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Control of PWM Inverter using a Discrete-time Sliding Mode Controller

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Abstract - PWM inverters are widely used in UPS systems and driving induction motors. They give AC sinusoidal voltage under linear or non-linear loads which is a basic tracking control problem. Since the duty cycle of a PWM inverter is unchanged within one switching period, inverters are very suitable to be represented by discrete-time state equations. A discrete-time sliding-mode controller based on an improved reaching condition has been proposed by Gao *et. al.* This paper will apply this controller to control PWM inverters with linear and non-linear loads. It is shown that good output responses are obtained in both cases.

I. INTRODUCTION

PWM inverters are widely used in uninterruptible power supplies (UPSs) and driving induction motors. In UPS systems, a 50Hz/60Hz sinusoidal output voltage is required. In driving induction motors, variable frequency and amplitude outputs are needed. Control of PWM inverters is a typical tracking problem. It is well-known that sliding mode control can give good tracking performance [6]. However, one major drawback of sliding mode control in PWM inverters is the varying switching frequency of the switch. This will generate a lot of high frequency noise and give a high THD (total harmonic distortion). A fixed frequency continuous-time sliding mode controller was proposed in [3]. his controller works like a bang-bang controller. The control signal is generated by applying a signal, which is a function of the output voltage, output current and inductor current, to a hysteric comparator.

However, fixed-frequency switching mode power inverters are more suitable to be represented by discrete-time state equations than continuous-time ones. It is because the control signal, the duty cycle, of the inverter, is unchanged within one switching period. A discrete-time sliding mode controller was proposed by Jung and Tzou [2]. The reaching condition used is: $|\sigma(k+1)| < |\sigma(k)|$. Gao *et. al.* have shown that this condition could not guarantee the states to move monotonically toward the switching plane and move across it in finite time [5]. They have derived a better sliding mode controller with a more complete reaching condition.

In this paper, a discrete-time sliding mode controller based on [5] will be applied to control a PWM inverter. It will be shown that this controller can give good output response under linear and non-linear loads. The model of the PWM inverter and the sliding mode controller will be detailed in section II and III respectively. Simulation results will be given in section IV and a conclusion will be drawn in section V.

II. MODELLING OF PWM INVERTERS

A half-bridge PWM inverter is shown in Fig. 1. It consists of an LC filter formed by an inductor L_f and a capacitor C_f, with series equivalent resistance R_{Lf} and R_{Cf} respectively. The bandwidth of this filter is designed to be much lower than the switching frequency. The load in Fig. 1 is a resistor of resistance $R_L = 10\Omega$ at this moment. Later, this load will be changed to a non-linear load to test the performance of the controller. It is assumed that the bi-directional switches S_1 and S_2 are ideal. When S_1 is turned on, S_2 is turned off such that v_i is equal to V_s. On the other hand, when S_1 is turned off, S_2 is turned on such that v_i is equal to $-V_s$. Let t_{on} and t_{off} be the turn-on and turn-off time of S_1 respectively, we define a duty cycle d as follows:

$$d = \frac{t_{on} - t_{off}}{t_{on} + t_{off}} \tag{1}$$

where $t_{on} + t_{off}$ is the switching period which is constant. It should be noted that the value of d is ranged from -1 to 1.

Assume the switching frequency is much higher than the bandwidth of the LC filter, by applying the time-averaging technique, the value of v_i is effectively equal to

$$\begin{aligned} v_i &= \frac{t_{on}V_s + t_{off}(-V_s)}{t_{on} + t_{off}} \\ &= dV_s. \end{aligned} \tag{2}$$

Then a system differential equation with d as the control input and v_o as the output can be written as follows:

$$\begin{aligned} &\frac{d}{dt}\begin{bmatrix} i_{Lf} \\ v_{Cf} \end{bmatrix} \\ &= \begin{bmatrix} -\dfrac{R_{Lf}}{L_f} - \dfrac{R_L R_{Cf}}{L_f(R_L + R_{Cf})} & -\dfrac{R_L}{L_f(R_L + R_{Cf})} \\ \dfrac{1}{C_f} - \dfrac{R_{Cf}}{C_f(R_L + R_{Cf})} & -\dfrac{1}{C_f(R_L + R_{Cf})} \end{bmatrix} \begin{bmatrix} i_{Lf} \\ v_{Cf} \end{bmatrix} \\ &+ \begin{bmatrix} \dfrac{V_s}{L_f} \\ 0 \end{bmatrix} d, \end{aligned} \tag{3}$$

$$v_o = \begin{bmatrix} \dfrac{R_L R_{Cf}}{R_L + R_{Cf}} & \dfrac{R_L}{R_L + R_{Cf}} \end{bmatrix} \begin{bmatrix} i_{Lf} \\ v_{Cf} \end{bmatrix}. \tag{4}$$

