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## Application of PGA on Optimization of Distribution of Shopping Centers

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**Abstract.** In this study, the distribution of shopping centers is optimized in terms of realizing the shortest car-based shopping trips in an urban area. Modal split is performed between road and public traffic networks is calculated, and then the interaction between land-use and transportation in the context of choice of shopping destinations is modeled to build the optimal function. Parallel genetic algorithm (PGA) is applied to solve the optimal problem on distribution of the area of shopping centers. Several problems in application of PGA are discussed. A case study is undertaken in order to examine the effectiveness of this method.

### 1 Introduction

Before the popularity of motorization, mass transit, bikes and walking were the main traffic modes and urban center had the best mass transit facilities and densest residences. Therefore, shopping centers (SC) were mainly constructed in the centers. With extension of the mass transit network, SCs also extended to terminals or interchanges of railways or buses. This location pattern can be seen even now. However, with the advance of the motorization, merits of central business district (CBD) or main stations for SCs' location decreased [1]. Now households can enjoy the door-to-door transportation service offered by private cars, which is much more attractive than mass transit. However when they drive to SCs near CBD or large terminals, they will suffer from heavy congestion and expensive parking cost. In order to satisfy the demand, large-scale shopping centers tend to appear in the suburb along artery roads where land price is cheaper and traffic flow is less. Today large scale SC with huge parking facilities in developed countries' suburbs is common and popular [2]. Locally, it increased citizen's quality of life in terms of shopping convenience. However, it made the city to sprawl and induced more car traffic. It even caused some deterioration of CBD.

Even though spatial distribution of SCs affects the behaviors of household location and shopping destination, this study does not tackle the location behavior of SCs but analyzes the behavior of selection of shopping destination with land-use and

transportation model. Factors such as business spaces and transportation convenience between SCs and houses are used to model the behavior in selecting shopping destination. An optimal function, which minimizes total length of car-based shopping trips, is put forth, where business spaces of SCs in zones are set as dependent variables and total business space of SCs is the constraint. Through solving the optimal problem with GA, the optimal spatial distribution of SCs in an urban area can be determined.

## 2 Land-Use and Transportation Model

In general, during a time period, the total purchasing ability of a city can be considered as unchanged. It is reasonable to say that the total business space of SCs in a city is nearly a constant. However, spatial distribution of SCs changes according to geographical characteristics, house location and transportation network. This spatial distribution affects distribution and modal split of shopping trips seriously. In a city if other aspects are kept the same, the total length of car-based shopping trip changes according to the distribution.

The analysis of the distribution of the business space is based on the following two hypotheses: along with the perfect shape of the earth's surface, the business space around the center of a city will be distributed like a circle; the distribution and evolvement of the business space of the city will tend to the biggest entropy.

The known data are the business space in a certain area, and the aim is to simulate detail space distribution model. The steps are as follows: (1) Divide the researched space into grid with certain accuracy; (2) Set a center in each space and link its business area to the center; (3) With an interpolation method, insert the business space density to the gridding surface.

There are a lot of interpolation methods in common use, and the kernel estimation method is selected here. If  $s$  is used as a random point of the space  $R$ ,  $s_i$  as the  $i$ th observation point and  $y_i$  as the  $i$ th observation value, then the definition of  $\lambda_\tau(s)$  is as follows:

$$\lambda_\tau(s) = \frac{1}{\sigma_\tau(s)} \sum_{i=1}^n \frac{1}{\tau^2} k\left(\frac{s-s_i}{\tau}\right) y_i \quad (1)$$

Herein,  $k(\cdot)$  is a double-variable probability density function, which is named Kernel function; the parameter  $\tau(\tau>0)$  is used to define the size of smoothing, which is named bandwidth, and in fact,  $\tau$  is the radius of a circle whose center is  $s$ , so each  $s_i$  will affect  $\lambda_\tau(s)$ . If a bandwidth is given, a typical Kernel function is obtained as follows:

$$k(u) = \begin{cases} \frac{3}{\pi} (1-u^T u)^2, & u^T u \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

So the following can be obtained:

$$\lambda_{\tau}(s) = \sum_{h_i \leq \tau} \frac{3}{\pi \tau^2} \left(1 - \frac{h_i^2}{\tau^2}\right)^2 y_i \quad (3)$$

Herein,  $h_i$  is the distance between the point  $s$  and the observation point  $s_i$ . The range of the points affecting  $\lambda_{\tau}(s)$  is just the circle whose center is the point  $s$  and the radius is  $\tau$ . No matter what kernel function is selected, if the bandwidth is added, the area around  $s$  point will be leveled. For bigger bandwidth,  $\lambda_{\tau}(s)$  will tend to be flat and the local character will be blurred.

