A Combined Wavelet-FE Method for Transient Electromagnetic-Field Computations

S. Y. Yang[1], S. L. Ho[2], P. H. Ni[1], and G. Z. Ni[1]

[1]EE College of Zhejiang University, Zhejiang, Hangzhou 310027, China
[2]Department of Electrical Engineering, the Hong Kong Polytechnic University, Kowloon, Hong Kong

A combined wavelet-finite-element (wavelet-FE) method is proposed for the computation of transient electromagnetic (EM) fields. In order to retain the essential mathematical properties, such as consistency and linear independence of the shape functions for the proposed method, bridge scales are used to modify the wavelets. To consider the influence of the external voltage supplies, the proposed wavelet-FE method is coupled with a state-space model to develop an integrated simulation approach. Computer simulation results are reported to demonstrate the feasibility of the proposed method for computations of both steady-state and transient EM fields.

***Index Terms*—Meshless method, multiresolution analysis, transient electromagnetic (EM) field, wavelet.**

I. Introduction

RECENTLY, the wavelet method has become an appealing alternative for finding the solutions of partial differential equations of different engineering disciplines by virtue of their localization properties in both special and spectral domains, their vanishing moment properties, and their multiresolution analytical ability. In essence, the salient merits of the wavelet-based method are: 1) it is truly meshless; 2) the multiresolution analytical ability of wavelets is serving as the local means in the development of a hierarchical solution procedure; and 3) the tradeoff between continuity and compact support properties is well balanced. Also, the wavelet has great potential in solving typical electromagnetic (EM) problems [1]–[4]. However, most works related to wavelet methods hitherto are focused on steady-state problems, and it is highly desirable to extend wavelets to the study of transient EM-field problems.

When wavelets are used as shape functions in numerical methods, they do not satisfy the Kronecker delta property and do not vanish on boundaries where the essential boundary conditions are imposed. Consequently, some special strategies are required to enforce the essential boundary conditions. On the other hand, the finite-element (FE) method is very efficient and simple in dealing with boundary conditions. Therefore, it is desirable to combine wavelet and FE methods in the development of a new alternative which inherits the advantages of both methods. Hence, a combined wavelet-FE method is proposed. Of course, the idea of combining FE method with meshless methods that incorporate wavelet-based ones is not new. The novelty of the proposed combination is that the bridge scales are generalized and used to retain the required mathematical properties, such as consistency and linear independence, of the entire shape functions.

II. Combined Wavelet-FE Model and Method

A. Interpolation Using FE Shape Functions and Wavelets

To fully exploit the advantages of wavelet-based methods (i.e., their truly meshless feature) and those of the FE methods (i.e., their simplicity and efficiency in enforcing essential boundary conditions), it is proposed that the FE method be used only to enforce boundary conditions. Therefore, the entire solution domain is divided into two subregions Ω_w and Ω_f^w. Most of the solution domains belong to Ω_w where only wavelets contribute to the approximation of the solution variable. Ω_f^w is the domain corresponding to the very thin layers near the essential boundaries where both FE and wavelets have influences. In the subregion Ω_w, the interpolation of the solution variable is the standard form of the wavelet method. To develop a general interpolation formula in the region Ω_f^w for the solution variable $A(x,y)$ in two dimensions using both FE shape functions and wavelets, one begins with

$$A(x,y)=\sum_i A_i N_i^{\text{fem}}(x,y)+\sum_{i,j} c_{i,j}\varphi_i^J(x)\varphi_j^J(y) \quad (1)$$

where $N_i^{\text{fem}}(x,y)$ is the FE shape function, and φ_i^J is a one-dimensional (1-D) wavelet basis at scale J.

To guarantee the required mathematical properties of the entire bases of wavelets and FE shape functions, such as consistency and linear independence, the bridging scale concept is used to modify the wavelets [5], and (1) becomes

$$A(x,y)=\sum_i A_i N_i^{\text{fem}}(x,y)+\sum_{i,j} c_{i,j}\tilde{\varphi}_{i,j}^J(x,y) \quad (2)$$

where $\tilde{\phi}_{i,j}^J(x,y)$ is the modified wavelets based on the introduction of bridging scales, and are defined as

$$\tilde{\varphi}_{j,k}^J(x,y)=\varphi_j^J(x)\varphi_k^J(y)-\sum_i N_i^{\text{fem}}(x,y)\varphi_j^J(x_i)\varphi_k^J(y_i). \quad (3)$$

