Rainfall-Runoff Correlation with Particle Swarm Optimization Algorithm

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Abstract. A reliable correlation between rainfall-runoff enables the local authority to gain more amble time for formulation of appropriate decision making, issuance of an advanced flood forewarning, and execution of earlier evacuation measures. Since a variety of existing methods such as rainfall-runoff modeling or statistical techniques involve exogenous input and different assumptions, artificial neural networks have the potential to be a cost-effective solution, provided that their drawbacks can be overcome. Usual problems in the training with gradient algorithms are the slow convergence and easy entrapment in a local minimum. This paper presents a particle swarm optimization model for training perceptrons. It is applied to forecasting real-time runoffs in Siu Lek Yuen of Hong Kong with different lead times on the basis of the upstream gauging stations or at the specific station. It is demonstrated that the results are both more accurate and faster to attain, when compared with the benchmark backward propagation algorithm.

1   Introduction

Precise prediction of rainfall-runoff is an important research topic in hydrologic engineering since it enables the local authority to gain more amble time for formulation of appropriate decision making, issuance of an advanced flood forewarning, and execution of earlier evacuation measures. However, the relationship between rainfall and runoff is not definite due to many pertinent factors such as ambient conditions, soil infiltration capacity, evapo-transpiration, etc. Existing rainfall-runoff modeling or statistical techniques require exogenous input and embrace different assumptions. In numerical modeling, the physical problem is represented by a highly coupled, non-linear, partial differential equation set. The involving processes are highly complex and uncertain which may demand huge computing cost and time. The representation by a deterministic or statistical model is not completely satisfactory.

  Recently, owing to various advantages (built-in dynamism, data-error tolerance and no need to have exogenous input), artificial neural networks (ANN), and in particular, the feed forward back-propagation (BP) perceptrons, have been widely applied in water resources engineering [1]. However, the commonly used BP

algorithm has the drawbacks of slow training convergence speed and easy entrapment in a local minimum.

In this paper, the particle swarm optimization (PSO) algorithm is employed to train multi-layer perceptrons for rainfall-runoff prediction in Shatin catchment of Hong Kong with different lead times and input precipitation data at adjacent or that stations.

2 PSO Algorithm

PSO algorithm is initially developed as a tool for modeling social behavior and is able to optimize hard numerical functions [2-3]. It is currently adapted as a computational intelligence technique intimately related to evolutionary algorithms [4]. It is an optimization paradigm that mimics the ability of human societies to process knowledge. It has roots in two main component methodologies: artificial life on bird swarming; and, evolutionary computation.

Its principle is founded on the assumption that potential solutions will be flown through hyperspace with acceleration towards more optimum solutions. PSO is a populated search method for optimization of continuous nonlinear functions resembling the movement of organisms in a bird flock or fish school. Each particle adjusts its flying according to the flying experiences of both itself and its companions. In doing so, it keeps track of its coordinates in hyperspace which are associated with its previous best fitness solution, and also of its counterpart corresponding to the overall best value acquired thus far by any other particle in the population. Vector, as a convenient form for optimization problems, is used as the variable presentation to represent particles.

Its major advantages are relatively simple coding and hence computationally inexpensive. A similarity between PSO and a genetic algorithm is the initialization of the system with a population of random solutions and the employment of the fitness concept. However, the evolution of generations of a population of these individuals in such a system is by cooperation and competition among the individuals themselves. The population is responding to the quality factors of the previous best individual values and the previous best group values. The allocation of responses between the individual and group values ensures a diversity of response. The principle of stability is adhered to since the population changes its state if and only if the best group value changes. It is adaptive corresponding to the change of the best group value. The capability of stochastic PSO algorithm to determine the global optimum with high probability and fast convergence rate has been shown in other cases. In the following, it is adopted to train the multi-layer perceptrons.