Different  $\tau$  is generally adopted to simulate  $\lambda_{\tau}(s)$  of different surfaces. If a  $\tau$  is set which fits for a sparse area, it will also blur dense area. So, in order to optimize the kernel estimation to different points, its bandwidth should be adjusted. The more observation values will lead to more information. Based on this information, the smaller bandwidth can be adopted. Adaptive kernel function is the method that can rectify  $\tau$  with different areas at any moment.

Based on a certain distribution of SCs, a land-use and transportation model can be employed to analyze the origin-destination (OD) traffic distribution and modal split of car in the context of shopping traffic.

In order to apply traffic demand model to analyze the selection behavior of shopping destination and traffic mode, it is supposed that a city is divided into a certain number of zones, in each of which there are certain SCs and households. First, the modal split model between car and railway is estimated for each zone in terms of shopping trips. A traditional method, namely stochastic utility function method, is applied to estimate the probability of each mode as follows:

$$P_{c,ij} = \frac{GT_{c,ij}^{\alpha}}{GT_{c,ij}^{\alpha} + GT_{b,ij}^{\alpha}} \quad (4)$$

where  $GT_{c,ij}$  and  $GT_{b,ij}$  represent generalized travel cost between zones  $i, j$  by car and railway, respectively.

The behavior of selection of shopping destination can also be analyzed in this way. Here shopping utility is defined by the accessibility of households to SCs, with its definition described in eq. (5). The probability of household in zone  $i$  shop in SC in zone  $j$  can be estimated with eq. (6). The total car-based shopping trip length from zone  $i$  to other zones can be estimated with eq. (7).

$$U_{ij} = q_j^{\beta} [P_{c,ij} \times f(GT_{c,ij}) + (1 - P_{c,ij}) \times f(GT_{b,ij})] + \xi \quad (5)$$

$$P_{ij} = \frac{q_j^{\beta} [P_{c,ij} \times f(GT_{c,ij}) + (1 - P_{c,ij}) \times f(GT_{b,ij})]}{\sum_{k=1}^m q_k^{\beta} [P_{c,ik} \times f(GT_{c,ik}) + (1 - P_{c,ik}) \times f(GT_{b,ik})]} \quad (6)$$

where  $U_{ij}$  : Utility of household living in zone  $i$  shopping in zone  $j$ ;  $q_j$  : Business space of SC in zone  $j$ ; and,  $m$  : Number of traffic analysis zone.

$$\sum_j^m h_i P_{ij} P_{c,ij} d_{ij} \quad (7)$$

where  $h_i$  : Households in zone  $i$ ; and,  $d_{ij}$  : Distance between zones  $i, j$  along the road network.

### 3 Optimal Model and its Algorithm

In term of environmental sustainability, the optimal distribution of SCs should help to realize the smallest length of car-based shopping trip in the city, since an effective method to reduce car emission is to decrease car mileages. Therefore, the optimal location model of SCs can be described with eq. (8).

$$\text{Min} \quad \sum_{i=1}^m \sum_{j=1}^m h_i P_{c,ij} \frac{q_j^\beta [P_{c,ij} \times f(GT_{c,ij}) + (1 - P_{c,ij}) \times f(GT_{b,ij})]}{\sum_{k=1}^m q_k^\beta [P_{c,ik} \times f(GT_{c,ik}) + (1 - P_{c,ik}) \times f(GT_{b,ik})]} d_{ij} \quad (8)$$

$$\text{S.T.} \quad \sum_j^m q_j = Q$$

$$q_j \geq q_{j,\min}$$

$$q_j \geq q_{j,\max}$$

where  $Q$  : total business space of SCs in the city; and,  $q_{j,\max}, q_{j,\min}$  : upper and lower limits of SCs' spaces in a zone, respectively.

