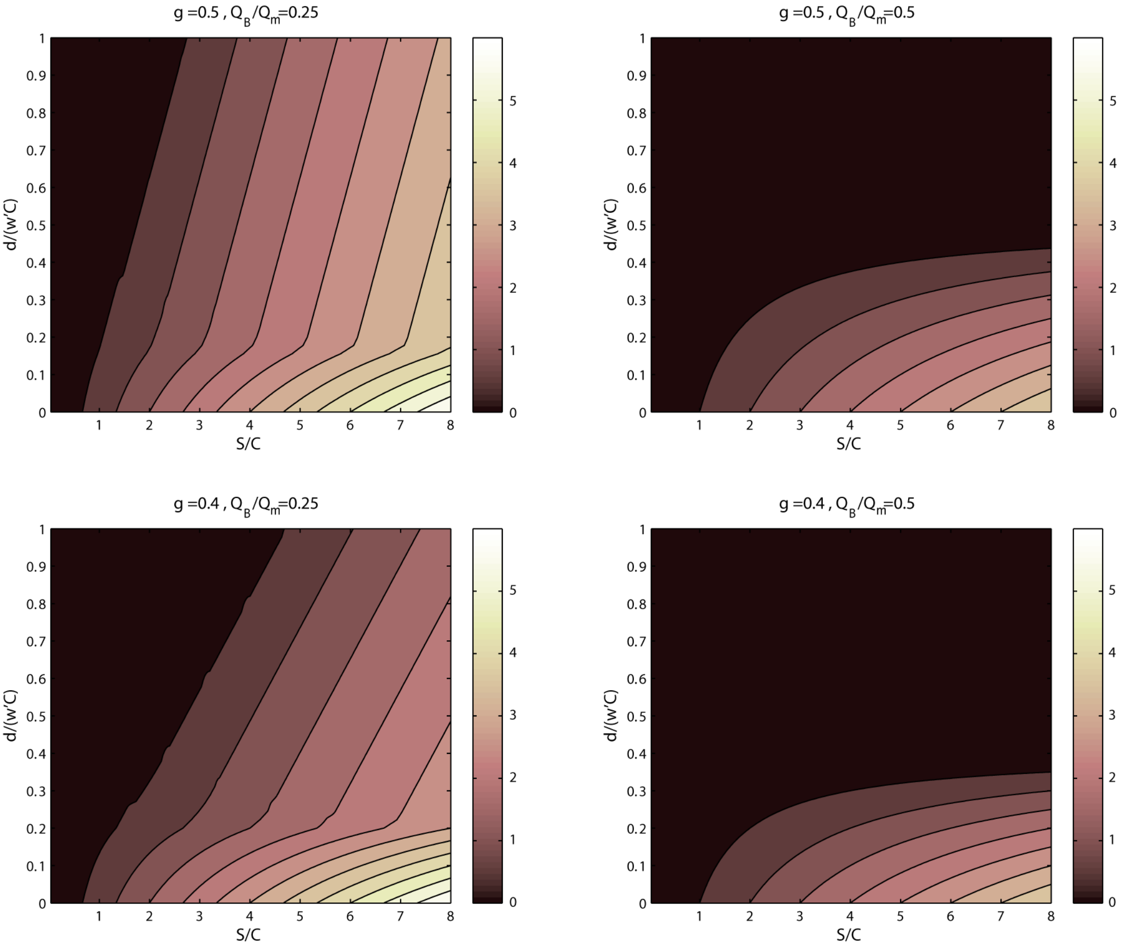


Figure 8.: Numerical results for capacity reduction over a 10-cycle period.

5.3. *An adaptive signal control strategy to improve capacity*

The analyses presented in Section 5.1 and 5.2 demonstrate that the location and timing of an obstruction can significantly impact the capacity of a signalized intersection, and that these impacts can be reduced or eliminated if harmful obstructions can be moved in space and time. Unfortunately, this may not always be possible or feasible for certain types of obstructions. Instead, it might be more practical to adjust the signal timing dynamically when an obstruction is present to mitigate some of its negative impacts. We propose here a simple strategy to demonstrate how the recipe presented in this paper can inform this dynamic signal control and illustrate the potential benefits.

