On Design of a Switching Controller for Nonlinear Systems with Unknown Parameters based on a Model Reference Approach[1]

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Abstract

This paper deals with nonlinear plants subject to unknown parameters based on a model reference approach. The nonlinear plant is represented by a fuzzy plant model. A switching controller is designed based on the fuzzy plant model to drive the system states of the nonlinear plant to follow those of a reference model. An application example on controlling an inverted pendulum on a cart will be given to illustrate the design procedure of the proposed switching controller.

I. INTRODUCTION

Control of nonlinear systems is difficult because we do not have systematic mathematical tools to help finding a necessary and sufficient condition to guarantee the stability and performance. The problem will become more complex if some of the parameters of the plant are unknown. By using a TSK fuzzy plant model [1], a nonlinear system can be expressed as a weighted sum of some simple sub-systems. This model gives a fixed structure to some of the nonlinear systems and thus facilitates the analysis of the systems. There are two ways to obtain the fuzzy plant model: 1) by system identification based on the input-output data of the plant [1], and 2) by direct derivation from the mathematical model of the nonlinear plant.

Stability of fuzzy systems formed by a fuzzy plant model and a fuzzy controller has been investigated recently. Different stability conditions have been obtained [2-3, 5-7]. Most of the fuzzy controllers proposed depend on the membership functions of the fuzzy plant model. Hence, the membership functions of the fuzzy plant model, or the plant parameters, must be known. Practically, the parameters of many nonlinear plants will change during the operation, e.g. the load of a dc-dc power converter. In these cases, the robustness property of the fuzzy controller is an important concern. Moreover, [2-3, 5-7] tackled only a regulation problem such that the controllers drive all the system states to zero. In practice, we may face a non-zero set-point regulation problem or a tracking problem.

In this paper, a switching controller is proposed to control nonlinear plants subject to unknown parameters within known bounds. The nonlinear plant is represented by a fuzzy plant model. This switching controller is able to drive the system states to follow those of a reference model. The switching controller consists of a number of linear controllers. One of the linear controllers will be employed at a time according to a switching scheme. The switching scheme will be derived based on the Lyapunov stability theory.

II. FUZZY MODEL BASED SYSTEM

The nonlinear plant to be tackled is of the following form,

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t) \tag{1}$$

where $\mathbf{A}(\mathbf{x}(t)) \in \Re^{n\times n}$ and $\mathbf{B}(\mathbf{x}(t)) \in \Re^{n\times m}$ are the system matrix and input matrix respectively, both of them have known structure but subject to unknown parameters, $\mathbf{x}(t) \in \Re^{n\times 1}$ is the system state vector and $\mathbf{u}(t) \in \Re^{m\times 1}$ is the input vector. The system of (1) is represented by a fuzzy plant model, which expresses the multivariable nonlinear system as a weighted sum of linear systems. A switching controller is to be designed to close the feedback loop of the nonlinear plant such that the system states follow those of a reference model.

A. Reference Model

A reference model is a stable linear system given by,

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{H}_m\hat{\mathbf{x}}(t) + \mathbf{B}_m\mathbf{r}(t) \tag{2}$$

where $\mathbf{H}_m \in \Re^{n\times n}$ is a constant stable system matrix, $\mathbf{B}_m \in \Re^{n\times m}$ is a constant input vector, $\hat{\mathbf{x}} \in \Re^{n\times 1}$ is the system state vector of this reference model and $\mathbf{r}(t) \in \Re^{m\times 1}$ is the bounded reference input.

B. Fuzzy Plant Model

Let p be the number of fuzzy rules describing the multivariable nonlinear plant of (1), the i-th rule is of the following format,

Rule i: IF $f_1(\mathbf{x}(t))$ is M_1^i and … and $f_\Psi(\mathbf{x}(t))$ is M_Ψ^i
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$ (3)

where M_α^i is a fuzzy term of rule i corresponding to the function $f_\alpha(\mathbf{x}(t))$ in terms of the system states and unknown parameters of the nonlinear plant, $\alpha = 1, \ldots, \Psi$, $i = 1, \ldots, p$, Ψ is a positive integer; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$ are known system and input matrices respectively of the i-th rule sub-system. The system dynamics are described by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))\big(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\big), \tag{4}$$

where $\sum_{i=1}^{p} w_i(\mathbf{x}(t)) = 1$, $w_i(\mathbf{x}(t)) \in [0 \quad 1]$ for all i (5)