In order to avoid estimating the reference value of i_{Lf}, a new state variable is defined as

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} v_{Cf} \\ \dot{v}_{Cf} \end{bmatrix}. \tag{5}$$

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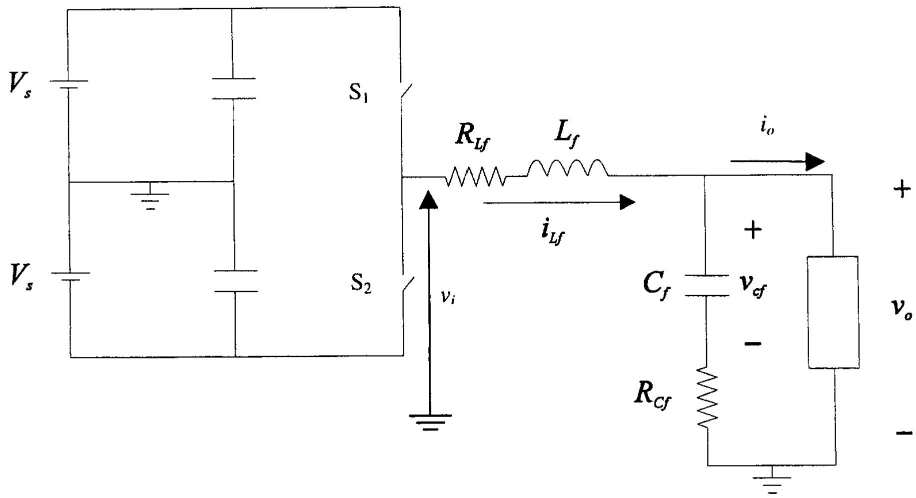

Fig. 1 A PWM inverter

Furthermore, for simplicity, we assume that R_{Lf} and R_{Cf} are equal to zero. The state equations of (3) and (4) can then be transformed into the following form:

$$\dot{x} = A_c x + B_c d, \tag{6}$$

$$v_o = [1 \;\; 0]\, x. \tag{7}$$

where $A_c = \begin{bmatrix} 0 & 1 \\ \dfrac{-1}{L_f C_f} & \dfrac{-1}{R_L C_f} \end{bmatrix}$,

$B_c = \begin{bmatrix} 0 \\ \dfrac{V_s}{L_f C_f} \end{bmatrix}$.

By applying the discrete-time state-space transformation [4], the above continuous time system can be transformed into the following discrete-time system:

$$x(k+1) = A_z x(k) + B_z d(k) \tag{8}$$

$$v_o(k) = [1 \;\; 0]\, x(k) \tag{9}$$

where $A_z = e^{A_c T_s}$ and $B_z = \int_0^{T_s} e^{A_c \tau} B_c d\tau$, T_s is the switching period (also the sampling period of the controller).

III. DISCRETE-TIME SLIDING MODE CONTROLLERS

Let the reference state of $x(k)$ be

$$x_r(k) = \begin{bmatrix} x_{r1}(k) \\ x_{r2}(k) \end{bmatrix} = \begin{bmatrix} V_o \sin(2\pi f_o k T_s) \\ 2\pi f_o V_o \cos(2\pi f_o k T_s) \end{bmatrix} \tag{10}$$

where V_o and f_o are the amplitude and the frequency of the reference sinusoidal output voltage respectively. An error state e is defined as

$$e(k) = x_r(k) - x(k) \tag{11}$$

From (8), (10) and (11),

V_s	250V
L_f	500μH
C_f	10μF
R_L	50Ω

TABLE 1. INVERTER PARAMETERS

$$e(k+1) = A_z e(k) - B_z d(k) + w(k) \tag{12}$$

where $w(k) = x_r(k+1) - A_z x_r(k)$.