This is a relatively complicated optimal problem which is difficult to be solved by conventional methods. In this paper, GA is used to solve the problem.

In recent years, GA has been shown to have advantages over classical optimization methods [3-9]. It has become one of the most widely used techniques for solving optimal problems in land-use planning. GA are heuristic iterative search techniques that attempt to find the best solution in a given decision space based on a search algorithm that mimics Darwinian evolution and survival of the fittest in a natural environment. The standard GA may not be good enough to solve this problem. A coarse-grained parallel GA, which not only accelerates search speed of GA, but also enlarges the population size and insulates child populations from one another, is employed here. It keeps the variety and decrease the probability of the premature convergence, and thereby it can improve the solution's quality and operation speed.

## 4 Genetic Algorithm Design

### 4.1 Parametric Study

There are four GA parameters, namely  $p_c$ ,  $p_m$ ,  $P_{size}$  and  $T_{max}$ .  $p_c$  means the crossover probability parameter that is typically set so that crossover is performed on most, but not all, of the population. It varies from 0.3 to 0.9.  $p_m$  is the mutation probability parameter that controls the probability of selecting a gene for mutation, which varies usually from 0.01 to 0.1.  $P_{size}$  is the population size parameter that provides sufficient sampling of the decision space while limiting the computational burden, which varies from 500 to 1000.  $T_{max}$  is the maximum number of generation, which varies from 1000 to 2000.

### 4.2 Coding

Population or business spaces of SCs are relatively large figures. So, decimal system (real value) is used to code the population of chromosomes. It means the located SCs' business spaces are used as the genes in chromosomes directly so that each chromosome will directly come into as decimal vector in the solution space. In other words, chromosome is the problem variable and genetic space is the problem space.

The total number of chromosomes is controlled by  $P_{size}$ . Each chromosome is a finite-length string of numbers that represents the values of the decision variables for that chromosome. Here the decision variables are the business spaces of SCs in zones. Even though it is tried to locate SCs to decrease the car-based shopping trips length, it is reasonable to think that business space in a zone has an upper and lower limits,  $q_i \in [q_{i,\min}, q_{i,\max}]$ , and they can be estimated based on current SCs' location pattern as shown in eqs. (9), (10) and (11).

$$q_{i,\max} = q_{\max} \times (h_i / h_{q_{\max}}) \quad (9)$$

$$\text{where } q_{\max} = \max(q_1, \dots, q_i, \dots, q_m)$$

$$\text{where } h_{\max} = \max(h_1, \dots, h_i, \dots, h_m)$$

$$q_{i,0}^0 = \text{Random}[q_{i,\min}, q_{i,\max}] \quad (10)$$

$$\text{where } i = 1, \dots, M, p = 1, \dots, P_{size}, q_{i,\min} = 0$$

$$q_{i,0} = Q \times \frac{q_{i,0}^0}{\sum_k q_{k,0}^0} \quad (11)$$

### 4.3 Fitness Function

Here the variable penalty method is used to build fitness function. The fitness function  $F$  is an inverse of the original function  $f$ , with the deduction of the penalty function, and hence is to be maximized. The penalty coefficient  $M$  will become larger with the increment of the generation  $t$ , and its formation is shown in eq. (12), with coefficients  $\beta_1$  and  $\beta_2$  as constants.

