Stability Design of TS Model Based Fuzzy Systems

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Abstract

An approach for designing TS model based fuzzy systems using output feedback with guaranteed closed-loop stability is proposed in this paper. The complex process on finding a common Lyapunov function to guarantee the system stability can be omitted. This can significantly simplify the design procedure. Moreover the overall closed-loop system behaves like a linear system and the system responses can be designed by properly choosing the coefficients of the closed-loop transfer function. An illustrative example will be given to demonstrate the ability of the proposed approach.

1. Introduction

A fuzzy model called Takagi-Sugeno (TS) fuzzy model for non-linear systems was proposed in [1]. This TS fuzzy model is widely accepted as a powerful modeling tool and applications of the TS models to various kinds of non-linear systems [3-5] can be found. Based on the TS fuzzy model, a stability design approach is proposed in [2]. Under this approach, with reference to the consequent part's linear system in each fuzzy rule of the TS model, one control strategy can be designed to give a stable linear system. On combining the rules in the fuzzy model and the fuzzy logic controller (FLC), many fuzzy sub-systems [2] are formed. A sufficient condition to ensure the stability of the overall system is obtained by finding a common Lyapunov function which can satisfy all fuzzy sub-systems.

However, a major drawback of this stability design approach is the difficulty in finding a common Lyapunov function. When combining the TS fuzzy model of the plant and the FLC, the number of sub-systems generated can be $\frac{r(r+1)}{2}$, where r is the number of rules. The difficulty in finding a common Lyapunov function increases when r becomes large. Recently, many researches [3-5] apply different methods to find the common Lyapunov function. Nevertheless, the problem of finding the common Lyapunov function remains open. Even worse, the existence of the common Lyapunov function is not guaranteed. Besides the problem on finding a common Lyapunov function, the performance of the closed-loop system is also difficult to design. Although each fuzzy rule in the FLC can control each linear system in the consequent part of each fuzzy model at a desired performance when applying it alone, the performance of the overall system is difficult to predict. In general, the reported design approaches can only tackle the closed-loop stability problem, but the system performance cannot be designed.

In view of these weaknesses, a new FLC for TS model based control systems is proposed in [6] which applies state-feedback technique. Its major advantage over the existing approach is that the stability of the overall system is ensured without the need of finding a common Lyapunov function. The design is thus very simple. Moreover, the closed-loop system response can be predicted. However, this approach restricts the consequent part of the TS model rules to be in phase-variable canonical form. Also it requires a full state feedback which may be expensive or even impossible to implement in some systems.

The proposed approach in this paper has the same advantages stated in [6]. But the restrictions in [6] are released. The consequent part of the TS model can be in any form and output feedback is used instead of full state feedback. The TS fuzzy model will be reviewed in section 2. In section 3, the proposed stability design approach will be detailed. In section 4, an application example will be given to show the advantages of the proposed approach. Finally, conclusions will be drawn in section 5.

2. The TS fuzzy model

In this paper, the i-th IF-THEN rule of the TS fuzzy model of an n-th order system is of the following form [1, 2]:

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Model Rule i: IF $y(k)$ is A_{i1} and ...... and $y(k-n+1)$ is $A_{i(n-1)}$
THEN $y(k+1)$
$= y_i(k+1)$
$= a_{i0}\, y(k) + a_{i1}\, y(k-1) + \ldots\ldots + a_{i(n-1)}\, y(k-n+1) + b_{i0}\, u(k) + b_{i1}\, u(k-1) + \ldots\ldots + b_{im}\, u(k-m)$ (1)

where $y(k+1)$, $y(k)$, $y(k-1)$, ..., $y(k-n+1)$ are state variables, $u(k)$, $u(k-1)$, ..., $u(k-m)$ are input variables, $y_i(k+1)$ is the output of rule i, A_{i0}, ..., $A_{i(n-1)}$ are fuzzy levels, a_{i0}, a_{i1}, ..., $a_{i(n-1)}$ and b_{i0}, b_{i1}, ..., b_{im} are coefficients of the linear system in the consequent part. The overall output of the TS fuzzy model is inferred as follows:

$$y(k+1) = \frac{\sum_{i=1}^{r} w_i y_i(k+1)}{\sum_{i=1}^{r} w_i} \quad (2)$$

where r is the total number of fuzzy rules in the TS model
$w_i = \mu(A_{i0}) \times \cdots \times \mu(A_{i(n-1)})$ for $i = 1, 2, \ldots\ldots, r$
$\times$ denotes the "and" operation
$\mu(A_{ij})$ is the degree of membership of "$y(k-j)$ is A_{ij}" for $j = 0, 1, 2, \ldots\ldots, n-1$

3. Proposed stability design approach

The proposed design approach is highly related to two properties of the fuzzy model based control system. They are called the *changing coefficient property* and the *double summation property.*

3.1 Changing coefficient property

From (1) and (2), it can be derived that

$$y(k+1) = \frac{1}{\sum_{i=1}^{r} w_i} \Big[\sum_{i=1}^{r} w_i\, a_{i0}\, y(k) + \sum_{i=1}^{r} w_i\, a_{i1}\, y(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, a_{i(n-1)}\, y(k-n+1) + \sum_{i=1}^{r} w_i\, b_{i0}\, u(k) + \sum_{i=1}^{r} w_i\, b_{i1}\, u(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, b_{im}\, u(k-m) \Big] \quad (3)$$

$$= a_{d0}\, y(k) + a_{d1}\, y(k-1) + \ldots\ldots + a_{d(n-1)}\, y(k-n+1) + b_{d0}\, u(k) + b_{d1}\, u(k-1) + \ldots\ldots + b_{dm}\, u(k-m) \quad (4)$$

where $a_{dj} = \frac{\sum_{i=1}^{r} w_i a_{ij}}{\sum_{i=1}^{r} w_i}$ for $j = 0, 1, 2, \ldots\ldots, n-1$

$b_{dl} = \frac{\sum_{i=1}^{r} w_i b_{il}}{\sum_{i=1}^{r} w_i}$ for $l = 0, 1, 2, \ldots\ldots, m$

It can be seen from (4) that the overall system is a quasi-linear system with fast changing coefficients a_{dj} and b_{dl}. We call it the *changing coefficient property*. These fast changing coefficients effectively make the system highly non-linear. Each coefficient, say a_{d0}, is the weighted sum of a_{10}, a_{20}, ..., a_{r0}. Thus each coefficient a_{dj}, $j = 0, 1, 2, \ldots\ldots, n-1$ is changing according to w_i, and w_i depends on the state variables.

To alleviate the problem, we can make $a_{i0} = a_{d0}$ for all i (that is for all rules) by the control signal of each rule such that a_{d0} will be unchanged in spite of the changing w_i. On applying the same approach to other coefficients, the overall system of (4) is effectively reduced to a linear system. Hence the control signal for each rule is as follows:

Control Rule i: IF *<premise i>*
THEN $\text{num}[u_i(k)] = [a_{di0}\, y(k) + a_{di1}\, y(k-1) + \ldots\ldots + a_{din}\, y(k-n-1) + b_{di1}\, u(k-1) + \ldots\ldots + b_{dim}\, u(k-m)]$
$\text{den}[u_i(k)] = b_{i0}$ (5)

where *<premise i>* is the short form of "$y(k)$ is A_{i0} and ...... and $y(k-n+1)$ is $A_{i(n-1)}$" in (1)
$\text{num}[u_i(k)]$ is the numerator of $u_i(k)$
$\text{den}[u_i(k)]$ is the denominator of $u_i(k)$
$a_{dij} = a_{cj} - a_{ij}$ for all $j = 0, 1, 2, \ldots\ldots, n-1$
$b_{dik} = b_{cl} - b_{il}$ for all $l = 1, 2, \ldots\ldots, m$
a_{cj} and b_{cl} are coefficients to be designed later and are independent of i.

