On the Advantage of Quantity Leadership When Outsourcing Production to a Competitive Contract Manufacturer

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This paper investigates a supply chain comprising an original equipment manufacturer (OEM) and a contract manufacturer (CM), in which the CM acts as both upstream partner and downstream competitor to the OEM. The two parties can engage in one of three Cournot competition games: a simultaneous game, a sequential game with the OEM as the Stackelberg leader and a sequential game with the CM as the Stackelberg leader. On the basis of these three basic games, this paper investigates the two parties' Stackelberg leadership/followership decisions. When the outsourcing quantity and wholesale price are exogenously given, either party may prefer Stackelberg leadership or followership. For example, when the wholesale price or the proportion of production outsourced to the CM is lower than a threshold value, both parties prefer Stackelberg leadership and, consequently, play a simultaneous game in the consumer market. When the outsourcing quantity and wholesale price are decision variables, the competitive CM sets a wholesale price sufficiently low to allow both parties to coexist in the market, and the OEM outsources its entire production to this CM. This study also examines the impact of the supply chain parties' bargaining power on contract outcomes by considering a wholesale price that is determined via the generalized Nash bargaining scheme, finding a Stackelberg equilibrium to be sustained when the CM's degree of bargaining power is great and the non-competitive CM's wholesale price is high.

Keywords: Outsourcing; Contract Manufacturing; Competitive CM; Quantity Leadership; Cournot Competition

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1 Introduction

Outsourcing the manufacturing function to contract manufacturers (CMs) is common practice today for many original equipment manufacturers (OEMs). In the personal computer industry, for example, Apple and Hewlett-Packard outsource all of their assembly functions to Foxconn, Flextronics and other CMs in Taiwan and mainland China (Smith 2008). Due to the intense competition among CMs, the services they provide now go beyond the pure manufacturing function. In the electronics industry, increasing numbers of classic CMs (which have no design capabilities) are becoming original design manufacturers (ODMs) that offer value-added services in addition to product manufacturing. Foxconn and Flextronics, for instance, have built large R&D centers to offer product design services to OEMs (Baljko 2006), a welcome development that allows OEMs to shorten new product development lead-times and introduce greater product variety.

However, allowing CMs to handle an increasing number of business functions, from innovation and design to production and even logistics, can prove a double-edged sword for OEMs, as the former are becoming increasingly capable of producing and selling their own self-branded products. A number of interesting business cases have been observed in which CMs act as both *upstream partner* and *downstream competitor* to OEMs. For example, BenQ, Motorola' s CM, produced its first own-brand cellular phone in 2005 (Hilmola et al. 2005). Asustek, a Taiwan-based CM for Apple, Dell, Sony and Toshiba, designs, produces and sells its own Asus brand of notebook computers (Shilov 2007). Acer Inc., originally a CM for IBM and Apple, actually became the third largest computer manufacturer in the world (by sales) in 2007 (Nystedt 2007).

If a CM performs a single role, whether upstream partner or downstream competitor, then its relationship with the OEM is relatively simple. In the former case, the OEM decides the production quantity as a monopoly, and the CM is responsible only for manufacturing. In the latter case, according to traditional oligopoly theory, the party that first decides the production quantity is able to capture a larger share of the market and obtain a higher profit, thereby exhibiting first-mover advantage (see, e.g., Vives 2001). However, a competitive CM is not only an OEM's competitor, but also its business partner. Its revenue is generated both from producing and selling its own self-branded products and from contract manufacturing. The outcome of Cournot competition between an OEM and its competitive CM and the incentives of both in choosing quantity leadership/followership remain unclear, which provides the motivation for this study.

In practice, it is common for an OEM to act as a Stackelberg leader in contracting with a competitive CM, although there are cases in which the latter assumes the leadership role. For example, at Computex 2007 in Taipei, Asustek announced its production of a low-cost sub-notebook based on Intel's Classmate PC reference design. It also reported a sales target of 200,000 units by the end of 2007 and of between three and five million by 2009 (Laptops 2007, Vilches 2007). One year later, one of its OEMs, Dell, entered the same market with a target production of more than 3.6 million (Dannen 2008).