$$F(q) = 1.0 / (f(q) - M(t)(Q - \sum_{i=1} q_i)) \quad (12)$$

$$M(t) = \beta_1 t^{\beta_2}$$

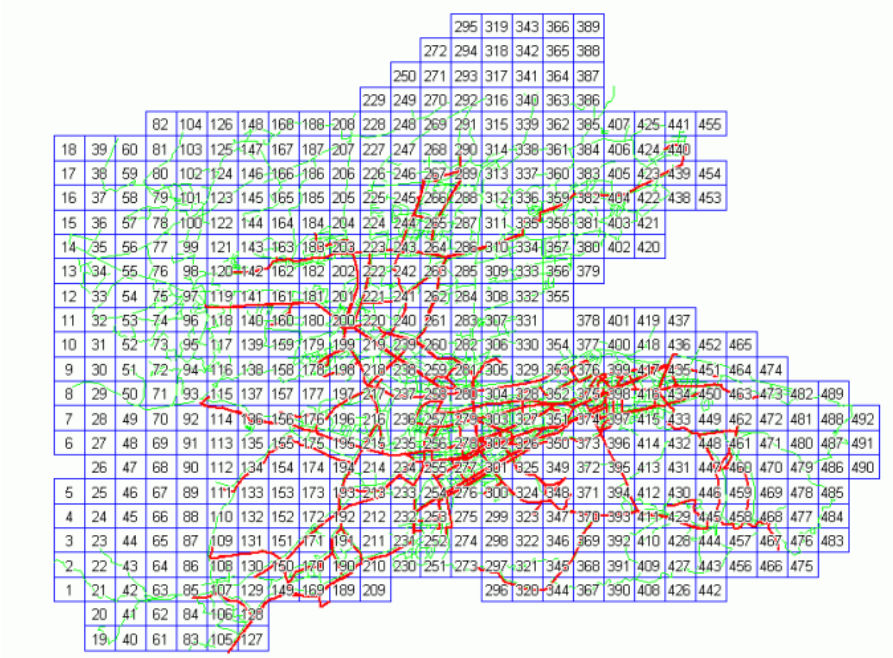


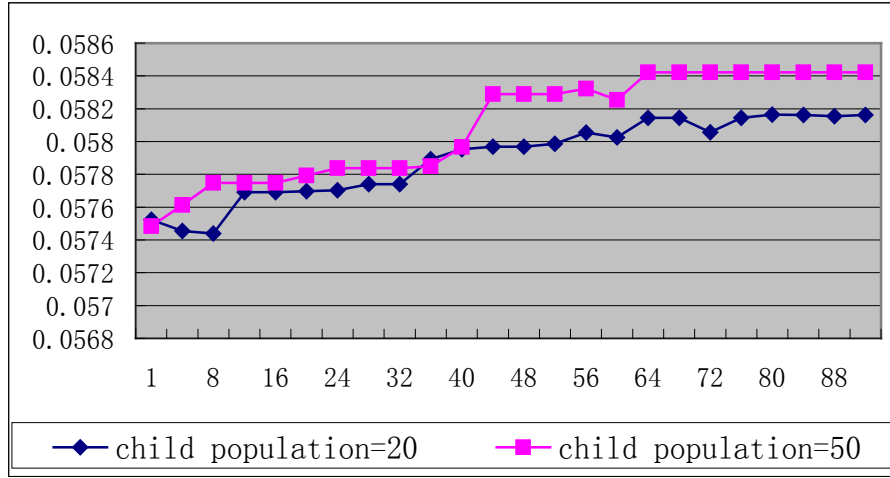
Fig. 1. Gridding Distribution of the city

### 4.4 Genetic Operators

Chromosomes are selected from the parent generation: The chromosomes are ranked according to the fitness and then based on the predetermined selected probability  $p_s$ ,  $p_s \times P_{size}$  are selected from the parent generation, whose fitness are relatively high.

while the sifted out ones  $(1 - p_s) \times P_{size}$  are replaced by chromosomes with higher fitness.

Genes are crossed over between two parent chromosomes: Parent chromosomes are paired. Even random selection method is adopted to select the crossing parent chromosomes and to generate child generation by linear crossover method. The genetic operation of crossover is performed on each mated pair based on the crossover probability.



**Fig. 2.** Performances of PGA under different sizes of child population

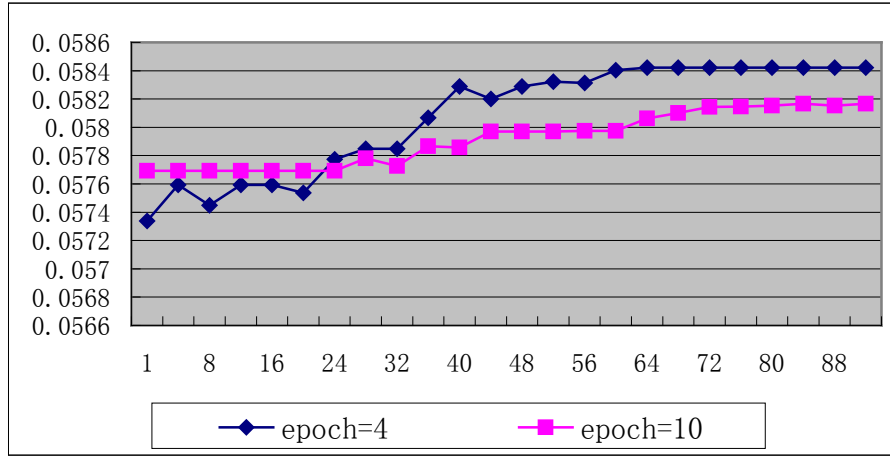
Mutation operation is applied with a mutation rate to avoid being trapped in local optimal: A non-uniform mutation to the mutation operation is designed. If  $\bar{Q} = (q_1, q_2, \dots, q_n)$  is a chromosome and the gene  $q_k$  is selected for the mutation, the result of mutation of  $q_k$  is shown in eq. (13).