Consequently, the interpolation of the solution variable in the entire domain is expressed as

$$A(x,y)=\sum_{i,j} c_{i,j}\varphi_i^J(x),\varphi_j^J(y) \text{ in } \Omega_w$$

$$A(x,y)=\sum_i A_i N_i^{\text{fem}}(x,y)+\sum_{i,j} c_{i,j}\tilde{\varphi}_{i,j}^J(x,y) \text{ in } \Omega_f^w. \quad (4)$$

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B. Spatial and Temporal Discretization

For the convenience of explanation, the following two-dimensional (2-D) transient eddy current problem is considered

$$v\frac{\partial^2 A(x,y,t)}{\partial x^2}+v\frac{\partial^2 A(x,y,t)}{\partial y^2}=-J+\sigma\frac{dA(x,y,t)}{dt}$$
$$\Gamma_D : A=A_0. \quad (5)$$

Based on the weak form of (5) and using the approximation of (4) and a Galerkin approach, the spatial discretization equation for problem (5) is

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22}\end{bmatrix}\begin{Bmatrix}\dot{A}\\ \dot{C}\end{Bmatrix}+\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22}\end{bmatrix}\begin{Bmatrix}A\\ C\end{Bmatrix}=\begin{Bmatrix}F\\ G\end{Bmatrix} \quad (6)$$

where $\{\dot{U}\}=d\{U\}/dt$ and

$$(k_{i,j})_{11}=\iint_\Omega\left\{\nu[\left(N_j^{\rm fem}\right)_x\left(N_i^{\rm fem}\right)_x+\left(N_j^{\rm fem}\right)_y\left(N_i^{\rm fem}\right)_y\right\}dxdy \quad (7)$$

$$(k_{ij,mn})_{22}=\iint_\Omega\nu\left[(\tilde{\varphi}_{i,j}^J)_x\left(\tilde{\varphi}_{m,n}^J\right)_x+\left(\tilde{\varphi}_{i,j}^J\right)_y\left(\tilde{\varphi}_{m,n}^J\right)_y\right]dxdy \quad (8)$$

$$(k_{i,jl})_{12}=\iint_\Omega\nu\left[\left(N_i^{\rm fem}\tilde{\varphi}_{j,l}^J\right)_x+\left(N_i^{\rm fem}\tilde{\varphi}_{j,l}^J\right)_y\right]dxdy \quad (9)$$

$$(k_{jl,i})_{21}=\iint_\Omega\nu\left[\left(\tilde{\varphi}_{j,l}^J N_i^{\rm fem}\right)_x+\left(\tilde{\varphi}_{j,l}^J N_i^{\rm fem}\right)_y\right]dxdy \quad (10)$$

$$(n_{i,j})_{11}=\iint_\Omega\left\{\sigma\left(N_i^{\rm fem}N_j^{\rm fem}\right)\right\}dxdy \quad (11)$$

$$(n_{ij,mn})_{22}=\iint_\Omega\left[\sigma\left(\tilde{\varphi}_{i,j}^J\tilde{\varphi}_{m,n}^J\right)\right]dxdy \quad (12)$$

$$(n_{i,jl})_{12}=\iint_\Omega\sigma\left[N_i^{\rm fem}\tilde{\varphi}_{j,l}^J)\right]dxdy \quad (13)$$

$$(n_{jl,i})_{21}=\iint_\Omega\sigma\left[\tilde{\varphi}_{j,l}^J N_i^{\rm fem}\right]dxdy \quad (14)$$

$$f_i=\iint_\Omega JN_i^{\rm fem}dxdy \quad (15)$$

$$g_{p,q}=\iint_\Omega J\tilde{\phi}_{p,q}^J(x,y)dxdy \quad (16)$$

where $(\cdot)_z$ denotes the partial differential with variable z.

