Decoding Generalized Joint Channel Coding and Physical Network Coding in the LLR Domain

Pingping Chen, Yi Fang, Lin Wang, *Senior Member, IEEE* and Francis C. M. Lau, *Senior Member, IEEE*

***Abstract*—This paper develops a log-likelihood-ratio-based decoder for a generalized joint channel-coding-and-physical-network coding used in a two-way relay system. Assuming that the same low-density parity-check codes are used at the source nodes, the proposed decoder shows identical error performance as the probability-domain-based decoder but with a much lower computational complexity.**

***Index Terms*—Complexity, joint channel-coding-and-physical-network coding, log-likelihood ratio, two-way relay**

I. Introduction

Physical network coding (PNC) has been recently proposed as an effective capacity-boosting approach for two-way relay systems [1], in which two source nodes A and B wish to exchange information with each other via a relay R. First, both sources transmit their information to the relay simultaneously. Based on the superpositioned signal received, the relay estimates the XOR value between the information from A and that from B. Then, the relay broadcasts the result to the sources.

In the presence of channel coding, several joint channel-and-physical-network-coding (JCNC) schemes have been developed for such systems [2], [3]. We assume that the same linear channel coding scheme is employed at the two sources. The XOR value of the codewords from the two sources is still a valid codeword. Consequently, the XOR-codeword can be directly decoded from the received signal using the same decoding algorithm. For example, a generalized JCNC (GJCNC) scheme has been proposed to decode such codewords at the relay [4], [5], [6]. In addition, the performance of the GJCNC scheme using quadrature-phase-shift-keying (QPSK) modulation has been investigated [7]. When the sum-product algorithm (SPA) [8] in the probability domain (PD) is used in the decoding process, superior bit-error-rate (BER) performance of the GJCNC scheme has been demonstrated. However, to the best of our knowledge, all the previous GJCNC schemes are based on SPA in the PD, which is known to involve lots of complex multiplications.

In this paper, we apply the SPA in the log-likelihood-ratio (LLR) domain to the decoding of the GJCNC scheme. We consider both the additive-white-Gaussian-noise (AWGN) channel and the Rayleigh fading channel. We show that the BER performance of the GJCNC scheme remains the same whether decoding is performed in the PD (GJCNC-PD) or in the LLR domain (GJCNC-LLR). We further compare the decoding complexity of the GJCNC scheme in the two different domains under the aforementioned channel conditions. We conclude that the GJCNC scheme has a considerably lower decoder complexity in the LLR domain compared with the PD, especially under the Rayleigh channels. We expect that when higher-order modulation is used [7], the decoder complexity in the LLR domain will be significantly lower than that in the PD.

II. Two-way Relay System

We consider a two-way relay system with two sources (A and B) and one relay (R). Both sources are to obtain information from each other with the assistance of the relay. Let $\mathbf{b}_{\mathrm{A}}$ and $\mathbf{b}_{\mathrm{B}}$, each of length M, denote the binary information vectors of A and B, respectively. Assume that $\mathbf{b}_{\mathrm{A}}$ and $\mathbf{b}_{\mathrm{B}}$ are encoded by the same linear channel code to form the codewords $\mathbf{c}_{\mathrm{A}}$ and $\mathbf{c}_{\mathrm{B}}$, each of length N, i.e., $\mathbf{c}_{\mathrm{A}}, \mathbf{c}_{\mathrm{B}} \in \{0,1\}^N$. Hence the code rate equals $R_c = M/N$. Afterward, the codewords are binary-phase-shift-keying (BPSK) modulated, i.e., $0 \to 1, 1 \to -1$, into $\mathbf{x}_{\mathrm{A}}$ and $\mathbf{x}_{\mathrm{B}}$, which are transmitted to the relay simultaneously over AWGN channels or Rayleigh channels. In the following section, we briefly review the GJCNC-PD scheme over AWGN channels.