Assume now that the obstruction can be detected when it starts to impede traffic. An example of this might be a GPS-enabled bus dwelling at a curbside stop. Also assume that the obstruction lasts at most one cycle ($S \leq C$). Equations (4) and (5) can be used to verify that for many realistic situations the obstruction's impact will be fully described by the modified signal path under very mild but realistic assumptions.[6]

As discussed in Section 4.2, capacity reductions will occur only if the obstruction penetrates the critical region shown in Figure 5a. By changing the signal timing, the negative effects of these capacity reductions can be mitigated. Many different strategies can be developed considering different objectives. We now develop one simple strategy

[6]Here we consider cases in which one lane of a multi-lane intersection approach is blocked by the obstruction (*i.e.*, $Q_B/Q > 1/2$) and $g \approx 0.5$.

for illustration purposes.

If the cross-street approach is undersaturated, then the adaptive signal control could focus on minimizing the capacity lost on the approach of interest. To achieve this goal, the entire green phase of the cycle after the obstruction begins can be delayed by some amount δ to ensure that the the critical region is never penetrated by the obstruction; see Figure 9. By moving the green period by this amount, the critical region moves forward in time by the same amount. Delaying the green time in this way would eliminate the capacity reduction caused during the cycle after the obstruction begins.

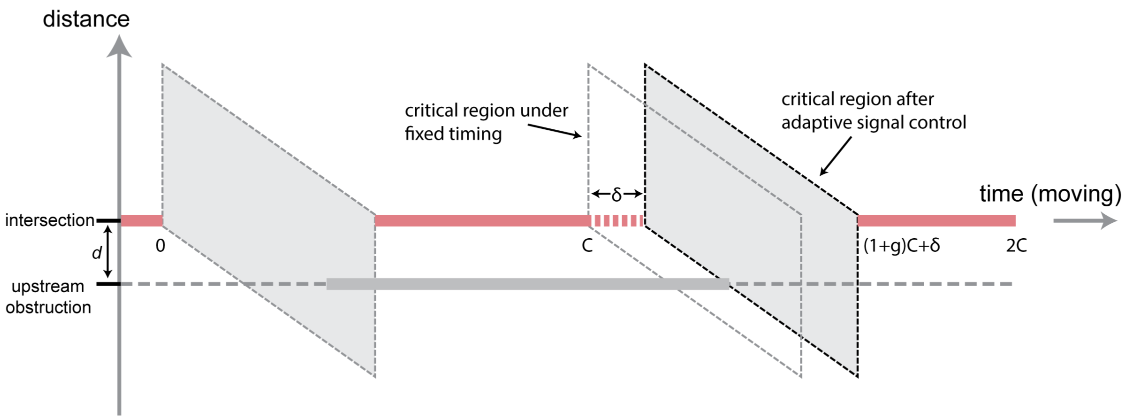


Figure 9.: Time-space diagrams showing the simple adaptive signal control scheme. When the end of an obstruction may penetrate the critical region (unshaded dotted area), signal timing is changed to move the critical region to minimize this penetration.

In most realistic cases, the value of S will never be known a priori; thus, the value of δ required to minimize the obstruction's impact could not be exactly determined. However, if the maximum duration of the obstruction, S_{max}, was known, the same strategy could be applied by delaying the green period to account for the longest-lasting obstruction expected. Using the maximum duration accounts for the worst-case obstruction, and ensures that the adaptive signal control scheme eliminates any capacity loss incurred due to the obstruction during the second cycle. Of course, such a strategy might result in unrealistically short red periods after the delayed green if a constant cycle length is maintained. Realistically, a minimum red period might be required to serve vehicles in the cross-street direction. Thus, the strategy can be further modified by including a minimum time for the shortened red period, r_{min}. Using this logic, the amount of delay to the green period for an upstream obstruction is:[7]

$$\delta = \begin{cases} \min\left[\max\left(t_a + S_{max} - d/w - C, 0\right), (1-g)C - r_{min}\right] & \text{upstream} \\ \\ \min\left[\max\left(t_a + S_{max} - C, 0\right), (1-g)C - r_{min}\right] & \text{downstream} \end{cases} \tag{7}$$

The primary drawback to accounting for a minimum red duration is that some obstructions might still impact capacity during the second cycle. Thus, a trade-off exists between the flexibility of the signal timings (i.e., r_{min}) and the increase in capacity if a constant cycle length is to be maintained[8]. For a given upstream obstruction, the capac-

[7]A similar equation can be developed for a downstream obstruction, but this is omitted for brevity.
[8]Additional signal timing strategies could be developed using flexible cycle lengths. However, for simplicity, we maintain this assumption here.

