Optimal and Stable Fuzzy Controllers for Nonlinear Systems subject to parameter uncertainties using Genetic Algorithm[1]

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Abstract

This paper tackles the control problem of nonlinear systems subject to parameter uncertainties based on a fuzzy logic approach and the genetic algorithm (GA). In order to achieve a stable controller, TSK fuzzy plant model is employed to describe the dynamics of the uncertain nonlinear plant. A fuzzy controller and the corresponding stability conditions will be derived. The parameters of the fuzzy controller and the solution to the stability conditions are determined using GA. In order to obtain the optimal performance, the membership functions of the fuzzy controller are obtained automatically by minimizing a defined fitness function using GA.

I. INTRODUCTION

Fuzzy control is one of the useful control techniques for uncertain and ill-defined nonlinear systems. Control actions of the fuzzy controller are described by some linguistic rules. This property makes the control algorithm to be understood easily. The early design of fuzzy controllers is heuristic. It incorporates the experience or knowledge of the designer into the rules of the fuzzy controller, which is fine-tuned based on trial and error. Genetic algorithm (GA) is a powerful searching algorithm [3] having been applied to fuzzy systems. By using GA, the membership functions and/or rule sets of the fuzzy system can be generated systematically [5]. These methodologies make the design simple and the system to have optimal performance; however, the design does not guarantee the system stability and robustness. In order to investigate the system stability, the TSK fuzzy plant model [1] was proposed to represent a nonlinear plant as a weighted sum of some simple sub-systems. This model gives a fixed structure to some of the nonlinear plants and thus facilitates the analysis of the systems. Stability of the fuzzy system formed by a fuzzy plant model and a fuzzy controller has been investigated recently. Different stability conditions have been obtained by employing Lyapunov stability theory]. Most of the fuzzy controllers reported depend on membership of the fuzzy plant model. Hence, the membership functions of the fuzzy plant model, or effectively the plant parameters must be known. Practically, the parameters may change during the operation. In these cases, the robustness property of the fuzzy controller is an important concern. Robust analysis of fuzzy control systems can be found [6-9]. However, these works only provided a stability and robustness testing conditions. The determination of the parameters (e.g., the gains and membership functions) of the fuzzy controller and the system performance have seldom been discussed. In order to have a systematic method to obtain a fuzzy controller which guarantees the system stability, robustness, and optimally good performance, a fuzzy controller designed and tuned with GA is proposed.

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II. TSK FUZZY PLANT MODEL AND FUZZY CONTROLLER

In order to obtain a fuzzy controller, a TSK fuzzy plant model is employed to describe the dynamics of the nonlinear plant subject to parameter uncertainties.

A. TSK Fuzzy Plant Model with Parameter Uncertainties

Let p be the number of fuzzy rules describing the uncertain nonlinear plant. The i-th rule is of the following format,

Rule i: IF $f_1(\mathbf{x}(t))$ is M_1^i and … and $f_\Psi(\mathbf{x}(t))$ is M_Ψ^i

THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$ (1)

where M_α^i is a fuzzy term of rule i corresponding to the function $f_\alpha(\mathbf{x}(t))$ containing parameter uncertainties of the nonlinear plant, $\alpha = 1, 2, \ldots, \Psi$, $i = 1, 2, \ldots, p$, Ψ is a positive integer; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$ are known constant system and input matrices respectively; $\mathbf{x}(t) \in \Re^{n\times 1}$ is the system state vector and $\mathbf{u}(t) \in \Re^{m\times 1}$ is the input vector. The inferred system is given by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))\big(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\big), \quad (2)$$

where $\sum_{i=1}^{p} w_i(\mathbf{x}(t)) = 1$, $w_i(\mathbf{x}(t)) \in [0,\ 1]$ for all i (3)

$$w_i(\mathbf{x}(t)) = \frac{\mu_{\mathrm{M}_1^i}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^i}(f_2(\mathbf{x}(t))) \times \cdots \times \mu_{\mathrm{M}_\Psi^i}(f_\Psi(\mathbf{x}(t)))}{\sum_{k=1}^{p}\Big(\mu_{\mathrm{M}_1^k}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^k}(f_2(\mathbf{x}(t))) \times \cdots \times \mu_{\mathrm{M}_\Psi^k}(f_\Psi(\mathbf{x}(t)))\Big)} \quad (4)$$

is a nonlinear function of $\mathbf{x}(t)$ and $\mu_{\mathrm{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$ is the membership function corresponding to M_α^i. The value of $\mu_{\mathrm{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$ is unknown as $f_\alpha(\mathbf{x}(t))$ is related to parameter uncertainties of the nonlinear plant.