A. System Model of PNC

The received signal $\mathbf{y}_{\mathrm{R}}$ at the relay is the linear superposition of the two transmitted signals plus noise and is thus given by

$$\mathbf{y}_{\mathrm{R}} = \mathbf{x}_{\mathrm{A}} + \mathbf{x}_{\mathrm{B}} + \eta \tag{1}$$

where η denotes the noise vector whose elements are independent and identically distributed (i.i.d.) zero-mean Gaussian random variables with variance σ_η^2. Based on $\mathbf{y}_{\mathrm{R}}$, the relay estimates $\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}} = \mathbf{c}_{\mathrm{A}} \oplus \mathbf{c}_{\mathrm{B}}$, i.e., the XOR output of the two source codewords $\mathbf{c}_{\mathrm{A}}$ and $\mathbf{c}_{\mathrm{B}}$. Since the same linear encoding scheme is used at the two sources, $\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}$ is a valid codeword and can be decoded with the same algorithm used for decoding $\mathbf{c}_{\mathrm{A}}$ and $\mathbf{c}_{\mathrm{B}}$. Subsequently, $\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}$ is BPSK-modulated, which is broadcasted to the sources in the broadcast stage. The decoding performance at the relay is critical to the error performance of the two-way transmission mechanism [4].

B. GJCNC-PD Decoding over AWGN Channels

Recall that $\mathbf{c}_{\mathrm{A}}, \mathbf{c}_{\mathrm{B}} \in \{0,1\}^N$ and $\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}} = \mathbf{c}_{\mathrm{A}} \oplus \mathbf{c}_{\mathrm{B}}$. We further define $\mathbf{c} = \mathbf{c}_{\mathrm{A}} + \mathbf{c}_{\mathrm{B}}$ where $+$ represents the mathematical addition operator. We also use $n = 1, 2, \ldots, N$ as the index of an element in a vector. Consequently, $\mathbf{c}(n) = 0$ or 2 implies $\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}(n) = 0$ while $\mathbf{c}(n) = 1$ implies $\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}(n) = 1$. As explained in [4], the a-priori probabilities of $\mathbf{c}(n) = 0, 1$ and 2 are $1/4, 1/2$ and $1/4$, respectively. For conciseness, we denote g_i as the probability that a certain $\mathbf{c}(n)$ equals $0, 1$ and 2. Further, we define $\mathbf{g} = [g_0, g_1, g_2]$. Assuming that the same low-density parity-check (LDPC) code is used at the two sources, $\mathbf{g}$ will be passed between the variable nodes (VNs) and the check nodes (CNs) during the decoding process at the relay [4], [5], [6]. In the PD, the decoding process is proceeded as follows.

1) Initialization: For the received signal $\mathbf{y}_{\mathrm{R}}(n)$, the channel messages for the n-th VN are computed by [4]

$$g_i = \begin{cases} \frac{1}{4\sqrt{2\pi}\beta} \exp\left(-\frac{[\mathbf{y}_{\mathrm{R}}(n)+2(i-1)]^2}{2\sigma_\eta^2}\right) & \text{for } i = 0, 2 \\ \frac{1}{2\sqrt{2\pi}\beta} \exp\left(-\frac{[\mathbf{y}_{\mathrm{R}}(n)]^2}{2\sigma_\eta^2}\right) & \text{for } i = 1 \end{cases} \tag{2}$$

where β is a normalization factor that ensures $\sum_{i=0}^{2} g_i = 1$. Note that for a punctured VN, the channel-message vector is set to $\mathbf{g} = [0.25, 0.5, 0.25]$. The message vectors are then passed to the connected CNs. We denote the updating rules at the CNs and at the VNs by CHK and VAR, respectively.

2) Message out of check nodes: Suppose a CN is connected to three VNs. Denote the incoming messages from two VNs by $\mathbf{p} = [p_0, p_1, p_2]$ and $\mathbf{q} = [q_0, q_1, q_2]$. Then the outgoing message to the third VN, denoted by $\mathbf{u} = [u_0, u_1, u_2]$, is given by [4, Eq.(21)]

$$\begin{aligned} \mathbf{u} =& \,\mathrm{CHK}(\mathbf{p}, \mathbf{q}) \\ =& [p_0q_0 + p_2q_2 + \frac{1}{2}p_1q_1, p_1q_2 + p_2q_1 + p_1q_0 + p_0q_1, \\ & p_0q_2 + p_2q_0 + \frac{1}{2}p_1q_1] \end{aligned} \tag{3}$$

The outgoing messages for the other VNs can be computed in a similar way.