The derivation of the discrete-time sliding mode controller proposed in [5] is summarized as follows. Define a sliding plane

$$\sigma(k) = s\, e(k) = 0 \tag{13}$$

where $s = [s_1 \;\; s_2]$ is a constant vector. Consider the incremental change of $\sigma(k)$,

$$\sigma(k+1) - \sigma(k) = s A_z e(k) - s B_z d(k) + s w(k) - s e(k) \tag{14}$$

Compare (14) with the following reaching law in [5], that is

$$\sigma(k+1) - \sigma(k) = -q T_s \sigma(k) - \varepsilon T_s \operatorname{sgn}[\sigma(k)]$$

where q and ε are designed constants, we have the control signal of the discrete-time sliding mode controller given as follows:

$$d(k) = -(s B_z)^{-1}[-s A_z e(k) - s w(k) + s e(k) - q T_s \sigma(k) - \varepsilon T_s \operatorname{sgn}[\sigma(k)]] \tag{15}$$

IV. EXAMPLES

A PWM inverter with parameter values listed in Table 1 is used as an illustrative example. The switching period is T_s = 50μs. Let $s = [1 \;\; 10^{-9}]$, $qT_s = 0.25$, $\varepsilon T_s = 0.1$, V_o = 150V, f_o = 50Hz. The simulation response of the output voltage with the nominal load of 50Ω under the sliding mode control is shown in Fig. 2. The simulation is done by MATLAB. The simulation responses of the inverter with a phase-controlled load are shown in Fig. 3 and Fig. 4. The load is effectively a resistor of 25Ω during the conducting phase, which is form 90° to 180° and 270° to 360° in this example. Otherwise, the load only draws a little leakage current, and is effectively a resistor of 1MΩ. Fig. 3 shows the output voltage and load current of an inverter with phase-controlled load under the control of the sliding mode controller. As a comparison, Fig. 4 shows the response of the inverter under the control of a proportional controller. It can be seen that the sliding mode controller can give better responses in the output voltage and load current.

V. CONCLUSION

This paper has reported a successful application of a discrete-time sliding mode controller to control a PWM inverter. We first model the PWM inverter as a discrete-time state equation. Since the control signal, duty cycle, of a PWM inverter is constant during one switching period, inverters can be modelled accuracely by discrete-time state equations. Then, based on the discrete-time sliding mode controller proposed by Gao *et. al.*, a controller is obtained. This controller has been applied to a PWM inverter with a linear resistance load and a non-linear phase controlled load. It is shown that the sliding mode controller can give good responses in both kinds of load. As compared with a proportional controller when using the phase-controlled load, the proposed sliding mode controller shows a better performance.

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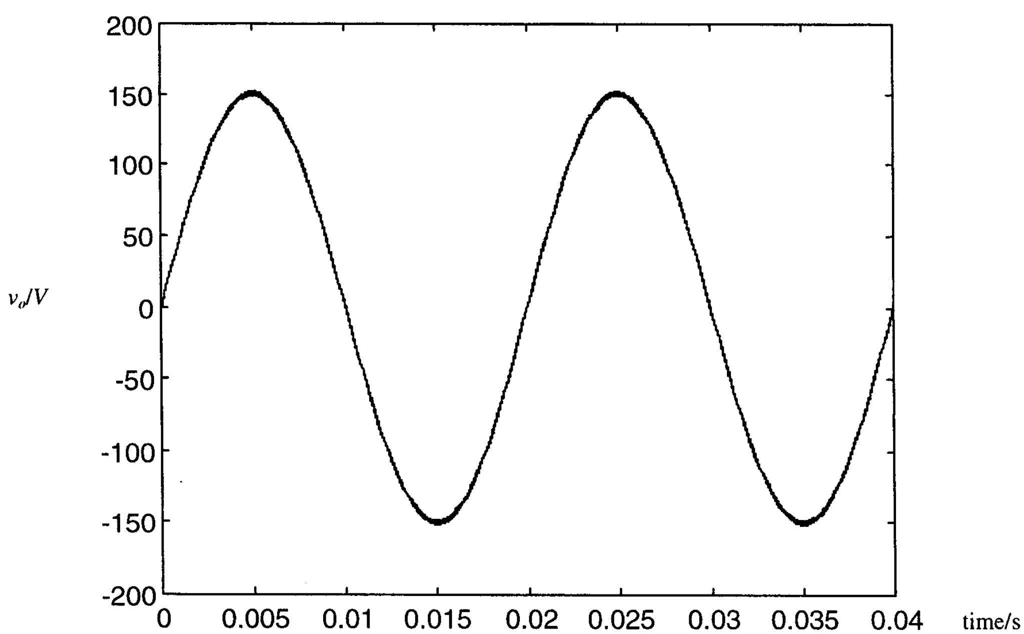