3.2 Double summation property

The reason for dividing $u_i(k)$ into a numerator and a denominator part is to tackle the *double summation property.* Conventionally [1-5], the control signal of a FLC has only the numerator part in (5), then the overall control signal is

$$u(k) = \frac{\sum_{i=1}^{r} w_i u_i(k)}{\sum_{i=1}^{r} w_i}$$

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When this is put in (3), the term corresponding to $u(k)$ in (3) becomes $\dfrac{\sum_{i=1}^{r} w_i b_{i0}}{\sum_{i=1}^{r} w_i}\dfrac{\sum_{i=1}^{r} w_i u_i(k)}{\sum_{i=1}^{r} w_i}$. It involves a multiplication of two summation signs which is known as the double summation property. In this case, the number of fuzzy sub-systems is r^2.

Therefore the control signal is now divided into two parts as given in (5) in order to eliminate the double summation. Form (5) we have

$$u(k) = \frac{\text{num}[u_i(k)]}{\text{den}[u_i(k)]}$$

$$u(k) = \frac{\sum_{i=1}^{r} w_i u_i(k)}{\sum_{i=1}^{r} w_i} \Bigg/ \frac{\sum_{i=1}^{r} w_i b_{i0}}{\sum_{i=1}^{r} w_i} \tag{6}$$

Put (6) into (3), we obtain

$$\begin{aligned}
y(k+1) = \frac{1}{\sum_{i=1}^{r} w_i} \Big[& \sum_{i=1}^{r} w_i\, a_{i0}\, y(k) + \sum_{i=1}^{r} w_i\, a_{i1}\, y(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, a_{i(n-1)}\, y(k-n-1) + \\
& \sum_{i=1}^{r} w_i\, a_{di0}\, y(k) + \sum_{i=1}^{r} w_i\, a_{di1}\, y(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, a_{di(n-1)}\, y(k-n-1) + \\
& \sum_{i=1}^{r} w_i\, b_{di1}\, u(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, b_{dim}\, u(k-m) + \\
& \sum_{i=1}^{r} w_i\, b_{i1}\, u(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, b_{im}\, u(k-m) \Big] \\
= \frac{1}{\sum_{i=1}^{r} w_i} \Big[& \sum_{i=1}^{r} w_i\, (a_{i0} + a_{di0})\, y(k) + \sum_{i=1}^{r} w_i\, (a_{i1} + a_{di1})\, y(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, (a_{i(n-1)} + a_{di(n-1)})\, y(k-n-1) + \\
& \sum_{i=1}^{r} w_i\, (b_{i1} + b_{di1})\, u(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, (b_{i1} + b_{di1})\, u(k-m) \Big] \\
= \frac{1}{\sum_{i=1}^{r} w_i} \Big[& \sum_{i=1}^{r} w_i\, a_{c0}\, y(k) + \sum_{i=1}^{r} w_i\, a_{c1}\, y(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, a_{c(n-1)}\, y(k-n-1) + \\
& \sum_{i=1}^{r} w_i\, b_{c1}\, u(k-1) + \ldots\ldots + \sum_{i=1}^{r} w_i\, b_{cm}\, u(k-m) \Big] \\
= {} & a_{c0}\, y(k) + a_{c1}\, y(k-1) + \ldots\ldots + a_{c(n-1)}\, y(k-n-1) + b_{c1}\, u(k-1) + \ldots\ldots + b_{cm}\, u(k-m)
\end{aligned}$$

Furthermore, we choose $b_{cl} = 0$ for all $l = 1, 2, \ldots\ldots, m$. Then the overall system becomes

$$y(k+1) = a_{c0}y(k) + a_{c1}y(k-1) + \ldots\ldots + a_{c(n-1)}\, y(k-n-1) \tag{7}$$

The closed-loop system in (7) becomes a linear system on applying the FLC described in (5). By properly choosing the coefficients a_{c0}, a_{c1}, ..., $a_{c(n-1)}$, the stability and performance of the closed-loop system can be ensured. This design approach does not require an extra and complex process to find a common Lyapunov function to guarantee the system stability. Moreover, the system performance can also be designed.