Suppose the CN has a degree of $d_c > 3$. Denoting the first d_c-1 incoming messages by $\mathbf{p}, \mathbf{q}, \mathbf{r}, \ldots$, the outgoing message to the d_c-th connected VN can then be computed recursively using

$$\mathbf{u} = \mathrm{CHK}(\mathbf{p}, \mathbf{q}, \mathbf{r}, \ldots) = \mathrm{CHK}(\mathbf{p}, \mathrm{CHK}(\mathbf{q}, \mathrm{CHK}(\mathbf{r}, \ldots))). \tag{4}$$

3) Message out of variable nodes: Assume that the n-th VN is connected to two CNs. Denote the channel message by $\mathbf{g}$ and the incoming message from the first CN by $\mathbf{p}$. The outgoing message to the second CN, denoted by $\mathbf{v} = [v_0, v_1, v_2]$, is given by [4, Eq.(17)]

$$\mathbf{v} = \mathrm{VAR}(\mathbf{g}, \mathbf{p}) = \gamma[4g_0p_0,\ 2g_1p_1,\ 4g_2p_2] \tag{5}$$

where γ is a normalization factor that ensures $\sum_{i=0}^{2} v_i = 1$.

Suppose the VN has a degree of $d_v > 2$. Denoting the first $d_v - 1$ incoming messages by $\mathbf{p}, \mathbf{q}, \ldots$, the outgoing message to the d_v-th connected CN can then be computed recursively using

$$\mathbf{v} = \mathrm{VAR}(\mathbf{g}, \mathbf{p}, \mathbf{q}, \ldots) = \mathrm{VAR}(\mathbf{g}, \mathrm{VAR}(\mathbf{p}, \mathrm{VAR}(\mathbf{q}, \ldots))). \tag{6}$$

4) PNC mapping: After a fixed number of iterations, the elements in $\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}$ are determined to be 0 or 1. For the n-th VN, the a-posteriori probability vector, denoted by $\mathbf{\Xi} = [\xi_0, \xi_1, \xi_2]$, is evaluated by taking all incoming messages from the CNs as well as the channel message into consideration and substituting them into (6). Then the PNC mapping is performed using the following rule.

$$\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}(n) = \begin{cases} 1 & \text{if } \xi_1 > (\xi_0 + \xi_2) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

III. GJCNC-LLR Decoding over AWGN Channels

In the GJCNC-PD scheme, many multiplication operations are involved in the decoding process and hence the decoding speed is limited. In this section, we describe a GJCNC-LLR decoding scheme that is mathematically equivalent to the GJCNC-PD scheme.

For a given probability vector $\mathbf{g} = [g_0, g_1, g_2]$, we define a corresponding LLR vector

$$\mathbf{g}' = [g_0', g_2'] = \left[\ln\left(\frac{g_0}{g_1}\right), \ln\left(\frac{g_2}{g_1}\right)\right]. \tag{8}$$

Then the proposed GJCNC-LLR decoding scheme can be proceeded as follows.

1) Initialization: Based on (2) and (8), it can be readily shown that the channel LLR messages for the n-th VN can be obtained using

$$g_i' = \frac{-2 - 2(i-1)\mathbf{y}_{\mathrm{R}}(n)}{\sigma_\eta^2}, \quad i = 0, 2. \tag{9}$$

Moreover, for a punctured VN, the channel LLR message vector is set to $\mathbf{g}' = [g_0', g_2'] = [\ln(0.5), \ln(0.5)]$. We denote the updating rules at the VNs and at the CNs by VAR' and CHK', respectively.

2) LLR message out of CNs: We consider a CN of degree 3. We denote the input LLR messages from two VNs by $\mathbf{p}' = [p_0', p_2'] = \left[\ln\left(\frac{p_0}{p_1}\right), \ln\left(\frac{p_2}{p_1}\right)\right]$ and $\mathbf{q}' = [q_0', q_2'] = \left[\ln\left(\frac{q_0}{q_1}\right), \ln\left(\frac{q_2}{q_1}\right)\right]$. Using (3) and (8), the output LLR message $\mathbf{u}'$ from this CN to the third VN can be calculated using

$$\mathbf{u}' = \mathrm{CHK}'(\mathbf{p}', \mathbf{q}') = [u_0', u_2'] \tag{10}$$

where

$$\begin{aligned} u_0' &= \ln\left(\frac{p_0q_0 + p_2q_2 + 0.5p_1q_1}{p_1q_2 + p_2q_1 + p_1q_0 + p_0q_1}\right) \\ &= \ln(e^{p_0'+q_0'} + e^{p_2'+q_2'} + 0.5) - \ln(\Omega) \end{aligned} \tag{11}$$

