Stability Analysis of Systems with Non-symmetric Dead Zone under Fuzzy Logic Control

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Abstract – Many existing stability analysis methods for fuzzy logic control systems like TS fuzzy model based methods usually tackle plants that are linear with respect to control. However, these methods may be difficult to apply to plants with dead zones. This paper presents an improved stability analysis method to resolve this difficulty. Our proposed method employs a Lyapunov function to prove the stability of non-linear fuzzy logic control systems with asymmetric dead zone and saturation. An illustrative example will be given to demonstrate the ability of the method.

I. INTRODUCTION

Non-linearities in actuators like dead zone and saturation are common in industrial control systems. Applying linear controllers or compensators to such systems may give limit cycles [1]. The non-linearity caused by a simple dead zone (Fig. 1a) can affect the steady-state performance of closed-loop systems. If a system involves non-asymmetric dead zones (Fig. 1b), and the function outside the dead zone is not a linear function (Fig. 1c), the system becomes more non-linear, and its control becomes a challenging task. Recently, Kim *et al* have developed a two-layered fuzzy logic controller (FLC) for systems with a dead zone [4]. The controller consists of a fuzzy logic based pre-compensator followed by a usual PD-type FLC. It has been demonstrated that the controller gives good transient and steady-state performance, but the system stability has not been proved. Other dead zone compensation techniques by inserting an inverse to cancel out the dead zone effects have been developed. These techniques results in adaptive FLCs [2-3] and neural networks [1]. Beside compensating the dead zones adaptively, a robust FLC can be applied. Such a robust FLC need not be adaptively tuned so its structure is simpler. The implementation and the stability analysis are also easy to realize.

System stability is one major concern on applying FLCs. Common approaches to tackle the stability problem of fuzzy logic control systems include those that are based on TS fuzzy plant models [6-12]. With the help of this TS model, an FLC is designed. Linear state feedback techniques are usually used in these designs. For example, Cao *et al* [6-8] developed a design method based on many sub-systems of the plant. Tanaka *et al* [9-10] applied LMI techniques to find a common Lyapunov function to ensure the system stability. TS fuzzy models work well for plants that are linear with respect to control [13]: a plant, represented by $\dot{x} = f(x) + b(x)g(u)$, has a restriction that $g(u)$ is linear. However, they are not suitable for plants with a dead zone and saturation. It is because a plant with a non-linear $g(u)$ may lead to a premise depending on u (in addition to the system state x) in the fuzzy control rules. Then the design of the FLC and the stability analysis may become very difficult.

Recently, we have proposed a stability analysis method which employs Lyapunov direct method for fuzzy logic control systems without fuzzy plant models [14-15]. The proposed method handles every fuzzy sub-system (that will be defined in section 2.1) individually instead of the system as a whole in order to avoid dealing with the complex non-linear control signal generated by the FLC. We have further improved this stability analysis method [16-17] so that it is not restricted to plants that are linear with respect to control [13]. If $g(u)$ is a non-linear function of u, and is either a monotonic increasing or decreasing function, our proposed stability analysis method can be applied. Plants with dead zones can be represented by a non-linear $g(u)$.

In Section II of this paper, the fuzzy logic control system considered in this paper will be introduced. Some definitions and useful properties will be stated. The proposed stability analysis method will be discussed in Section III. In Section IV, a plant with nonsymmetric dead zone and saturation (Fig. 1c) will be controlled by a simple FLC. The stability of the closed-loop system will be proved using the proposed stability analysis method. It will be shown that both the FLC and the stability proof are simple. Finally a conclusion will be drawn in Section V.

II. FUZZY LOGIC CONTROL SYSTEMS

A fuzzy logic control system comprising a plant and an FLC is shown in Fig. 2. The FLC consists of r fuzzy rules. Each fuzzy rule generates an output u_i and a degree of membership $\mu_i \in [0, 1]$, $i = 1, 2, \ldots, r$. The overall control signal applying to the plant is a function of u_i and μ_i. Let the plant be of single input, n-th order and non-linear type, which can be described by the following equation:

$$\dot{x} = f(x) + b(x)g(u) \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T$ is a state-vector, $f(x) = [f_1(x), f_2(x), \ldots, f_n(x)]^T$ and $b(x) = [b_1(x), b_2(x), \ldots, b_n(x)]^T$ are vectors describing the dynamics of the plant, $g(u)$ is a linear or non-linear function of u, and u is a control signal generated by an FLC. The i-th fuzzy rule of the FLC is of the following form:

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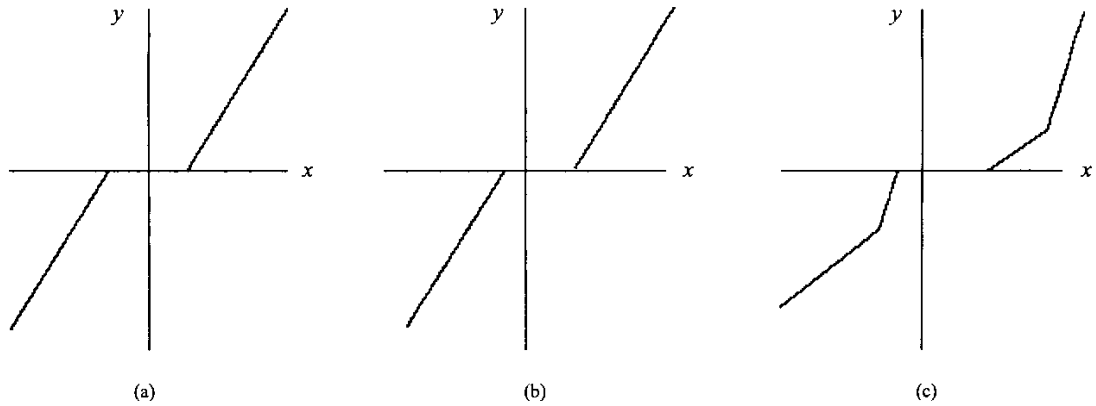

Fig. 1.(a) Simple dead zone, (b) different slopes, (c) asymmetric type.

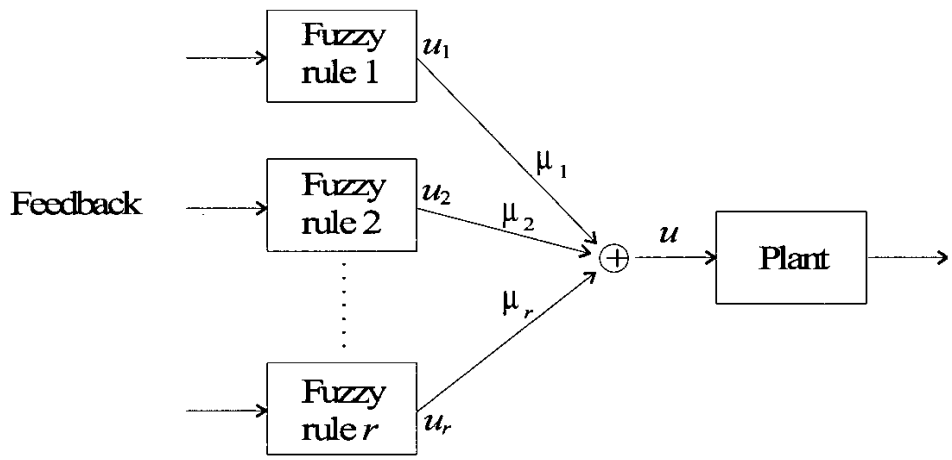

Fig. 2. A general fuzzy logic control system

Rule i: **IF** *<premise i>* **THEN** $u = u_i$ (2)

where *<premise i>* is the premise of fuzzy rule i with an input variable vector $\boldsymbol{x}$. The control u_i can be a constant or a function, either linear or non-linear, of $\boldsymbol{x}$. It is assumed that for any input of the FLC, there exists at least one fuzzy rule giving a non-zero μ_i. The membership functions of the output fuzzy sets are singletons. The center of gravity defuzzification method is applied and the control signal of the FLC is given by:

$$u = \frac{\sum_{i=1}^{r} \mu_i u_i}{\sum_{i=1}^{r} \mu_i}. \tag{3}$$

2.1 Some definitions and properties

We shall first define some terms and highlight some useful properties in this sub-section.

Definition 1: For any input $\boldsymbol{x}_o \in \boldsymbol{X}$, which is the input universe of discourse, if the degree of membership $\mu_i(\boldsymbol{x}_o)$ of fuzzy rule i is zero, this fuzzy rule is called an *inactive fuzzy rule for* $\boldsymbol{x}_o$; otherwise, it is called an *active fuzzy rule for* $\boldsymbol{x}_o$. Note that we describe a fuzzy rule is active or inactive for only one input vector $\boldsymbol{x}_o$, but not for all input values. An *active region* of a fuzzy rule i is defined as the region $\boldsymbol{X}_r \subset \boldsymbol{X}$ such that the fuzzy rule i is active for any $\boldsymbol{x} \in \boldsymbol{X}_r$.

