1997 IEEE International Symposium on Circuits and Systems, June 9-12, 1997, Hong Kong

Regularized restoration of VQ compressed images with Constrained Least Squares approach

Steven S. O. Choy, C. N. Yue, S. W. Hong and Yuk-Hee Chan

Department of Electronic Engineering
The Hong Kong Polytechnic University
Hong Kong

Abstract

In this paper, an iterative algorithm is proposed to restore VQ-encoded images. This algorithm incorporates adaptivity into simple CLS restoration algorithm by weighting every pixel according to its expected derivation from the original. This algorithm is fully compatible with any VQ codec to improve codec's coding performance. Computer simulations showed that the proposed scheme was superior to other conventional schemes in terms of SNR improvement. Besides, the image quality could also be improved subjectively by reducing the blocking effect.

Introduction

Most popular image compression techniques applied nowadays such as transform coding techniques[1] and vector quantization techniques[2] are basically lossy approaches. These compression techniques typically allow a tolerable distortion to appear after the compression in order to achieve a higher compression rate. When the compression rate is not too high, the distortion introduced will do little harm to the quality of encoded images since a small distortion of the image quality is generally unobservable under our human visual system. However, the story will be completely different when the compression rate is sufficiently high. This is the reason why one can see the so-called blocking effect in block-based coding techniques such as JPEG and VQ.

Image restoration refers to the problem of estimating the original image from its degraded observation. Though this technique does not aim at improving the subjective look of an image as image enhancement does, it can be very helpful for one to recover the original image from the encoded image and in turn improve the image appearance to a certain extent as well. Amount of research has been carried out recently to eliminate blocking effect of JPEG-encoded images with various image restoration techniques[3-5]. However, by far little effort has been appeared in the literature to address the restoration of VQ-encoded images.

In this paper, the restoration of VQ-encoded images is formulated as a regularized image recovery problem. This work makes use of the prior knowledge of the VQ quantization distortion to recover a VQ-encoded image. Simulations shown that this proposed approach can provide a better SNR improvement compared with some other approaches adopted from those approaches for JPEG-decoded images.

Algorithm

In image restoration, an image degradation process can be generally formulated by the following matrix-vector equation

$$\boldsymbol{g} = \mathbf{H}\boldsymbol{f} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{f}$ and $\boldsymbol{g}$ are the lexicographically ordered original and degraded images, $\boldsymbol{n}$ is a noise vector and $\mathbf{H}$ represents a linear degradation operator[6]. However, this model cannot directly applied to a VQ process. Unlike those classical problems tackled in image restoration such as deblurring, vector quantization is not a linear degradation operation.

In VQ compression, the source image $\boldsymbol{f}$ is partitioned into a number of subimages of equal size. Each subimage is then treated as a vector and is represented with a particular codeword selected from a codebook. The selection is based on the minimum

0-7803-3583-X/97 $10.00 ©1997 IEEE

Euclidean distance criterion, where the Euclidean distance measure between vectors $\vec{v_1}$ and $\vec{v_2}$ is defined as $d(\vec{v_1},\vec{v_2}) \equiv \| \vec{v_1} - \vec{v_2} \|$. Here, without losing the generality, we lexicographically order these two-dimensional subimage vectors as one-dimensional column vectors of size D each.

The corresponding VQ-encoded image, say $\boldsymbol{g}$, will certainly be distorted to a certain extent. To restore this distorted image, one possible approach is to estimate $\boldsymbol{f}$ by seeking a $\hat{\boldsymbol{f}}$ which approximates $\boldsymbol{g}$ in a least mean squares sense under a certain constraint based on the *a priori* knowledge about the smoothness of typical pictures.