ity recovered when using the adaptive signal control scheme (as compared to when the obstruction limits capacity), measured in units of additional vehicle discharge, is:

$$\Delta N_g = (Q - Q_B) \begin{cases} \text{mid}\,(0, (1+g)C + \delta - t_a - S, gC - d/w) - \\ \text{mid}\,(0, (1+g)C - t_a - S, gC - d/w) - & \text{upstream} \\ \\ \text{mid}\,(0, (1+g)C + \delta - d/w - t_a - S, gC - d/w) - \\ \text{mid}\,(0, (1+g)C - d/wt_a - S, gC - d/w) - & \text{upstream} \end{cases} \tag{8}$$

Figure 10 presents numerical results that quantify the fraction of the capacity loss that may be recovered using the adaptive signal control scheme described for various obstruction locations and values of r_{min}. The following parameters were used to create this figure: $g/C = 0.5$, $S/C \sim$ U$(0.8, 1.0)$ and $Q_B/Q = 0.5$, and the obstruction start time was uniformly distributed over the length of a cycle. Note that the capacity benefits can be significant: up to 20% of the capacity of a single cycle. The expected benefits decrease linearly with the obstruction's distance from the intersection, and the fraction of a cycle required as a minimum red period. As expected, when the minimum red time is equal to the red time used when signal timings are fixed (in this case, when $r_{min}/C = 1 - g = 0.5$), no benefits can be provided as there is no signal flexibility.

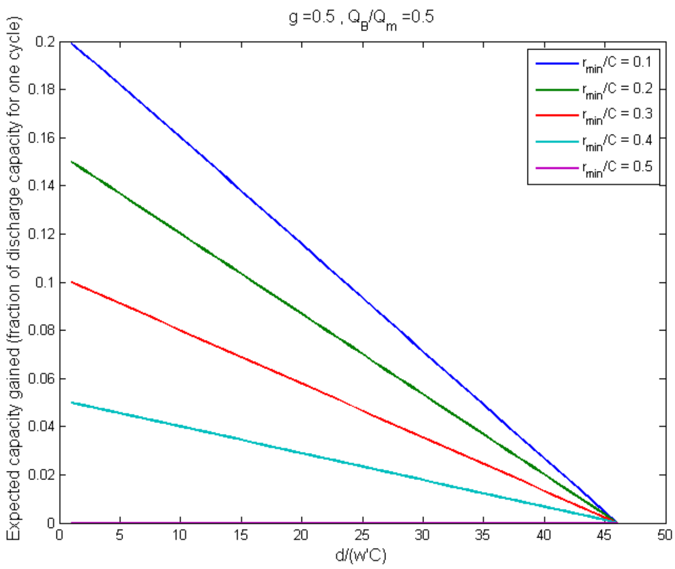


Figure 10.: Fraction of capacity loss avoided using adaptive signal control scheme.

6. Concluding Remarks

We have applied the variational theory of kinematic waves in a moving-time coordinate system to examine the impact of obstructions on the capacities of nearby signalized intersections. A simple recipe to calculate the capacity of an intersection when a roadway obstruction is present was developed. The recipe is general, requires very few calculations, and can be applied to obstructions of any type, duration and location, including both upstream and downstream of the intersection. Thus, it provides a more complete picture of how intersection capacity might be affected by nearby obstructions. From the recipe, we further develop capacity formulae for an arbitrary obstruction. These formulae can be readily used by practitioners for estimating an intersection's vehicle discharge capacity when a nearby obstruction is present, and perhaps eventually even be incorporated into future versions of the Highway Capacity Manual software and methodology. Furthermore, these formulae can be used to assess the impacts of randomly occurring capacity disruptions, such as on-street parking maneuvers. Monte Carlo techniques can be applied where stochastic parameters (e.g., start time of obstruction, location and duration) can be randomly drawn from known or assumed distributions to estimate the expected capacity loss at these locations.

The recipe and resulting capacity formulae can also be used to determine the optimal locations for various obstructions to provide the highest signal capacity while simultaneously allowing the obstruction to be as close to the intersection as possible. The latter is often desirable; e.g., transit agencies prefer to place bus stops near intersections to facilitate passenger transfers between bus lines and protected street crossings (Fitzpatrick et al. 1996). For short obstructions, the variational theory methodology can be used to devise strategies to place obstructions even closer to the intersection if their start times can be controlled. If they cannot be controlled, signal timings may be adjusted dynamically to reduce the impacts of these obstructions. These strategies can easily be extended to obstructions with longer durations.