Fig. 2. Output voltage with nominal load under the sliding mode control

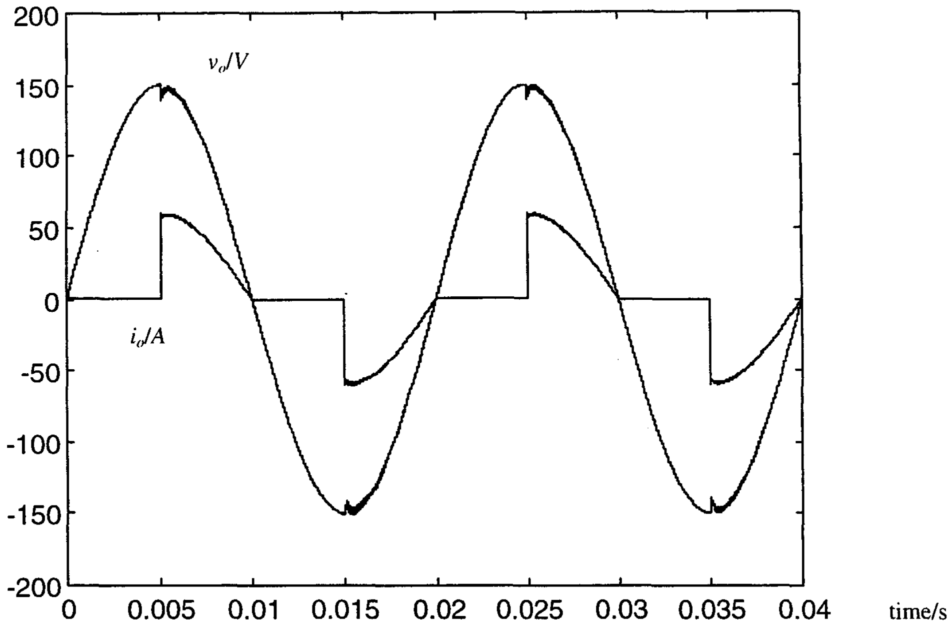

Fig. 3. Output voltage and load current of phase-controlled load under the sliding mode control

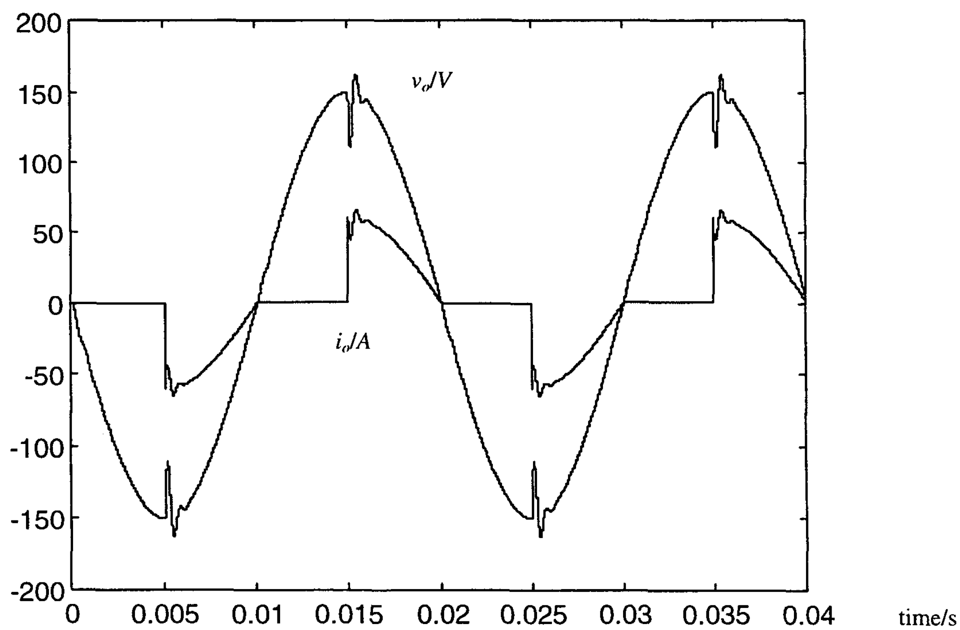

Fig. 4. Output voltage and load current of phase-controlled load under a proportional control