This paper considers a setting in which the end market includes an OEM and a competitive CM. The OEM outsources part of its production to the competitive CM and the remainder to non-competitive CMs. All of the CMs, whether competitive or non-competitive, are capable of both design and manufacture.[1] There is no intellectual property (IP) conflict between the OEM and competitive CM's products in this paper. Further, the products offered by the competitive CM and the OEM are imperfectly substitutable; that is, the OEM's products can be fully substituted for those of the CM, but the reverse does not hold true. There exist three basic Cournot (quantity) competition games between these two parties: a simultaneous game, a sequential-move Stackelberg game with the OEM as the leader and a sequential-move Stackelberg game with the competitive CM as the leader. To explore the endogenous quantity leadership issue, this paper adopts the extended two-stage game in Hamilton and Slutsky (1990). In the first stage, the two players simultaneously choose a leadership or followership role. In the second stage, they play a simultaneous game if both players choose leadership or followership in the first stage, and a sequential game otherwise.

To provide a full picture of the outcomes of the three games, the paper first considers a scenario in which the wholesale price and the proportion outsourced from the OEM to

[1] With regard to the electronics industry, the CMs investigated in this paper are considered to be ODMs. Although ODMs are capable of both design and manufacture, they do not necessarily constitute OEM competitors. In this industry, some ODMs have successfully launched self-branded businesses, whereas others, such as BenQ (Wang 2006), have tried but failed to do so. Others, such as Foxconn and Flextronics, enjoy large accumulative profits from contract manufacturing and have thus decided not to enter the consumer market. (Note that Foxconn and Flextronics do produce self-branded components, but they do not produce self-branded end products for the consumer market.)

the competitive CM are *exogenously* given. This scenario is realistic in certain settings. For example, intense price wars among CMs can result in price alliances or associations among them and lead to an industry standard price, which can be deemed as given. At the same time, an OEM may have multiple reasons to outsource production to several CMs, one of the most important of which is to avoid supply risks (see Tomlin (2006) for details of supply chain disruptions). The proportion of production the OEM outsources in these settings can be deemed exogenous. Both first- and second-mover advantages may exist for the OEM and the competitive CM. The advantage of quantity leadership depends on multiple factors, such as the market size, the wholesale price, the product substitution rates and the percentage of production that the OEM outsources to this competitive CM. When the wholesale price or the proportion of production outsourced to the CM is lower than a threshold value, both parties prefer Stackelberg leadership and, consequently, play a simultaneous game in the consumer market. As the degree of homogeneity between the OEM and competitive CM's products increases, the more difficult it becomes to keep the CM as the follower and the more likely it is that a simultaneous game appears.

The second scenario this paper considers is one in which both the wholesale price and the proportion of production outsourced to the competitive CM are *endogenized*. In this scenario, the OEM determines the proportion of production that it outsources to the competitive CM, whereas the competitive CM endogenously determines the wholesale price.

Interestingly, the OEM is found to prefer outsourcing *entirely* to the competitive CM as long as its wholesale price is no more than that of non-competitive CMs. Further, when the competitive CM sets the wholesale price, it always sets it sufficiently low to allow both parties to *coexist* in the market.

This finding implies that a rational competitive CM will not readily give up its contract manufacturing business and that a rational OEM will be *cautious* about employing the outsourcing quantity as a weapon against a competitive CM. A win-win solution for both may be to allow coexistence in the market. Otherwise, the loss of orders from OEMs may actually spur CMs to develop and sell their own-brand products, thereby turning them into aggressive competitors. Many CMs in Taiwan and the Pearl River Delta region of China were reportedly forced to create their own brands to compensate for lost OEM orders following the global financial crisis that began in September 2008 (Liu 2009). At the same time, however, there are many examples of OEMs and competitive CMs coexisting in harmony. Some competitive CMs even put considerable effort into retaining long-term relationships with OEMs, such as by dividing themselves into two companies, with one responsible for self-branded business and the other for contract manufacturing business (Chung 2004). Other CMs, such as Arima, Clevo, Elite, TPV Technology and Twinhead, choose to maintain their self-branded and ODM businesses within the same organization. In the case of any conflict, they place priority on the latter, satisfying outsourced orders first by reducing the output of their own branded products (Yang 2006, Wang 2008). As a result, many OEMs choose to retain long-term relationships with competitive CMs rather than terminate their business with them, especially when those CMs have accumulated special expertise, such as trained workers and good production control systems and policies.