4. Examples

Consider a TS fuzzy model based system as follows:

Model Rule 1: IF $y(k)$ is M_1

THEN $y_1(k+1) = -2y(k) - 3y(k-1) - y(k-2) + 0.2u(k) + 0.1u(k-1)$

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Model Rule 2: IF $y(k)$ is M_2

$$\text{THEN } y_2(k+1) = -y(k) - 2\,y(k-1) - 2\,y(k-2) + 2\,u(k) + 2\,u(k-1)$$

M_1 and M_2 are membership functions as shown in Fig. 1. An FLC is going to be designed such that the desired closed-loop system response is

$$y(k+1) = 0.3y(k) + 0.2y(k-1) - 0.35y(k-2) \quad (8)$$

Hence the corresponding control rules are as follows:

Control Rule 1: IF $y(k)$ is M_1

$$\text{THEN } \text{num}[u_1(k)] = 2.3y(k) + 3.2y(k-1) + 0.65y(k-2) - 0.1u(k-1)$$

$$\text{den}[u_1(k)] = 0.2$$

Control Rule 2: IF $y(k)$ is M_2

$$\text{THEN } \text{num}[u_2(k)] = 1.3y(k) + 2.2y(k-1) + 1.65y(k-2) - 2u(k-1)$$

$$\text{den}[u_2(k)] = 2$$

The output response of the closed-loop system is shown in Fig. 2. The initial value of $y(k)$ is 1 for all $k \le 0$. The response matches exactly with the response of the linear system of (8).

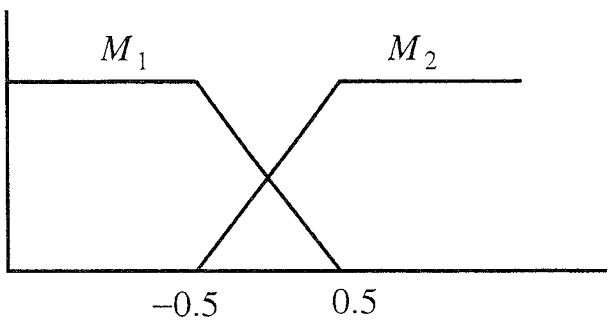

Fig. 1. Membership functions

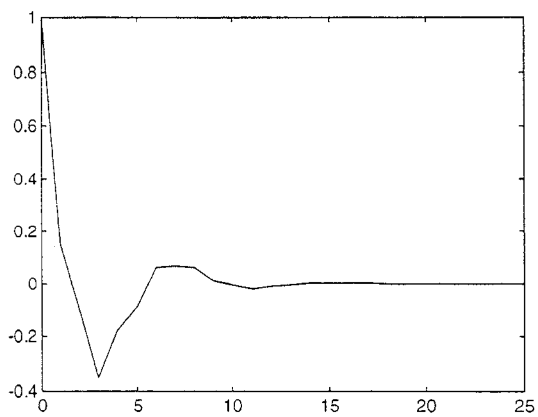

Fig. 2. Output response of the example

5. Conclusion

A stability design approach for TS fuzzy model based systems is proposed in this paper. The closed-loop response of the system behaves like a linear system. The coefficients of the transfer function can be chosen such that system performance can be designed as desired. Since the overall system is effectively a linear system, the system stability can be easily guaranteed without applying a complex process to find a common Lyapunov function as in the existing approaches. Also only output feedback is required; as full state feedback is usually expensive or even impossible for some systems.

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