Fuzzy Model Reference Control of Wheeled Mobile Robots[1]

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Abstract – **This paper presents the control of a two-wheeled mobile robot (WMR) using a fuzzy model approach. A fuzzy controller will be designed based on a TS fuzzy plant model of the WMR. This proposed controller is capable of driving the system states of the WMR to follow those of a stable reference fuzzy model with the stability of the closed-loop system guaranteed. Simulation results will be presented to illustrate the merits of the proposed fuzzy controller.**

I. Introduction

Wheeled mobile robots (WMRs) are widely applied in the real world. However, the control of WMRs is a challenging task because WMRs are nonholonomic systems which could not be input-state linearizable [5-6]. Hence, linear control theory cannot be applied. Different methods such as input-output linearization [11], state-feedback linearization [8], sliding mode control [9-10], have been proposed to control WMRs. Most of these methods have a high demand on the computational power of the controllers, and the structures of the controllers are relatively complex. As a high-gain switching signal is used, chattering will appear in the system output when the sliding controller is used.

Inspired by the fuzzy set theory established by Zadeh in 1965, Mamdani proposed fuzzy controllers to tackle nonlinear systems [1]. Since then, fuzzy control has become a promising research platform. A fuzzy controller has the ability to incorporate human knowledge into itself. Many successful applications of fuzzy control were reported in various areas such as sludge wastewater treatment [2], control of cement kiln [3], etc.. In order to facilitate design of the fuzzy controllers, a TS (Takagi-Sugeno) fuzzy plant model [4] was proposed. The TS fuzzy plant model is a powerful tool to describe the behavior of ill-defined systems. It was proved to be a universal approximator [8]. In this paper, the TS fuzzy plant model will be used to represent the WMR as a weighted sum of some linear sub-systems. Then, we propose a fuzzy controller which is realized as a weighted sum of some state feedback controllers. As a result, the fuzzy controller is a nonlinear system with a simpler structure as compared with other published nonlinear controllers.

The contributions of this paper are twofold. First, a TS fuzzy plant model is derived based on the transformed kinematic model of the WMR. Second, based on this TS fuzzy plant model, a fuzzy controller is designed to drive the system states of the WMR to follow those of a stable reference fuzzy system. Design of the fuzzy controller will be presented. Simulation results are obtained to verify the design.

This paper is organized as follows. Section II presents the kinematic model of the WMR. Section III presents the fuzzy plant model, the fuzzy controller and the reference fuzzy plant model. Section IV presents the design of the fuzzy controller. Simulation results will be given in Section V. A conclusion will be drawn in section VI.

II. Kinematic Model of the Wheeled Mobile

The schematic diagram of a two-wheeled mobile robot is shown in Fig. 1. Its kinematic model [11] can be written as,

$$\dot{\mathbf{x}}(t) = \mathbf{B}(\mathbf{x}(t))\overline{\mathbf{u}}(t) \tag{1}$$

where,

$$\mathbf{B}(\mathbf{x}(t)) = \begin{bmatrix} R\cos(x_3(t)) & 0 \\ R\sin(x_3(t)) & 0 \\ 0 & R \end{bmatrix} \tag{2}$$

$$\overline{\mathbf{u}}(t) = \begin{bmatrix} \overline{u}_1(t) \\ \overline{u}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(\omega_r + \omega_l) \\ \frac{1}{2}(\omega_r - \omega_l) \end{bmatrix} \tag{3}$$

$\mathbf{x}(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^{\mathrm{T}}$ is the system state vector, $x_1(t)$ and $x_2(t)$ denote the center position of the WMR in meter (m), $x_3(t)$ is the heading angle in radian (rad); $2D = 0.07$m is the distance between the two wheels, $R = 0.02$m is the radius of the wheels; ω_r and ω_l are the angular velocities of the right and left wheels respectively. Referring to Fig. 1, the control objective is to control the position of the point Q of the WMR to track a reference path. The position Q is represented by,

$$\mathbf{z}(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t) + h\sin(x_3(t)) \\ x_2(t) - h\cos(x_3(t)) \end{bmatrix} \tag{4}$$

From (1) and (4), we have,

$$\dot{\mathbf{z}}(t) = \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} Rz_3(t)\cos(x_3(t)) \\ Rz_3(t)\sin(x_3(t)) \end{bmatrix} \tag{5}$$