3 Paradigm for Training of Network

If a three-layered preceptron is considered, $W^{[1]}$ and $W^{[2]}$ represent the connection weight matrix between the input layer and the hidden layer, and that between the hidden layer and the output layer, respectively. During training of the multi-layer preceptrons, the i-th particle is denoted by $W_i = \{W^{[1]}, W^{[2]}\}$ whilst the velocity of

particle i is denoted by V_i. The position representing the previous best fitness value of any particle is denoted by P_i whilst the best matrix among all the particles in the population is recorded as P_b. Let m and n represent the index of matrix row and column, respectively, the following equation represents the computation of the new velocity of the particle based on its previous velocity and the distances of its current position from the best experiences both in its own and as a group.

$$V_i^{[j]}(m,n) = V_i^{[j]}(m,n) + r\alpha[P_i^{[j]}(m,n) - W_i^{[j]}(m,n)] \tag{1}$$

$$+ s\beta[P_b^{[j]}(m,n) - W_i^{[j]}(m,n)]$$

where j = 1, 2; m = 1, …, M_j; n= 1, …, N_j; M_j and N_j are the row and column sizes of the matrices W, P, and V; r and s are positive constants; α and β are random numbers in the range from 0 to 1. In the context of social behavior, the cognition part $r\alpha[P_i^{[j]}(m,n) - W_i^{[j]}(m,n)]$ represents the private thinking of the particle itself whilst the social part $s\beta[P_b^{[j]}(m,n) - W_i^{[j]}(m,n)]$ denotes the collaboration among the particles as a group. The new position is then determined based on the new velocity as follows.

$$W_i^{[j]} = W_i^{[j]} + V_i^{[j]} \tag{2}$$

The following equation is used to determine the fitness of the i-th particle in term of an output mean squared error of the neural networks

$$f(W_i) = \frac{1}{S} \sum_{k=1}^{S} \left[\sum_{l=1}^{O} \{t_{kl} - p_{kl}(W_i)\}^2 \right] \tag{3}$$

where f is the fitness value, t_{kl} is the target output; p_{kl} is the predicted output based on W_i; S is the number of training set samples; and, O is the number of output neurons.

4 Application Case

The usefulness and applicability of any modeling system can only be affirmed by verifying its capability to mimic a particular case study with accurate depiction of real phenomena. This system has been verified and validated by applying to study the rainfall-runoff correlation in the Shatin catchment of Hong Kong [5-12]. Discharge at Siu Lek Yuen is forecasted with a lead time of 1 and 2 days based on the measured daily precipitations there and at Tate's Cairn. The data comprises continuous precipitations from 1998 to 2002 with 1460 pairs of daily records, of which two-third and one-third were used for training and validation, respectively. Data preprocessing is performed so that high and low discharge periods of the year and also rapid changes in runoffs are contained in both data sets.

Figure 1 shows the perceptron which has an input layer with one neuron, a hidden layer with three neurons, and output layer with two neurons. The input neuron represents the rainfall at the current day whilst the output nodes include the runoffs

after 1 day and 2 days, respectively. All source data are normalized into the range between 0 and 1, by using the maximum and minimum values of the variable over the whole data sets. The number of population is set to be 30 whilst the maximum and minimum velocity values are 0.3 and -0.3 respectively.
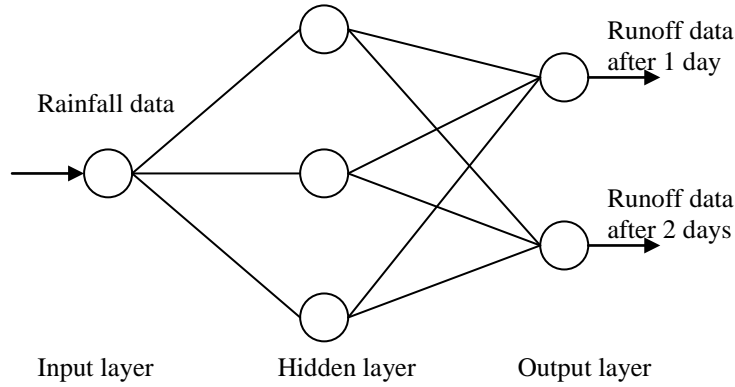
**Fig. 1.** Forecasting schema of PSO-based perceptrons network

Table 1. Normalized mean square errors at various fitness evaluation times during training for PSO-based and BP-based perceptrons