Our models are limited in that they assume the fundamental diagram is triangular, ignore the impact of turning traffic and the interaction of multiple signals in series (as would occur along a signalized arterial with closely spaced intersections). The former is assumed to obtain simple cost functions in a moving-time coordinate system. However, numerous studies have suggested that the triangular fundamental diagram is appropriate to describe traffic on links (e.g., see Newell 1993; Cassidy 1998). Additionally, turning traffic at the intersection would have little impact on the solution for obstructions upstream of the intersection if dedicated turn lanes are not present. For obstructions downstream, vehicles turning off of the segment of interest might reduce the capacity loss incurred during the green phase of the signal while vehicles turning onto the segment during the red phase might increase the capacity loss. These effects are likely to be small and cancel each other out, especially for the critical cases when the obstruction is near the intersection. Furthermore, the variational theory method can be easily extended to consider two (or more) signalized intersections in series with an obstruction between them. However, by considering an isolated location the impacts of obstructions at signalized intersections can be analyzed in depth and the different interactions between obstruction location and capacity can be fully understood. This approach of initially focusing on an isolated intersection to fully comprehend the problem is often taken in the literature (Guler and Cassidy 2012; Gu et al. 2013; Guler and Menendez 2014; Gu et al. 2014). Using this comprehensive understanding of the problem, current work is being performed to consider intersections in series to provide insights on the impact of obstructions in more general settings. Additionally, this method can be used to model left- and right-turning lanes that might exist by treating these as a separate (independent) approach. Further work is required to model how congestion on through lanes might impact the capacities of left- and right-turning lanes, and vice versa.

7. Appendix A: List of variables

Variable	Meaning
C	cycle length [time]
δ	additional red time applied in adaptive signal strategy [time]
ΔN_g	recovered vehicle discharge when applying adaptive signal strategy [veh]
$\Delta N'_O$	cost reduction on modified obstruction path over signal path [veh]
$\Delta N'_S$	cost reduction on modified signal path over signal path [veh]
$\Delta Z'_O$	cost of modified obstruction path [veh]
ΔZ_S	cost of modified signal path [veh]
ΔZ_S	cost of signal path [veh]
d	distance of obstruction from intersection [dist]
$d*$	critical distance over which capacity is not improved by moving obstruction further away from signal [dist]
g	green ratio
k_j	jam density [veh/dist]
n	number of cycles considered in anaylsis period
Q_b	reduced capacity at obstruction [veh/time]
q_c	capacity of signal considering impacts of obstruction [veh/time]
Q_m	roadway capacity [veh/time]
$r'(u')$	cost function used in variational theory, applied in moving-time coordinate system
$r(u)$	cost function used in variational theory
S	obstruction duration [time]
S_{max}	maximum duration of obstruction
t_a	starting time of obstruction measured in moving-time coordinate system [time]
t_e	ending time of obstruction measured in moving-time coordinate system [time]
u	speed of observer path [dist/time]
u'	speed of observer path in moving-time coordinate system [dist/time]
v_f	free flow speed [dist/time]
$-w$	backward wave speed [dist/time]
$-w'$	backward wave speed in moving-time coordinate system [dist/time]

References

Box, P., 2004. Curb-parking problems: Overview. ASCE Journal of Transportation Engineering Practitioners Forum 130(1), 1–5.

Cao, J., Vasileios, N., Menendez, M., 2013. On-street parking near intersections: Effects on traffic, in: 13th Swiss Transport Research Conference.

Cassidy, M.J., 1998. Bivariate relations in nearly stationary highway traffic. Transportation Research Part B 32(1), 49–59.

Chiabaut, N., Xie, X., Leclercq, L., 2012. Road capacity and travel times with bus lanes and intermittent priority activation. Transportation Research Record 2315, 182–190.

Christofa, E., Skabardonis, A., 2011. Traffic signal optimization with application of transit signal priority to an isolated intersection. Transportation Research Record 2259, 192–201.

Daganzo, C., 2005a. A variational formulation of kinematic waves: Basic theory and complex boundary conditions. Transportation Research Part B 39(2), 187–196.

Daganzo, C., 2005b. A variational formulation of kinematic waves: Solution methods. Transportation Research Par B 39(10), 934–950.

Daganzo, C., Geroliminis, N., 2008. An analytical approximation for the macroscopic fundamental diagram of urban traffic. Transportation Research Part B 42, 771–781.