It should be noted that an inactive fuzzy rule for $\boldsymbol{x}_o$ will not affect the controller output u (for this input). By letting $\boldsymbol{I}_A$ be the set containing the rule numbers of the active fuzzy rules for $\boldsymbol{x}_o$ ($\boldsymbol{I}_A$ is different for different $\boldsymbol{x}_o$), (3) can be rewritten so as to consider only the active fuzzy rules for $\boldsymbol{x}_o$ as follows:

$$u = \frac{\sum_{i \in I_A} \mu_i u_i}{\sum_{i \in I_A} \mu_i}. \tag{4}$$

Property 1: For any input $\boldsymbol{x}_o \in \boldsymbol{X}$, there exists p, $q \in \boldsymbol{I}_A$ such that $u_p \le u_i \le u_q$ for all $i \in \boldsymbol{I}_A$.

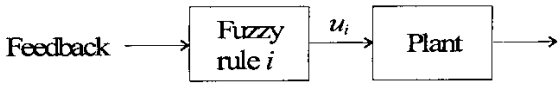

Fig. 3. A fuzzy sub-system

Definition 2: A *fuzzy sub-system* (Fig. 3) associated with fuzzy rule i is a system with a plant of (1) controlled by only u_i, which is the output of the fuzzy rule i in the form of (2).

III. STABILITY ANALYSIS METHOD

One major difficulty in analysing the stability of the whole fuzzy logic control system is that we have to tackle a very complex non-linear control signal u. Fig. 2 shows that u is a function of u_i and μ_i for $i \in [1, r]$. Moreover, the μ_i's are functions of $\boldsymbol{x}$. The idea of the proposed stability analysis method is to break down the problem of analysing the stability of the whole fuzzy logic control system into analysing every fuzzy sub-system (as defined in *Definition 2*) individually. In this case, only one u_i instead of many u_i's has to be considered; and the highly complex non-linear function μ_i is not involved. Hence the complexity of the stability analysis is drastically decreased. Here, the plant is not restricted to be linear with respect to control. The proposed stability analysis method requires $g(u)$ to be a monotonic increasing or decreasing function only. The detail about the stability analysis method is given in the following theorem.

Theorem 1: If

M1: $\boldsymbol{P}$ is a quadratic, symmetric and positive definite matrix, and $V = \boldsymbol{x}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{x} \to \infty$ as $\|\boldsymbol{x}\| \to \infty$;

M2: $\dot{V}$ is negative definite in every fuzzy sub-system's active region;

M3: $g(u)$ is either a monotonic increasing or a monotonic decreasing function;

M4: the defuzzification method of (3) is applied,

then the equilibrium point at the origin is globally asymptotically stable.

Proof:

From M1,

$$V = \boldsymbol{x}^T \boldsymbol{P} \boldsymbol{x} \tag{5}$$

$$\dot{V} = \dot{\boldsymbol{x}}^T \boldsymbol{P} \boldsymbol{x} + \boldsymbol{x}^T \boldsymbol{P} \dot{\boldsymbol{x}}. \tag{6}$$

From (1),

$$\dot{V} = (\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{b}(\boldsymbol{x}) g(u))^T \boldsymbol{P} \boldsymbol{x} + \boldsymbol{x}^T \boldsymbol{P} (\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{b}(\boldsymbol{x}) g(u)).$$

Let
$$F = \boldsymbol{f}^T(\boldsymbol{x}) \boldsymbol{P} \boldsymbol{x} + \boldsymbol{x}^T \boldsymbol{P} \boldsymbol{f}(\boldsymbol{x}), \tag{7}$$

$$B = \boldsymbol{b}(\boldsymbol{x})^T \boldsymbol{P} \boldsymbol{x} + \boldsymbol{x}^T \boldsymbol{P} \boldsymbol{b}(\boldsymbol{x}), \tag{8}$$

then
$$\dot{V} = F + Bg(u).$$

From M2 and *Property 1*,

$$\dot{V}\Big|_{u=u_q} = F + Bg(u_q) \tag{9}$$

is negative definite, and

$$\dot{V}\Big|_{u=u_p} = F + Bg(u_p) \tag{10}$$

is also negative definite. On the other hand, from M4, *Property 1*, and (4), we have

$$\frac{\sum_{i \in I_A} \mu_i u_p}{\sum_{i \in I_A} \mu_i} \le \frac{\sum_{i \in I_A} \mu_i u_i}{\sum_{i \in I_A} \mu_i} \le \frac{\sum_{i \in I_A} \mu_i u_q}{\sum_{i \in I_A} \mu_i}$$

$$\Rightarrow u_p \frac{\sum_{i \in I_A} \mu_i}{\sum_{i \in I_A} \mu_i} \le u \le u_q \frac{\sum_{i \in I_A} \mu_i}{\sum_{i \in I_A} \mu_i}$$

$$\Rightarrow u_p \le u \le u_q, \text{ equality holds when } u_i = u_p = u_q. \tag{11}$$

