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Robust Filtering for Uncertain Discrete-time Systems: an Improved LMI Approach

Lihua Xie, Lilei Lu
School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore 639798
e-mail: elhxie@ntu.edu.sg

David Zhang, Huanshui Zhang
Department of Computing
Hong Kong Polytechnic University

***Abstract*— This paper discusses the robust filtering problems for linear discrete-time systems with polytopic parameter uncertainty under the H_2 and H_∞ performance. We aim to derive a less conservative design than existing sufficient linear matrix inequality (LMI) based conditions. It is shown that a more efficient evaluation of robust H_2 or H_∞ performance can be obtained by a matrix inequality condition which contains additional free parameters as compared to existing characterizations. When applying this new matrix inequality condition to the robust filter design, these parameters give additional freedoms in optimizing the guaranteed H_2 or H_∞ performance. The optimization will then lead to a less conservative design. The results will recover the existing robust H_2 and H_∞ filtering ones when the additional free parameters are set to be zero. We also propose an iterative algorithm to further refine the suboptimal filter. Examples are given to demonstrate the less conservatism of the proposed approaches.**

I. INTRODUCTION

Linear estimation has significant applications in engineering such as communication, control and signal processing. For systems with known model and noise statistics, linear estimation with minimum variance was the focus of research in the 1960s and the 1970s; see, for example, [1]. However, exact modeling of systems is usually difficult if not impossible. Furthermore, system parameters may vary with time. Hence, it was realized in the 1980s that there is a need to consider system uncertainty in controller/filter design [25].

Filtering with guaranteed error for uncertain systems was first addressed in [12]. A Riccati equation based approach was adopted in [20], [21], [18], [16], [26] to deal with parameter uncertainty of norm-bounded type. The results of these works involve searching for appropriate scaling parameters such that the associated Riccati equation has a solution and the guaranteed error variance is minimized. This is not an easy task in general. Another drawback of the Riccati based approach is that it assumes a fixed Lyapunov function for the entire family of systems characterized by the norm-bounded uncertainty, which is unavoidably conservative.

An alternative based on the linear matrix inequality approach has gained popularity since the development of the interior point algorithm for convex optimization. In [13], [4], [7], [23], the LMI approach has been applied to solve the robust H_2 or H_∞ filtering for systems with norm-bounded uncertainty or integral quadratic constraints (IQCs). These works do not require searching for scaling parameters but still apply a fixed Lyapunov function.

To reduce the design conservatism, there have been many attempts in the past few years. In [3], a Lyapunov function which is a linear or quadratic function of the uncertain parameter vector has been developed. In [14], [11], an additional parameter-dependent matrix variable has been introduced which allows the Lyapunov function to be vertex-dependent. This technique has been effective in alleviating the design conservatism significantly. Indeed, a less conservative design using this technique has been recently demonstrated in [17], [10]. On the other hand, it is noted that the result of [14], [11] is a special case of the robust stabilization result given in [15]. This means that there is still much potential to be explored for achieving better performance than that by [17], [10].

In this paper we extend the result of [15] to deal with robust filtering problems for discrete-time uncertain systems. It is shown that as compared to existing results, two additional scaling parameters can be introduced for robust H_2 filtering and three additional scaling parameters and one matrix variable for robust H_∞ filtering. These additional freedoms indeed lead to a less conservative design than those of [17], [10]. Our proposed approach will reduce to the results of [17], [10] when those free parameters are set to be zero. Hence, it is clear that a better filter can be designed by optimization over these parameters. An iterative approach is also proposed for further refinement of the robust H_2 filter. Numerical examples clearly demonstrate the less conservatism of the proposed design. In particular, the iterative procedure indeed gives much improvement on filtering performance in some applications.

