A Simple Large-Signal Nonlinear Modeling Approach for Fast Simulation of Zero-Current-Switch Quasi-Resonant Converters

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***Abstract*— A nonlinear modeling approach for zero-current-switch (ZCS) quasi-resonant converters (QRC) is proposed which can be derived easily using simple analytical techniques. The converter model obtained is readily absorbed by MATLAB for analysis and design of both the open- and closed-loop configurations in fast speed. Simulations have shown its accuracy, even for large-signal transient responses. Applications of this modeling approach to the three basic topologies of buck, boost, and buck–boost converters are given as illustrative examples. The condition for zero-current switching is identified from the model. The feasibility of applying this proposed modeling approach to the extended period QRC topologies is to be discussed. Simulation results for the three basic topologies are given to show the merits of the proposed modeling approach.**

***Index Terms*—Modeling, simulation, zero-current-switch quasi-resonant converters.**

I. Introduction

ZERO-CURRENT-SWITCH (ZCS) quasi-resonant converters (QRC's) [3] have been kinds of popular topologies for dc–dc power conversion due to their inherent soft-switching characteristic and circuit simplicity. The properties and characteristics of ZCS QRC's have been detailed in [1]–[3]. In general, a ZCS is formed by adding a pair of resonant inductors and capacitors to the electronic switch so that the value of current flowing through the electronic switch is zero when it is turned on and off. The switching loss of the converter can thus be reduced and the operating frequency can be increased, resulting in a higher power density. However, the design of regulated switch-mode power supplies based on ZCS QRC's are difficult to realize without a good model of the open-loop converter. Still, due to its nonlinearities and complicated operating characteristics, ZCS QRC's are hard to model.

Steady-state analyses of QRC's were carried out, and relationships between their static voltage conversion ratios and the switching frequencies were reported [3]. This information is useful in getting an understanding of the operation of the QRC in steady state, but cannot be used to predict the transient behavior. A dynamic model of QRC was reported based on graph theory [4]. Nevertheless, an expert knowledge on circuit theory is needed to achieve this model. On the other hand, Chan and Chau [7] used the finite element method to obtain a dynamic model for QRC's, but it is just a numerical method which cannot be easily applied to the problem of design and analysis of the closed-loop regulated power supply.

In this paper, a modeling approach for ZCS QRC's is proposed through which the converter model can be derived by simple analytical techniques. The result is a nonlinear differential equation model readily absorbed by some analytical tools, such as MATLAB, for analysis and simulation. Inside the MATLAB environment with the Control Systems Toolbox, the open-loop converter can be simulated easily. For the problem of close-loop regulated converter design, such a model, along with a simple program, can easily be used to give accurate simulations on the behavior of the designed system. By virtue of this modeling approach, the simulation speed is so fast that accurate results can be obtained almost instantly from a common computer. Such a modeling approach takes the advantage of the operating characteristics of the ZCS, which can be utilized to obtain a model for the associated QRC topology. The resulting converter model is found to be accurate even for large-signal transient responses. In particular, the models of the three basic topologies of buck, boost, and buck–boost converters are to be investigated. All these details will be given in Sections II and III. It was reported in [6] that zero-current switching is governed by a given condition. This condition is verified on applying the proposed modeling approach and will be explained in Section IV. The feasibility of applying the proposed modeling approach to the extended period QRC topologies is to be discussed in Section V. Simulation results for the three basic topologies are to be given in Section VI as application examples.

II. The Zero-Current Switch

The circuit diagrams of a full-wave ZCS and a half-wave ZCS are shown in Fig. 1(a) and (b), respectively. A ZCS consists of an electronic switch S, diodes D_1 and D_2 (for full-wave only), resonant inductor L_r, and resonant capacitor C_r. The nodes at the two sides of the ZCS are denoted by A and K, respectively. When the ZCS is off, the voltage across the switch is V_{AK} and the current from A to K is zero. When the ZCS is fully on, the current from A to K is I. The value of I is obtained based on the QRC topology and is assumed to be constant *within one switching cycle*. This assumption can

Manuscript received April 8, 1996; revised January 7, 1997. This work was supported by the Earmarked Grant from Research Grants Council, UGC, Hong Kong, under Project HKP 62/94E.