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where,

$$z_3(t) = \bar{u}_1(t) + h\bar{u}_2(t) \tag{6}$$

is assumed to be a nonzero scalar. This can be achieved by rotating the WMR when it stays at a point. From (1), (5) and letting $\dot{z}_3(t) = v_1(t)$, we have,

$$\ddot{\mathbf{z}}(t) = \begin{bmatrix} \ddot{z}_1(t) \\ \ddot{z}_2(t) \end{bmatrix} = \begin{bmatrix} R\cos(x_3(t)) & -R^2 z_3(t)\sin(x_3(t)) \\ R\sin(x_3(t)) & R^2 z_3(t)\cos(x_3(t)) \end{bmatrix} \begin{bmatrix} v_1(t) \\ \bar{u}_2(t) \end{bmatrix} \tag{7}$$

From (7),

$$\dot{\bar{\mathbf{x}}}(t) = \begin{bmatrix} \dot{\bar{x}}_1(t) \\ \dot{\bar{x}}_2(t) \\ \dot{\bar{x}}_3(t) \\ \dot{\bar{x}}_4(t) \end{bmatrix} = \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \\ \ddot{z}_1(t) \\ \ddot{z}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \\ \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ R\cos(x_3(t)) & -R^2 z_3(t)\sin(x_3(t)) \\ R\sin(x_3(t)) & R^2 z_3(t)\cos(x_3(t)) \end{bmatrix} \begin{bmatrix} v_1(t) \\ \bar{u}_2(t) \end{bmatrix}$$

$$= \bar{\mathbf{A}}\bar{\mathbf{x}}(t) + \bar{\mathbf{B}}\mathbf{u}(t) \tag{8}$$

where $\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} v_1(t) \\ \bar{u}_2(t) \end{bmatrix}$. The design of the fuzzy controller will be based on (8) and discussed later.

III. Fuzzy Plant Model, Fuzzy Controller and Reference Fuzzy Model

A TS fuzzy plant model will be used to approximate the behavior of (8). Based on this TS fuzzy plant model, a fuzzy controller will be designed to control the WMR of (8) such that the system states of (8) will follow those of a stable reference model.

A. Fuzzy Plant Model

A 5-rule TS fuzzy plant model is used to represent the WMR of (8). The i-th rule is of the following format,

Rule i: IF $x_3(t)$ is M^i THEN $\dot{\bar{\mathbf{x}}}(t) = \mathbf{A}_i\bar{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t)$, $i = 1, 2, 3, 4, 5$ (9)

where M^i is a fuzzy term of rule i, i = 1, 2, 3, 4, 5; $\mathbf{A}_i \in \Re^{4\times4}$ and $\mathbf{B}_i \in \Re^{4\times2}$ are constant known system matrix and input vector respectively of the i-th rule sub-system. The system dynamics are described by,

$$\dot{\bar{\mathbf{x}}}(t) = \sum_{i=1}^{5} w_i(x_3(t))\big(\mathbf{A}_i\bar{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t)\big), \tag{10}$$

where $\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}_3 = \mathbf{A}_4 = \mathbf{A}_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$;

$\mathbf{B}_1 = \mathbf{B}_3 = -\mathbf{B}_5 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ R & 0 \\ 0 & R^2 \end{bmatrix}$, $\mathbf{B}_2 = -\mathbf{B}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & R^2 \\ -R & 0 \end{bmatrix}$;

$$\sum_{i=1}^{p} w_i(x_3(t)) = 1,\ w_i((x_3(t))) \in [0 \quad 1] \text{ for } i = 1, 2, 3, 4, 5. \tag{11}$$

$$w_i((x_3(t)) = \frac{\mu_{\mathrm{M}^i}(x_3(t)))}{\sum_{k=1}^{5}\mu_{\mathrm{M}^k}(x_3(t)))} \tag{12}$$

$\mu_{\mathrm{M}^i}(x_3(t)))$ is the membership functions of M^i, i =1, 2, 3, 4, 5. These membership functions are shown in Fig. 2. The fuzzy plant model can be obtained by using the method in [13].

B. Fuzzy Controller

A fuzzy controller having 5 fuzzy rules is used to control the WMR. The design will be based on the fuzzy plant model of (10) for the WMR of (8). The j-th rule of the fuzzy controller is of the following format,

Rule j: IF $x_3(t)$ is M^i THEN $\mathbf{u}(t) = \mathbf{G}_j(t)\bar{\mathbf{x}}(t) + \mathbf{r}_j(t)$, $j = 1, 2, 3, 4, 5$ (13)

where $\mathbf{G}_j(t) \in \Re^{2\times4}$, j = 1, 2, 3, 4, 5, are the constant feedback gains to be designed and $\mathbf{r}_j(t) \in \Re^{2\times1}$, j = 1, 2, 3, 4, 5, are the bounded reference inputs. The global output of the fuzzy controller is given by,

$$\mathbf{u}(t) = \sum_{j=1}^{5} w_j(x_3(t))\big(\mathbf{G}_j(t)\bar{\mathbf{x}}(t) + \mathbf{r}_j(t)\big) \tag{14}$$