B. Fuzzy Controller

A fuzzy controller with c fuzzy rules is to be designed for the plant. Its j-th rule is of the following format,

Rule j: IF $g_1(\mathbf{x}(t))$ is N_1^j and … and $g_\Omega(\mathbf{x}(t))$ is N_Ω^j

THEN $\mathbf{u}(t) = \mathbf{G}_j\mathbf{x}(t) + \mathbf{r}$ (5)

where N_β^j is a fuzzy term of rule j corresponding to the function $g_\beta(\mathbf{x}(t))$, $\beta = 1, 2, \ldots, \Omega$, $j = 1, 2, \ldots, c$, Ω is a positive integer; $\mathbf{G}_j \in \Re^{m\times n}$ is the feedback gain of rule j to be designed, $\mathbf{r} \in \Re^{n\times 1}$ is the reference input vector. The inferred output of the fuzzy controller is given by,

$$\mathbf{u}(t)=\sum_{j=1}^{c} m_j(\mathbf{x}(t))\big(\mathbf{G}_j\mathbf{x}(t)+\mathbf{r}\big) \tag{6}$$

where $\sum_{j=1}^{c} m_j(\mathbf{x}(t))=1$, $m_j(\mathbf{x}(t))\in[0,\ 1]$ for all j (7)

$$m_j(\mathbf{x}(t))=\frac{\mu_{\mathrm{N}_1^j}(g_1(\mathbf{x}(t)))\times\mu_{\mathrm{N}_2^j}(g_2(\mathbf{x}(t)))\times\cdots\times\mu_{\mathrm{N}_\Omega^j}(g_\Omega(\mathbf{x}(t)))}{\sum_{k=1}^{c}\left(\mu_{\mathrm{N}_1^k}(g_1(\mathbf{x}(t)))\times\mu_{\mathrm{N}_2^k}(g_2(\mathbf{x}(t)))\times\cdots\times\mu_{\mathrm{N}_\Omega^k}(g_\Omega(\mathbf{x}(t)))\right)} \tag{8}$$

is a nonlinear function of $\mathbf{x}(t)$ and $\mu_{\mathrm{N}_\beta^j}(g_\beta(\mathbf{x}(t)))$ is the membership function corresponding to N_β^j to be designed.

C. Fuzzy Control System

From (2) and (6), the closed-loop fuzzy system is given by,

$$\dot{\mathbf{x}}(t)=\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(t))m_j(\mathbf{x}(t))\big(\mathbf{H}_{ij}\mathbf{x}(t)+\mathbf{B}_i\mathbf{r}\big) \tag{9}$$

where $\mathbf{H}_{ij}=\mathbf{A}_i+\mathbf{B}_i\mathbf{G}_j$ (10)

III. Stability Analysis

To analyze the stability of the fuzzy control system of (9) subject to parameter uncertainties, we multiply a transformation matrix $\mathbf{T}\in\Re^{n\times n}$ of rank n to both sides,

$$\mathbf{T}\mathbf{x}(t+\Delta t)=\mathbf{T}\mathbf{x}(t)+\sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j\mathbf{T}\big(\mathbf{H}_{ij}\mathbf{x}(t)+\mathbf{B}_i\mathbf{r}\big)\Delta t+\mathbf{T}\mathrm{o}(\Delta t)$$

$$=\left(\mathbf{I}+\sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j\mathbf{T}\mathbf{H}_{ij}\mathbf{T}^{-1}\Delta t\right)\mathbf{T}\mathbf{x}(t)+\sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j\mathbf{T}\mathbf{B}_i\mathbf{r}\Delta t+\mathbf{T}\mathrm{o}(\Delta t)$$