$$w_i(\mathbf{x}(t)) = \frac{\mu_{\mathrm{M}_1^i}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^i}(f_2(\mathbf{x}(t))) \times \cdots \times \mu_{\mathrm{M}_\Psi^i}(f_\Psi(\mathbf{x}(t)))}{\sum_{k=1}^{p}\left(\mu_{\mathrm{M}_1^k}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^k}(f_2(\mathbf{x}(t))) \times \cdots \times \mu_{\mathrm{M}_\Psi^k}(f_\Psi(\mathbf{x}(t)))\right)} \tag{6}$$

is a nonlinear function of the system states and the unknown parameters. (Fuzzy modeling is discussed in [1].) In this paper, the fuzzy plant model of (4) to be tackled is assumed to have the following properties,

$$\overline{\mathbf{A}}_i = \mathbf{A}_i - \mathbf{H}_m = \mathbf{B}_m\mathbf{D}_i,\ i = 1, 2\ldots, p \tag{7}$$

[1]The work described in this paper was substantially supported by a grant from the Research Grants Council' of the Hong Kong Special Administration Region, China (Project No. B-Q394 PolyU1000/01E)

$$\mathbf{B}(\mathbf{x}(t))=\sum_{i=1}^{p}w_i(\mathbf{x}(t))\mathbf{B}_i=\alpha(\mathbf{x}(t))\mathbf{B}_m \tag{8}$$

where $\mathbf{D}_i\in\Re^{m\times n}$, i = 1, 2..., p, are constant matrices to be designed. $\alpha(\mathbf{x}(t))$ is an unknown non-zero scalar (because $w_i(\mathbf{x}(t))$ is unknown) but with known bounds and sign. It should be noted that because $\alpha(\mathbf{x}(t))\neq 0$ is required, $\mathbf{B}(\mathbf{x}(t))\neq\mathbf{0}$ is assumed.

C. Switching Controller

A switching controller is employed to control the nonlinear plant of (1). The switching controller consists of some simple sub-controllers. These sub-controllers will switch among each other to control the system of (1) according to a switching scheme. The switching controller is described by,

$$\mathbf{u}(t)=\sum_{j=1}^{p}m_j(\mathbf{x}(t))\big(\mathbf{G}_j\mathbf{x}(t)+\mathbf{r}\big) \tag{9}$$

where $m_j(\mathbf{x}(t))$, j = 1, 2, ..., p takes the value of $-\frac{1}{\alpha_{\min}}$ or $\frac{1}{\alpha_{\min}}$ according to the switching scheme to be discussed later, $\alpha_{\min}$ is the minimum value of $\alpha(\mathbf{x}(t))$. It can be seen that (9) is a linear combinations of 2^p linear state-feedback controllers. At each moment, one of the linear state-feedback controllers will be chosen to control the nonlinear plant according to the switching scheme.

III. Stability Analysis and Design

In this section, the switching controller will be designed under the consideration of the system stability. From (2) and (4), writing $w_i(\mathbf{x}(t))$ = w_i, $m_j(\mathbf{x}(t))$ as m_j and $\alpha(\mathbf{x}(t))$ as α, and using the property of (5) that $\sum_{i=1}^{p}w_i=1$, we have,

$$\dot{\mathbf{e}}(t)=\dot{\mathbf{x}}(t)-\dot{\hat{\mathbf{x}}}(t)=\sum_{i=1}^{p}w_i\big(\mathbf{A}_i\mathbf{x}(t)+\mathbf{B}_i\mathbf{u}(t)\big)-\mathbf{H}_m\hat{\mathbf{x}}(t)-\mathbf{B}_m\mathbf{r}(t)$$

$$=\mathbf{H}_m\mathbf{x}(t)-\mathbf{H}_m\mathbf{x}(t)-\mathbf{H}_m\hat{\mathbf{x}}(t)+\sum_{i=1}^{p}w_i\big(\mathbf{A}_i\mathbf{x}(t)+\mathbf{B}_i\mathbf{u}(t)\big)-\mathbf{B}_m\mathbf{r}(t)$$

$$=\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}w_i\big[\overline{\mathbf{A}}_i\mathbf{x}(t)+\mathbf{B}_i\mathbf{u}(t)\big]-\mathbf{B}_m\mathbf{r}(t) \tag{10}$$