The final scenario considered here is that the CM and the OEM negotiate the wholesale price via a generalized Nash bargaining (GNB) scheme. Paradoxically, a weak CM is found to behave aggressively in the end-product market and, consequently, to play a simultaneous game with the OEM, whereas a powerful CM is rather cooperative, prompting a sequential-move game. The explanation is that greater bargaining power allows the CM to obtain a larger revenue share from contract manufacturing, which weakens its incentives to sell its own-brand products.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model notations and assumptions. Section 4 analyzes the way in which the OEM's outsourcing decision affects the production quantity and leadership preference of both itself and the competitive CM, and carries out sensitivity analysis of the product substitutability parameter. Section 5 extends the discussion to a setting with endogenized quantity and wholesale price, and Section 6 investigates the endogenous wholesale price via a GNB scheme. Section 7 concludes the paper. All of the proofs are relegated to online Appendix A.

2 Literature Review

The issue of subcontracting to a rival/potential entrant has been discussed in the economics literature. Spiegel (1993), for example, shows that if the transfer payment can be shared via Nash bargaining, then outsourcing production to a potential rival always renders both the incumbent and the potential rival better off, meaning the latter has fewer incentives to build up its self-branded business. However, the issue of outsourcing to a competitive CM is relatively new to the operations management literature. Arruñada and Vázquez (2006) provide a number of business cases of competition between an OEM and a competitive CM. Horng and Chen (2007) empirically examine why some Taiwanese CMs have shifted toward own-brand management, and Arya et al. (2007) investigate a Cournot competition model between a retailer and its supplier. In an encroachment setting, they assume that the supplier has the right to set the wholesale price and that the retailer maximizes its profit by choosing the retail quantity. In a non-encroachment setting, they assume the wholesale price to be exogenously given. By comparing encroachment and non-encroachment settings, they demonstrate that supplier encroachment can achieve Pareto improvement by inducing lower wholesale prices and increasing downstream competition. Ozkan and Wu (2009a) explore the market entry timing problem from the perspective of a competitive CM by adopting a product life-cycle model, and they further consider such a CM's capacity allocation issue (Ozkan and Wu 2009b). Lim and Tan (2010) investigate an OEM's make, buy, and make-and-buy decisions by considering its interactions with its supplier (a CM in our context) over two periods. They show that the OEM's high degree of brand equity can prevent the potential market entry of its CM. Chen et al. (2010) examine the OEM's component sourcing decision in the face of a competitive CM, i.e., the decision concerning whether to buy and resell components or delegate the procurement function to the competitive CM. This paper, in contrast, investigates how the OEM's outsourcing decisions affect the Stackelberg leadership/followership preferences of both itself and a competitive CM, and also considers the endogenous wholesale price and outsourcing proportion decisions.

This work is closely related to the study of firms' outsourcing decisions. Elmaghraby (2000) presents a survey of the operational issues related to outsourcing, and Cachon and Harker (2002) consider two competitive firms facing economies of scale. McGovern and Quelch (2005) summarize the reasons that firms engage in outsourcing, and discuss what should be outsourced and the responsibility borne by marketing managers. Ülkü et al. (2007) investigate whether the OEM/CM should bear the inventory/capacity risk. Arya et al. (2008a) are concerned with a firm's make-or-buy decision, in which the firm can either produce inputs internally or outsource them to a monopoly supplier. Gray et al. (2009a) explore the impact of cost-reduction ability and the OEM's outsourcing decision in a two-period game setting. They (Gray et al. 2009b) further test the OEM's outsourcing propensity by jointly considering cost and quality issues. Kaya and Özer (2009) discuss the quality risks of outsourcing, and Feng and Lu (2009) characterize OEMs' design-related outsourcing decisions. In the marketing field, Stremersch et al. (2003), Leiblein and Miller

(2003), Hoetker (2005) and Parmigiani (2007) empirically investigate the OEM's make-or-buy decision from the transaction cost perspective.

This work is also related to studies of multi-channel distribution and dual sales. Chiang et al. (2003) consider a setting in which the manufacturer can open a direct channel to compete with its retailers. They investigate the impact of that channel on supply chain performance, and show that it can benefit the manufacturer even when no direct sales occur. Tsay and Agrawal (2004b) study the channel conflict issue between existing reseller partners and direct sales, and find that the addition of a direct channel is not necessarily detrimental to the reseller. Chen et al. (2008) assume consumer demand to be endogenously affected by the service level (delivery lead time and product availability), and investigate the optimal time for the manufacturer to establish a direct or retail channel if it is already selling through one or the other. Arya et al. (2008b) consider a dual distribution channel in which the manufacturer sells a product to a retailer and also competes with that retailer in the retail market. More work in this area can be found in the survey carried out by Tsay and Agrawal (2004a).