Hence, (11) shows that the control signal from the FLC lies between u_p and u_q (note that u_p and u_q vary according to FLC input, i.e. $\boldsymbol{x}$) if M4 is satisfied.

The system stability can be proved by employing the Lyapunov direct method. From M1, V is a positive definite function of $\boldsymbol{x}$ and $V \to \infty$ as $\|\boldsymbol{x}\| \to \infty$. If we can prove that $\dot{V}$ is negative definite for the whole state-space (M2 only assures that $\dot{V}$ is negative definite for only some regions), then the system can be proved as stable by the Lyapunov theorem. The following proof will show that the conditions M2 to M4 can guarantee that $\dot{V}$ is negative definite for the whole state-space.

Five cases will be considered. Case 1 and Case 2 consider a monotonic increasing $g(u)$. Case 3 and Case 4 consider a monotonic decreasing $g(u)$. All these four cases assume that $\boldsymbol{x} \neq \boldsymbol{0}$. The situation $\boldsymbol{x} = \boldsymbol{0}$ will be considered in Case 5. If $\dot{V}$ is negative definite in all these five cases, $\dot{V}$ is negative definite for the whole state-space.

Case 1: If $g(u)$ is monotonic increasing, $B > 0$, $\boldsymbol{x} \neq \boldsymbol{0}$

Let
$$\dot{V}_{q1} = F + Bg(u_q).$$

It should be noted that by M2 and (9), $\dot{V}_{q1}$ is negative definite. Now, $g(u)$ is monotonic increasing, $g(u_q) \ge g(u)\ \forall\ u \le u_q$. Consequently,

$$\dot{V} = F + Bg(u) \le \dot{V}_{q1} < 0 \ \ \forall\ u \le u_q. \tag{12}$$

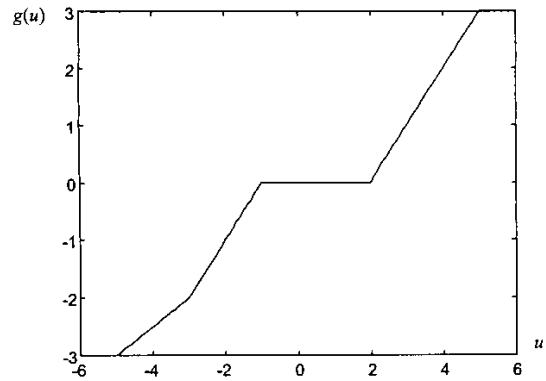

Fig. 4. Function of $g(u)$

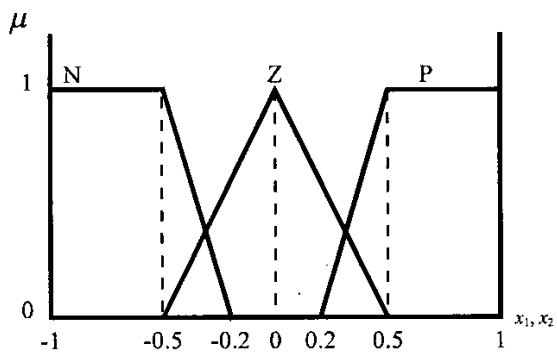

Fig. 5. Membership functions of x_1 and x_2

Rule	Antecedent		Consequent
	x_1	x_2	u
1	P	P	−5
2	N	N	5
3	Z	P	−3
4	Z	N	3
5	P	Z	−1
6	N	Z	1
7	P	N	0
8	N	P	0
9	Z	Z	0

Table 1. Fuzzy rule base

Case 2: If $g(u)$ is monotonic increasing, $B \le 0$, $\boldsymbol{x} \ne \mathbf{0}$