$$\begin{aligned} u_2' &= \ln\left(\frac{p_0q_2 + p_2q_0 + 0.5p_1q_1}{p_1q_2 + p_2q_1 + p_1q_0 + p_0q_1}\right) \\ &= \ln(e^{p_2'+q_0'} + e^{p_0'+q_2'} + 0.5) - \ln(\Omega) \end{aligned} \tag{12}$$

$$\ln(\Omega) = \ln(e^{p_0'} + e^{p_2'} + e^{q_0'} + e^{q_2'}). \tag{13}$$

The outgoing messages for the other VNs can be computed in a similar way. Furthermore, for CNs with degree larger than 3, the outgoing messages can be evaluated by using (10) recursively (see (4)).

We can see that the operation at the CNs is an extension of the box-plus operator introduced for channel decoding in GF(2) [10]. In addition, (11), (12) and (13) can be computed by using the Jacobi logarithm [11], i.e.,

$$\begin{aligned} \ln(e^{x_1} + e^{x_2}) &= \max{}^*(x_1, x_2) \\ &= \max(x_1, x_2) + \ln(1 + e^{-|x_1-x_2|}) \end{aligned} \tag{14}$$

TABLE I
LOOK-UP TABLE FOR $\ln(1+e^{-|x|})$

$\|x\|$	$\ln(1+e^{-\|x\|})$	$\|x\|$	$\ln(1+e^{-\|x\|})$
[0, 0.196)	0.65	[1.05, 1.508)	0.25
[0.196, 0.433)	0.55	[1.508, 2.252)	0.15
[0.433, 0.71)	0.45	[2.252, 4.5)	0.05
[0.71, 1.05)	0.35	[4.5, $+\infty$)	0.0

and by applying

$$\max{}^*(x_1,x_2,x_3) = \max{}^*(\max{}^*(x_1,x_2),x_3). \quad (15)$$

Note that the $\max^*$ operation includes a $\max$ term adding to a logarithm correction term. In some of our simulation results, we further reduce the computation complexity by realizing the $\ln(1+\bullet)$ term with a small look-up table (LUT) having an input of $|x_1 - x_2|$ [11], as shown in Table I. In such a case, the $\max^*$ operation is implemented using one comparison (for the $\max$ term), one table look-up (for the $\ln(1+\bullet)$ term) and one addition.

In such a case, the $\max^*$ operation is implemented using one comparison (for the $\max$ term), one table look-up (for the $\ln(1+\bullet)$ term) and one addition.

3) LLR message out of VNs: Suppose the n-th VN is connected to two CNs. Denote the channel message by $\mathbf{g}'$ and the incoming message from the first CN by $\mathbf{p}'$. Using (5) and (8), the outgoing LLR message to the second CN is given by

$$\begin{aligned}\mathbf{v}' &= \mathrm{VAR}'(\mathbf{g}',\mathbf{p}')\\ &= [v_0', v_2']\\ &= [\ln(2)+g_0'+p_0', \ln(2)+g_2'+p_2'].\end{aligned} \quad (16)$$

For VNs with degree larger than 2, the outgoing LLR message to each CN can be found by using (16) recursively (see (6)).

4) PNC mapping: For the n-th VN, we define the a-posteriori LLR vector as $\boldsymbol{\Xi}' = [\xi_0', \xi_2'] = \left[\ln\left(\frac{\xi_0}{\xi_1}\right), \ln\left(\frac{\xi_2}{\xi_1}\right)\right]$. Similar to $\boldsymbol{\Xi}$ in Sect. II-B-4), $\boldsymbol{\Xi}'$ is evaluated by taking all incoming LLR messages from the CNs as well as the channel LLR message into consideration and by using (16) recursively. Furthermore, we apply $\ln(e^{\xi_0'}+e^{\xi_2'}) = \max(\xi_0', \xi_2') + \ln(1+e^{-|\xi_0'-\xi_2'|})$ (see (14)) and we assume that $e^{-|\xi_0'-\xi_2'|} \approx 0$ at the end of the iterative process. Thus, combining the above equations with (7), the PNC mapping based on the LLR messages is given by

$$\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}(n) = \begin{cases}1 & \text{if } 0 > \max(\xi_0', \xi_2')\\ 0 & \text{otherwise}\end{cases} \quad (17)$$