From (8), (10) becomes,

$$\begin{aligned}\dot{\mathbf{e}}(t)&=\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}w_i\overline{\mathbf{A}}_i\mathbf{x}(t)+\sum_{i=1}^{p}w_i\mathbf{B}_i\mathbf{u}(t)-\mathbf{B}_m\mathbf{r}(t)\\&=\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}w_i\big(\overline{\mathbf{A}}_i+\mathbf{B}_m\mathbf{G}_i\big)\mathbf{x}(t)\\&\quad-\sum_{i=1}^{p}w_i\mathbf{B}_m\mathbf{G}_i\mathbf{x}(t)+\mathbf{B}(\mathbf{x}(t))\mathbf{u}(t)-\mathbf{B}_m\mathbf{r}(t)\end{aligned} \tag{11}$$

Putting (9) into (11),

$$\begin{aligned}\dot{\mathbf{e}}(t)&=\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}w_i\big(\overline{\mathbf{A}}_i+\mathbf{B}_m\mathbf{G}_i\big)\mathbf{x}(t)\\&\quad-\sum_{i=1}^{p}w_i\mathbf{B}_m\mathbf{G}_i\mathbf{x}(t)+\mathbf{B}(\mathbf{x}(t))\sum_{j=1}^{p}m_j\big(\mathbf{G}_j\mathbf{x}(t)+\mathbf{r}\big)-\mathbf{B}_m\mathbf{r}(t)\\&=\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}w_i\big(\overline{\mathbf{A}}_i+\mathbf{B}_m\mathbf{G}_i\big)\mathbf{x}(t)\\&\quad-\sum_{i=1}^{p}w_i\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\big)+\sum_{i=1}^{p}m_i\mathbf{B}(\mathbf{x}(t))\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}\big)\end{aligned} \tag{12}$$

From (7), (8) and (12),

$$\begin{aligned}\dot{\mathbf{e}}(t)&=\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}w_i\big(\mathbf{B}_m\mathbf{D}_i+\mathbf{B}_m\mathbf{G}_i\big)\mathbf{x}(t)\\&\quad-\sum_{i=1}^{p}w_i\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\big)+\sum_{i=1}^{p}m_i\alpha\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}\big)\\&=\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}w_i\big(\mathbf{B}_m\mathbf{D}_i+\mathbf{B}_m\mathbf{G}_i\big)\mathbf{x}(t)\\&\quad+\sum_{i=1}^{p}\big(\alpha m_i-w_i\big)\,\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\big)\end{aligned} \tag{13}$$

Let

$$\mathbf{G}_i=-\mathbf{D}_i,\ i=1,2,\ldots,p \tag{14}$$

(13) becomes,

$$\dot{\mathbf{e}}(t)=\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}\big(\alpha m_i-w_i\big)\,\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\big) \tag{15}$$

To investigate the stability of (15), the following Lyapunov function is employed.

$$V=\frac{1}{2}\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{x}(t) \tag{16}$$

where $\mathbf{P}\in\Re^{n\times n}$ is a constant symmetric positive definite matrix. Differentiating (16), we have,

$$\dot{V}=\frac{1}{2}\big(\dot{\mathbf{x}}(t)^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)+\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\dot{\mathbf{x}}(t)\big) \tag{17}$$

Putting (15) into (17),

$$\begin{aligned}\dot{V}&=\frac{1}{2}\left(\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}\big(\alpha m_i-w_i\big)\,\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\big)\right)^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\\&\quad+\frac{1}{2}\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\left(\mathbf{H}_m\mathbf{e}(t)+\sum_{i=1}^{p}\big(\alpha m_i-w_i\big)\,\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\big)\right)\\&=-\frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{Q}\mathbf{e}(t)+\sum_{i=1}^{p}\big(\alpha m_i-w_i\big)\,\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\big)\end{aligned} \tag{18}$$

where,

$$\mathbf{Q}=-\big(\mathbf{H}_m\mathbf{P}+\mathbf{P}\mathbf{H}_m\big) \tag{19}$$

$\mathbf{Q}\in\Re^{n\times n}$ is a constant symmetric positive definite matrix. As α is bounded, we can consider that $|\alpha|\in[\alpha_{\min}\quad\alpha_{\max}]$ where $\alpha_{\max}$ > $\alpha_{\min}$ > 0; $\alpha_{\max}$ and $\alpha_{\min}$ are the minimum and maximum values of the absolute value of α respectively. We choose m_i, i = 1, 2, ..., p, as follows,

$$m_i=-\frac{sign\big(\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_m\big(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\big)\big)}{sign(\alpha)\alpha_{\min}},\ i=1,2,\ldots,p \tag{20}$$