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Publisher Item Identifier S 0885-8993(97)03286-9.

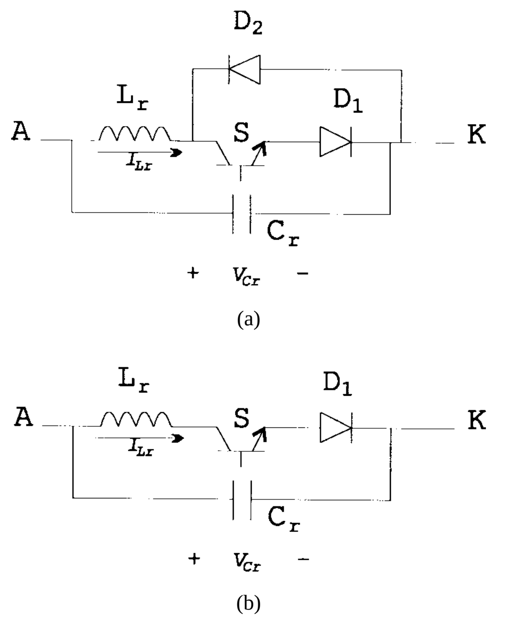


Fig. 1. ZCS quasi-resonant switch: (a) full-wave and (b) half-wave mode.

often be met because QRC's are operating at high frequency such that the period of one switching cycle is negligibly small with respect to the dynamics of the converter filters (input or output filters).

From the operation principle of the ZCS, a switching cycle is divided into four stages [1]–[3], namely, the inductor-charging stage, resonant stage, capacitor-charging stage, and free-wheeling stage. The waveforms of the resonant inductor current I_{L_r} and resonant capacitor voltage V_{Cr} for full-wave mode of operation and half-wave mode of operation are shown in Fig. 2(a) and (b), respectively. The state equations governing the ZCS and time durations of these stages can be expressed as follows.

A. Stage I: Inductor-Charging Stage $[t_0, t_1]$

In this stage, the electronic switch S turns on and the inductor current I_{L_r} rises linearly from zero to I. The state equation is

$$L_r \frac{dI_{L_r}}{dt} = V_Z \tag{1}$$

where $V_Z = V_s$ for the ZCS quasi-resonant (QR) buck converter, $V_Z = V_o$ for the ZCS QR boost converter, $V_Z = V_s - V_o$ for the ZCS QR buck–boost converter, and V_s and V_o are the input and output voltages of the converters, respectively.

The time duration for this stage is

$$T_1 = \frac{L_r I}{V_Z}. \tag{2}$$

B. Stage II: Resonant Stage $[t_1, t_2]$

In this stage, L_r and C_r resonate. The state equations are as follows:

$$C_r \frac{dV_{C_r}}{dt} = I - I_{L_r} \tag{3}$$

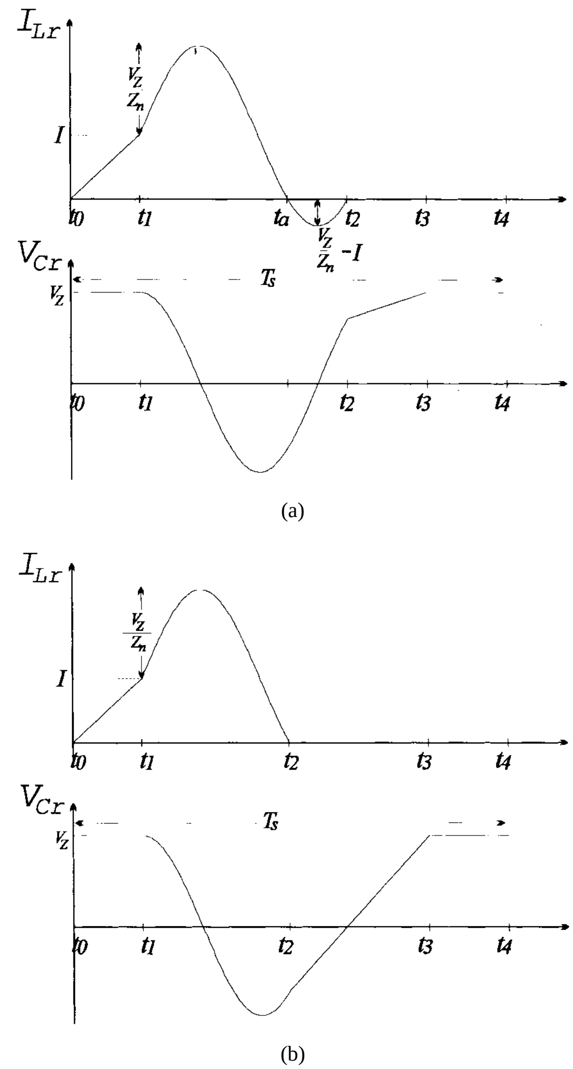


Fig. 2. Current and voltage waveforms of the resonant inductor and capacitor: (a) for full-wave mode of operation and (b) half-wave mode of operation.

$$L_r \frac{dI_{L_r}}{dt} = V_{C_r}. \tag{4}$$

By applying the initial conditions $I_{L_r} = I$ and $V_{C_r} = V_Z$ at $t = t_1$ and the final condition $I_{L_r} = 0$ at $t = t_2$, the time duration of this stage is

$$T_2 = \frac{\theta}{\omega} \tag{5}$$

where $\theta = \sin^{-1}(-Z_n I/V_Z)$, $Z_n = \sqrt{L_r/C_r}$, and $\omega = 1/\sqrt{L_r C_r}$, $\pi < \theta < 3\pi/2$ for half-wave mode of operation and $3\pi/2 < \theta < 2\pi$ for full-wave mode of operation.

C. Stage III: Capacitor-Charging Stage $[t_2, t_3]$

For full-wave mode of operation, after I_{L_r} has gone negative, the switch S turns off between t_a and t_2. For both modes of operation, S is off at t_2 and I_{L_r} is zero so that the capacitor C_r is linearly charged by I. The state equation is

$$C_r \frac{dV_{C_r}}{dt} = I_{L_r}. \tag{6}$$

TABLE I
COMPONENT VALUE OF TEST CIRCUITS

	ZCS Buck	ZCS Boost	ZCS Buck- Boost
V_s	15V	15V	15V
L_r	1.6μH	0.16μH	1.6μH
C_r	0.064μF	0.64μF	0.064μF
L	100μH	100μH	100μH
C	1μF	10μF	3.3μF
R	10Ω	20Ω	20Ω

By applying the initial condition $V_{Cr} = V_Z \cos\theta$ at $t = t_2$ and the final condition $V_{C_r} = V_Z$ at $t = t_3$, the time duration of this stage can be found to be

$$T_3 = \frac{C_r V_Z (1 - \cos\theta)}{I}. \tag{7}$$

D. Stage IV: Free-Wheeling Stage $[t_3, t_4]$

After V_{C_r} returns to V_Z at t_3, the whole ZCS turns off and the free-wheeling stage begins. The time duration of this stage is

$$T_4 = T_s - T_1 - T_2 - T_3 \tag{8}$$

where T_s is the period of one switching cycle.

III. THE QUASI-RESONANT CONVERTER MODEL

The idea of the proposed modeling approach is to reduce the operation of the ZCS from four stages to two stages, viz. on- and off-stage. When the current from A to K is zero, the ZCS is regarded as off. When the current from A to K is I, the ZCS is regarded as fully on. From this point of view, a ZCS is very similar to an ideal switch. The derivation of the model is based on the fact that the ZCS is turned fully on in Stages II and III, during which the current from A to K is I. In Stage IV, the current flowing through the ZCS is zero, so that it is fully turned off. In Stage I, both the ZCS and the associated passive diode switch supply or sink current I to or from the filter inductor. The current flowing through the ZCS is linearly raised from zero to I during T_1 [3]. As I is the summation of the current of the ZCS and that of the passive diode switch, on average, it is equivalent to say that the ZCS and the associated diode switch each take half of T_1 to supply or sink the current I. Let t_{on} be the time duration in one switching period that the ZCS is on, then