$$\begin{aligned} \bar{Q}' &= (q_1^{t-1}, q_2^{t-1}, \dots, q_{k-1}^{t-1}, q_k^t, q_{k+1}^{t-1}, \dots, q_m^{t-1}) \\ q_k^t &= \begin{cases} q_k^{t-1} + \Delta(t, q_{k_{\max}} - q_k^{t-1}) & \text{if } \text{random}(0,1) = 0 \\ q_k^{t-1} + \Delta(t, q_k^{t-1} - q_{k_{\min}}) & \text{if } \text{random}(0,1) = 1 \end{cases} \end{aligned} \quad (13)$$

The function  $\Delta(t, y)$  returns a value in between  $[0, y]$  such that the value for the probability of  $\Delta(t, y)$ , which is given with (14).

$$\Delta(t, y) = y \times (1 - r^{(1-t/T_{\max})^\lambda}) \quad (14)$$

Here  $r$  is a random number between  $[0,1]$ .  $T_{max}$  and  $t$  are maximum number of generations and current generation, respectively.  $\lambda$  ( $\lambda = 2 \rightarrow 5$ ) is a parameter, which determines the degree of dependency with the number of iterations. This property causes this operator to make a uniform search in the initial space when  $t$  is small, and a very local one in later stages. The stopping criterion is defined as follows: if iteration reaches maximum times, then the calculation is terminated and the maximum fitness is output; otherwise the algorithm returns to step 2 to continue the calculation.



**Fig. 3.** Performances of PGA under different migration rates

## 5 Parallel Genetic Algorithm Design

The coarse-grained parallel genetic algorithm (PGA) is the fittest and the most widely used PGA which divides the initial population into several child populations by the number of processor [10-13]. Each child population operates independently and subsequently in different processor, while through certain evolution generations, each child population exchanges several excellent chromosomes from others.

### 5.1 Link topology

The link among child populations adopts double tuning round and each one exchanges chromosome with both sides.



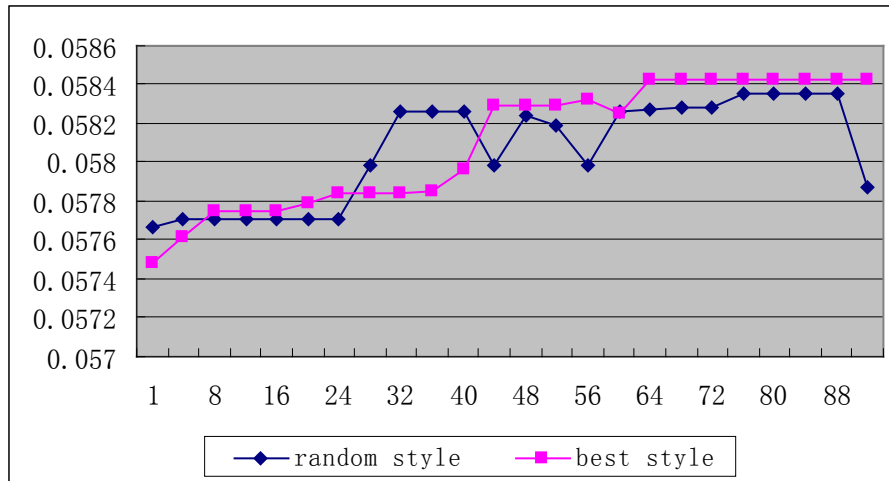
## 5.2 Migration Strategy

In coarse-grained parallel genetic algorithm, the most popular migration strategy is the fittest replace the worst. This strategy is adopted here, with the exchange number ( $n_m$ ) being one. Some previous researches showed that it would destroy the child population variety and disturb solution quality if the migration epoch was too short. However, based on simulation results and the objective in this problem, the least migration epoch adopted is 4. Under this condition, the precondition of the solution quality can be ensured and the fastest convergence speed can be obtained.