where,

$$sign(z)=\begin{cases}1 & z>0\\-1 & z\leq 0\end{cases} \tag{21}$$

From (18) and (20),

$$\dot{V} = -\frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{Q}\mathbf{e}(t)$$
$$+\sum_{i=1}^{p}\left(\frac{\alpha sign\left(\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_{m}\left(\mathbf{G}_{i}\mathbf{x}(t)+\mathbf{r}(t)\right)\right)}{sign(\alpha)\alpha_{\min}} - w_{i}\right)\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_{m}\left(\mathbf{G}_{i}\mathbf{x}(t)+\mathbf{r}(t)\right)$$
$$\leq -\frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{Q}\mathbf{e}(t) - \sum_{i=1}^{p}\frac{|\alpha|}{\alpha_{\min}}\left|\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_{m}\left(\mathbf{G}_{i}\mathbf{x}(t)+\mathbf{r}(t)\right)\right|$$
$$+\sum_{i=1}^{p} w_{i}\left|\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_{m}\left(\mathbf{G}_{i}\mathbf{x}(t)+\mathbf{r}(t)\right)\right|$$
$$= -\frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{Q}\mathbf{e}(t) - \sum_{i=1}^{p}\left(\frac{|\alpha|}{\alpha_{\min}} - w_{i}\right)\left|\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_{m}\left(\mathbf{G}_{i}\mathbf{x}(t)+\mathbf{r}(t)\right)\right| \quad (22)$$

Since $\frac{|\alpha|}{\alpha_{\min}} \geq 1 \geq w_i$, $i = 1, 2, \ldots, p$, hence (22) becomes,

$$\dot{V} \leq -\frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{Q}\mathbf{e}(t) \leq 0 \quad (23)$$

Equality holds when $\mathbf{e}(t) = \mathbf{0}$. From (23), it can be concluded that $\mathbf{e}(t) \to \mathbf{0}$ or equivalently $\mathbf{x}(t) \to \hat{\mathbf{x}}(t)$ as $t \to 0$. The analysis results are summarized by the following lemma.

Lemma I: *The system states of the nonlinear plant of* (1) *represented by the fuzzy plant model of* (4) *will follow those of the reference mode of* (2), *if the fuzzy plant model satisfies the following conditions,*

(i) $\overline{\mathbf{A}}_i = \mathbf{A}_i - \mathbf{H}_m = \mathbf{B}_m\mathbf{D}_i$, *i = 1, 2..., p*

(ii) $\mathbf{B}(\mathbf{x}(t)) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))\mathbf{B}_i = \alpha(\mathbf{x}(t))\mathbf{B}_m$

and the switching controller is designed by choosing

(iii) $\mathbf{P} > \mathbf{0}$

(iv) $m_i = -\frac{sign\left(\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_m\left(\mathbf{G}_i\mathbf{x}(t)+\mathbf{r}(t)\right)\right)}{sign(\alpha)\alpha_{\min}}$, *i = 1, 2, ..., p*

(v) $\mathbf{Q} = -\left(\mathbf{H}_m\mathbf{P} + \mathbf{P}\mathbf{H}_m\right) > \mathbf{0}$

(vi) $\mathbf{G}_i = -\mathbf{D}_i$, *i = 1, 2, ..., p*

where $sign(z) = \begin{cases} 1 & z > 0 \\ -1 & z \leq 0 \end{cases}$, $|\alpha| \in [\alpha_{\min} \quad \alpha_{\max}]$ *and* $\alpha_{\max} > \alpha_{\min} > 0$.

The design procedure of the linear controller is summarized into the following steps.

Step I). Obtain the fuzzy plant model of a nonlinear plant, by means of methods in [1] or other ways.
Step II). Choose a reference model in the form of (2).
Step III). Check if the fuzzy plant model satisfying conditions (i) to (ii) of Lemma I.
Step IV). Design the switching controller according to condition (iii) to (vi) of Lemma I.