Consider the case that the encoded image is encoded with a codebook containing N codewords, namely, $C = \left\{ \vec{y_i} \mid i=1,2...N \right\}$. According to VQ theory, these N codewords partition the whole vector space concerned, say $\boldsymbol{V}$, into N non-overlapped Voronoi regions. For a particular Voronoi region $R_k = \{ \vec{x} \mid \vec{x} \in \boldsymbol{V}$ and $d(\vec{x}, \vec{y_k}) \le d(\vec{x}, \vec{y_j})$ for $j=1,2..N\}$, the associated variance vector is given as $\vec{\sigma_k} = (\sigma^2_{k,1}, \sigma^2_{k,2}, .. \sigma^2_{k,D})^T$, where $\sigma_{k,m}{}^2 = \frac{1}{N_{o,k}} \sum_{\vec{x} \in R_k} \left(x_m - y_{k,m} \right)^2$. Here, $N_{o,k}$ is the total number of vectors in R_k, x_m is the m^{th} element of $\vec{x}$ and $y_{k,m}$ is the m^{th} element of $\vec{y_k}$. In practice, in order to reduce the realization complexity, one may select some typical images to form a training set Ω_t and then approximate $\vec{\sigma_k}$ with vectors in $\Omega_t \cap R_k$ instead of R_k. Note that all $\vec{\sigma_k}$'s are solely determined by the codebook and hence can be precomputed once the codebook is given and stored as an extension of the codebook for future application.

Let $\vec{g_b}$ and $\vec{f_b}$ be the b^{th} subimages of $\boldsymbol{g}$ and $\boldsymbol{f}$ respectively. Suppose $\vec{y_k}$ is the codeword used to represent $\vec{f_b}$. In that case, we have $\vec{g_b}=\vec{y_k}$ and the distance between $\vec{g_b}$ and $\vec{f_b}$ is bounded by the boundary of the Voronoi region R_k. For any particular pixel $g_{b,i}$, its deviation from $f_{b,i}$ is also bounded. In formulation, we have

$$(g_{b,i} - f_{b,i})^2 \le \varepsilon_{b,i} \tag{2}$$

where $g_{b,i}$ and $f_{b,i}$ are respectively the i^{th} elements of $\vec{g_b}$ and $\vec{f_b}$ respectively, and, $\varepsilon_{b,i}$ is the bound for the uncertainty of pixel $g_{b,i}$ in the observed image. Since we have $\vec{g_b}=\vec{y_k}$, $g_{b,i}=y_{k,i}$ holds and the bound $\varepsilon_{b,i}$ can be estimated to be a function of $\sigma^2_{k,i}$. In this paper, that $\varepsilon_{b,i}$ is proportional to $\sigma^2_{k,i}$ is assumed. In that case, we have $\varepsilon_{b,i} = \mathrm{K}\, \sigma^2_{k,i}$, where K is a constant. Note that this assumption can easily be satisfied as long as K is sufficiently large. In practice, by assuming a normal distribution, $\mathrm{K} \approx 10$ is a reasonable estimate as the range of $[\, y_{k,i} - 3\,\sigma_{k,i},\ y_{k,i} + 3\,\sigma_{k,i} \,]$ covers more than 99% of the possible values of $f_{b,i}$.

Adaptivity is then introduced to weight the contribution of a particular pixel to the total expected image distortion according to its amount of uncertainty. To achieve this, we first rewrite eqn (2) as

$$\alpha^2_{b,i} \, (g_{b,i} - f_{b,i})^2 \le \mathrm{K} \tag{3}$$

where $\alpha^2_{b,i} = \sigma^{-2}_{k,i}$. The overall distortion can then be obtained by

$$J_d = \sum_{b=1}^{N_s} \sum_{i=1}^{D} \alpha^2_{b,i} \left(g_{b,i} - f_{b,i} \right)^2 \le N_s D\, \kappa \equiv \varepsilon_d \tag{4}$$

where N_s is the total number of subimages partitioned. In matrix form, we have

$$J_d = \| A(\boldsymbol{g'} - \boldsymbol{f'}) \|^2 \tag{5}$$

where $\boldsymbol{f'} = \left(\vec{f_1}, \vec{f_2}, .. \vec{f_{N_s}} \right)^T$, $\boldsymbol{g'} = \left(\vec{g_1}, \vec{g_2}, .. \vec{g_{N_s}} \right)^T$,

$$A = \begin{bmatrix} A_1 & & 0 \\ & A_2 & \\ & & \ddots \\ 0 & & A_{N_s} \end{bmatrix} \quad \text{and} \quad A_b = \begin{bmatrix} \alpha_{b1} & & 0 \\ & \alpha_{b2} & \\ & & \ddots \\ 0 & & \alpha_{bD} \end{bmatrix}.$$

Both A and A_b are diagonal matrixes.

