An Adaptive Digital Controller for Switching Dc-dc Converters

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Abstract: *The control of PWM-type switching dc-dc converters is reviewed. With the difficulties of controller design inherent from the non-linear nature of the converters properly addressed, an innovative adaptive digital controller is proposed. This adaptive controller will make use of a grid-point concept to characterize the operation space of the controlled dc-dc converter. In conjunction with a novel on-line parameter estimation scheme - the on-line simplex algorithm - and a robust control law, the objectives of adaptivity to wide operating range, disturbance rejection, stability and satisfactory transient responses are achieved. Simulation results on a published Ćuk converter are provided to illustrate the feasibility and performance of such an adaptive control scheme.*

I INTRODUCTION

With the sophistication of complicated electronic systems, the demands for lightweight and highly efficient dc power sources will become more and more severe. The switched-mode power supply (SMPS) promises to be a solution to these demands. During the past three decades, many different aspects of SMPSs have been explored. These aspects include the development of topologies,[1] the modelling and analysis,[2] the control[3-5] of switching dc-dc converters, etc.. Among them, the control of switching dc-dc converters has attracted much research interest due to the inherent difficulties in achieving the desired characteristics of SMPSs. These characteristics can be summarized into three control objectives as follows:

Objective 1: stiff line and load regulation (dc responses).

Objective 2: low output impedance and audio susceptibility (ac responses).

Objective 3: robustness to uncertainties in plant parameters.

By properly controlling the switching dc-dc converter, a zero steady-state error between the output voltage and the reference value can be reached even when sustained deviations from the standard values of line voltage and load current are present (Objective 1). Besides, satisfactory transient responses can be achieved when the system is subjected to perturbations which will bring about instantaneous variations in load current or line voltage (Objective 2). It should be noted that the model of the switching dc-dc converter is usually only a small-signal one. The acquisition of such a model often involves many approximations. Even when the model is accurate, it is an accurate representation of the converter for small signal consideration at one particular operating point only. When the switching dc-dc converter is subjected to significant disturbances, or the application of the SMPS demands a wide operating range, the operating point will not be fixed at one nominal position. So the modelling parameters of the plant (dc-dc converter) will always be subjected to uncertainties, and the controller so designed must be robust enough to accommodate these uncertainties (Objective 3).

In this paper, a general adaptive control scheme is proposed for controlling the PWM-type switching dc-dc converters. The main idea is to select a set of grid-points throughout the operation space of the converter, and each grid-point will be associated with a control law robust enough within a certain operation sub-space. This is akin to the partitioning approach of non-linear control.[6] By properly choosing the grid-points, the operation sub-spaces associated with them will cover the whole operation space concerned. Adaptivity can be achieved when the controller is capable of estimating the system parameters of the open-loop plant and properly controlling the system based on a predefined control law. The whole control problem will thus be divided into two parts: the determination of the grid-points and the derivation of the control law corresponding to each grid-point. The development and evaluation of such a control scheme are thus the main themes of this paper. In section II, the problem of control for switching dc-dc converters will be stated, and the proposed adaptive control scheme will be outlined. Then, the method of determining the grid-points, which involves a novel on-line parameter estimation algorithm, and the derivation of the control law(s) will be discussed in section III. In section IV, with a Ćuk converter as an example, simulation results will be presented to illustrate the feasibility of the proposed adaptive control scheme. The performance will be evaluated in accordance with the extent of achieving the objectives stated above. A conclusion to the whole paper will be given in section V.

II DESIGN CONSIDERATION

A. Problem of control

As far as the control is concerned, a switching dc-dc converter can be regarded as a single-input single-output (SISO) plant. The output is the output voltage v_o of the converter. The control input is the signal represented by the PWM duty ratio d, where d is the ratio of the power switch

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ON time to the switching period. The converter parameters are affected by the input line voltage v_{in} and output load R_L, which in conjunction with other minor factors such as EMI and stray effects constitute the external disturbances to the system. The steady-state values of d, v_o, v_{in} and R_L constitute the operating point of the switching dc-dc converter. When there are sustained deviations in the steady-state values of d, v_o, v_{in} and R_L, the switching dc-dc converter will be described as having multiple operating points. In accordance with the application of the SMPS so designed, the range of operating points of the dc-dc converters should be specified, which will in turn define a space of parameters describing the plant. Such a space of parameters describing the plant is termed the operation space of the switching dc-dc converter.