C. Reference Fuzzy Model

A reference fuzzy model is a stable fuzzy system with rules given by,

Rule k: IF $x_3(t)$ is M^k

THEN $\dot{\hat{\mathbf{x}}}(t) = \mathbf{H}_{m_k}\hat{\mathbf{x}}(t) + \mathbf{B}_{m_k}\mathbf{u}(t)$, $k = 1, 2, 3, 4, 5$ (15)

Then the dynamics of the reference model is defined as follows,

$$\dot{\hat{\mathbf{x}}}(t) = \sum_{k=1}^{5} w_k(x_3(t))\big(\mathbf{H}_{m_k}\hat{\mathbf{x}}(t) + \mathbf{B}_{m_k}\mathbf{r}_m(t)\big) \tag{16}$$

where $\mathbf{H}_{m_k} \in \Re^{4\times4}$, $k = 1, 2, 3, 4, 5$, are constant matrices, $\mathbf{B}_{m_k} \in \Re^{4\times2}$, k = 1, 2, 3, 4, 5, are constant input vectors, $\hat{\mathbf{x}}(t) \in \Re^{4\times1}$ is the state vector of this reference model and $\mathbf{r}_m(t) \in \Re^{2\times1}$ is the bounded reference input. The stability of the reference fuzzy model can be checked by applying the stability analysis in [12-13]. (16) becomes a linear system when all $\mathbf{H}_{m_k}$, $k = 1, 2, 3, 4, 5$, are the same and all $\mathbf{B}_{m_k}$, k = 1, 2, 3, 4, 5, are the same. The reason of using this fuzzy model instead of a linear model as the reference is that it can give a better performance. For instance, the reference fuzzy model can be designed such that it has a faster response when $x_3(t)$ is close to its reference value.

IV. Design of Fuzzy Controller

In this section, we present the design of the fuzzy controller of (14) based on the fuzzy plant model of (10) such that the system states of the WMR will follow those of the reference model of (16). From (9), (16) and the property of (11) that $\sum_{i=1}^{p} w_i(x_3(t)) = 1$, we have,

$$
\begin{aligned}
\dot{\mathbf{e}}(t) &= \dot{\bar{\mathbf{x}}}(t) - \dot{\hat{\mathbf{x}}}(t) \\
&= \sum_{i=1}^{5} w_i(x_3(t))\big(\mathbf{A}_i\bar{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t)\big) \\
&\quad - \sum_{k=1}^{5} w_k(x_3(t))\big(\mathbf{H}_{m_k}\hat{\mathbf{x}}(t) + \mathbf{B}_{m_k}\mathbf{r}_m(t)\big) \\
&= \sum_{i=1}^{5} w_i(x_3(t))\big(\mathbf{A}_i\bar{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t) - \mathbf{H}_{m_i}\hat{\mathbf{x}}(t) - \mathbf{B}_{m_i}\mathbf{r}_m(t)\big)
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{i=1}^{5} w_i(x_3(t))\mathbf{H}_{m_k}\big(\bar{\mathbf{x}}(t) - \hat{\mathbf{x}}(t)\big) + \sum_{i=1}^{5} w_i(x_3(t))[\mathbf{A}_i\bar{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t) \\
&\quad - \mathbf{H}_{m_i}\hat{\mathbf{x}}(t) - \mathbf{B}_{m_i}\mathbf{r}_m(t) - \mathbf{H}_{m_i}\big(\bar{\mathbf{x}}(t) - \hat{\mathbf{x}}(t)\big)]
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{i=1}^{5} w_i(x_3(t))\mathbf{H}_{m_i}\mathbf{e}(t) \\
&\quad + \sum_{i=1}^{5} w_i(x_3(t))\big[\big(\mathbf{A}_i - \mathbf{H}_{m_i}\big)\bar{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t) - \mathbf{B}_{m_i}\mathbf{r}_m(t)\big]
\end{aligned} \tag{17}
$$

