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On Fuzzy Model Reference Adaptive Control Systems: Full-State Feedback and Output Feedback

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***Abstract* — The problem of model reference adaptive control (MRAC) for nonlinear time-varying plants based on a fuzzy model is addressed. It is known that this kind of plants can readily be described by a fuzzy model. To tackle the plant parameter variations, an adaptive algorithm is derived to tune a designed fuzzy controller such that the system output follows a desired output from a reference fuzzy plant model. The stability of the closed-loop system is guaranteed. Cases of full-state feedback and output feedback are to be investigated. The results and merits of the proposed algorithms are illustrated by an example for each case.**

I. INTRODUCTION

FUZZY logic control have become a promising research platform during the past decade. Uncertain or ill-defined plants are readily handled by fuzzy controllers of which the design need not rely on accurate models of the plants. The endowment with human spirit have made fuzzy controller be successfully applied in many areas [1, 2]. Still, the conventional design of fuzzy controllers are based on a heuristic approach which cannot guarantee the closed-loop system's stability and dynamic behaviour. This has stimulated much research work on the analysis of the stability of the fuzzy system [5-8]. However, the parameter uncertainties were not put into account in these designs. When the plants are nonlinear with inherent parameter uncertainties or even time-varying parameters, a systematic design approach is lacked.

In this paper, a solution is offered on the problem of design of adaptive fuzzy logic controller (FLC) for the aforementioned plants. Cases of control with full-state feedback and with output feedback are investigated. It is known that any nonlinear plant can readily be represented by a fuzzy model proposed by Takagi and Sugeno [3] (TS-model). Due to the time-varying or uncertain plant parameters, adaptivity has to be introduced by a fuzzy controller to tune the TS-model representing the plant during on-line operation.

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The objective is to drive the system output to follow the one come from a reference TS-model, with guaranteed closed-loop stability and design of dynamic behaviour. During the control process, the parameters of the fuzzy model representing the plant is estimated simultaneously. Hence, a model for the nonlinear plant is readily obtainable without resort to the tedious work for deriving the physical model. The design of the closed-loop system is thus eased.

II. FUZZY MRAC WITH FULL-STATE FEEDBACK

A nonlinear plant is represented by a TS-model of which the i-th rule is given by:

$$\text{IF } x_1 \text{ is } \mathrm{M}_1^i \text{ AND } \ldots \text{ AND } x_n \text{ is } \mathrm{M}_n^i \text{ THEN}$$
$$\dot{\mathbf{x}}^i = \mathbf{A}^i\mathbf{x} + \mathbf{b}u \tag{1}$$

where M_k^i is a fuzzy term of the rule i corresponding to the state x_k; $\mathbf{A}^i \in \Re^{n\times n}$, $\mathbf{b} \in \Re^{n\times 1}$ are in phase variable canonical form,

$$\mathbf{A}^i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ a_1^i & a_2^i & \cdots & a_n^i \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \tag{2}$$

The inferred output is defined as,

$$\mathbf{x} = \sum_{i=1}^{p} w^i\mathbf{x}^i, \sum_{j=1}^{p} w^i = 1 \tag{3}$$

where w^i is the normalized weight for rule i, p is the number of rules. An FLC is to be designed based on this fuzzy model of the plant. The j-th rule of the FLC is given by,

$$\text{IF } x_1 \text{ is } \mathrm{M}_1^j \text{ AND } \ldots \text{ AND } x_n \text{ is } \mathrm{M}_n^j \text{ THEN}$$
$$\text{control is } u^j \tag{4}$$

where u^j is the control corresponding to the j-th rule which is obtained from the adaptive control law to be discussed later. The number of rules of the FLC is the same as that of the fuzzy plant model. The inferred control output is defined as,

$$u = \sum_{j=1}^{p} w^j u^j, \sum_{j=1}^{p} w^j = 1 \tag{5}$$

The adaptive control is to drive $\mathbf{x}$ to follow that from a reference TS-model of the plant. The i-th rule of the reference model is given by:

$$\text{IF } x_{m1} \text{ is } \mathrm{M}_1^i \text{ AND } \ldots \text{ AND } x_{mn} \text{ is } \mathrm{M}_n^i$$
$$\text{THEN } \dot{\mathbf{x}}_m^i = \mathbf{A}_m^i \mathbf{x}_m + \mathbf{b}r \quad (6)$$

where r is the reference input; $\mathbf{A}_m^i \in \Re^{n\times n}$ is in phase variable canonical form,

$$\mathbf{A}_m^i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ a_{m1}^i & a_{m2}^i & \cdots & a_{mn}^i \end{bmatrix} \quad (7)$$

The inferred output of the reference model is defined as,

$$\mathbf{x}_m = \sum_{k=1}^{p} w_m^k \mathbf{x}_m^k, \ \sum_{k=1}^{p} w_m^k = 1 \quad (8)$$

where w_m^k is the normalized weight for rule k.