In region Ω_w, only the wavelets contribute to the approximation; hence, they do not need any modification. Consequently, (8), (12), and (16) become, respectively, as

$$(k_{ij,mn})_{22}=\iint_\Omega\nu\left[\left(\varphi_{i,j}^J\right)_x\left(\varphi_{m,n}^J\right)_x+\left(\varphi_{i,j}^J\right)_y\left(\varphi_{m,n}^J\right)_y\right]dxdy \quad (17)$$

$$(n_{ij,mn})_{22}=\iint_\Omega\left[\sigma(\varphi_{i,j}^J\varphi_{m,n}^J)\right]dxdy \quad (18)$$

$$g_{p,q}=\iint_\Omega J\phi_{p,q}^J(x,y)dxdy. \quad (19)$$

It should be pointed out that due to the compactly supported property of the wavelets, the coefficient matrices (stiffness matrices in FE context) are bounded sparse ones. Moreover, since both the compactly supported nature of the wavelets and the fact that very thin layers near the essential boundaries are meshed as Ω_f^w, most of the quantities in the submatrices $K_{12}(K_{21})$ and $N_{12}(N_{21})$ are zeros.

Using the Crank–Nicolson scheme to approximate the differential with respect to the time variable in (6), the temporal discretization of (6) becomes

$$\begin{bmatrix} N_{11}+\frac{h}{2}K_{11} & N_{12}+\frac{h}{2}K_{12} \\ N_{21}+\frac{h}{2}K_{21} & N_{22}+\frac{h}{2}K_{22}\end{bmatrix}\begin{Bmatrix}A\\ C\end{Bmatrix}_{n+1}=\begin{bmatrix} N_{11}-\frac{h}{2}K_{11} & N_{12}-\frac{h}{2}K_{12} \\ N_{21}-\frac{h}{2}K_{21} & N_{22}-\frac{h}{2}K_{22}\end{bmatrix}\begin{Bmatrix}A\\ C\end{Bmatrix}_n+\begin{Bmatrix}\bar{F}\\ \bar{G}\end{Bmatrix} \quad (20)$$

where h is the time step length $\bar{P}=h(P_{n+1}+P_n)/2$, and n is the time step number.

C. State-Space Model

Generally speaking, the current density in (5) is an unknown quantity to be evaluated in a transient process. Therefore, it is necessary to couple the proposed combined wavelet-FE model with the circuit equations of the system. Consequently, the state-space model as proposed in [6] is used. The state-space model of the system is formulated as

$$[R]\{i\}+\frac{d}{dt}([L]\{i\})=\{u\} \quad (21)$$

where $[R]$ is the resistance matrix; and $\{u\}$ and $\{i\}$ are, respectively, the vectors of the instantaneous current and terminal voltage of the sources; and $[L]$ is the inductance matrix which is determined from the proposed wavelet-FE solutions of the EM fields.

Using the Crank–Nicolson scheme to approximate the differential with respect to the time variable in (21), the time stepping formulation of the state-space model becomes

$$\left[L+\frac{h}{2}R\right]\{i\}^{n+1}=\left[L-\frac{h}{2}R\right]\{i\}^{n+1}+\{\bar{u}\}. \quad (22)$$

D. Solution Procedure of Different Model Systems

In general, the two equation systems of the proposed combined wavelet-FE and the circuit models are coupled together. Because these two equation sets are causes and effects of each other, it is difficult to solve them simultaneously. Consequently, an iterative procedure as described below is proposed for solving the equation sets of (20) and (22).

Step 1) Predicate the initial electric current for the nth step of the combined wavelet-FE system from those of its previous steps $\{i\}^{n-k}(k=1,2,\cdots,l)$ using a response surface model, and denoted it as $\{i\}_k^n$.

Step 2) Solve (20) using $\{i\}_k^n$ as the known quantity; then compute the corresponding inductance matrix $[L]_k^{n+1}$.

Step 3) Solve (22) using matrix $[L]_k^{n+1}$ as the known parameters and let the new solution of currents be $\{i\}_{k+1}^n$.

Step 4) Compare $\{i\}_k^n$ and $\{i\}_{k+1}^n$. If significant errors exist, modify $\{i\}_k^n$ and then go to Step 2 to begin the subsequent cycles of iterations. Otherwise, proceed to iterations of the next time step.

In order to predicate the currents of the present time step from those of their previous values as elucidated in Step 1, the interpolating moving least-squares approximation technique as introduced by the authors [7] is used in this paper.