Let $\dot{V}_{p1} = F + Bg(u_p)$

which is negative definite from M2 and (10). Now, $g(u)$ is monotonic increasing, $g(u_p) \le g(u)$ $\forall$ $u \ge u_p$. Consequently,

$$\dot{V} = F + Bg(u) \le \dot{V}_{p1} < 0 \ \ \forall\, u \ge u_p. \tag{13}$$

Case 3: If $g(u)$ is monotonic decreasing, $B > 0$, $\boldsymbol{x} \ne \mathbf{0}$

Let $\dot{V}_{p2} = F + Bg(u_p)$

which is negative definite from M2 and (10). Now, $g(u)$ is monotonic decreasing, $g(u_p) \ge g(u)$ $\forall$ $u \ge u_p$. Consequently,

$$\dot{V} = F + Bg(u) \le \dot{V}_{p2} < 0 \ \ \forall\, u \ge u_p. \tag{14}$$

Case 4: If $g(u)$ is monotonic decreasing, $B \le 0$, $\boldsymbol{x} \ne \mathbf{0}$

Let $\dot{V}_{q2} = F + Bg(u_q)$

which is negative definite from M2 and (9). Now, $g(u)$ is monotonic decreasing, $g(u_q) \le g(u)$ $\forall$ $u \le u_q$. Consequently,

$$\dot{V} = F + Bg(u) \le \dot{V}_{q2} < 0 \ \ \forall\, u \le u_q. \tag{15}$$

Case 5: If $\boldsymbol{x} = \mathbf{0}$

From (7) and (8), $F = B = 0$ if $\boldsymbol{x} = \mathbf{0}$ irrespective of the values of $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{b}(\boldsymbol{x})$. Therefore

$$\dot{V} = 0 \tag{16}$$

From M4, we know that $u_i \in [u_p, u_q]$, $i \in I_A$. Then from *Case 1* to *Case 5*, (12) to (16), we have:

$$\dot{V} \begin{cases} < 0 \text{ if } \boldsymbol{x} \ne 0 \\ = 0 \text{ if } \boldsymbol{x} = 0 \end{cases} \tag{17}$$

which means that that $\dot{V}$ is negative definite. By the Lyapunov theorem, *Theorem 1* is proved.

QED

IV. ILLUSTRATIVE EXAMPLE

Consider a plant of the following form:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} g(u) \tag{18}$$

where $g(u)$ is a function with non-symmetric dead zone, and the function outside the dead zone is piecewise linear with different slopes and saturation as shown in Fig. 4. An FLC with fuzzy rules described by Table 1 and membership functions shown in Fig. 5 are applied to control the plant, and the defuzzification method of (3) is used. The proof of system stability on applying the method proposed in Section III is given as follows:

Proof:

Let $\boldsymbol{x} = [x_1 \ \ x_2]^T$. We select a quadratic and positive definite Lyapunov function as follows:

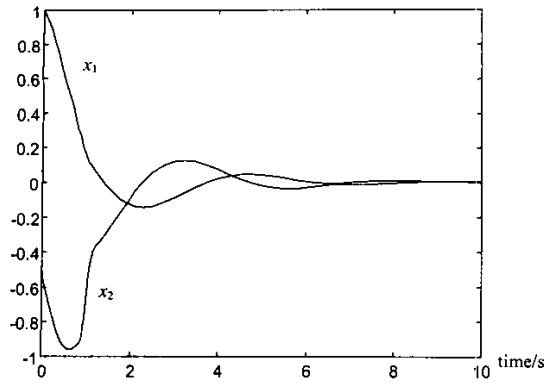

Fig. 6. Responses of the system with non-linear in control

$$V = \frac{1}{2}\boldsymbol{x}^T\begin{bmatrix}100 & 2\\ 2 & 50\end{bmatrix}\boldsymbol{x} \qquad (19)$$
$$= 50x_1^2 + 25x_2^2 + 2x_1x_2.$$

Note that $V(\boldsymbol{x}) \to \infty$ as $\|\boldsymbol{x}\| \to \infty$. The first derivative of V is given by

$$\dot{V} = 50x_1\dot{x}_1 + 25x_2\dot{x}_2 + 2\dot{x}_1x_2 + 2x_1\dot{x}_2. \qquad (20)$$

From (18),

$$\begin{aligned}\dot{V} &= 50x_1x_2 + 25x_2[-2x_1 - x_2 + 2g(u)]\\ &\quad + 2x_2^2 + 2x_1[-2x_1 - x_2 + 2g(u)]\\ &= 50x_1x_2 - 50x_1x_2 - 25x_2^2 + 50x_2g(u)\\ &\quad + 2x_2^2 - 4x_1^2 - 2x_1x_2 + 4x_1g(u)\\ &= -4x_1^2 - 23x_2^2 - 2x_1x_2 + 4x_1g(u) + 50x_2g(u).\end{aligned} \qquad (21)$$