II. PROBLEM FORMULATION

Consider the following asymptotically stable system:

$$x_{k+1} = Ax_k + Bw_k, \quad x_0 = 0 \tag{1}$$

$$y_k = Cx_k + Dw_k \tag{2}$$

$$z_k = L_1x_k + L_2w_k \tag{3}$$

where $x_k \in \mathcal{R}^n$ is the system state vector, $y_k \in \mathcal{R}^r$ is the measurement, $z_k \in \mathcal{R}^p$ is the signal to be estimated and $w_k \in \mathcal{R}^m$ is the noise input.

Note that for the case when the process noise and input noise are different (usually so in practice), say w_{1k} and w_{2k}, we can simply put $B = [B_1 \quad 0]$, $D = [0 \quad D_1]$ and let $w_k = [w_{1k}^T \quad w_{2k}^T]^T$ in the system model (1)-(2).

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The matrices A, B, C, D, L_1 and L_2 are appropriately dimensioned with partially unknown parameters. They belong to the following uncertainty polytope:

$$\begin{aligned}\Omega &= \{(A,B,C,D,L_1,L_2) \mid (A,B,C,D,L_1,L_2) = \\ &\sum_{i=1}^{N} \alpha_i (A^{(i)}, B^{(i)}, C^{(i)}, D^{(i)}, L_1^{(i)}, L_2^{(i)}),\ \alpha_i \geq 0, \\ &\left.\sum_{i=1}^{N} \alpha_i = 1\right\} \end{aligned} \tag{4}$$

We consider a filter of the form for the system (1)-(3):

$$\hat{x}_{k+1} = \hat{A}\hat{x}_k + \hat{B}y_k, \quad \hat{x}_0 = 0; \tag{5}$$

$$\hat{z}_k = \hat{C}\hat{x}_k + \hat{D}y_k \tag{6}$$

where the matrices $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ are to be determined.

The robust H_2 and H_∞ filtering problems to be investigated are stated as follows.

Robust H_2 filtering problem: Assume that the noise input w_k is a Gaussian white noise with zero-mean and unit covariance. Design a filter of the form (5)-(6) such that for all uncertainties belonging to the polytope (4) the filtering error variance, $\mathcal{E}\{[z(t)-\hat{z}(t)]^T[z(t)-\hat{z}(t)]\}$, has a minimum possible upper bound.

Robust H_∞ filtering problem: Assume that $w \in \ell_2[0,\infty)$. Given a prescribed scalar γ, design a filter of the form (5)-(6) such that for all non-zero $w \in \ell_2[0,\infty)$,

$$\|z - \hat{z}\|_2 \leq \gamma \|w\|_2$$

over the entire polytope Ω.

Remark 1: Robust H_2 and H_∞ filtering problems have received a lot of interest in the past decades. There are basically two approaches to the problems, namely, the Riccati equation approach and the LMI approach. The former was commonly adopted in dealing with norm-bounded uncertainty in the early stage of development; see [20], [16], [18], [21]. Recently, there have been many interests in the LMI approach mainly due to its numerical capability in handling more general type of uncertainty such as polytopic uncertainty and solving multi-objective filtering problems; see [13], [10], [11], [17], [19]. In particular, a parameter dependent Lyapunov function based approach has been proposed for the robust H_2 filtering in [17], [10] for the case where a strictly proper filter is considered. Since these results are all sufficient, attempts are being made towards improving the conservativeness of the design. Motivated by the work of [15], in this paper we present an improved design method for the problems.

III. Robust H_2 Filter

This section first presents a less conservative analysis result for evaluating the upper bound of the H_2 norm of uncertain discrete-time systems. Additional free parameters (slack variables) are introduced in the result which help reduce the conservatism of the evaluation. We then apply the result to derive a less conservative design for the robust H_2 filter.