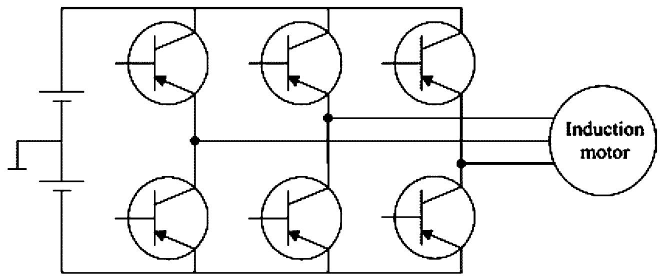

Fig. 1. Schematic diagram of a typical open-loop PWM inverter-fed cage induction motor drive system.

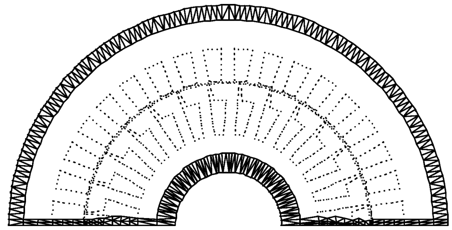

Fig. 2. Meshes and integration cells being used for the proposed combined wavelet-FE method.

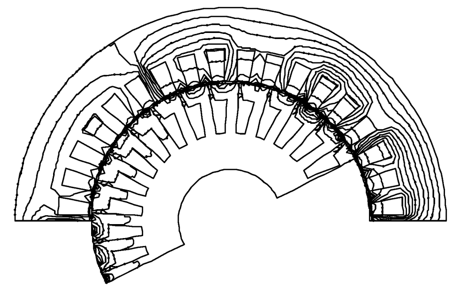

Fig. 3. Computed flux distribution at a specific time step during the no-load starting process using the proposed combined wavelet-FE method.

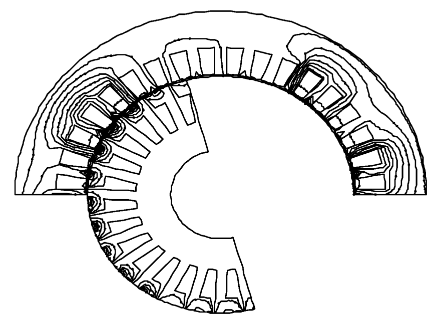

Fig. 4. Computed flux distribution at another time step during the no-load starting process using the proposed combined wavelet-FE method.

III. Numerical Examples

To validate and demonstrate the effectiveness of the proposed combined wavelet-FE method in the study of both steady-state and transient EM phenomena, the proposed algorithm is used to simulate the transient behavior of a prototype induction motor drive fed by a pulsewidth-modulation (PWM) inverter as shown in Fig. 1. In this case study, the induction motor is powered by a voltage source with a PWM inverter having five switching angles within one-quarter period.

In the numerical implementation to be reported, the transient inductances of the motor used in the circuit model are determined based on numerical solutions of the 2-D transient EM field of the motor using the proposed method. The wavelets used in this application are Daubechies' scale functions with $L = 6$ [8]. For the proposed combined wavelet-FE method to work, some thin layers near the essential boundaries of the 2-D solution domain of the motor are first meshed into triangular elements as represented in solid lines in Fig. 2, and some integration cells, which are coincidental to the division lines of different materials, as represented by dotted lines in Fig. 2. The coefficient matrices and the quantities of (6) related to the (modified) wavelets are then subsequently generated. Numerical calculation of the integrals of (6) related to (modified) wavelets is determined using the Gaussian formula and the aforementioned integration cells. Under these mesh conditions and with the scale parameter of the wavelets being set to 4, the total number of degrees of freedom of the proposed method is 1925.

The transient performances of the system are simulated under the following operating conditions. 1) The drive starts under no-load conditions for 0.1 s. 2) It then operates at half rated load for the next 0.2 s. 3) Finally, it runs at its rated load condition for another 0.2 s.

Using a PC with an Intel 2.0-GHz processor and a time step length of 1.39×10^{-4} s, it was found that it takes about 2 h and 45 min of CPU time for the proposed combined wavelet-FE method to emulate the behavior of the prototype drive over the entire transient operating sequence as stipulated above. For performance comparison purposes, this case study is also studied using a time-stepping FE method coupled to an external circuit model [9], and the total CPU time it takes for the simulation of the same transient sequence is about 58 min and 45 s with the same time step length of 1.39×10^{-4} s and with meshes comprising of 3631 first-order triangular elements and 2045 nodes. Figs. 3 and 4 depict the computed magnetic flux distribution at two different time steps during the no-load starting process using the proposed method. Fig. 5 gives the computed magnetic flux distribution at the same time step as that of Fig. 3 using the time-stepping FE method. Fig. 6 shows the computed stator current profiles using the proposed combined wavelet-FE method.