From (14), (17) and the property of (11) that $\sum_{i=1}^{p} w_i(x_3(t)) = 1$,

$$
\begin{aligned}
\dot{\mathbf{e}}(t) &= \sum_{i=1}^{5} w_i(x_3(t))\mathbf{H}_{m_i}\mathbf{e}(t) + \sum_{i=1}^{5} w_i(x_3(t))[\big(\mathbf{A}_i - \mathbf{H}_{m_i}\big)\bar{\mathbf{x}}(t) \\
&\quad + \mathbf{B}_i \sum_{j=1}^{5} w_j(x_3(t))\big(\mathbf{G}_j(t)\bar{\mathbf{x}}(t) + \mathbf{r}_j(t)\big) - \mathbf{B}_{m_i}\mathbf{r}_m(t)] \\
&= \sum_{i=1}^{5} w_i(x_3(t))\mathbf{H}_{m_i}\mathbf{e}(t) + \sum_{i=1}^{5} w_i(x_3(t))[\big(\mathbf{A}_i - \mathbf{H}_{m_i}\big)\bar{\mathbf{x}}(t) \\
&\quad + \left(\sum_{j=1}^{5} w_j(x_3(t))\mathbf{B}_j\right)\big(\mathbf{G}_i(t)\bar{\mathbf{x}}(t) + \mathbf{r}_i(t)\big) - \mathbf{B}_{m_i}\mathbf{r}_m(t)]
\end{aligned} \tag{18}
$$

From (18), letting,

$$\left(\sum_{j=1}^{5} w_j(x_3(t))\mathbf{B}_j\right)\mathbf{G}_i(t) = -\big(\mathbf{A}_i - \mathbf{H}_{m_i}\big) \tag{19}$$

$$\left(\sum_{j=1}^{5} w_j(x_3(t))\mathbf{B}_j\right)\mathbf{r}_i(t) = \mathbf{B}_{m_i}\mathbf{r}_m(t) \tag{20}$$

we have,

$$\dot{\mathbf{e}}(t) = \sum_{i=1}^{5} w_i(x_3(t))\mathbf{H}_{m_i}\mathbf{e}(t) \tag{21}$$

By using the results in [12-13], both the reference fuzzy model and the system of (21) are guaranteed to be stable if there exists a symmetric positive definite matrix $\mathbf{P} \in \Re^{4\times4}$ such that the following conditions are satisfied,

$\mathbf{H}_{m_i}{}^{\mathrm{T}}\mathbf{P} + \mathbf{PH}_{m_i}$ are negative definite, i = 1 2, 3, 4, 5. (22)

When (21) is stable, the designed fuzzy controller will drive the system states of the WRM to follow those of the reference fuzzy model. The results are summarized by the following lemma.

Lemma 1: *The fuzzy controller of* (14) *can drive the system states of the WMR to follow those of the reference fuzzy model of* (10) *if the following conditions are satisfied.*

(i). The feedback gains of the fuzzy controller are designed such that:

$$\left(\sum_{j=1}^{5} w_j(x_3(t))\mathbf{B}_j\right)\mathbf{G}_i(t) = -\big(\mathbf{A}_i - \mathbf{H}_{m_i}\big)$$

$$\left(\sum_{j=1}^{5} w_j(x_3(t))\mathbf{B}_j\right)\mathbf{r}_i(t) = \mathbf{B}_{m_i}\mathbf{r}_m(t)$$

(ii). There exists a symmetric positive definite matrix $\mathbf{P} \in \Re^{4\times4}$ *such that,*

$\mathbf{H}_{m_i}{}^{\mathrm{T}}\mathbf{P} + \mathbf{PH}_{m_i}$ *are negative definite, i = 1, 2, 3, 4, 5.*

V. Simulation Results

A fuzzy controller will be designed for the WMR according to Lemma 1. The designed fuzzy controller will then be applied to the WMR of (8) to close the feedback loop for obtaining the simulation results. The system matrices of the reference fuzzy plant model are chosen to be

$$\mathbf{H}_{m_1} = \mathbf{H}_{m_3} = \mathbf{H}_{m_5} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -3.4641 & 0 \\ 0 & -1 & 0 & -3.4641 \end{bmatrix},$$

$$\mathbf{H}_{m_2} = \mathbf{H}_{m_4} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -2.6458 & 0 \\ 0 & -1 & 0 & -2.6458 \end{bmatrix};$$

$\mathbf{B}_{m_1} = \mathbf{B}_{m_2} = \mathbf{B}_{m_3} = \mathbf{B}_{m_4} = \mathbf{B}_{m_5} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$. The feedback gains, $\mathbf{G}_j$, j = 1, 2, 3, 4, 5, are designed according to Lemma 1. We use

$\mathbf{P} = \begin{bmatrix} 32.4240 & 0 & 8.6697 & 0 \\ 0 & 32.4240 & 0 & 8.6697 \\ 8.6697 & 0 & 7.3073 & 0 \\ 0 & 8.6697 & 0 & 7.3073 \end{bmatrix}$ such that