IV. GJCNC-LLR DECODING ALGORITHM OVER RAYLEIGH CHANNELS

In this section, we describe the GJCNC-LLR decoding algorithm for the PNC systems over Rayleigh fading channels. We denote the fading coefficient between Source A and Relay R as h_A and that between Source B and R as h_B. In a noiseless environment, four possible signals, i.e., $\mathbf{S}(0) = h_A + h_B, \mathbf{S}(1) = -h_A + h_B, \mathbf{S}(2) = h_A - h_B, \mathbf{S}(3) = -h_A - h_B$, can be received at the relay. As explained in [5], the a-priori probabilities of $\mathbf{S}(i)$ are the same, i.e., $\mathbf{S}(i) = 1/4$ for $i = 0,1,2,3$. When noise η with zero mean and variance σ_η^2 is added, the received signal $\bar{\mathbf{y}}_{\mathrm{R}}(n)$ is given by

$$\bar{\mathbf{y}}_{\mathrm{R}} = h_{\mathrm{A}}\mathbf{x}_{\mathrm{A}} + h_{\mathrm{B}}\mathbf{x}_{\mathrm{B}} + \eta. \quad (18)$$

Given that $\bar{\mathbf{y}}_{\mathrm{R}}(n)$ is received, we denote g_i as the probability that the received signal should be $\mathbf{S}(i)$ ($i = 0,1,2,3$) in the absence of noise. Based on the PD decoding algorithm for Rayleigh channels [5], we derive the GJCNC-LLR decoding algorithm as follows.

1) Message Initialization: Denoting the initial LLR channel message of the n-th VN by $\bar{\mathbf{g}} = [\bar{g}_0, \bar{g}_2, \bar{g}_3] = [\ln(\frac{g_0}{g_1}), \ln(\frac{g_2}{g_1}), \ln(\frac{g_3}{g_1})]$, we have

$$\bar{g}_i = \frac{2(\mathbf{S}(i)-\mathbf{S}(1))\bar{\mathbf{y}}_{\mathrm{R}}(n) + \mathbf{S}^2(1) - \mathbf{S}^2(i)}{2\sigma_\eta^2}, \quad i = 0,2,3. \quad (19)$$

Note that for punctured VNs, the initial LLR message is given by $\bar{\mathbf{g}} = [0,0,0]$.

2) LLR message out of CNs: We consider a degree-3 CN with the first two input LLR messages denoted by $\bar{\mathbf{p}} = [\bar{p}_0, \bar{p}_2, \bar{p}_3]$ and $\bar{\mathbf{q}} = [\bar{q}_0, \bar{q}_2, \bar{q}_3]$. Based on the updating rule in the PD [5, eq.(17)], the outgoing LLR message from the CN to the third VN, denoted by $\bar{\mathbf{u}} = [\bar{u}_0, \bar{u}_2, \bar{u}_3]$, equals

$$\bar{u}_0 = \ln(e^{\bar{p}_0+\bar{q}_0} + e^{\bar{p}_2+\bar{q}_2} + e^{\bar{p}_3+\bar{q}_3} + 1) - \ln(\Lambda) \quad (20)$$
$$\bar{u}_2 = \ln(e^{\bar{p}_2+\bar{q}_0} + e^{\bar{p}_3} + e^{\bar{p}_0+\bar{q}_2} + e^{\bar{q}_3}) - \ln(\Lambda) \quad (21)$$
$$\bar{u}_3 = \ln(e^{\bar{p}_3+\bar{q}_0} + e^{\bar{p}_2} + e^{\bar{p}_0+\bar{q}_3} + e^{\bar{q}_2}) - \ln(\Lambda) \quad (22)$$

where $\Lambda = e^{\bar{p}_0} + e^{\bar{q}_0} + e^{\bar{q}_2+\bar{p}_3} + e^{\bar{q}_3+\bar{p}_2}$. The outgoing LLR messages to the other two VNs can be calculated in a similar manner. Moreover, for CNs with degree larger than 3, we can apply the above calculations recursively. Note that (20) to (22) can also be implemented by the Jacobi logarithm.