## 6 Case Study

In order to examine the effectiveness of the method, a case study is carried out with the data of Dalian city in China (business space is over 1000m<sup>2</sup>). There are 34 SCs and the total business space is about 2,238,417m<sup>2</sup>. Zones, public transit and road networks are obtained from 2001 PT survey as Fig 1.

The study area is divided into 492 grids (1km\*1km), and 34 observation values. Owing to the relatively small size of observation values, a bigger bandwidth ( $\tau=7$ ) is adopted. In the computation of the affected area of observation points, a similar area is adopted. When the distance between each grid to observation point is calculated, the road networks distance between two points, rather than the beeline, is adopted. It is found to be suitable for this practice. The SCs distribution can be obtained from eq. (1).



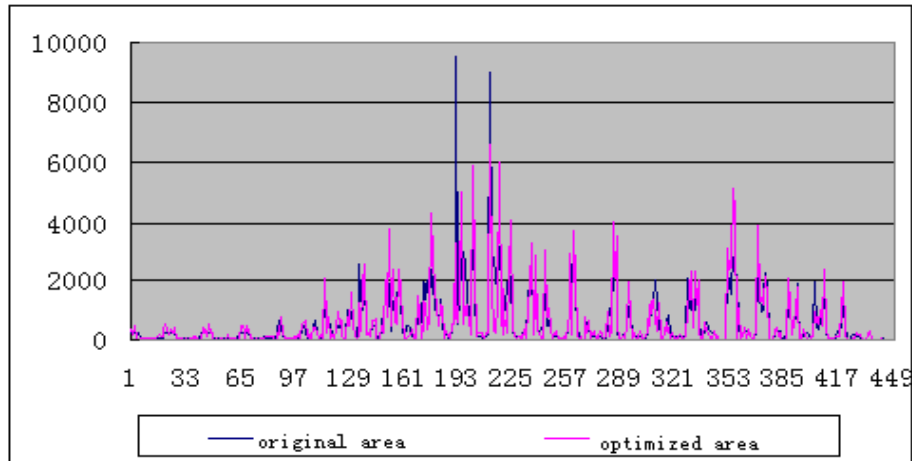
**Fig. 4.** Performances of PGA under different migration styles

In order to estimate the modal split for each zone, the car OD traffic time is computed using the Dijkstra algorithm. The railway OD traffic time is obtained from

former study [14]. Taking  $\alpha = 2$ , the modal split is calculated with eq. (4). It is set that  $\beta = 1.0$  when calculating with eq. (6). It is further supposed that the transportation impedance function follows eq. (15).

$$f(GT) = 1/GT^2 \quad (15)$$

PGA is then used to solve the optimal model. It can be observed from Fig 2 and Fig 3 that, if the size of population is maintained steady, the quality of optimization can be improved by adding the size of child population or migration rate. Fig 4 depicts PGA's capability in different migration styles. In general, random style overmatches the best style. However, the best style is adopted here, due to its better result and convergence speed. Fig 5 depicts comparison between the solution and distributed SCs. It can be seen that the solution results accord with practice. Compared to the existing situation, SCs distribution tends to be consistent with population density distribution and is more concentrated along public transit lines.



**Fig. 5.** Comparison between the solution and distributed SCs

## 7 Conclusions

Through analyzing the behavior of selection of shopping destination based on traffic zone and transportation network, an optimal model is established in terms of environmental sustainability. The objective function in the model is to minimize the total length of car-based shopping trips subject to satisfying the purchasing ability of the households. The modal split model for railway and road modes is integrated into the optimal model. Parallel Genetic algorithm is used to solve this optimal problem. Some techniques, such as generation of initial population, methods of crossover and

mutation, and coding chromosomes with real values, are suggested. These suggestions might be useful for further application of PGA in land-use and transportation model.

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