$\mathbf{H}_{m_i}{}^{\mathrm{T}}\mathbf{P} + \mathbf{PH}_{m_i}$, i = 1 2, 3, 4, 5, are negative definite. Fig. 3 to Fig. 4 show the state responses of the WMR when the initial states $\mathbf{x}(0) = [1 \;\; 1 \;\; 2]^{\mathrm{T}}$ and $\mathbf{r}_m(t) = \left[2 + \frac{\sin(t/10)}{3} \quad 2 + \frac{\cos(t/10)}{3}\right]^{\mathrm{T}}$. The initial states of the reference model are $\hat{\mathbf{x}}(0) = [0 \;\; 0 \;\; 0 \;\; 0]^{\mathrm{T}}$.

VI. Conclusion

A fuzzy controller has been proposed to control a two-wheeled mobile robot. Design of the fuzzy controller based on the TS fuzzy plant model of the WMR has been presented. The proposed fuzzy controller has an ability to drive the system states of the WMR to follow those of a stable reference fuzzy model. Simulation results have been presented.

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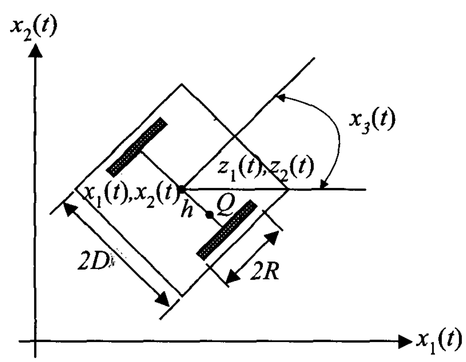

Fig. 1. Schematic of the two-wheeled mobile robot.

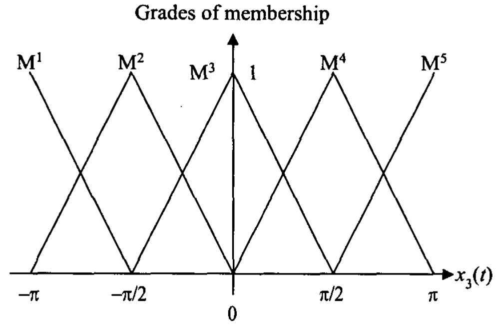

Fig. 2. Membership functions of the wheeled mobile robot.

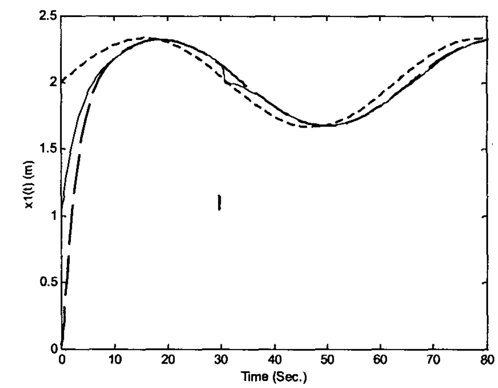

Fig. 3 Responses of $\bar{x}_1(t)$ (solid line) of the WMR with $\mathbf{x}(0)=[1\ \ 1\ \ 2]^T$, $\hat{x}_1(t)$ (dash line) with $\hat{\mathbf{x}}(0)=[0\ \ 0\ \ 0\ \ 0]^T$ under $\mathbf{r}_m(t)=\left[2+\frac{\sin(t/10)}{3}\ \ \ 2+\frac{\cos(t/10)}{3}\right]^T$ (dotted line).

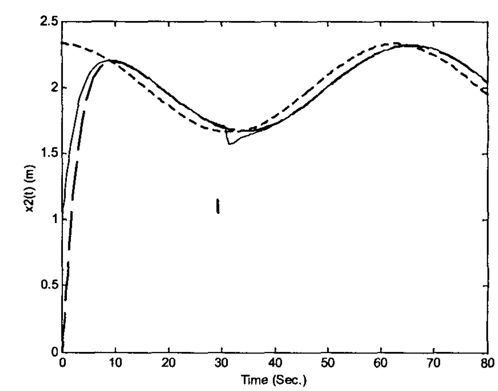

Fig. 4 Response of $\bar{x}_2(t)$ (solid line) of the WMR with $\mathbf{x}(0)=[1\ \ 1\ \ 2]^T$, $\hat{x}_2(t)$ (dash line) with $\hat{\mathbf{x}}(0)=[0\ \ 0\ \ 0\ \ 0]^T$ under $\mathbf{r}_m(t)=\left[2+\frac{\sin(t/10)}{3}\ \ \ 2+\frac{\cos(t/10)}{3}\right]^T$ (dotted line).