3) LLR message out of VNs: Suppose a VN has a degree of 2. If the initial channel LLR message is denoted by $\bar{\mathbf{g}} = [\bar{g}_0, \bar{g}_2, \bar{g}_3]$ and the incoming LLR message from the first CN is denoted by $\bar{\mathbf{p}} = [\bar{p}_0, \bar{p}_2, \bar{p}_3]$, the outgoing LLR message to the second CN, denoted by $\bar{\mathbf{v}} = [\bar{v}_0, \bar{v}_2, \bar{v}_3]$, is given by

$$\bar{v}_i = \bar{g}_i + \bar{p}_i, \; i = 0,2,3. \quad (23)$$

The outgoing LLR message to the other CN can be calculated in a similar manner. Moreover, for VNs with degree larger than 2, we can apply the above calculations recursively.

4) PNC mapping: Let $\bar{\boldsymbol{\Xi}} = [\bar{\xi}_0, \bar{\xi}_2, \bar{\xi}_3]$ denote the a-posteriori LLR message for the n-th VN. Similar to $\boldsymbol{\Xi}$ in Sect. II-B-4), $\bar{\boldsymbol{\Xi}}$ is evaluated by taking all incoming LLR messages from the CNs as well as the channel LLR message into consideration and by using (23) recursively. Finally, $\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}(n)$ is determined using the following rule.

$$\hat{\mathbf{c}}_{\mathrm{A}\oplus\mathrm{B}}(n) = \begin{cases}1 & \text{if } \max(0, \bar{\xi}_2) > \max(\bar{\xi}_0, \bar{\xi}_3)\\ 0 & \text{otherwise}\end{cases} \quad (24)$$

V. COMPUTATION COMPLEXITY AND ERROR PERFORMANCE

Based on the algorithms described in Sect. II-B to Sect. IV, we evaluate the number of operations required in each decoding iteration in the PD and in the LLR domain under

TABLE II
NUMBER OF OPERATIONS REQUIRED PER ITERATION IN THE DECODING OF THE GJCNC SCHEME. PD: PROBABILITY DOMAIN; LLR: LOG-LIKELIHOOD DOMAIN; VNU: VARIABLE-NODE UPDATING; CNU: CHECK-NODE UPDATING.

	Operation	Addition($+$)	Multiplication($*$)	$\max^*$
AWGN Channels	**PD/VNU**	0	$3N(\bar{d}_v - 2)$	0
	PD/CNU	$7M(\bar{d}_c - 2)$	$10M(\bar{d}_c - 2)$	0
	LLR/VNU	$2N(\bar{d}_v - 2)$	0	0
	LLR/CNU	$6M(\bar{d}_c - 2)$	0	$7M(\bar{d}_c - 2)$
Rayleigh Channels	**PD/VNU**	0	$4N(\bar{d}_v - 2)$	0
	PD/CNU	$12M(\bar{d}_c - 2)$	$16M(\bar{d}_c - 2)$	0
	LLR/VNU	$3N(\bar{d}_v - 2)$	0	0
	LLR/CNU	$10M(\bar{d}_c - 2)$	0	$12M(\bar{d}_c - 2)$

AWGN channels as well as Rayleigh channels. The results are shown in Table II, where $\bar{d}_v$ and $\bar{d}_c$ denote the average degrees of the VNs and the CNs, respectively. We can observe that the GJCNC-PD decoder involves additions and multiplications whereas the GJCNC-LLR decoder involves additions and $\max^*$ operations. In practice, the $\max^*$ operation can be implemented by a comparator (for the $\max$ operation), a small LUT (for the $\ln(1+\bullet)$ function) and an addition. Thus, the computation complexity of the GJCNC-LLR decoder is dominated by the $\mathcal{O}(M\bar{d}_c)$ additions. On the other hand, the computation complexity of the GJCNC-PD decoder is determined by $\mathcal{O}(M\bar{d}_c)$ multiplications. Thus we conclude that our proposed GJCNC-LLR decoder has a much lower implementation complexity than the GJCNC-PD decoder in a fixed point architecture. Note also that the LLR-based decoders are less sensitive to the quantization effect compared to the PD-based decoders [9].

In Fig. 1, we present the simulated BER performance of the GJCNC-PD decoder, GJCNC-LLR decoder and GJCNC-LLR(LUT) decoder. The codes used in our simulations are the accumulate-repeat-by-3-accumulate (AR3A) protograph LDPC codes with punctured VNs [12]. Moreover, the parameters used are $[M,N] = [512, 1024]$ and $[1024, 2048]$. Figure 1 shows that the proposed GJCNC-LLR decoder performs the same as the GJCNC-PD decoder in terms of BER under AWGN channels or Rayleigh channels. In addition, the GJCNC-LLR decoder with a LUT performs equally well as the other decoders under both types of channels.