IV. Application Example

An application example will be given to show the design of the switching controller. A cart-pole type inverted pendulum system [3] is shown in Fig. 1. The design follows the procedure given in previous section.
Step I). The dynamic equation of the cart-pole type inverted pendulum system is given by,

$$\ddot{\theta}(t) = \frac{g\sin(\theta(t)) - aml\dot{\theta}(t)^2\sin(2\theta(t))/2 - a\cos(\theta(t))u(t)}{4l/3 - aml\cos^2(\theta(t))} \quad (24)$$

where θ is the angular displacement of the pendulum, $g = 9.8\text{m/s}^2$ is the acceleration due to gravity, $m \in [0.5 \quad 2]kg$ is the mass of the pendulum, $a = 1/(m + M)$, $M \in [8 \quad 80]kg$ is the mass of the cart, $2l = 1$m is the length of the pendulum, and u is the force applied to the cart. The objective is to design a linear controller to close the feedback loop of (25) such that $\theta = 0$ at steady state. (25) can be modeled by a fuzzy plant model having four rules. The i-th rule can be written as follows,

Rule i: IF $f_1(\mathbf{x}(t))$ is M_1^i AND $f_2(\mathbf{x}(t))$ is M_2^i

THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t)$ for $i = 1, 2, 3, 4$ (25)

so that the system dynamics is described by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{4} w_i\left(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t)\right) \quad (26)$$

where $\mathbf{x}(t) = [x_1(t) \quad x_2(t)]^{\mathrm{T}} = [\theta(t) \quad \dot{\theta}(t)]^{\mathrm{T}}$, $\theta(t) \in [\theta_{\min} \quad \theta_{\max}] = \left[-\frac{22\pi}{45} \quad \frac{22\pi}{45}\right]$ and $\dot{\theta}(t) \in [\dot{\theta}_{\min} \quad \dot{\theta}_{\max}] = [-5 \quad 5]$; $f_1(\mathbf{x}(t)) = \frac{g - amlx_2(t)^2\cos(x_1(t))}{4l/3 - aml\cos^2(x_1(t))}\left(\frac{\sin(x_1(t))}{x_1(t)}\right)$ and $f_2(\mathbf{x}(t)) = -\frac{a\cos(x_1(t))}{4l/3 - aml\cos^2(x_1(t))}$; $\mathbf{A}_1 = \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ f_{1_{\min}} & 0 \end{bmatrix}$ and $\mathbf{A}_3 = \mathbf{A}_4 = \begin{bmatrix} 0 & 1 \\ f_{1_{\max}} & 0 \end{bmatrix}$; $\mathbf{B}_1 = \mathbf{B}_3 = \begin{bmatrix} 0 \\ f_{2_{\min}} \end{bmatrix}$ and $\mathbf{B}_2 = \mathbf{B}_4 = \begin{bmatrix} 0 \\ f_{2_{\max}} \end{bmatrix}$; $f_{1_{\min}} = 9$ and $f_{1_{\max}} = 20$, $f_{2_{\min}} = -2$ and $f_{2_{\max}} = -0.001$; $w_i = \frac{\mu_{\mathrm{M}_1^i}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^i}(f_2(\mathbf{x}(t)))}{\sum_{l=1}^{4}\left(\mu_{\mathrm{M}_1^l}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^l}(f_2(\mathbf{x}(t)))\right)}$;

$\mu_{\mathrm{M}_1^\beta}(f_1(\mathbf{x}(t))) = \frac{-f_1(\mathbf{x}(t)) + f_{1_{\max}}}{f_{1_{\max}} - f_{1_{\min}}}$ for $\beta = 1, 2$ and $\mu_{\mathrm{M}_1^\delta}(f_1(\mathbf{x}(t))) = 1 - \mu_{\mathrm{M}_1^1}(f_1(\mathbf{x}(t)))$ for $\delta = 3, 4$; $\mu_{\mathrm{M}_2^\varepsilon}(f_2(\mathbf{x}(t))) = \frac{-f_2(\mathbf{x}(t)) + f_{2_{\max}}}{f_{2_{\max}} - f_{2_{\min}}}$ for $\varepsilon = 1, 3$ and $\mu_{\mathrm{M}_2^\phi}(f_2(\mathbf{x}(t))) = 1 - \mu_{\mathrm{M}_2^1}(f_2(\mathbf{x}(t)))$ for $\phi = 2, 4$ are the membership functions which are shown in Fig. 2.