When the switching dc-dc converters are modelled and analyzed, it is found that they are highly non-linear systems. The non-linearities can be generalized into the following three groups:

1) topology changes due to, perhaps, high temperature or component failure,

2) non-linear characteristics of the electronic switches (fast dynamics), and

3) non-linear parameter variations within the operation space of the converter due to external 'instantaneous' and/or sustained disturbances (slow dynamics).

In order to tackle these non-linearities, most of the previous work concerning the control of switching dc-dc converter adopted the following approach.

Approach 1: Assuming that the variations in line voltage and load current are infrequent and small, and other disturbances are insignificant, the switching dc-dc converter can be regarded as having only one operating point subjected to small perturbations. The control law can then be designed based on a small-signal linearized model of the converter.

When the specifications of the SMPS are not so demanding such that the assumptions stated remain to be valid, *Approach 1* promises to be a simple and inexpensive way of achieving the control objectives as discussed in section I. However, when the validities of the assumptions are in doubt, *Approach 1* may no longer be applicable. In view of these weaknesses, a more general approach for controlling the switching dc-dc converter is proposed as follows:

Approach 2: Design a high-quality adaptive controller that is capable of adapting to significant non-linearities and catering to multi-operating point situations.

Under *Approach 2*, the various aspects of adaptive control have to be considered. It should be noted that +despite the extensive research on the topic, no complete theory of adaptive control has been established yet. Major contributions on sub-topics like model reference adaptive system (MRAS) and self-tuning regulator (STR)[7] remain to be mainly on the theoretical and academic basis. The applicability of these theories depends very much on the satisfaction of various constraints, e.g. the availability of a proper model representation (deterministic or stochastic) of the plant. These constraints may introduce adverse effects on the successful applications to plants with high uncertainties in parameters, such as the switching dc-dc converters.

B. *Adaptive control scheme*

In this paper, an innovative adaptive control scheme is proposed as shown in Fig.1. This scheme may be regarded as an extension of *Approach 1*. Under such a control scheme, the whole operation space of system parameters is characterized by a set of pre-defined grid-points. A direct search optimization technique, known as the *on-line simplex algorithm*, is applied to minimize the errors between the actual output and the estimated output in order to reach the optimal estimation of the model parameters. However, the feedback control law is not directly dependent upon the values of the estimated parameters, but is dependent on the chosen 'optimal' grid-point (i.e. the best estimated model parameters within a pre-defined set). Each grid-point will be associated with a control law derived under *Approach 1* which is robust enough within an operation sub-space of model parameters. In this way, the desired adaptivity can be achieved if (1) the parameter estimation scheme is capable of determining the best grid-point, and (2) the control law associated with each grid-point is so robust that the union of the operation sub-spaces corresponding to all grid-points will cover the whole operation space of the system model parameters.

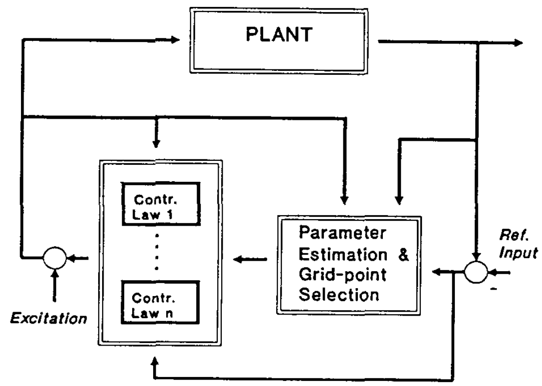

Fig. 1: A block diagram for the proposed adaptive control scheme.

It can be seen that this proposed adaptive control scheme possesses the following three advantages when compared with other common adaptive control schemes:

1) The proposed on-line simplex algorithm for system parameter estimation does not demand for a well-understood actual system model representation (e.g. the modelling of noise).

2) The parameter estimation scheme is not used for determining the accurate model parameters, but only for a mild objective of determining the best grid-point.

3) The estimated system parameters are not used directly to derive the final control law. So the convergence of the adaptive control scheme depends only on the convergence of the parameter estimation algorithm for determining the correct grid-point. The problem of convergence will thus be simplified and many constraints inherent in common adaptive schemes can be removed.

III THE ADAPTIVE CONTROLLER

With reference to the discussion about the proposed adaptive control scheme in the previous section, it can be seen that the design of the adaptive controller can be divided into the following two aspects:

1) the location of the grid-points within the operation space and the design of the control laws associated with the grid-point;

2) the algorithm for the on-line system parameter estimation to determine the best grid-point describing the plant.

These two aspects will be discussed more deeply in this section.