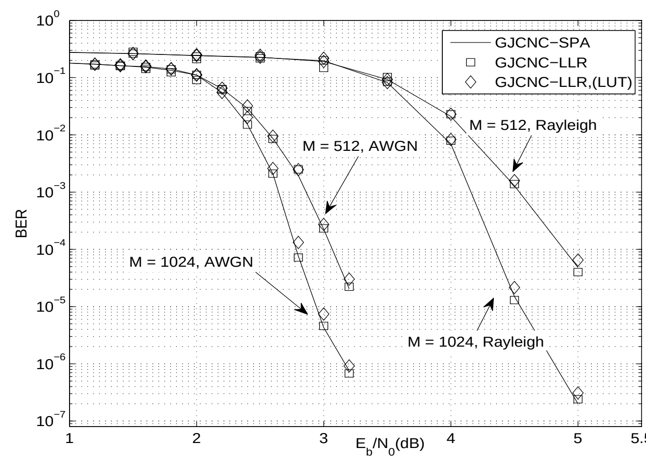

Fig. 1. The BER performance of the GJCNC-PD decoder and the GJCNC-LLR decoder over AWGN channels and Rayleigh channels. Accumulate-repeat-by-3-accumulate (AR3A) protograph LDPC codes with punctured VNs and $[M,N] = [512, 1024]$ and $[1024, 2048]$ are used. In the GJCNC-LLR(LUT) decoder, a look-up table (LUT) is used to implement the $\max^*$ operation.

VI. CONCLUSION

In this paper, a LLR-based decoding algorithm for the GJCNC scheme over two-way relay networks has been developed. The proposed LLR-based decoder can be easily implemented by using hardware to realize the addition operations, $\max$ operations and simple table look-up operations. The error performance of the proposed decoder has been considered and simulated over AWGN channels and Rayleigh channels.

REFERENCES

[1] S. Zhang, S. C. Liew, and P. Lam, "Physical layer network coding," in *Proc. MobiCom*, Los Angeles, USA, 2006, pp. 358–365.

[2] A. Zhan and C. He, "Joint design of channel coding and physical network coding for wireless network," in *Proc. IEEE Int. Conf. on Neural Networks and Signal Processing*, Zhejiang, China, 2008.

[3] S. Zhang and S. C. Liew, "Channel coding and decoding in a relay system operated with physical-layer network coding," *IEEE J. Sel. Areas Commun.,* vol. 27, no. 5, pp. 788–796, Oct. 2009.

[4] Y. Lang and D. Wubben, "Generalized joint channel coding and physical network coding for two-way relay systems," in *Proc. IEEE VTC,* Taipei, Taiwan, May 2010.

[5] D. Wubben and Y. Lang, "Generalized sum-product algorithm for joint channel decoding and physical-layer network coding in two-way relay systems," in *Proc. IEEE GLOBECOM,* Miami, USA, Nov. 2010.

[6] L. Lu and S. C. Liew, "Asynchronous physical-layer network coding," *IEEE Trans. Wireless Commun.,* vol. 11, no. 2, pp. 819–831, Feb. 2012.

[7] D. Wubben, "Joint channel decoding and physical-layer network coding in two-way QPSK relay systems by a generalized sum-product algorithm," in *Proc. ISWCS*, York, United Kingdom, 2010, pp. 576–580.

[8] F. R. Kschischang, B. J. Frey, and H. A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inf. Theory,* vol. 47, no. 2, pp. 498–519, Feb. 2001.

[9] L. Ping and W. K. Leung, "Decoding low density parity check codes with finite quantization bits," *IEEE Comm. Letters,* vol. 4, no. 2, pp. 62–64, Feb. 2000.

[10] J. Hagenauer, E. Offer, C. Measson, and M. Morz, "Decoding and equalization with analog non-linear networks," *European Trans. Comm.,* vol. 10, pp. 659–680, Nov. 1999.

[11] X. Hu, E. Eleftheriou, D. M. Arnold, and A. Dholakia, "Efficient implementations of the sum-product algorithm for decoding LDPC codes," *in Proc. IEEE GLOBECOM,* San Antonio, USA, 2001, pp. 1036–1036.

[12] A. Abbasfar, D. Divsalar, and K. Yao, "Accumulate repeat accumulate codes," in *Proc. IEEE ISIT,* Chicago, USA, June 2004, p. 505.