$$t_{\text{on}} = 0.5T_1 + T_2 + T_3 \tag{9}$$

and the turn-off time is

$$t_{\text{off}} = 0.5T_1 + T_4. \tag{10}$$

The state equations of the converter for the on- and off-stage can then be found. Since the period of each switching cycle is relatively short, time-averaging within one switching cycle can be applied so that the two state equations are combined to reach a nonlinear state equation model relating the output voltage to the switching frequency.

For a quasi-resonant buck converter, let L, C, and R be the inductance, capacitance, and load resistance of the output filter, respectively. The state equations of the converter during t_{on} are given by

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} V_s \tag{11}$$

where I_L is the current flowing through the filter inductor ($I_L = I$), V_o is the output voltage, and V_s is the input voltage. The state equations of the quasi-resonant buck converter operating in t_{off} are given by

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_s. \tag{12}$$

By using time-averaging, the weighted average of (11) and (12), with respect to t_{on} and t_{off}, respectively, is given by

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{t_{\text{on}}}{T_s L} \end{bmatrix} V_s \tag{13}$$

where $T_s = t_{\text{on}} + t_{\text{off}}$.

From (2), (5), (7), and (9), it can be shown that t_{on} is a function of I_L only:

$$t_{\text{on}} = 0.5\frac{L_r I_L}{V_s} + \sqrt{L_r C_r}\,\sin^{-1}\left(\frac{-\sqrt{\dfrac{L_r}{C_r}}\,I_L}{V_s}\right) + \frac{C_r V_s\left\{1 - \cos\left[\sin^{-1}\left(\dfrac{-\sqrt{\dfrac{L_r}{C_r}}\,I_L}{V_s}\right)\right]\right\}}{I_L}. \tag{14}$$

Hence, (13) and (14) constitute a nonlinear differential equation model for the ZCS quasi-resonant buck converter relating its state vector $[V_o\ I_L]^T$ to the switching frequency $1/T_s$. Although the expressions look fairly complicated, such a model can easily be handled by some software tools, such as MATLAB and its Toolboxes, for analysis and simulation. In addition, this model can be used as a basis for the design of the compensating network for optimal closed-loop behavior. On using this model, all simulations can be done in a much faster speed than those based on circuit simulators.

For a ZCS quasi-resonant boost converter, the state equations corresponding to t_{on} and t_{off}, respectively, are

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} V_s \tag{15}$$

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{-1}{C} \\ \dfrac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} V_s. \tag{16}$$

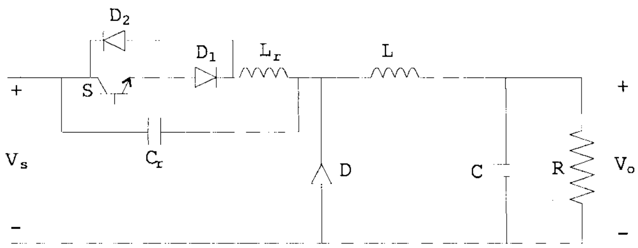


Fig. 3. A full-wave ZCS QR buck converter.

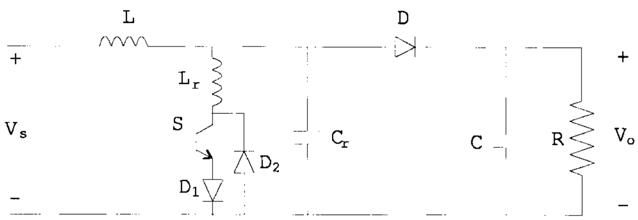


Fig. 4. A full-wave ZCS QR boost converter.