First, denote $\xi = [x^T \ \ \hat{x}^T]^T$. It follows from (1)-(3) and (5)-(6) that

$$\xi_{k+1} = \bar{A}\xi_k + \bar{B}w_k \tag{7}$$

$$z_k - \hat{z}_k = \bar{C}\xi_k + \bar{D}w_k \tag{8}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ \hat{B}C & \hat{A} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ \hat{B}D \end{bmatrix} \tag{9}$$

$$\bar{C} = \begin{bmatrix} L_1 - \hat{D}C & -\hat{C} \end{bmatrix}, \quad \bar{D} = L_2 - \hat{D}D \tag{10}$$

Recall that when the matrices $(\bar{A}, \bar{B}, \bar{C}, \bar{D})$ are known, the H_2 norm of the system (7)-(8) can be computed by the following minimization:

$$\min_P trace(\bar{C}P\bar{C}^T + \bar{D}\bar{D}^T) \tag{11}$$

subject to

$$\bar{A}P\bar{A}^T - P + \bar{B}\bar{B}^T < 0 \tag{12}$$

Note that (12) is equivalent to

$$\begin{bmatrix} \bar{A} & \bar{B} \end{bmatrix} diag\{P, I\} \begin{bmatrix} \bar{A}^T \\ \bar{B}^T \end{bmatrix} - P < 0 \tag{13}$$

or

$$\begin{bmatrix} \bar{A} & \bar{B} \end{bmatrix}^T Q \begin{bmatrix} \bar{A} & \bar{B} \end{bmatrix} - diag\{Q, I\} < 0 \tag{14}$$

where $Q = P^{-1}$.

The following lemma can be established using a similar argument as in [15].

Lemma 1: There exists a matrix $Q = Q^T > 0$ to (14) if and only if there exists a solution (F, Q, G) with $Q = Q^T$ such that

$$\begin{bmatrix} -diag\{Q, I\} + \begin{bmatrix} \bar{A}^T \\ \bar{B}^T \end{bmatrix} F + F^T[\bar{A} \ \ \bar{B}] & * \\ -F + G^T[\bar{A} \ \ \bar{B}] & Q - G - G^T \end{bmatrix} < 0 \tag{15}$$

Proof The proof is rather straightforward. First, if (14) holds for some $Q > 0$, by setting $F = 0$ and $G^T = G = Q$ and applying the Schur complement, (15) is satisfied. On the other hand, if (15) holds for some (F, Q, G), multiplying (15) from the left and from the right by Γ^T and Γ, respectively, where $\Gamma^T = \begin{bmatrix} I & [\bar{A} \ \ \bar{B}]^T \end{bmatrix}$, (14) follows.

Remark 2: When the matrices $(\bar{A}, \bar{B})$ are known, the above result implies the equivalence between (14) and (15). However, if the matrices $(\bar{A}, \bar{B})$ are from an uncertain polytope, (15) would render a less conservative evaluation of the upper-bound of the H_2 norm of the system (7)-(8) due to the freedom given by the slack variables F and Q and the fact that Q is allowed to vertex-dependent in (15). We note that when setting $F = 0$, Lemma 1 reduces to that in [17], [10]. This additional matrix variable will enable us to derive a less conservative design than that of [17], [10].

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Lemma 2: Given the filter $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$, an upper bound of the H_2 norm of the error system (7)-(8) can be evaluated by the following optimization

$$\min_{(F,G,Q^{(i)},i=1,2,\cdots,N)} trace(S)$$

subject to

$$\begin{bmatrix} -diag\{Q^{(i)}, I\} + \begin{bmatrix} \bar{A}^{(i)T} \\ \bar{B}^{(i)T} \end{bmatrix} F + F^T[\bar{A}^{(i)} \quad \bar{B}^{(i)}] & -F^T + \begin{bmatrix} \bar{A}^{(i)T} \\ \bar{B}^{(i)T} \end{bmatrix} G \\ -F + G^T[\bar{A}^{(i)} \quad \bar{B}^{(i)}] & Q^{(i)} - (G + G^T) \end{bmatrix} < 0 \quad (16)$$

and

$$\begin{bmatrix} S & \bar{C}^{(i)} & \bar{D}^{(i)} \\ \bar{C}^{(i)T} & Q^{(i)} & 0 \\ \bar{D}^{(i)T} & 0 & I \end{bmatrix} > 0 \quad (17)$$

for $i = 1, 2, \cdots, N$, where $\bar{A}^{(i)}$, $\bar{B}^{(i)}$, $\bar{C}^{(i)}$ and $\bar{D}^{(i)}$ are the matrices in (9)-(10) at the $i-th$ vertex of the polytope Ω.