The adaptive control law is derived based on the Lyapunov's stability theorem. From (1) to (5), the closed loop system is,

$$\dot{\mathbf{x}} = \sum_{i=1}^{p} w^i (\mathbf{A}^i \mathbf{x} + \mathbf{b}u^i) \quad (9)$$

Let $\mathbf{e} \equiv \mathbf{x} - \mathbf{x}_m$, from (8) and (9),

$$\dot{\mathbf{e}} = \sum_{i=1}^{p} w^i (\mathbf{A}^i \mathbf{x} + \mathbf{b}u^i) - \sum_{k=1}^{p} w_m^k (\mathbf{A}_m^k \mathbf{x}_m + \mathbf{b}r) \quad (10)$$

Let $w_m^i \equiv w^i + \Delta w^i$, from (3) and (8),

$$\sum_{i=1}^{p} w_m^i = \sum_{i=1}^{p} (w^i + \Delta w^i) = 1 \quad (11)$$

$$\sum_{i=1}^{p} \Delta w^i = 0 \quad (12)$$

Hence,

$$\dot{\mathbf{e}} = \sum_{i=1}^{p} w^i (\mathbf{A}^i \mathbf{x} + \mathbf{b}u^i - \mathbf{A}_m^i \mathbf{x}_m - \mathbf{b}r - \sum_{k=1}^{p} \Delta w^k \mathbf{A}_m^k \mathbf{x}_m) \quad (13)$$

The control signal u of (5) is designed such that

$$u^i = -\hat{\mathbf{a}}^i \mathbf{x} + \mathbf{a}_m^i \mathbf{x}_m + r + \sum_{k=1}^{p} \Delta w^k \mathbf{a}_m^k \mathbf{x}_m + \mathbf{a}_o \mathbf{e} \quad (14)$$

where $\hat{\mathbf{a}}^i = [\hat{a}_1{}^i \ \ \hat{a}_2{}^i \ \cdots \ \hat{a}_n{}^i]$ is a vector of estimated parameters which is governed by an adaptive law to be derived later; $\mathbf{a}_m^i = [a_{m1}^i \ \ a_{m2}^i \ \cdots \ a_{mn}^i]$ is the vector of the last row of $\mathbf{A}_m^i$; and $\mathbf{a}_o = [a_{o1} \ \ a_{o2} \ \cdots \ a_{on}]$ is the last row of a matrix $\mathbf{A}_o$ such that

$$\mathbf{A}_o = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 \\ a_{o1} & a_{o2} & \cdots & a_{on} \end{bmatrix} \quad (15)$$

which is to be designed to govern the dynamics of $\mathbf{e}$. From (13) and (14),

$$\dot{\mathbf{e}} = \sum_{i=1}^{p} w^i (\mathbf{A}^i \mathbf{x} + \mathbf{b}(-\hat{\mathbf{a}}^i \mathbf{x} + \mathbf{a}_m^i \mathbf{x}_m + r + \sum_{k=1}^{p} \Delta w^k \mathbf{a}_m^k \mathbf{x}_m + \mathbf{a}_o \mathbf{e})$$
$$- \mathbf{A}_m^i \mathbf{x}_m - \mathbf{b}r - \sum_{k=1}^{p} \Delta w^k \mathbf{A}_m^k \mathbf{x}_m)$$
$$= \mathbf{A}_o \mathbf{e} - \sum_{i=1}^{p} w^i \mathbf{b} \tilde{\mathbf{a}}^i \mathbf{x} \quad (16)$$

where $\tilde{\mathbf{a}}^i = \hat{\mathbf{a}}^i - \mathbf{a}^i$. The dynamics of $\mathbf{e}$ is governed by (16).