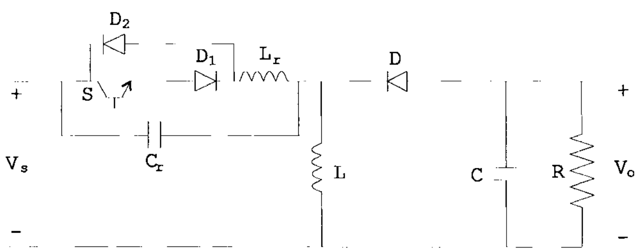


Fig. 5. A full-wave ZCS QR buck–boost converter.

Similarly, by applying time-averaging, the nonlinear model of the converter can be represented as follows:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{-(1-t_{\text{on}})}{T_s C} \\ \dfrac{-(1-t_{\text{on}})}{T_s L} & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} V_s \tag{17}$$

$$t_{\text{on}} = 0.5\,\frac{L_r I_L}{V_o} + \sqrt{L_r C_r}\,\sin^{-1}\left(\frac{-\sqrt{\dfrac{L_r}{C_r}}\,I_L}{V_o}\right) + \frac{C_r V_o\left\{1-\cos\left[\sin^{-1}\left(\dfrac{-\sqrt{\dfrac{L_r}{C_r}}\,I_L}{V_o}\right)\right]\right\}}{I_L}. \tag{18}$$

For a ZCS quasi-resonant buck–boost converter, the state equations corresponding to t_{on} and t_{off}, respectively, are as follows:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} V_s \tag{19}$$

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{-1}{C} \\ \dfrac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_s. \tag{20}$$

By applying time-averaging, the nonlinear model of the converter can be represented as follows:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{-(1-t_{\text{on}})}{T_s C} \\ \dfrac{1-t_{\text{on}}}{T_s L} & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{t_{\text{on}}}{L} \end{bmatrix} V_s \tag{21}$$

$$t_{\text{on}} = 0.5\,\frac{L_r I_L}{V_s - V_o} + \sqrt{L_r C_r}\,\sin^{-1}\left(\frac{-\sqrt{\dfrac{L_r}{C_r}}\,I_L}{V_s - V_o}\right) + \frac{C_r (V_s + V_o)\left\{1-\cos\left[\sin^{-1}\left(\dfrac{-\sqrt{\dfrac{L_r}{C_r}}\,I_L}{V_s - V_o}\right)\right]\right\}}{I_L}. \tag{22}$$

IV. Condition for Zero-Current Switching

It can be seen from (5) that for θ to be a real number, the following condition must be satisfied:

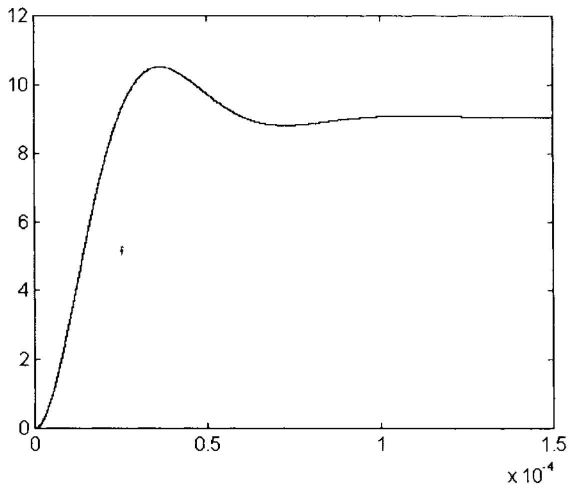
$$\left|\frac{Z_n I}{V_Z}\right| < 1. \tag{23}$$

Physically, it represents a condition for zero-current switching. If (23) fails, T_2 becomes undefined. Under this situation, the current of the electronic switch S does not return to zero when it is turned off. This can also be seen from the current waveform of L_r in Fig. 2(a) and (b). For zero-current switching, the switch S should be turned off when I_{Lr} is not positive during $[t_a, t_2]$ for full-wave mode of operation and after t_2 for half-wave mode of operation. However, when (23) fails such that $|Z_n I/V_Z| > 1$, which implies $V_Z/Z_n - I < 0$, the minimum turning point of I_{Lr} will be greater than zero. This means the resonant inductor as well as the switch S always conducts after the resonant stage has commenced. Consequently, the current of the switch S cannot be zero during turning off.