The procedure of stability analysis can be found in [6] and will not be repeated here. The result can be summarized by the following lemma:

Lemma 1. *The fuzzy control system, subject to parameter uncertainties, as given by* (9) *is exponentially stable for* $\mathbf{r}=\mathbf{0}$ *or input-to-state stable for* $\mathbf{r}\neq\mathbf{0}$ *if* $\mathbf{T}\mathbf{H}_{ij}\mathbf{T}^{-1}$ *is designed such that,*

$\mu[\mathbf{T}\mathbf{H}_{ij}\mathbf{T}^{-1}]\leq-\varepsilon$ *for all i and j*

where ε *is a nonzero positive constant scalar,* $\mu[\mathbf{T}\mathbf{H}_{ij}\mathbf{T}^{-1}]=$

$$\lim_{\Delta t\to 0^+}\frac{\|\mathbf{I}+\mathbf{T}\mathbf{H}_{ij}\mathbf{T}^{-1}\Delta t\|-1}{\Delta t}=\lambda_{\max}\left(\frac{\mathbf{T}\mathbf{H}_{ij}\mathbf{T}^{-1}+(\mathbf{T}\mathbf{H}_{ij}\mathbf{T}^{-1})^{*}}{2}\right).$$

It should be noted that with the use of a suitable $\mathbf{T}$, any Hurwitz matrix having a positive or zero matrix measure can be transformed into another matrix having a negative matrix measure (see (18)). The stability conditions derived can then be applied. The problem left is how to find such a matrix $\mathbf{T}$ for a given system. This will be discussed later.

IV. Genetic Algorithm

GA is a powerful searching algorithm [3]. In this paper, GA is used to solve the solution of the stability conditions (Lemma 1) and optimize the performance by choosing the membership functions of the fuzzy controller. The traditional GA process is very simple; however, it does not guarantee that a superior offspring can be produced in each reproduction process. In this paper, this weakness is removed. The modified GA is shown in Fig. 1. Its details will be given as follows.

A. Initial Population

Initial population is the potential solution set, P. The first set of population is usually generated randomly. The potential solution set is defined as,

$$P=\{p_1, p_2, \cdots, p_{pop\_size}\} \tag{11}$$

$$p_i=\begin{bmatrix}p_{i_1} & p_{i_2} & \cdots & p_{i_{no\_vars}}\end{bmatrix},\ i=1,2,\ldots,pop\_size \tag{12}$$

$$p_{i_j}=\begin{bmatrix}p_1^{i_j} & p_2^{i_j} & \cdots & p_{b_{i_j}}^{i_j}\end{bmatrix},\ i=1,2,\ldots,pop\_size, j=1,2,\ldots,no\_vars \tag{13}$$

where pop_size denotes the population size, no_vars denotes the number of variables used in the fitness function, b_{i_j} denotes the number of bits to encode the variable p_{i_j}. It can be seen from (11) to (13) that the potential solution set P consists of some candidate solutions p_i (chromosome). The chromosome p_i consists of some variables p_{i_j} (genes) used in the fitness function. The gene p_{i_j} is then encoded by a b_{i_j}-bit binary number.

B. Evaluation

Each chromosome in the population will be evaluated by a defined fitness function. The better chromosomes will return higher values in this process. The fitness function to evaluate the i-th chromosomes p_i, i = 1,2, …, pop_size, in the population is defined as follows,

$$fitness=f(\mathbf{y}_i),\ i=1,2,\ldots,pop\_size \tag{14}$$

$$\mathbf{y}_i=\begin{bmatrix}y_{i_1} & y_{i_2} & \cdots & y_{i_{no\_vars}}\end{bmatrix},\ i=1,2,\ldots,pop\_size \tag{15}$$

$$y_{i_j}=y^i_{j_{\min}}+decimal(p_{i_j})\frac{y^i_{j_{\max}}-y^i_{j_{\min}}}{2^{b_{i_j}}-1},\ i=1,2,\ldots,pop\_size, j=1,2,\ldots,no\_vars \tag{16}$$

where $decimal(p_{i_j})$ denotes the decimal value of the binary number p_{i_j}, $y^i_{j_{\max}}$ and $y^i_{j_{\min}}$ are the maximum and minimum values the parameter corresponding to the gene p_{i_j} respectively. It should be noted that (16) effectively convert a binary number to a decimal floating number. The definition of $f(\mathbf{y}_i)$ depends on the application.