Proof: First, observe that a convex combination of (16) at all the vertices of Ω implies (15) for $Q = \sum \alpha_i Q^{(i)}$ by considering the fact that $[\bar{A} \quad \bar{B}] = \sum \alpha_i [\bar{A}^{(i)} \quad \bar{B}^{(i)}]$. Hence, by Lemma 1, (14) holds for $Q = \sum \alpha_i Q^{(i)}$. Similarly, (17) implies by the Schur complement that $\bar{C}(\sum \alpha_i Q^{(i)})^{-1}\bar{C}^T + \bar{D}\bar{D}^T < S$. Hence, the optimization in Lemma 2 is equivalent to that in (11) subject to (14) with $P = (\sum \alpha_i Q^{(i)})^{-1}$.

The proof of Lemma 2 clearly demonstrates the use of a parameter-dependent Lyapunov function.

While the above is useful for evaluating the H_2 norm bound for the error system (7)-(8) when a filter (5)-(6) is given, it may not be directly applicable to the robust H_2 filter design problem due to the presence of the products of F with $\bar{A}^{(i)}$ and G with $\bar{A}^{(i)}$. To enable the sub-optimal robust H_2 filter design, we specialize the matrix F as follows:

$$F = \begin{bmatrix} \Lambda G & 0_{2n\times m} \end{bmatrix} \quad (18)$$

where $\Lambda = diag\{\lambda_1 I_n, \lambda_2 I_n\}$ with λ_1 and λ_2 being real scalars.

Using the above F, (16) can be rewritten as

$$\begin{bmatrix} -Q^{(i)} + \bar{A}^{(i)T}\Lambda G + G^T\Lambda\bar{A}^{(i)} & G^T\Lambda\bar{B}^{(i)} & -G^T\Lambda + \bar{A}^{(i)T}G \\ \bar{B}^{(i)T}\Lambda G & -I & \bar{B}^{(i)T}G \\ -\Lambda G + G^T\bar{A}^{(i)} & G^T\bar{B}^{(i)} & Q^{(i)} - (G + G^T) \end{bmatrix} < 0 \quad (19)$$

The following result gives a solution to the robust H_2 filtering problem.

Theorem 1: Consider the system (1)-(3) over the polytope (4). A filter of the form (5)-(6) that gives a suboptimal guaranteed filtering error covariance bound can be derived from the following optimization

$$\min_{(R,W,S_A,S_B,S_C,T,\hat{D},\hat{Q}_{11}^{(i)},\hat{Q}_{12}^{(i)},\hat{Q}_{22}^{(i)},i=1,2,\cdots N,\lambda_1,\lambda_2)} trace(S)$$

subject to

$$\begin{bmatrix} \lambda_1(A^{(i)T}R + R^TA^{(i)}) - \hat{Q}_{11}^{(i)} & * & * & * & * \\ \lambda_1 W^TA^{(i)} + \lambda_2(S_BC^{(i)} + S_A) - \hat{Q}_{12}^{(i)T} & -\lambda_2(S_A + S_A^T) - \hat{Q}_{22}^{(i)} & * & * & * \\ \lambda_1 B^{(i)T}R & \lambda_1 B^{(i)T}W + \lambda_2 D^{(i)T}S_B^T & -I & * & * \\ R^TA^{(i)} - \lambda_1 R & -\lambda_1 W - \lambda_2 T^T & R^TB^{(i)} & \hat{Q}_{11}^{(i)} - (R + R^T) & * \\ W^TA^{(i)} + S_BC^{(i)} + S_A & -S_A + \lambda_2 T^T & W^TB^{(i)} + S_BD^{(i)} & \hat{Q}_{12}^{(i)T} - (W^T + T) & \hat{Q}_{22}^{(i)} + (T + T^T) \end{bmatrix} < 0 \quad (20)$$