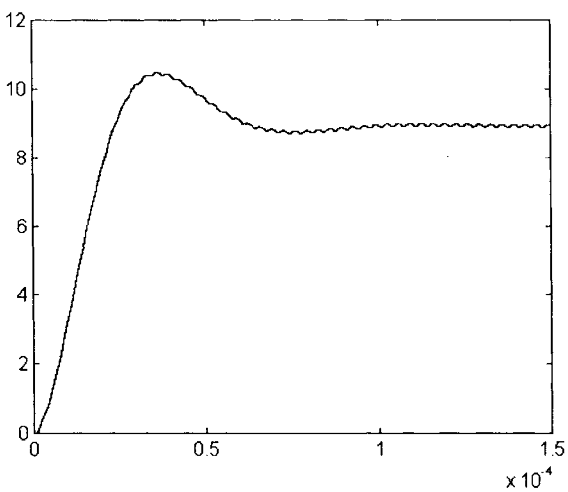
From (23), it can be seen that the cause of this failure of zero-current switching condition is due to a large current I or a low voltage V_Z. This will easily happen during the transient responses of the converter when large dynamics are involved, even though zero-current switching is maintained in the steady state. A poor choice of Z_n, that is, the values of L_r and C_r, with respect to the operation of the converter, may also be the cause.

V. Extended Period Quasi-Resonant Converter

A QRC topology with constant switching frequency during operation, called extended period QRC, was developed by Cheng [8]. The technique involves dividing the resonant stage (Stage II), discussed in Section II, into two parts and adding an extended stage between them. By varying the duration of this extended stage, the QRC can be controlled with constant switching frequency. Basically, the quasi-resonant switch of

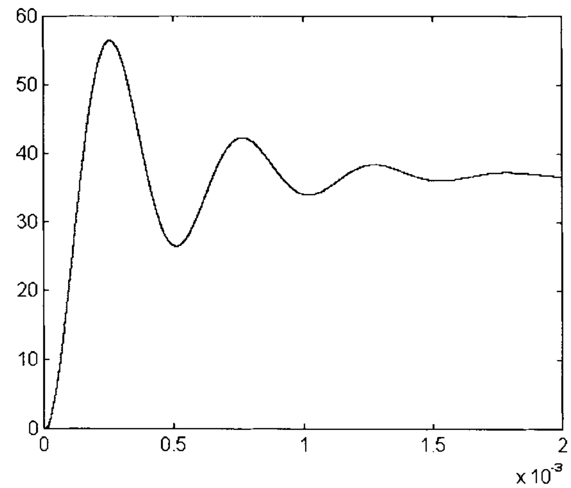


(a)

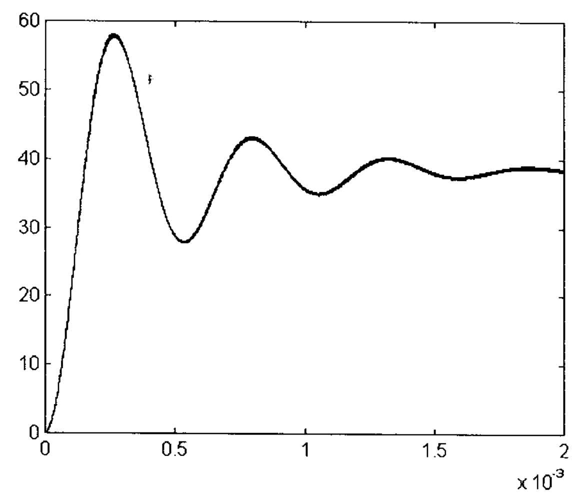


(b)

Fig. 6. Transient response of output voltage of a full-wave ZCS buck QRC: (a) based on the proposed model and (b) by a circuit simulator.



(a)



(b)

Fig. 7. Transient response of output voltage of a full-wave ZCS boost QRC: (a) based on the proposed model and (b) by a circuit simulator.