C. Selection

In the selection process, some of the chromosomes will be selected to undergo the genetic operations for reproduction. The chromosomes having higher fitness values will have a higher chance to be selected. It is believed that the high potential parents will produce better offspring, which is similar to the survival of the best ones in the nature. The selection can be done by assigning a probability q_i to the chromosome p_i:

$$q_i=f(\mathbf{y}_i)\Big/\sum_{j=1}^{pop\_size} f(\mathbf{y}_j),\ i=1,2,\ldots,pop\_size \tag{17}$$

The cumulative probability $\hat{q}_i$ is defined as,

$$\hat{q}_i=\sum_{j=1}^{i} q_j,\ i=1,2,\ldots,pop\_size \tag{18}$$

The selection process starts by randomly generating a nonzero floating-point number, $d\in[0\ \ 1]$, for each chromosome. Then, the chromosome p_i is chosen if $\hat{q}_{i-1}<d\leq\hat{q}_i$, i = 1,2,

..., *pop_size*, where $\hat{q}_0 = 0$. It can be observed from this selection process that some chromosomes will be selected more than once. Consequently, the best chromosomes will get more copies, the average will stay and the worst will die off.

D. Genetic Operations

The genetic operations are to generate some new chromosomes (offspring) from their parents after the selection process. They include the crossover and the mutation operations. The crossover operation is mainly for exchanging information from two parents in order to produce a better offspring. To perform this, a floating-point number, $d_c \in [0 \quad 1]$, is generated randomly for each chromosome. Then, the chromosome is chosen for undergoing crossover if its $d_c < p_c$ where $p_c \in [0 \quad 1]$ is the probability of crossover. Each chosen chromosome will then be selected randomly to form a pair for crossover. A random integer, $pos \in [1 \quad num\_bits - 1]$, will be generated for each pair of the chromosomes where num_bits denotes the number of bits of the chromosomes, pos denotes the position of the crossing point. The contents of the pair of chromosomes after this crossing point will be exchanged to generate their pair of offspring. This pair of offspring will replace their parents.

These offspring will undergo the mutation operation, which changes the genes (bits) of the chromosomes. Consequently, the features of the chromosomes inherited from their parents can be changed. To perform the mutation operation, a floating-point number, $d_m \in [0 \quad 1]$, is generated randomly for each bit of all chromosomes in the population. The bit value is complemented if $d_m < p_m$ where $p_m \in [0 \quad 1]$ is the probability of mutation.

After the processes of selection, crossover, and mutation, a new population is generated. This new population will repeated the same processes to obtain a better result. Termination occurs when the result reaches a defined condition. For the simple GA process, the offspring generated may not be better than their parents. To improve this situation, we modify the simple GA process to that shown Fig. 1. After each iteration, we compare the previous and current fitness values. If the current fitness value is better, the new offspring are used; otherwise, the parents are used again to generate other offspring. A better searching result is then obtained.

V. GA for Stability Conditions, Feedback Gains

A. Stability Conditions and Feedback Gains

From Lemma 1, the system is stable if there exists a transformation matrix $\mathbf{T}$ satisfying the following conditions,

$$\mu\left[\mathbf{T}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)\mathbf{T}^{-1}\right] \leq -\varepsilon,\ i = 1, 2, \ldots, p, j = 1, 2, \ldots, c \quad (19)$$