Theorem 1: The dynamics of $\mathbf{e}$ *will converge to zero if the following adaptive law for* $\hat{\mathbf{a}}^i$ *is used:*

$$\dot{\hat{\mathbf{a}}}^{i\mathrm{T}} = w^i \gamma^i \mathbf{e}^{\mathrm{T}} \mathbf{P} \mathbf{b} \mathbf{x} \quad (17)$$

Proof: Let a Lyapunov's function be

$$V = \frac{1}{2}(\mathbf{e}^{\mathrm{T}} \mathbf{P} \mathbf{e} + \sum_{i=1}^{p} \frac{\tilde{\mathbf{a}}^i \tilde{\mathbf{a}}^{i\mathrm{T}}}{\gamma^i}) > 0 \quad (18)$$

where $\mathbf{P} \in \Re^{\mathrm{n}\times\mathrm{n}}$ is positive definite matrix and γ_i is a design constant. Taking the derivative of V, and from (16)

$$\dot{V} = \frac{1}{2}(\dot{\mathbf{e}}^{\mathrm{T}} \mathbf{P} \mathbf{e} + \mathbf{e}^{\mathrm{T}} \mathbf{P} \dot{\mathbf{e}}) + \sum_{i=1}^{p} \frac{\tilde{\mathbf{a}}^i \dot{\tilde{\mathbf{a}}}^{i\mathrm{T}}}{\gamma^i}$$
$$= \frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{A}_o{}^{\mathrm{T}} \mathbf{P} \mathbf{e} + \frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{P} \mathbf{A}_o \mathbf{e} - \sum_{i=1}^{p} w^i \mathbf{e}^{\mathrm{T}} \mathbf{P} \mathbf{b} \tilde{\mathbf{a}}^i \mathbf{x} + \sum_{i=1}^{p} \frac{\tilde{\mathbf{a}}^i \dot{\tilde{\mathbf{a}}}^{i\mathrm{T}}}{\gamma^i}$$
$$= \frac{1}{2} \mathbf{e}^{\mathrm{T}} (\mathbf{A}_o{}^{\mathrm{T}} \mathbf{P} + \mathbf{P} \mathbf{A}_o) \mathbf{e} + \sum_{i=1}^{p} \frac{1}{\gamma^i} \tilde{\mathbf{a}}^i (\dot{\tilde{\mathbf{a}}}^{i\mathrm{T}} - w^i \gamma^i \mathbf{e}^{\mathrm{T}} \mathbf{P} \mathbf{b} \mathbf{x})$$
$$= -\frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{Q} \mathbf{e} < 0 \quad (19)$$

if $\dot{\hat{\mathbf{a}}}^{i\mathrm{T}} = \dot{\tilde{\mathbf{a}}}^{i\mathrm{T}} = w^i \gamma^i \mathbf{e}^{\mathrm{T}} \mathbf{P} \mathbf{b} \mathbf{x}$ and $-\mathbf{Q} = \mathbf{A}_o{}^{\mathrm{T}} \mathbf{P} + \mathbf{P} \mathbf{A}_o$ which is a positive definite matrix QED

The control system can be represented by a diagram as shown in Fig. 1.

III. Fuzzy MRAC with Output State Feedback

The rules for the plant model, the FLC and the reference model are respectively given by:

Rule i: IF y is M^i THEN $\dot{\mathbf{x}}^i = \mathbf{A}^i \mathbf{x} + \mathbf{B}\mathbf{u}$ (20)

Rule j: IF y is M^j THEN control is $\mathbf{u}^j$ (21)

Rule k: IF y is M^k THEN $\dot{\mathbf{x}}_m^k = \mathbf{A}_m^k \mathbf{x} + \mathbf{B}_m \mathbf{r}$ (22)

where $\mathbf{A}^i, \mathbf{A}_m^k, \mathbf{B}, \mathbf{B}_m \in \Re^{\mathrm{n}\times\mathrm{n}}$; $\mathbf{u}^j$, $\mathbf{r} \in \Re^{\mathrm{n}\times 1}$