Fitness valuation time	Algorithm	Normalized MSE
5000	BP-based	0.21
	PSO-based	0.12
10000	BP-based	0.14
	PSO-based	0.09
20000	BP-based	0.11
	PSO-based	0.09

5   Analysis of Results

The performance of the PSO-based multi-layer ANN is evaluated in comparison with the benchmarking standard BP-based network. In order to provide a fair and common initial ground for comparison purpose, the training process of the BP-based perceptron commences from the best initial population of the corresponding PSO-based perceptron. Table 1 shows the normalized mean square errors (MSE) at various fitness evaluation times during training for PSO-based and BP-based perceptrons. The fitness evaluation time here for the PSO-based perceptron is equal to the product of the population with the number of generations. It can be observed that the PSO-based perceptron exhibits much better and faster convergence performance in the training

process as well as better prediction ability in the validation process than those by the BP-based perceptron.
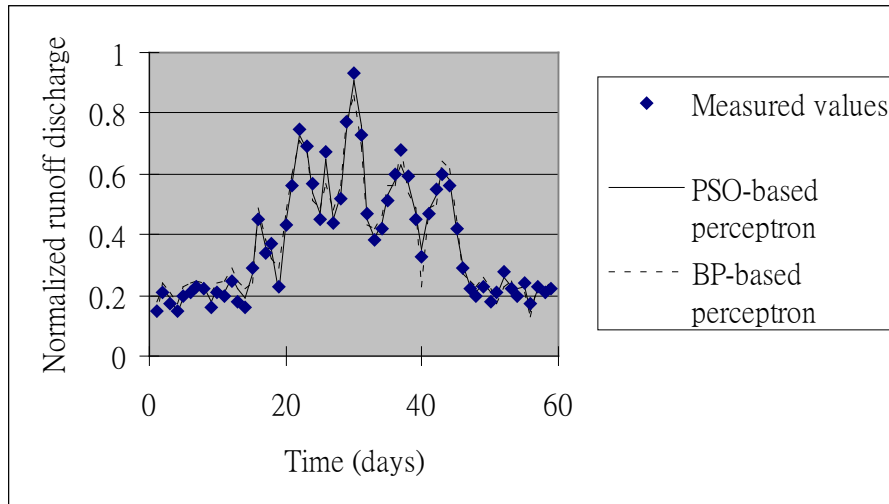
Fig. 2. 1 day lead time water discharge prediction by both perceptrons in the validation process

Figure 2 shows the 1 day lead time normalized water discharge prediction by both perceptrons in the validation process. Table 2 shows comparisons of the results for runoff forecasting at Siu Lek Yuen with both 1 day and 2 day lead times based on precipitation data at the same station (Siu Lek Yuen) and adjacent station (Tate's Cairn). It should be noticed that runoff forecasting at Siu Lek Yuen made by using the data collected at Tate's Cairn is generally better compared to the data collected at Siu Lek Yuen. From these analyses, as a final remark, it can also be observed that the performance of PSO-based perceptron for both training and verification simulations is better than its counterparts of BP-based perceptron.

Table 2. Results for runoff forecasting at Siu Lek Yuen based on precipitation data at the same and adjacent stations

Input data	Algorithm	Coefficient of correlation			
		Training		Validation	
		1 day ahead	2 day ahead	1 day ahead	2 day ahead
Siu Lek Yuen	BP-based	0.956	0.911	0.937	0.893
	PSO-based	0.975	0.964	0.953	0.941
Tate's Cairn	BP-based	0.973	0.945	0.957	0.907
	PSO-based	0.991	0.981	0.985	0.977

6   Conclusions

In this paper, a perceptron approach based on particle swarm optimization (PSO) paradigm is employed for real-time prediction of runoff discharge at Siu Lek Yuen in Shatin catchment with different lead times based on precipitation gauging stations at Tate's Cairn or at Siu Lek Yuen. The algorithm is shown to be capable to furnish model-free estimates in deducing the runoff output from the precipitation input, and hence is demonstrated to be a robust forewarning and decision-support aid. It is noticed from the training and verification simulation that, when compared with the benchmarking BP-based perceptron, the rainfall-runoff prediction results are apparently more accurate and at the same time consume less computational cost.

7   Acknowledgement

References

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