Since the numerical computation of a transient EM process at one time step is equivalent to that of a steady EM problem, the advantages and shortcomings of the proposed method for solving a steady-state EM problem can be evaluated using the averaged performances of transient simulation results. As can be observed from the results given previously, the averaged CPU time required to obtain the numerical solution of one time step for the proposed combined wavelet-FE and the time stepping FE methods are, respectively, 2.75 s and 1 s. Also, referring to Fig. 2 for the meshes being used in the proposed method and bearing in mind that about three to four iterations are generally needed for the proposed wavelet-FE and circuit models to converge in each time step, one will have the following observations on the proposed method regarding the study of steady-state problems.

1) Compared with the FE method, the most salient advantage of the proposed one is that it is nearly meshless despite the need to have integration cells in the numerical implementations. Indeed, it is the flexibility of being meshless that

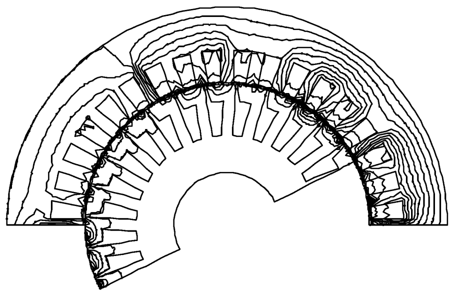

Fig. 5. Computed flux distribution at the same time step of Fig. 3 during the no-load starting process using a time-stepping FE method.

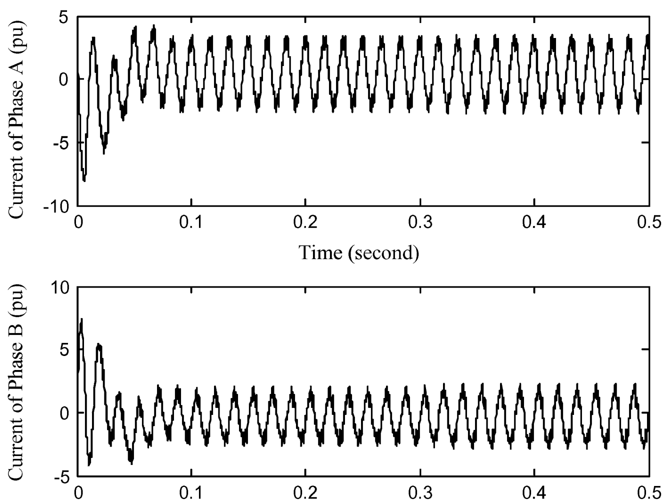

Fig. 6. Profiles of the computed phase currents of the motor using the proposed combined wavelet-FE method.

makes the proposed algorithm a good candidate for being used in the study of three-dimensional (3-D) problems.

2) Compared with FE methods, the shortcoming of the proposed method is that it is less efficient because this is a relatively new method and the solver for this method is not as well developed as those for the FE methods.
3) Compared with other meshless methods, the proposed one does not require any burdensome techniques for enforcing the essential boundary conditions.

On the other hand, also considering the fact that three to four iterations are generally needed for the proposed wavelet-FE and circuit models to converge at one time step, the total CPU time required by the proposed algorithm would be reduced from 2 h and 45 min to around 41 to 55 min if a simultaneous solution technique, which is the same as that for time-stepping FE and external circuit model, is developed for the wavelet-FE and circuit model, provided that the additional time for coupling these two equation system is negligible. Once such a solver is developed, the proposed algorithm would become a strong contender to the well-developed time-stepping FE methods in terms of both CPU time and accuracies in regards to transient EM-field study.

IV. Conclusion

A combined wavelet-FE method is developed and applied to study the transient performances of a prototype inverter-fed motor drive. The primary numerical results demonstrate that the proposed wavelet-FE algorithm has good potential in terms of robustness for transient EM-field computations. Even though there are very few applications of meshless methods, including the proposed one in the study of transient electromagnetic problems, the authors will strive, as their long-term goal, to develop a simultaneous solution technique for the wavelet-FE and circuit model in order to enhance the computation speed of the proposed algorithm.

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Manuscript received June, 20 2005; revised November 20, 2005 (e-mail: eeslho@inet.polyu.edu.hk).