and

$$\begin{bmatrix} S & * & * & * \\ L_1^{(i)T} - C^{(i)T}\hat{D}^T - S_C^T & \hat{Q}_{11}^{(i)} & * & * \\ S_C^T & \hat{Q}_{12}^{(i)T} & \hat{Q}_{22}^{(i)} & * \\ L_2^{(i)T} - D^{(i)T}\hat{D}^T & 0 & 0 & I \end{bmatrix} > 0 \quad (21)$$

for $i = 1, 2, \cdots, N$. The suboptimal filter is given by

$$\hat{A} = T^{-1}S_A, \quad \hat{B} = T^{-1}S_B, \quad \hat{C} = S_C, \quad \hat{D} = \hat{D} \quad (22)$$

Remark 3: Observe that for given λ_1 and λ_2, (20) and (21) are linear in $R, W, S_A, S_B, S_C, T, \hat{D}, \hat{Q}_{11}^{(i)}, \hat{Q}_{12}^{(i)}$ and $\hat{Q}_{22}^{(i)}$, and hence can be solved by employing the LMI Tool [8]. The problem is then how to find the optimal values of λ_1 and λ_2 in order to minimize the filtering error variance bound. One way is to apply numerical optimization such as the program **fminsearch** in the optimization toolbox of Matlab [2]. This can be done by first solving a feasibility problem of (20)-(21). Then, apply the **fminsearch** to obtain local optimal scaling parameters λ_1 and λ_2.

Remark 4: It should be mentioned that when $\lambda_1 = \lambda_2 = 0$, Theorem 1 recovers the existing results in [17], [10] where the signal to be estimated does not explicitly contain the input noise w and a strictly proper filter is adopted. It is thus expected that the result in Theorem 1 should be less conservative due to the extra degrees of freedom in optimization, which will be verified through a number of numerical examples in Section 5.

Note that Theorem 1 has been derived with a specialized matrix F of the form (18) in order to linearize the matrix inequality. This, however, is restrictive. In the following we will develop an iterative algorithm which can be applied to refine the filter designed using Theorem 1.

To this end, we denote

$$\overline{AB}^{(i)} = \begin{bmatrix} A^{(i)} & 0 & B^{(i)} \\ 0 & 0 & 0 \end{bmatrix}, \quad \widehat{AB} = [\hat{A} \quad \hat{B}],$$
$$\overline{CD} = \begin{bmatrix} 0 & I & 0 \\ C^{(i)} & 0 & D^{(i)} \end{bmatrix}. \quad (23)$$

Then, (16) can be rewritten as

$$\begin{bmatrix} -diag\{Q^{(i)}, I\} + \Xi + \Xi^T & * \\ -F + G^T\overline{AB} + G^T\begin{bmatrix} 0 \\ I \end{bmatrix}\widehat{AB}\overline{CD}^{(i)} & Q^{(i)} - (G + G^T) \end{bmatrix} < 0 \quad (24)$$

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where $\Xi = \overline{AB}^{(i)T} F + \overline{CD}^{(i)T} \widehat{AB}^{T} [0 \quad I] F$. The following iterative procedure can be applied:

Step 1: Given the filter parameters $(\hat{A}, \hat{B})$, F, G and $Q^{(i)}$ may be found by minimizing $tr(S)$ subject to (16) and (17). The initial $(\hat{A}, \hat{B})$ can be the suboptimal filter designed by Theorem 1.

Step 2: With the F, G and $Q^{(i)}$ obtained in Step 1, an improved filter can be obtained by minimizing $tr(S)$ subject to (24) and (17).