A. *Grid-point and the associated control law*

Since the operation space of the system parameters is characterized by a set of grid-points, one question that will be asked naturally is on how to determine the locations of the grid-points. From the theoretical point of view, a possible solution is to perform sensitivity analysis, investigating theoretically the effects of the parameter (mathematical or physical) variations on the system behaviour. The information thus obtained may help to define the locations of the grid-points. This method, though applicable, may be too complicated and exhaustive. It should be repeated that the major concern for defining the locations of the grid-points is to ensure that the control law associated with each grid-point is robust enough that the union of the operation sub-space is equal to the complete operation space of the dc-dc converter. So a heuristic and experimental approach can be used for defining the locations of the grid-points. By this approach, the first grid-point is defined based on the standard operating conditions of the dc-dc converter (or arbitrarily if no standard operating conditions are present). A robust control law is designed based on this grid-point. Experiments are then performed so as to investigate the extent of the operation space in which the control law remains to be valid. The information from the experiments is then used for the location of the next grid-point. The procedure is repeated until the whole operating range of the SMPS has been covered.

For the determination of the control law at each grid-point, the problem is reduced to that using *Approach 1* as mentioned in the last section. The problems involved in the design and the general design procedures for such a feedback control law have been discussed in another paper by the same authors[8] and will not be detailed here. In summary, the design is based on the theory of linear optimal control and involves state-space techniques and state-estimator design. Emphasis is placed on the robustness of the control law towards the uncertainties in plant parameters.

B. *On-line simplex algorithm*

During the past two decades, rapid expansions had been found in the use of various optimization techniques for system parameter identification. The well-known algorithms such as LMS, ELS and RML[9], which are widely adopted in the field of adaptive control, can be classified as particular cases of one type of optimization techniques known as the gradient methods. One significant drawback of the gradient methods is that they require the determination of partial derivatives of the performance index, which is sometimes difficult to achieve.[10] The particular algorithms mentioned early help to relieve this drawback by computing the partial derivatives indirectly based on a well-defined model structure for both the plant and the noise. As discussed in the previous section however, these algorithms are too theoretically based which will introduce many limitations on their successful applications.

An alternative method for system parameter estimation is proposed in this paper. This method does not require the determination of the partial derivatives of the performance index and a well-defined system model structure. Instead, the method is developed in a more application oriented direction - by a direct searching for the optimal parameters of an assumed model (which is not necessarily well-defined) such that the errors between the actual system outputs and the outputs based on the estimated parameters can be reduced to as small as possible. Since the objective is to determine the best grid-point representing the actual system, the values of the parameters are not significant. As long as the parameter estimation scheme converges, the optimal grid-point can be readily determined.

The proposed method for system parameter estimation is called the *on-line simplex algorithm*. It is developed from its off-line counter-part originally proposed by Nelder and Mead.[11] By its name, a simplex in an n-dimensional Euclidean space is a polyhedron defined by a set of (n+1) vertex points. When the (n+1) points are mutually equidistant, the simplex is regular. This method can be applied to the minimization of a function of n variables (the performance index), which depends on the comparisons of function values at the (n+1) vertices of a general simplex. It is then followed by the replacement of the vertex with the highest value by another point. In this way, the simplex adapts itself to the local landscape, and contracts on to the final minimum.

The on-line simplex algorithm, which is developed from the original off-line simplex method, is capable of performing real-time recursive parameter estimation. The main idea of the algorithm is to divide a large batch of input/output data (essentially having an infinite number of data items for a typical real-time application) into data slots each consisting of a fixed number of data items. For each data slot, the off-line simplex algorithm is executed until either the convergence criterion has been met or the number of iterations has reached a pre-defined maximum number. The necessary information is then stored and passed to the next data slot, which is formed by discarding the oldest data item and absorbing the most updated data, for improved convergence and estimation. The details about the algorithm have been discussed in another paper by the same authors[10] and a summary of it can be represented by the flow diagram as shown in Fig.2. However, before ending this section, several points concerning this on-line simplex algorithm have to be explained as follows:

1) The simplex algorithm is especially recommended for solving optimization problems with high dimensionality (or order of the system).[12]

2) The algorithm is robust when compared with other direct optimization techniques, i.e. the convergence ability is satisfactory even though only moderately reasonable starting values are used. However, as discussed previously, the success of the adaptive control scheme depends on the determination of the 'optimal' grid-point. The convergence of the parameter estimation scheme is not a sufficient condition for successful control. The reason is that the estimated system parameters may converge to a local minimum of the performance index only, rather than a global minimum. In order to ensure the convergence to the global minimum, parallel processing using a multi-processor system is proposed such that each processor performs the parameter estimation around each grid-point with different initial conditions. In this way, one of the minima around the grid-points should be the global minimum. Such a proposed parallel processing method can enhance the identifiability of the system, but it has the drawbacks of high cost and complexity.