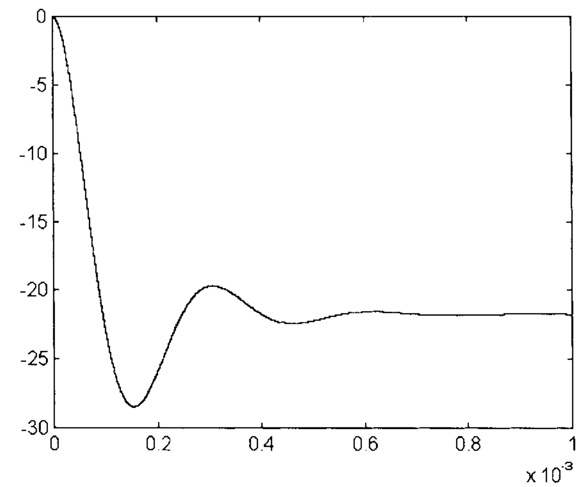
this converter topology is similar to the ZCS's shown in Fig. 1 with the addition of a unidirectional switch in parallel with the resonant inductor L_r. During the extended stage, the current from A to K of the extended period quasi-resonant switch is I, which means the switch is fully on effectively, as discussed in Section III. Thus, the modeling approach proposed in this paper can also be applied to this extended period QRC by simply adding the time duration of the extended stage to $t_{\rm on}$.

VI. Simulation Results

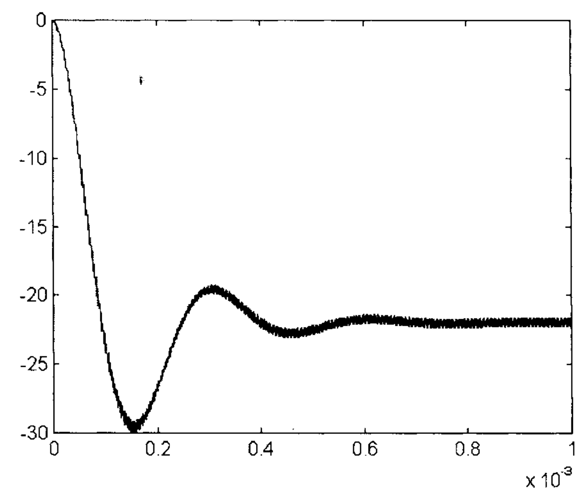
The ZCS quasi-resonant buck, boost, and buck–boost converters in full-wave mode of operation are used as illustrative examples. The schematic diagrams of the three topologies are shown in Figs. 3–5, respectively. The startup transient responses associated with these three converters obtained by MATLAB, based on the proposed modeling approach and by a circuit simulator, are shown in Figs. 6–8, respectively. The component values of each example circuit are listed in Table I. The switching frequency is set at 300 kHz for all topologies. The electronic switch and the diodes are assumed to be ideal. It can be seen that the responses obtained based on the proposed modeling approach match well with those from a circuit simulator, even though large-signal startup transient dynamics are involved. Still, the results offered by the proposed models can be obtained in much faster speed when compared with those provided by a circuit simulator.

VII. Conclusion

A modeling approach for ZCS QRC's is proposed, which can be derived using simple analytical techniques. A nonlinear differential equation model is reached, which can accurately predict the transient behavior of the QRC, even for large-signal operation. The responses of the QRC can be easily obtained by using MATLAB and its Toolboxes in fast speed. By virtue of this modeling approach, the design of regulated power converters can be realized efficiently and effectively, especially when tunings of the controller parameters and converter component values are necessary. The condition for maintaining zero-current switching is also identified from the model derived. During simulation, the nonzero-current



(a)



(b)

Fig. 8. Transient response of output voltage of a full-wave ZCS buck–boost QRC: (a) based on the proposed model and (b) by a circuit simulator.

switching can be observed when this condition fails. Designers of QRC's can then take note of this fact and try to avoid nonzero-current switching through proper choices of circuit component values and operation range. The feasibility of applying the proposed modeling approach to the extended period QRC topologies has also been discussed.

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