Repeat the above steps until $trace(S_{k-1} - S_k) < \mu$, where μ is a prescribed tolerance and S_k is the matrix S of (17) at the k-th iteration.

It should be emphasized that the above iteration always converges.

IV. Robust H_∞ Filtering

Recall that when the system (7) and (8) is known, it is stable with its H_∞ norm less than γ if and only if there exists a matrix $P = P^T > 0$ such that [24], [22]

$$\tilde{A}^T diag\{P, I\} \tilde{A} - diag\{P, \gamma^2 I\} < 0 \tag{25}$$

where

$$\tilde{A} = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}$$

Without loss of generality, we shall assume $m = p$, i.e. the input and the signal to be estimated have the same dimension. Note that, if this is not the case, some simple modification can render the requirement satisfied. For example, if $m < p$, the matrices B and D can be extended as $B' = [B \quad 0_{n\times(p-m)}]$ and $D' = [D \quad 0_{p\times(p-m)}]$.

Similar to the derivation of Lemma 1, we have the following result.

Lemma 3: When the system (7) and (8) is known, it is stable with its H_∞ norm less than γ if and only if there exist matrices (P, F, G) with $P = P^T$ such that

$$\begin{bmatrix} -diag\{P, \gamma^2 I\} + \tilde{A}^T F + F^T \tilde{A} & -F^T + \tilde{A}^T G \\ -F + G^T \tilde{A} & diag\{P, I\} - (G + G^T) \end{bmatrix} < 0 \tag{26}$$

Note that any solution P of (26) must be positive definite since (26) implies (25) which clearly indicates the positive definiteness of P.

In order to facilitate the robust H_∞ filter design, we need to consider a special case of the above lemma. To this end, we specify the matrices F and G as follows:

$$F = \begin{bmatrix} \Lambda\Phi & 0 \\ 0 & \varepsilon\Psi \end{bmatrix}, \quad G = \begin{bmatrix} \Phi & 0 \\ 0 & \Psi \end{bmatrix} \tag{27}$$

where $\Phi \in \mathcal{R}^{n\times n}$, $\Psi \in \mathcal{R}^{p\times p}$ and $\Lambda = diag\{\lambda_1 I_n, \lambda_2 I_n\}$ with λ_1 and λ_2 being any real numbers.

Remark 5: When setting $\lambda_1 = \lambda_2 = \varepsilon = 0$ (i.e. setting $F = 0$) and $\Psi = I$ and by some row-column exchanges, the above inequality reduces to

$$\begin{bmatrix} P - \Phi - \Phi^T & \Phi^T \bar{A} & \Phi^T \bar{B} & 0 \\ \bar{A}^T \Phi & -P & 0 & \bar{C}^T \\ \bar{B}^T \Phi & 0 & -\gamma^2 I & \bar{D}^T \\ 0 & \bar{C} & \bar{D} & -I \end{bmatrix} < 0$$

which is the result of [11]. Therefore, our result in Lemma 3 should lead to a less conservative result due to the additional freedoms given by the scalars λ_1, λ_2, ε and the matrix variable Ψ.

The following result gives a solution to the robust H_∞ filtering.

Theorem 2: Consider the system (1)-(3) over the polytope (4). A filter of the form (5)-(6) that solves the robust H_∞ filtering problem exists if for some λ_1, λ_2 and ε, there exists a solution $(\hat{P}_{11}^{(i)}, \hat{P}_{12}^{(i)}, \hat{P}_{22}^{(i)}, R, W, S_A, S_B, S_C, S_D, T, \Psi, \hat{D})$ to the following LMIs:

$$\begin{bmatrix} \lambda_1(A^{(i)T}R + R^T A^{(i)}) - \hat{P}_{11}^{(i)} & * & * & * & * & * \\ \lambda_1 W^T A^{(i)} + \lambda_2(S_B C^{(i)} + S_A) - \hat{P}_{12}^{(i)T} & -\lambda_2(S_A + S_A^T) - \hat{P}_{22}^{(i)} & * & * & * & * \\ \varepsilon(\Psi^T L_1^{(i)} - S_D C^{(i)} - S_C) + \lambda_1 B^{(i)T} R & \varepsilon S_C + \lambda_1 B^{(i)T} W + \lambda_2 D^{(i)T} S_B^T & \Pi_1 & * & * & * \\ -\lambda_1 R + R^T A^{(i)} & -\lambda_1 W - \lambda_2 T^T & R^T B^{(i)} & \hat{P}_{11}^{(i)} - (R + R^T) & * & * \\ W^T A^{(i)} + S_B C^{(i)} + S_A & \lambda_2 T^T - S_A & W^T B^{(i)} + S_B D^{(i)} & \hat{P}_{12}^{(i)T} - (W^T + T) & \Pi_2 & * \\ \Psi^T L_1^{(i)} - S_D C^{(i)} - S_C & S_C & -\varepsilon\Psi + \Psi^T L_2^{(i)} - S_D D^{(i)} & 0 & 0 & \Pi_3 \end{bmatrix} < 0 \tag{28}$$

for $i = 1, 2, \cdots, N$, where $\Pi_1 = -\gamma^2 I + \varepsilon(\Psi^T L_2^{(i)} + L_2^{(i)T}\Psi - S_D D^{(i)} - D^{(i)T} S_D^T)$, $\Pi_2 = \hat{P}_{22}^{(i)} + (T + T^T)$, $\Pi_3 = I - (\Psi + \Psi^T)$. In this situation, a suitable H_∞ filter is given by

$$\hat{A} = T^{-1} S_A, \ \hat{B} = T^{-1} S_B, \ \hat{C} = \Psi^{-T} S_C, \ \hat{D} = \Psi^{-T} S_D \tag{29}$$

Remark 6: Observe that for given λ_1, λ_2 and ε, (29) is linear in $(\hat{P}_{11}^{(i)}, \hat{P}_{12}^{(i)}, \hat{P}_{22}^{(i)}, R, W, S_A, S_B, S_C, S_D, T, \Psi, \hat{D})$ and can be solved by convex optimization. As for the problem of choosing appropriate scaling parameters λ_1, λ_2 and ε, a similar procedure as mentioned in Remark 3 can be applied.

V. Illustrative Examples

A. The example in [21]

Consider the example in [21]:

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 0 & -0.5 \\ 1 & 1+\delta \end{bmatrix} x_k + \begin{bmatrix} -6 \\ 1 \end{bmatrix} w_k && (30) \\ y_k &= [-100 \quad 10] x_k + v_k && (31) \\ z_k &= [1 \quad 0] x_k && (32) \end{aligned}$$

where w_k and v_k are uncorrelated zero-mean white noise signals with unit variances, respectively. δ is the uncertain parameter satisfying $|\delta| \le \delta_0$, where δ_0 is known to be a positive real number. We consider three cases $\delta_0 = 0.3, 0.4, 0.45$ respectively. The suboptimal upper bound of the filtering error variances are shown in Table 1.

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	$\delta_0 = 0.3$	$\delta_0 = 0.4$	$\delta_0 = 0.45$
First method in [17]	52.17	63.54	86.05
Iterative refinement in [17]	51.40	58.78	72.97
Proposed Theorem 1	51.59	58.95	65.39
Proposed iterative method	51.43	57.57	61.31