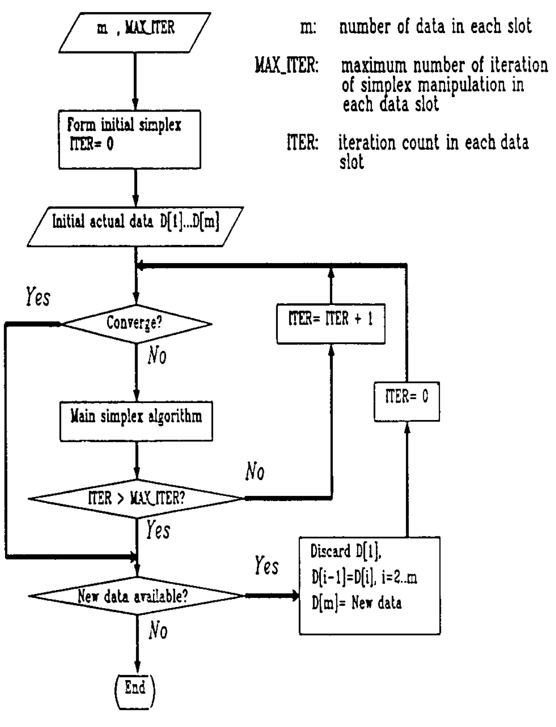

Fig. 2: A flow diagram for the on-line simplex algorithm.

3) It should be noted that the parameter estimation module need not be executed at all time. When the system is under control, the adaptive controller can be reduced to a fixed-parameter controller. The parameter estimation should be triggered when a large error between the output voltage and the reference value is detected. Also, for the sake of identifiability, external persistent excitation should be added to the system in estimating the parameters.[13] The amplitude of the excitation should be such that the ripples in the output caused by it are within the specifications allowed.

IV APPLICATION EXAMPLE

In this section, a dc-dc converter is used as the plant to illustrate how the control strategy discussed in this paper can be carried out. This plant is a Ćuk converter which, at the standard operating point, can be represented by the circuit as shown in Fig.3. It is assumed that the specifications require a load variation from $R_L = 10\Omega$ to 35Ω. Also, it is found that when the load changes from 30Ω to 32Ω or higher, two open-loop zeroes will be shifted outside the unit circle in the z-plane, implying significant changes of the system parameters. On the other hand, when the load decreases towards the value of 10Ω, the deviations of system parameters from the standard operating point will become more and more significant. The control strategy for each grid-point can be represented by the block diagram as shown in Fig.4. It is then logical to define at least three grid-points G1, G2 and G3 corresponding to the models with load values of 30Ω, 10Ω and 35Ω respectively. As discussed in the previous section, the control laws designed in accordance with the models of each grid-point should be sufficiently robust such that a reasonably large parameter sub-space around the grid-point can be found. Inside this sub-space, the control law associated with the grid-point can bring about stable and satisfactory control of the closed-loop system. Also, the union of the parameter sub-spaces corresponding to all the grid-points should cover the whole operation space of the Ćuk converter. (If this is not the case, more grid-points have to be added).

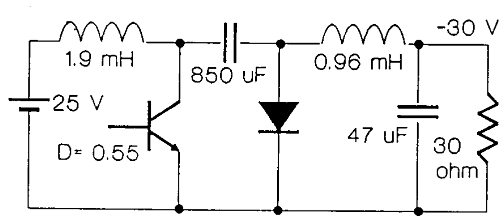

Fig. 3: An example Ćuk converter circuit at its standard operating point.

To illustrate the feasibility of the proposed adaptive control scheme, the approximate small-signal linear model of each grid-point is firstly found and the corresponding robust feedback control law designed. The complete control scheme as shown in Fig.1 is then subjected to computer simulation with a unit reference input. The parameter estimation scheme is triggered when the output of the plant is outside the tolerance band of 0.85 to 1.35. After the parameter estimation has been started, persistent excitations formed by a pseudo random binary sequence (PRBS) ranging from 0 to 0.0001 is added to the input of the plant. The performance index to be minimized in the on-line simplex algorithm is given by:

$$J = \sum_{k=1}^{N} \beta^{N-k+1} [y(k) - \hat{y}(k)]^2$$

where, N is the number of data items in each data slot, β is a time weighting factor less than unity such that the more recent data will have a higher weight in the performance index, y is the actual output of the plant, and $\hat{y}$ is the output computed based on the estimated model. In this example, N=50 and β=0.9990. When the plant output has been kept inside the tolerance band of 0.85 to 1.35 for a duration of 10ms or more, the system is assumed to be properly controlled and the optimal grid-point has been successfully identified. The parameter estimation scheme will then be turned off.