The objectives are to find $\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1n} \\ T_{21} & T_{22} & \cdots & T_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ T_{n1} & T_{n2} & \cdots & T_{nn} \end{bmatrix}$ and $\mathbf{G}_j = \begin{bmatrix} G_{11}^j & G_{11}^j & \cdots & G_{1n}^j \\ G_{21}^j & G_{22}^j & \cdots & G_{2n}^j \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}^j & G_{m2}^j & \cdots & G_{mn}^j \end{bmatrix}$ such that the above conditions are satisfied. The fitness function is defined as follows,

$$fitness = \sum_{i=1}^{p}\sum_{j=1}^{c} n_{ij}\mu\left[\mathbf{T}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)\mathbf{T}^{-1}\right] \quad (20)$$

where $n_{ij} \geq 0$, $i = 1, 2, \ldots, p, j = 1, 2, \ldots, c$, are constant scalar. The problems of finding $\mathbf{T}$ and $\mathbf{G}_j$ are now formulated to a minimization problem. The aim is to minimize the fitness function with $\mathbf{T}$ and $\mathbf{G}_j$ using GA. As $\mathbf{T}$ and $\mathbf{G}_j$ are the variables of the fitness function of (20), they will be encoded to binary numbers to form the genes of the chromosomes for the GA process. Under this minimization problem, the minimum fitness value does not necessarily satisfy (19). Hence, n_{ij}, $i = 1, 2, \ldots, p, j = 1, 2, \ldots, c$, are used to make the effects of some terms on the right hand side of (20) less significant.

B. Optimizing the System Performance

After $\mathbf{T}$ and $\mathbf{G}_j$ have been determined, what follows is to determine the membership functions of the fuzzy controller using GA such that the performance of the uncertain fuzzy control system is optimal subject to a defined performance index. The dynamics of the uncertain fuzzy control system is restated with modification as follows,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(t)) m_j(\mathbf{x}(t), \mathbf{z}_j)\left(\mathbf{H}_{ij}\mathbf{x}(t) + \mathbf{B}_i\mathbf{r}\right) \quad (21)$$

where $\mathbf{z}_j$, $j = 1, 2, \ldots, c$, are the parameter vectors governing the membership functions of the fuzzy controller, e.g., the values of the mean and standard deviation of a Gaussian membership function. The fitness function (performance index) is defined as follows,

$$fitness = \int \mathbf{x}(t)^{\mathrm{T}}\mathbf{W}_x\mathbf{x}(t) + \mathbf{u}(t)^{\mathrm{T}}\mathbf{R}_u\mathbf{u}(t)dt \quad (22)$$

where $\mathbf{W}_x \in \Re^{n\times n}$ and $\mathbf{R}_u \in \Re^{m\times m}$ are constant semi-positive or positive definite matrices. This fitness function is the performance index used in the conventional optimal control. The optimization problem formulated here will be handled by GA. $\mathbf{z}_j$, $j = 1, 2, \ldots, c$, will be encoded to binary numbers and form the genes of the chromosomes for the GA process.

The procedure to obtain the fuzzy controller using GA can be summarized by the following steps.

1) Obtain the mathematical model of the nonlinear system and convert it into the fuzzy plant model of (2).

2) Determine the number of rules of the fuzzy controller. Solve $\mathbf{T}$ and $\mathbf{G}_j$ with the fitness function defined in (20) and $n_{ij} = 1$, $i = 1, 2, \ldots, p, j = 1, 2, \ldots, c$ using GA. If $\mathbf{T}$ and $\mathbf{G}_j$ cannot be found, adjust n_{ij} accordingly.

3) Determine the membership functions of the fuzzy controller. Obtain the parameters of the membership functions using GA to optimize the system performance of the fuzzy control system.

VI. Application Example

An application example of a cart-pole inverted pendulum as shown in Fig. 2 will be given. The dynamic equation of the system is given by,

$$\ddot{\theta}(t) = \frac{g\sin(\theta(t)) - aml\dot{\theta}(t)^2\sin(2\theta(t))/2 - a\cos(\theta(t))u(t)}{4l/3 - aml\cos^2(\theta(t))} \quad (23)$$

where θ is the angular displacement of the pendulum, $g = 9.8\text{m/s}^2$ is the acceleration due to gravity, $m = 2$kg is the mass of the pendulum, $a = 1/(m + M)$, $M \in [8 \quad 80]$kg is the mass of the cart, $2l$ = 1m is the length of the pendulum, and u is the force applied to the cart. The fuzzy plant model has four rules.