Typical images would generally have weak high frequency components as the intensity of neighboring pixels is highly correlated. This feature can be used as an additional constraint to regularize the restoration solution. In practice, a smoothness measure J_s is defined as $J_s = \| Sf \|^2$, where S is generally a highpass filtering operator. The smoothness constraint can then be defined as $J_s \le \varepsilon_s$, where ε_s is the upper energy bound of the high frequency components. The computation of the regularized solution then reduces to the minimization of (4) subject to $J_s \le \varepsilon_s$. This is equivalent to the minimization of

$$J = \| AP(g - f) \|^2 + \alpha \| Sf \|^2 \quad (6),$$

where $\alpha = \varepsilon_d / \varepsilon_s$ and P is the permutation matrix that permutes a lexicographically ordered image f to f'.

The minimization of J with respect to f leads to the normal equation

$$\left[P^T A^T AP + \alpha S^T S \right] \hat{f} = P^T A^T AP g \quad (7)$$

In general, $\hat{f}$ cannot be evaluated directly from this equation as it requires the inversion of an huge matrix. An alternative approach is to use a steepest descent algorithm to obtain $\hat{f}$ iteratively. This approach leads to the following iterative equation:

$$\hat{f}_{k+1} = \hat{f}_k + \beta \left[P^T A^T AP \left(g - \hat{f}_k \right) - \alpha S^T S \hat{f}_k \right] \quad (8)$$

where $\hat{f}_k$ is the estimate of $\hat{f}$ after the k^{th} iteration and β is the contraction parameter for the iteration. Convergence can be guaranteed if β satisfies the following condition:

$$0 < \beta < \frac{2}{\lambda_{max}} \quad (9)$$

where λ_{max} is the largest eigenvalue of the matrix $\left(P^T A^T AP + \alpha S^T S \right)$. As matrix A is codebook-dependent and S can be predetermined, both $\alpha S^T S$ and $P^T A^T AP$ can be precomputed and hence β can also be well-defined. This saves quite a bit of effort in realizing (8). The initial estimate $\hat{f}_0$ = $P^T A^T AP g$ is generally used to obtain the solution.

Simulations

Simulation has been performed to evaluate the performance of the proposed restoration scheme on a set of 256 level gray-scale digital images of size 256×256 each. In our simulation, four standard images "House", "Girl", "Couple" and "Germany" were used as the training set to generate a codebook with LBG algorithm. The codebook is of size 256 and the codeword size is 16 (a block of 4×4 pixels). Images encoded with this codebook were then restored with the proposed scheme. In our simulation, variance vectors were approximated with the training vectors as we mentioned in previous section, and, α and β were set to be $\alpha = \varepsilon_d / (10 \| Sg \|^2)$ and 1 respectively.

Some other restoration schemes were also evaluated for comparison. Most of them were originally proposed for restoring JPEG-encoded images. Note that restoring a VQ-encoded image is more or less the same as restoring a JPEG-encoded image in view of their common interest in deblocking. Hence, as few schemes had been proposed for restoring VQ-encoded images, they were also adopted here and simulated for comparison study.

Table 1 shows the SNR improvement (ΔPSNR) obtained with various approaches, where $\Delta PSNR \equiv 10 \log_{10} (\| g - f \|^2 / \| \hat{f} - f \|^2)$. One can see that the proposed scheme is much superior to others in terms of this objective criterion. The first and second columns are the restoration results of filtering approaches[3]. A 3×3 spatial filter was applied to the entire image in spatial filtering but to the block boundaries only in block edges filtering. Narayan's algorithm[7] is a heuristic POCS algorithm. In our simulation, the convex sets ζ_3 and ζ_4 concerned in [7] were defined as $\zeta_3 = \{ b | B[u,v]=0 \text{ if } u+v>256 \}$ and $\zeta_4 = \{ b \mid 0 \le e_n < 256 \text{ for any } e_n \in b \}$ respectively, where e_n is a pixel, b is any image of size 256×256 and B[u,v] is a 2D transform coefficient of b as defined in [7]. The CLS1 approach were basically the same as that proposed in [4] and the CLS2 approach were adopted from [5]. In particular, solutions were obtained by minimizing objective functions $\| f - g \|^2 + \alpha_1 \| Sf \|^2$ and $\| R(f - g) \|^2 + \alpha_2 \| Sf \|^2$ respectively. Iterative approach was applied. Since direct implementation of [5] may cause convergence problem in restoring VQ-encoded images, the determination of R was modified in a way that all its diagonal elements were mapped to a range of [0.5,1.0] linearly with mapping f(r)=(r+1)/2 after following the procedures defined in [5] to get R. As usual, a 3×3 Laplican operator was used as