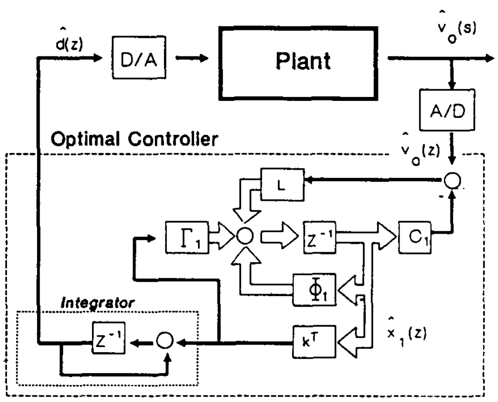

Fig. 4: *A block diagram showing the robust feedback control scheme for each grid-point.*

The transient response of the plant output when the load of the Ćuk converter is changed from 30Ω to 35Ω (G1 to G3) at time= 100ms is shown in Fig.5. The same response for a load change from 35Ω to 10Ω (G3 to G2) is shown in Fig.6. From these figures, it can be seen that after the changing of the operating point which causes large output errors, the adaptive control scheme is capable of identifying the optimal grid-point and changing the control law to bring the system back to proper control. With reference to the control objectives discussed in section II, it can be seen that satisfactory performance can be achieved.

V CONCLUSION

The problem of controlling PWM-type switching dc-dc converters with significant parameter uncertainties has been investigated. It is found that due to the inherent non-linearities of the converters, the usual approach based on an approximate small-signal linear model around a single operating point may not be adequate for satisfactory control of SMPSs with demanding applications. The problem is tackled with a more general approach by extending the original idea of single operating point to multiple operating points, characterized by a set of grid-points. The grid-point best describing the operation of the SMPS at a time is estimated by using an innovative parameter estimation scheme. A robust control law will be designed for each grid-point such that after the estimation of the optimal grid-point, the associated control law can be applied to control the switching dc-dc converter. Simulation results of an application example have been presented to illustrate the feasibility of such a control scheme.

As a highlight for further research, it must be admitted that the materials in this paper provide only an idea for tackling the problem of control to switching dc-dc converters. When the idea is considered for implementation, many problems remain to be unsolved. One major problem will be the limitation of computation speed. The reason is that the on-line simplex algorithm is quite computationally demanding and is not expected to be executed within a sample period of a digitally controlled switching dc-dc converter using reasonably priced DSPs. It is hoped that with the advance of technology in fast computation, this problem can be solved in the coming years.

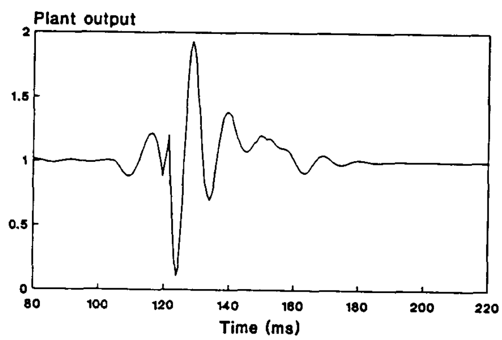

Fig. 5: *Transient response of the plant output when the load changes from 30Ω to 35Ω (G1 to G3) at time= 100ms.*

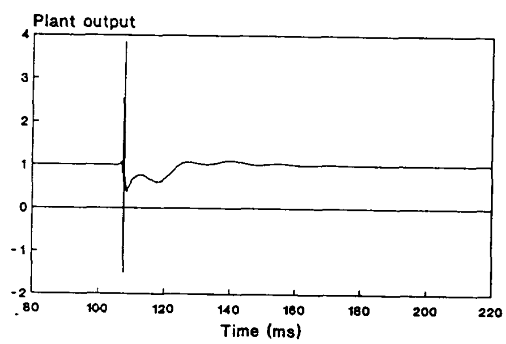

Fig. 6: *Transient response of the plant output when the load changes from 35Ω to 10Ω (G3 to G2) at time= 100ms.*

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