Daganzo, C., Menendez, M., 2005. A variational formulation of kinematic waves: Bottleneck properties and examples, in: 16th International Symposium on Transportation and Traffic Theory.

Dijkstra, E., 1959. A note on two problems in connexion with graphs. Numerische Mathematik 1, 269–271.

Fitzpatrick, K., Hall, K., Finley, M., Farnsworth, S., 2002. Alternative bus stop configuration: An analysis of the effect of bus bulbs. Journal of Public Transportation 5(1), 19–37.

Fitzpatrick, K., Hall, K., Perkinson, D., Nowlin, L., 1997. Location and design of bus stops. ITE Journal 67(5), 36–41.

Fitzpatrick, K., Hall, K., Perkinson, D., Nowlin, L., Koppa, R., 1996. Guidelines for the location and design of bus stops. TCRP Report 19.

Gayah, V., Daganzo, C., 2012. Analytical capacity comparison of one-way and two-way signalized street networks. Transportation Research Record 2301, 76–85.

Geroliminis, N., Boyaci, B., 2012. The effect of variability of urban systems characteristics in the network capacity. Transportation Research Part B 46(10), 1607–1623.

Girianna, M., Benekohal, R., 2002. Dynamic signal coordination for networks with oversaturated intersections. Transportation Research Record 1811, 122–130.

Gu, W., Cassidy, M., Gayah, V., Ouyang, Y., 2013. Mitigating negative impacts of near-side bus stops on cars. Transportation Research Part B 47(1), 42–56.

Gu, W., Gayah, V.V.,Cassidy, M.J., Saade, N., 2014. On the impacts of bus stops near signalized intersections: Models of car and bus delays. Transportation Research Part B, in press.

Guler, S.I., Cassidy, M., 2012. Strategies for sharing bottleneck capacity among buses and cars. Transportation Research Part B 46(10), 1334–1345.

Guler, S.I., Menendez, M., 2014. Analytical formulation and empirical evaluation of pre-signals. Transportation Research Part B 64, 41-53.

Kim, W., Rilett, L., 2005. Improved transit signal priority system for networks with nearside bus stops. Transportation Research Record 1925, 205–214.

Leclercq, L., Laval, J., Chiabaut, N., 2011. Capacity drops at merges: An endogenous model. Transportation Research Part B 45(9), 1302–1313.

Leclercq, L., Geroliminis, N., 2013. Estimating MFDs in simple networks with route choice, in: Procedia-Social and Behavioral Sciences 80.

Lighthill, M., Whitham, G., 1955. On kinematic waves. I: Flow movement in long rivers. II: A Theory of traffic flow on long crowded roads. Proceedings of the Royal Society 229, 281–345.

Miller, A., 1963. Computer control system for traffic networks, in: Proceedings of the 2nd International Symposium on Theory of Traffic Flow.

Newell, G., 1993. A simplified theory of kinematic waves in highway traffic, parts I-III. Transportation Research Part B 27(4), 281–313.

Richards, P., 1956. Shock waves on the highway. Operations research 4, 42–51.

Robertson, D., Bretherton, R., 1974. Optimal control of an intersection for any known sequences of vehicle arrivals, in: Proceedings of the 2nd IFAC-IFIP-IFORS Symposium on Traffic Control and Transport Systems.

Roess, R., Prassas, E., McShane, W., 2004. Traffic Engineering. Prentice. 4th edition.

Shelby, S., 2004. Single-intersection evaluation of real-time adaptive traffic signal control algorithms. Transportation Research Record 1867, 183–192.

Terry, D., Thomas, G., 1971. Farside bus stops are better. Traffic Engineering 41(3), 21–29.

TRB, 2010. Highway Capacity Manual. Transportation Research Board.

Wu, J., Hounsell, N., 1998. Bus priority using pre-signals. Transportation Research Part A 32(8), 563–583.

Xuan, Y., Gayah, V., Cassidy, M., Daganzo, C., 2012. Using a pre-signal to increase bus- and car-carrying capacity at intersections: Theory and experiment. Transportation Research Record 2315, 191–196.

Yannis, G., Golias, J., Antoniou, C., 2006. Effects of urban delivery restrictions on traffic movements. Transportation Planning and Technology 29(4), 295–311.

Ye, X., Chen, J., 2011. Traffic delay caused by curb parking set in the influenced area of signalized intersections, in: 11th International Conference of Chinese Transportation Professionals.