Step II). The stable system matrix and the input vector of the reference model are chosen as follows.

$$\mathbf{H}_m = \begin{bmatrix} 0 & 1 \\ -8 & -8 \end{bmatrix} \quad (27)$$

$$\mathbf{B}_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (28)$$

Step III). The conditions (i) and (ii) of Lemma I are satisfied if we choose,

$$\mathbf{D}_1 = \mathbf{D}_2 = \left[f_{1_{\min}} + 8 \quad 8\right] = [17 \quad 8] \quad (29)$$

$$\mathbf{D}_3 = \mathbf{D}_4 = \lfloor f_{2_{\min}} + 8 \quad 8 \rfloor = [28 \quad 8] \tag{30}$$

It can be seen that

$$\alpha(\mathbf{x}(t)) = f_2(\mathbf{x}(t)) < 0 \tag{31}$$

Step IV). The switching controller is designed as,

$$u(t) = \sum_{j=1}^{2} m_j(\mathbf{x}(t), \hat{\mathbf{x}}(t))(\mathbf{G}_j\mathbf{x}(t) + r) \tag{32}$$

where $m_i = -\dfrac{sign(\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}_m(\mathbf{G}_i\mathbf{x}(t) + r(t)))}{sign(\alpha)\alpha_{\min}}$, i = 1, 2;

$\mathbf{P} = \begin{bmatrix} 1.0625 & 0.0625 \\ 0.0625 & 0.0703 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$; $\mathbf{G}_i = -\mathbf{D}_i$, i = 1, 2;

and $|\alpha| \in [\alpha_{\min} \quad \alpha_{\max}] = [0.001 \quad 0.2]$.

Fig. 3 and Fig. 4 show the simulation results. The simulation is carried out by using the actual plant of (24) and the switching controller of (32). Fig. 3 shows the response of $x_1(t)$ and $\hat{x}_1(t)$ respectively under the initial condition of $\mathbf{x}(0) = [0.5 \quad 0.5]^{\mathrm{T}}$ and $\hat{\mathbf{x}}(0) = [0 \quad 0]^{\mathrm{T}}$, m = 2kg, M=8kg and $r(t) = 8$. Fig. 4 shows the response of $x_1(t)$ and $\hat{x}_1(t)$ respectively under the initial condition of $\mathbf{x}(0) = [0.5 \quad 0.5]^{\mathrm{T}}$ and $\hat{\mathbf{x}}(0) = [0 \quad 0]^{\mathrm{T}}$, m = 2kg, M=8kg and $r(t) = \sin(t)$.

V. Conclusion

A switching controller has been designed for nonlinear plants subject to unknown parameters. Under some conditions, this switching controller has the ability to drive the system states to follow those of a reference model. An application example of an inverted pendulum on a cart has been given.

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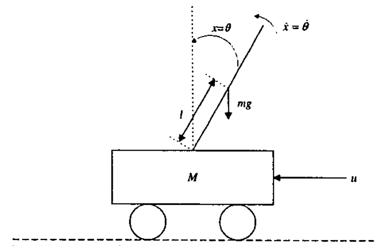

Fig. 1. A cart-pole type inverted pendulum system.

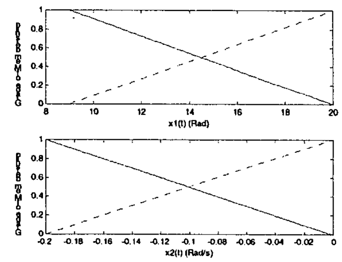

Fig. 2. Membership functions of the fuzzy plant model of the inverted pendulum in a cart.

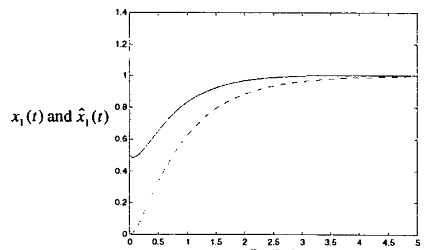

Fig. 3. Response of $x_1(t)$ (solid line) and $\hat{x}_1(t)$ (dotted line) under the initial condition of $\mathbf{x}(0) = [0.5 \quad 0.5]^{\mathrm{T}}$ and $\hat{\mathbf{x}}(0) = [0 \quad 0]^{\mathrm{T}}$, m = 2kg, M=8kg and $r(t) = 8$.

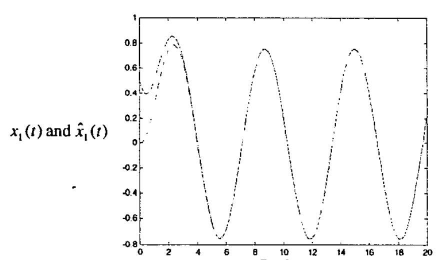

Fig. 4. Response of $x_1(t)$ (solid line) and $\hat{x}_1(t)$ (dotted line) under the initial condition of $\mathbf{x}(0) = [0.5 \quad 0.5]^{\mathrm{T}}$ and $\hat{\mathbf{x}}(0) = [0 \quad 0]^{\mathrm{T}}$, m = 2kg, M=8kg and $r(t) = \sin(t)$.