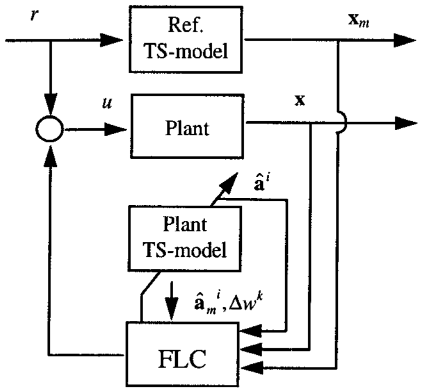

Fig. 1. A block diagram representing the fuzzy MRAC with full-state feedback.

$$\mathbf{A}^i = \begin{bmatrix} a_1^i & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n-1}^i & 0 & \cdots & 1 \\ a_n^i & 0 & \cdots & 0 \end{bmatrix},\ \mathbf{A}_m{}^i = \begin{bmatrix} a_m^i & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{mn-1}^i & 0 & \cdots & 1 \\ a_{mn}^i & 0 & \cdots & 0 \end{bmatrix} \tag{23}$$

$\mathbf{B}$ is arbitrary but invertible. The output equation is given by:

$$y = \mathbf{cx} \tag{24}$$

where $\mathbf{c} = [1 \ \cdots \ 0 \ \ 0]$. Let $\mathbf{A}^i\mathbf{x} = \mathbf{A}_m^i\mathbf{x} + \mathbf{G}^i y$,

$$\mathbf{G}^i = [g_1^i \ \cdots \ g_{n\text{-}1}^i \ \ g_n^i]^{\mathrm{T}}$$

From (3), (11), (12), (21) to (24),

$$\dot{\mathbf{e}} = \sum_{i=1}^{p} w^i(\mathbf{A}_m^i\mathbf{x} + \mathbf{G}^i y + \mathbf{B}\mathbf{u}^i - \mathbf{A}_m^i\mathbf{x}_\mathbf{m} - \mathbf{B}_\mathbf{m}\mathbf{r} - \sum_{k=1}^{p} \Delta w^k\, \mathbf{A}_m^k\mathbf{x}_m) \tag{25}$$

The control signal $\mathbf{u}$ is designed such that

$$\mathbf{u}^i = \mathbf{B}^{-1}(-\hat{\mathbf{G}}^i y + \mathbf{B}_\mathbf{m}\mathbf{r} + \sum_{k=1}^{p} \Delta w^k \mathbf{A}_m^k \mathbf{x}_m + \mathbf{v}^i) \tag{26}$$

where $\hat{\mathbf{G}}^i$ is the estimated vector of $\mathbf{G}^i$ for rule i and constitute the adaptive control law which also govern the value of the vector $\mathbf{v}^i$. Under this design of $\mathbf{u}^i$, and from (25) and (5), the error dynamics is given by:

$$\dot{\mathbf{e}} = \mathbf{A}_m^i\mathbf{e} - \tilde{\mathbf{G}}^i y + \mathbf{v}^i \tag{27}$$

where $\tilde{\mathbf{G}}^i = \hat{\mathbf{G}}^i - \mathbf{G}^i$ The derivations of $\hat{\mathbf{G}}^i$ and $\mathbf{v}^i$ are based on the following theorem and lemma.

Theorem 2: The system $\dot{\mathbf{e}} = \mathbf{A}_m^i\mathbf{e} - \mathbf{d}^i\tilde{\mathbf{G}}^{i\mathrm{T}}\boldsymbol{\omega}^i$ *is stable and convergent to zero if the adaptive law for* $\tilde{\mathbf{G}}^i$ *is given by*

$$\dot{\tilde{\mathbf{G}}} = \dot{\hat{\mathbf{G}}} = \gamma^i y \boldsymbol{\omega}^i \tag{28}$$

where $\mathbf{d}^i = [1 \ \cdots \ d_{n-1}^i \ \ d_n^i]^{\mathrm{T}}$ *which is chosen such that* $\mathbf{c}(s\mathbf{I} - \mathbf{A}_m^i)^{-1}\mathbf{d}^i$ *is strictly positive real, and* $\boldsymbol{\omega}^i = [\omega_1^i \ \cdots \ \omega_{n-1}^i \ \ \omega_n^i]^{\mathrm{T}}$