From rule 1 of Table 1, x_1 is P, x_2 is P, $u = -5$. This implies that $x_1 \in [0.2, 1]$, $x_2 \in [0.2, 1]$ and $g(u) = -3$. From (21),

$$\begin{aligned}\dot{V} &= -4x_1^2 - 23x_2^2 - 2x_1x_2 + 4x_1g(u) + 50x_2g(u)\\ &= -4x_1^2 - 23x_2^2 - 2x_1x_2 - 12x_1 - 150x_2\\ &< 0\end{aligned}$$

since x_1 and x_2 are positive.

For rule 2, x_1 is N, x_2 is N, $u = 5$. This implies that $x_1 \in [-1, -0.2]$, $x_2 \in [-1, -0.2]$ and $g(u) = 3$. From (21),

$$\begin{aligned}\dot{V} &= -4x_1^2 - 23x_2^2 - 2x_1x_2 + 4x_1g(u) + 50x_2g(u)\\ &= -4x_1^2 - 23x_2^2 - 2x_1x_2 + 12x_1 + 150x_2\\ &< 0\end{aligned}$$

since x_1 and x_2 are negative.

For rule 3, x_1 is Z, x_2 is P, $u = -3$. This implies that $x_1 \in [-0.5, 0.5]$, $x_2 \in [0.2, 1]$ and $g(u) = -2$. From (21),

$$\begin{aligned}\dot{V} &= -4x_1^2 - 23x_2^2 - 2x_1x_2 - 8x_1 - 100x_2\\ &\le -4x_1^2 - 23x_2^2 + 1 + 4 - 100x_2\\ &< 0\end{aligned}$$

since $\max(5-100x_2) = -15$.

For rule 4, x_1 is Z, x_2 is N, $u = 3$. This implies that $x_1 \in [-0.5, 0.5]$, $x_2 \in [-1, -0.2]$ and $g(u) = 1$. From (21),

$$\begin{aligned}\dot{V} &= -4x_1^2 - 23x_2^2 - 2x_1x_2 + 4x_1 + 50x_2\\ &\le -4x_1^2 - 23x_2^2 + 1 + 2 + 50x_2\\ &< 0\end{aligned}$$

since $\max(3+50x_2) = -7$.

For rule 5, x_1 is P, x_2 is Z, $u = -1$. This implies that $x_1 \in [0.2, 1]$, $x_2 \in [-0.5, 0.5]$ and $g(u) = 0$. From (21),

$$\begin{aligned}\dot{V} &= -4x_1^2 - 23x_2^2 - 2x_1x_2\\ &= -3x_1^2 - 22x_2^2 - (x_1 + x_2)^2\\ &< 0.\end{aligned}$$

For rule 6 to 8, since $g(u) = 0$, the analysis for rule 5 can also be applied to give the result $\dot{V} < 0$.

For rule 9, if $[x_1 \;\; x_2] \neq [0 \;\; 0]$, the analysis of rule 5 can be applied to give the result $\dot{V} < 0$. If $[x_1 \;\; x_2] = [0 \;\; 0]$, from (21), $\dot{V} = 0$.

Hence, $\dot{V}$ is negative definite in every fuzzy sub-system's active region, $g(u)$ is monotonic increasing and the defuzzification method of (3) is applied. From *Theorem 1*, the equilibrium point at the origin is globally asymptotically stable.

QED

Simulation results of the zero-input responses of the closed-loop system with initial values $\boldsymbol{x}(0) = [1 \;\; -0.5]^T$ are shown in Fig. 6. The stability of the fuzzy logic control system is verified.

V. CONCLUSION

A stability analysis method has been proposed in this paper to prove the stability of fuzzy logic control systems with non-symmetric dead zone, and the function outside the dead zone being piecewise linear with different slopes and saturation. The plant in the system is controlled by an FLC. Since a dead zone is involved, the stability analysis becomes difficult because the plant is no longer linear with respect to control. To model such plants with a TS fuzzy model, its fuzzy rules may have a premise that depends on the control. This makes the stability analysis based on a TS fuzzy plant model difficult to apply. Our proposed stability analysis

method can tackle the system easily by examining every fuzzy rule individually instead of the system as a whole. An example has been given to illustrate the application of the proposed method.

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