Rule i: IF $f_1(\mathbf{x}(t))$ is M_1^i AND $f_2(\mathbf{x}(t))$ is M_2^i

$$\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t) \text{ for } i = 1, 2, 3, 4 \qquad (24)$$

All $\mathbf{A}_i, \mathbf{B}_i$ are known. A 4-rule fuzzy controller is designed:

Rule j: IF $x_1(t)$ is N_1^j and $x_2(t)$ is N_2^j

$$\text{THEN } u(t) = \mathbf{G}_j\mathbf{x}(t), j = 1, 2, 3, 4 \qquad (25)$$

30-bit binary numbers are used to encode each element of $\mathbf{T}$ and $\mathbf{G}_j$, j = 1, 2, 3, 4. The minimum and maximum values of each element $\mathbf{T}$ are chosen to be –20 and 20 respectively. The minimum and maximum values of each element of $\mathbf{G}_1$ and $\mathbf{G}_3$ are chosen to be 0 and 4500 respectively. The minimum and maximum values of each element of $\mathbf{G}_2$ and $\mathbf{G}_4$ are chosen to be 0 and 10000 respectively. The fitness function is defined as follows,

$$fitness = \sum_{i=1}^{4}\sum_{j=1}^{4} n_{ij}\,\mu\left[\mathbf{T}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)\mathbf{T}^{-1}\right] \qquad (26)$$

where $n_{ij} = 1$, i = 1, 2, 3, 4, j = 1, 2, 3, 4.

After applying the GA process, we obtain $\mathbf{T} = \begin{bmatrix} 15.0698 & -2.2784 \\ -17.9689 & -3.3757 \end{bmatrix}$ and $\mathbf{G}_1 = [4376.7841 \quad 3549.4041]$, $\mathbf{G}_2 = [4770.5758 \quad 3020.2127]$, $\mathbf{G}_3 = [4096.6674 \quad 2730.6881]$, $\mathbf{G}_4 = [9346.4048 \quad 5772.3017]$. The population size is 100 and the initial values of $\mathbf{T}$ and $\mathbf{G}_j$, j = 1, 2, 3, 4, are randomly generated. The stability analysis result is tabulated in Table I. According to Lemma 1, the fuzzy control system is stable. The parameters of the membership functions of the fuzzy controller are tuned by GA. (population size = 10). The fitness function is chosen as follows,

$$fitness = \int_0^{6} \mathbf{x}(t)^{\mathrm{T}}\mathbf{W}_x\mathbf{x}(t) + u(t)^{\mathrm{T}} R_u u(t)\mathrm{d}t \qquad (27)$$

$$\text{where } \mathbf{W}_x = \begin{bmatrix} 10000 & 0 \\ 0 & 0 \end{bmatrix},\ R_u = 0.01 \qquad (28)$$

After the GA process, $\overline{m}_1 = 0.2415$, $\overline{\sigma}_1 = 0.5085$, $\overline{m}_2 = 4.8303$ and $\overline{\sigma}_2 = 0.7461$ under the nominal $M = 8\text{kg}$.

VII. Conclusion

Fuzzy control of nonlinear systems subject to parameter uncertainties has been presented. Stability conditions have been derived for this class of fuzzy control system. GA has been proposed to help finding a solution of the stability conditions, determining the feedback gains and the membership functions of the fuzzy controller.

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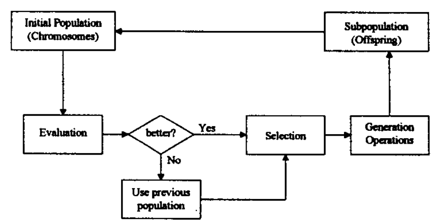

Fig. 1. Modified GA.

TABLE I
Stability Analysis Result of the Inverted Pendulum

i,j	$\mu[\mathbf{TH}_{ij}\mathbf{T}^{-1}]$
1, 1	1.4229
1, 2	3.1220
1, 3	2.9617
1, 4	3.1736
2, 1	1.2696
2, 2	1.4951
2, 3	1.1917
2, 4	2.3968
3, 1	1.2992
3, 2	3.0967
3, 3	2.9105
3, 4	3.1711
4, 1	0.4704
4, 2	0.6630
4, 3	0.2639
4, 4	1.9710