Proof: Let the Lyapunov's function be,

$$V^i = \frac{1}{2}(\mathbf{e}^{\mathrm{T}}\mathbf{P}^i\mathbf{e} + \frac{\tilde{\mathbf{G}}^{i\mathrm{T}}\tilde{\mathbf{G}}^i}{\gamma^i}) > 0 \tag{29}$$

$$\begin{aligned} \dot{V}^i &= \frac{1}{2}\dot{\mathbf{e}}^{\mathrm{T}}\mathbf{P}^i\mathbf{e} + \frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{P}^i\dot{\mathbf{e}} + \frac{\tilde{\mathbf{G}}^{i\mathrm{T}}\dot{\tilde{\mathbf{G}}}^i}{\gamma^i} \\ &= \frac{1}{2}\mathbf{e}^{\mathrm{T}}(\mathbf{A}_m^{i\ \mathrm{T}}\mathbf{P}^i + \mathbf{P}^i\mathbf{A}_m^i)\mathbf{e} - \mathbf{e}^{\mathrm{T}}\mathbf{P}^i\mathbf{d}^i\tilde{\mathbf{G}}^i\boldsymbol{\omega}^i + \frac{\tilde{\mathbf{G}}^{i\mathrm{T}}\dot{\hat{\mathbf{G}}}^i}{\gamma^i} \end{aligned} \tag{30}$$

By the Kalman-Yakubovich lemma, if $\mathbf{c}(s\mathbf{I} - \mathbf{A}_m^i)^{-1}\mathbf{d}^i$ is strictly positive real, there exist symmetric positive-definite matrices $\mathbf{P}^i$ and $\mathbf{Q}^i$ such that

$$\mathbf{A}_m^{i\ \mathrm{T}}\mathbf{P}^i + \mathbf{P}^i\mathbf{A}_m^i = -\mathbf{Q}^i \tag{31}$$

$$\mathbf{P}^i\mathbf{d}^i = \mathbf{c}^{\mathrm{T}} \tag{32}$$

Then (30) becomes

$$\begin{aligned} \dot{V}^i &= -\frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{Q}^i\mathbf{e} - \mathbf{e}^{\mathrm{T}}\mathbf{c}^{\mathrm{T}}\tilde{\mathbf{G}}^{i\mathrm{T}}\boldsymbol{\omega} + \frac{\tilde{\mathbf{G}}^i\dot{\hat{\mathbf{G}}}^i}{\gamma^i} \\ &= -\frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{Q}^i\mathbf{e} + \frac{\tilde{\mathbf{G}}^{i\mathrm{T}}}{\gamma^i}(\dot{\hat{\mathbf{G}}}^i - \gamma^i y\boldsymbol{\omega}^i) \\ &= -\frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{Q}^i\mathbf{e} \le 0 \end{aligned} \tag{33}$$

when adaptive law (28) is used. QED

Lemma 1: The system as given by (27) *is the same as that in Theorem 2 if*

$$\mathbf{v}^i = [0 \ \ \Lambda_2^i\boldsymbol{\omega}^i \ \cdots \ \Lambda_n^i\boldsymbol{\omega}^i]\dot{\tilde{\mathbf{G}}}^i \tag{34}$$

where $\omega_q^i \equiv F_q^i(s)y$, *and*

$$F_q^i(s) = \frac{s^{n-q}}{s^{n-1} + d_2^i s^{n-2} + \cdots + d_n^i},\ q = 2, 3, \ldots, n \tag{35}$$

$\Lambda_q^i \in \Re^{n \times n}$ *is defined in* (36):

$$\Lambda_q^i = \begin{bmatrix} 0 & -d_q^i & -d_{q+1}^i & \cdots & -d_n^i & 0 & \cdots & \cdots & 0 \\ 0 & 0 & -d_q^i & -d_{q+1}^i & \cdots & -d_n^i & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & -d_q^i & -d_{q+1}^i & \cdots & -d_n^i \\ 0 & 1 & d_2^i & \cdots & d_{q-1}^i & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & 1 & d_2^i & \cdots & d_{q-1}^i \end{bmatrix} \tag{36}$$

The proof for *Lemma 1* can be found in [4].