TABLE I
PERFORMANCE COMPARISON BETWEEN DIFFERENT METHODS

From the results shown in Table 1, it can be seen that the guaranteed filtering error bounds based upon the improved method of this paper are smaller than those based upon the first method in [17] for all the three cases. And the bounds obtained by the iterative method of this paper are also smaller than those by the iterative method in [17]. Our methods give better upper bounds for the guaranteed error covariance than the methods in [17] especially when the bound of the uncertainty becomes larger. For the case of $\delta_0 = 0.45$, the minimum bound of 65.39 is obtained by using Theorem 1 for $\lambda_1 = -0.8$ and $\lambda_2 = 0.2$ and the filter parameters are given by

$$\hat{A} = \begin{bmatrix} -0.2506 & 0.0062 \\ -1.5257 & 0.8484 \end{bmatrix}, \hat{B} = \begin{bmatrix} -0.0108 \\ -0.0039 \end{bmatrix},$$
$$\hat{C} = [0.6818 \quad -0.0172].$$

Starting from the above filter parameters, we can employ the iterative method and get the minimum bound of 61.31.

It should be pointed out that the DLMI method in [17] can get a better performance. It, however, only guarantees that the bound will be valid at the vertices of the uncertain polytope and the estimator performance should be checked at all the points within the ploytope.

B. The example in [10]

Consider the discrete-time system in the form of (1)-(3) with [10]

$$A = \begin{bmatrix} 0.9 & 0.1 + 0.06\alpha \\ 0.01 + 0.05\beta & 0.9 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$C = [1 \quad 0], D = [0 \quad 0 \quad 1.414], L_1 = [1 \quad 1], L_2 = 0$$

where $|\alpha| \leq 1$ and $|\beta| \leq 1$. The value of the H_2 guaranteed cost based upon the method in [10] is 44.0039. Using Theorem 1, we can achieve the H_2 guaranteed cost value of 24.7750 for $\lambda_1 = -0.99$ and $\lambda_2 = -0.99$. The resultant filter is given by

$$\hat{A} = \begin{bmatrix} -0.0829 & 0.0031 \\ 3.0133 & 0.4404 \end{bmatrix}, \hat{B} = \begin{bmatrix} 1.0542 \\ -2.5541 \end{bmatrix},$$
$$\hat{C} = [1.3263 \quad 0.2377].$$

Based upon the above filter and the iterative algorithm, a much less conservative minimum bound of 14.64 can be obtained and the filter is given by

$$\hat{A} = \begin{bmatrix} -0.0823 & 0.0030 \\ 3.0097 & 0.4410 \end{bmatrix}, \hat{B} = \begin{bmatrix} 1.0536 \\ -2.5510 \end{bmatrix},$$
$$\hat{C} = [1.3263 \quad 0.2377].$$

The actual performance of the resultant filter obtained by the iterative method is shown in Figure 2. It is clear from the figure that the obtained upper bound is not conservative.

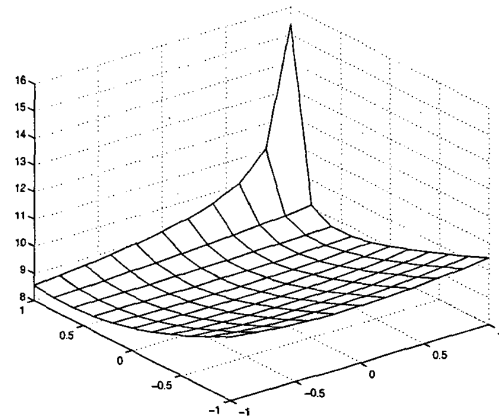

Fig. 1. Actual bound versus the uncertain parameters α and β

VI. CONCLUSION

This paper has addressed the robust filtering of discrete-time linear uncertain systems with polytopic uncertainty. Based on a parameter dependent Lyapunov function approach, we present less conservative designs of H_2 and H_∞ filters in terms of improved LMIs than existing approaches. The improved LMIs contain a number of slack variables which offer additional flexibility in optimization. The solution of LMIs, if exists, provides a robust filter with a minimum upper bound to the variance of the filtering error or a guaranteed H_∞ level of noise attenuation over the entire uncertainty polytope. We also proposed an iterative approach to further improve the filter performance. A comparison has been made with existing results.

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