Image	Signal-to-Noise ratio improvement (ΔPSNR) of Various approaches (Unit : dB)					
	Filtering		Constraint Least Squares Approach			POCS
	Spatial Filtering[3]	Block Edges Filtering[3]	CLS1[4]	CLS2[5]	Proposed Algorithm	Narayan's Algorithm[7]
Baboon	0.0948	0.0259	0.0774	0.0927	0.0735	-0.0477
Cameraman	0.0347	0.0580	0.1062	0.0889	0.2413	0.0296
Couple	0.2777	0.0907	0.3168	0.4100	0.5470	0.1875
Germany	0.2732	0.0923	0.2539	0.3230	0.5497	-0.0203
Girl	0.4884	0.1120	0.4033	0.5711	0.9011	0.2409
House	0.1528	0.1183	0.2657	0.2407	0.6616	0.2217

Table 1. Comparison of the restoration performance of various restoration algorithms in restoring VQ-encoded images.

S in all CLS approaches including the proposed scheme. The terminating criterion for all iterative processes was $\|\hat{f}_k - \hat{f}_{k-1}\|^2 / \|\hat{f}_k\|^2 \leq 1\times10^{-6}$.

Conclusions

In this paper, a CLS scheme is proposed to restore VQ-encoded images. This scheme incorporates adaptivity into simple CLS restoration scheme[4] by weighting every pixel according to its expected derivation from the original. Computer simulations showed that the proposed scheme could achieve a much better restoration performance in terms of SNR improvement compared with other existing schemes[3-5,7]. Besides, the image quality could also be improved subjectively by reducing most of the blocking effect.

Since no extra information other than the codebook is required to carry out the restoration with the proposed scheme, no transmission overhead is necessary and hence it can be fully compatible with any VQ codec to improve its coding performance. Also note that the proposed algorithm is not tailor-made for a particular VQ scheme and hence no *a priori* knowledge about the construction of the codebook is required during the restoration. This makes it be always able to provide a reasonable restoration performance whatever VQ scheme with which it works, which is quite different from the schemes that are dedicated to restore images encoded with particular type of VQ schemes.

References

[1] W.B. Pennebaker and J.L. Mitchell, *JPEG still image data compression standard*, Van Nostrand Reinhold, New York, 1993.

[2] A. Gersho and R. M. Gray, *Vector quantization and signal compression*, Kluwer Academic Publishers, 1992.

[3] H. C. Reeve and J. S. Lim, "Reduction of blocking effect in image coding," Optical Engineering, Vol.23, No.1, pp.34-37, Jan 1984.

[4] Y. Yang, N. P. Galatsanos and A. K. Katsaggelos, "Regularized reconstruction to reduce blocking artifacts of block discrete cosine transform compressed images," IEEE Trans. CAS for Video Technology, Vol.3, No.6, pp.421-432, Dec 1993.

[5] S. W. Hong, Y. H. Chan and W. C. Siu, "An adaptive constrained least square approach for removing blocking effect," IEEE ISCAS'95, Vol.II, pp.773-776, 30 Apr - 3 May 1995, Seattle, USA.

[6] J. Biemond, R. L. Lagendijk and R. M. Mersereau, "Iterative methods for image deblurring," IEEE Proceedings of the IEEE, Vol.78, No.5, pp.856-883, May 1990.

[7] A. Narayan and J. F. Doherty, "Convex projections based edge recovery in low bit rate VQ", IEEE Signal Processing Letters, Vol.3, No.4, pp.97-99, Apr 1996.