IV. EXAMPLES

For fuzzy MRAC with full-state feedback, the reference fuzzy model is represented by one rule only such that

$$\dot{\mathbf{x}}_m = \mathbf{A}_m \mathbf{x}_m + \mathbf{b}r,\ \mathbf{A}_m = \begin{bmatrix} 0 & 1 \\ -5 & -3 \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ r = 5$$

The rules of the fuzzy model and the fuzzy controller are respectively given by (1) and (14). Two fuzzy terms, Positive (P) and Negative (N) are assigned of which the membership functions are shown in Fig. 2. The simulation results are shown in Fig. 3. In this figure, the solid lines and dotted lines are the responses $\mathbf{x}$ and $\mathbf{x}_m$ respectively. The parameters of the plant's TS-model are:

$$\mathbf{A}^1 = \begin{bmatrix} 0 & 1 \\ -0.72 & -1.87 \end{bmatrix},\ \mathbf{A}^2 = \begin{bmatrix} 0 & 1 \\ -5.23 & -2.73 \end{bmatrix},$$
$$\mathbf{A}^3 = \begin{bmatrix} 0 & 1 \\ 0.23 & 1.74 \end{bmatrix},\ \mathbf{A}^4 = \begin{bmatrix} 0 & 1 \\ -3.68 & 2.22 \end{bmatrix}$$

The adaptive gains γ^i are all chosen as 1000. The initial values of $\hat{\mathbf{a}}^i$ are all zero. Also,

$$\mathbf{P} = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

For MRAC with output feedback, a one rule reference fuzzy model is chosen as,

$$\mathbf{A}_m = \begin{bmatrix} -5 & 1 \\ -3 & 0 \end{bmatrix},\ \mathbf{B}_m = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},\ \mathbf{r} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The membership functions and γ^i are kept the same as the previous example. The results are shown in Fig. 4 in which the solid lines and dotted lines are the responses of y and y_m respectively. The parameters of the plant's fuzzy model are:

$$\mathbf{A}^1 = \begin{bmatrix} -0.72 & 1 \\ -1.87 & 0 \end{bmatrix},\ \mathbf{A}^2 = \begin{bmatrix} -5.23 & 1 \\ -2.73 & 0 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Also, $\mathbf{d} = [1 \quad 1]^T$. The initial values of $\hat{\mathbf{G}}^i$ are all zero. The successes of the fuzzy MRAC for both cases can be seen from the simulation results.

V. CONCLUSION

The fuzzy MRAC for the SISO system with full-state feedback and MISO system with output feedback are presented. The control laws and the adaptive algorithms for both cases are derived. With the use of these laws, the stability of the fuzzy systems is guaranteed. The performance of the fuzzy systems is governed by a defined reference fuzzy model. The control process and fuzzy model parameter estimation are being done simultaneously.

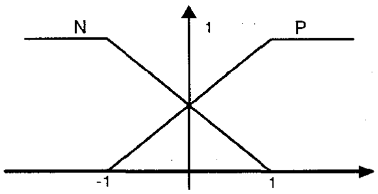

Fig. 2. Membership functions of the two fuzzy terms

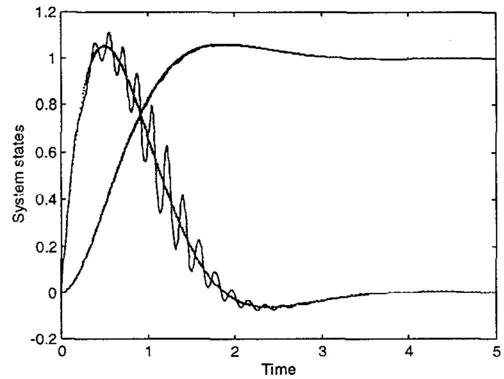

Fig. 3. Simulation results for MRAC with full-state feedback.

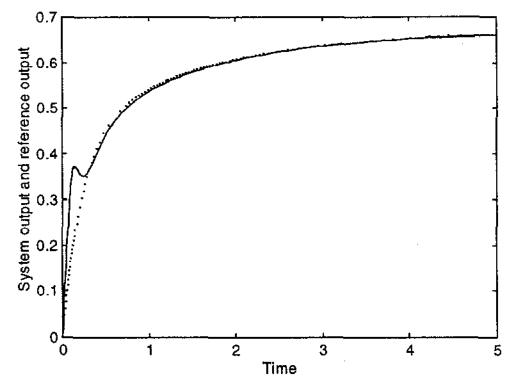

Fig. 4. Simulation results for MRAC with output state feedback.

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