Wang et al. (2009) adopt the endogenized timing game to investigate the production strategy choices of two competing firms, where each decides individually whether to be efficient (to begin production before demand realization) or responsive (to begin production after demand realization). They identify the conditions under which efficiency or responsiveness is a Nash equilibrium (NE).

3 Notation and Assumptions

This paper considers an OEM (labeled o) that outsources the entire manufacture of its products to CMs. One competitive CM (labeled c) both manufactures this OEM's products and produces and sells its own-brand products to the consumer market. The two parties' products are substitutable. Let θ, $\theta \in [0, 1]$, represent the proportion of production that the OEM outsources to this competitive CM. The OEM then purchases the remaining $(1 - \theta)$ proportion from non-competitive CMs. For simplicity, the CM incurs the same production cost in producing its own and the OEM's products. Let w represent the wholesale price that the OEM pays to all of the CMs for each unit of product they produce. w is first considered to be exogenously given and greater than each CM's unit production cost. Later, in §5 and §6, the analysis is extended to the cases in which the wholesale price is an endogenized decision variable that is either determined by the competitive CM or negotiated between it and the OEM.

The OEM and competitive CM engage in quantity-setting Cournot competition in the consumer market. Thus, the market prices of their products are jointly determined by their respective production quantities, i.e., via inverse demand functions. For tractability, this study adopts the commonly used inverse demand function for the differentiated product of firm i [2]:

$$p_i(q_i, q_j) = m - q_i - b_i q_j, \quad i, j = o, c; \quad i \neq j, \tag{1}$$

where p_i is firm i's market price, q_i is its production quantity, and b_i is a parameter that measures the cross-effect of the change in firm i's product demand caused by a change in that of firm j. Let $0 \leq b_i \leq 1$, and note that the limiting values $b_i = 0$ and $b_i = 1$ correspond to the cases of independent products and perfect substitutes, respectively. b_i is interpreted as the substitution rate of firm j's product over that of firm i, $i, j = o, c; \quad i \neq j$. As the OEM's products are usually regarded as superior to those of the CM (Arruñada and Vázquez 2006), the former are assumed to be perfect substitutes for the latter, but the reverse is not true; that is, $b_c = 1$. Further, $b_o = b \leq 1$. To omit cases in which no production occurs, m, the upper bound on market size, is assumed to be sufficiently large relative to the wholesale price w. For simplicity, the CM's marginal production cost is normalized to zero[3]. Then, the profit functions of the OEM and the competitive CM are, respectively,

$$\Pi_o = (m - q_o - bq_c)q_o - wq_o, \tag{2}$$

$$\Pi_c = (m - q_c - q_o)q_c + \theta w q_o, \tag{3}$$

which are concave and differentiable. Note that in these two functions the first term is the profit that each firm gains from selling products in the consumer market, whereas the second is the transferred outsourcing payments (this term is negative for the OEM and positive for the competitive CM).

The OEM and the competitive CM can play three basic games: a simultaneous game, an OEM-as-leader sequential game and an CM-as-leader sequential game. To explore the

[2]Linear (inverse) demand functions are widely used in the economics, marketing and operations fields to investigate product competition; see Bernstein and Federgruen (2004) and Farahat and Parakis (2011) and the references therein.

[3]As this paper considers both the competitive and non-competitive CMs to be ODMs, the difference between their cost structures is slight and can be ignored.

Stackelberg leadership preferences of the OEM and the competitive CM and the way in which those preferences affect the realization of the three aforementioned settings, this paper employs a two-stage extended game called the endogenous timing game (see, for example, Hamilton and Slutsky, 1990; van Damme and Hurkens, 2004; Amir and Stepanova, 2006). This extended game features a pre-play stage in which the OEM and competitive CM simultaneously, though independently, choose either to move first and be the Stackelberg leader (denoted as L) or to move second and be the Stackelberg follower (denoted as F). The players are then committed to this choice. $\alpha = (\alpha_o, \alpha_c)$ denotes the joint actions of the OEM and the competitive CM. Then, $\alpha \in \{(L, L), (L, F), (F, L), (F, F)\}$. Next, each player's timing choice is announced, and the next stage is played accordingly: a simultaneous play if both players decide to move first/second ($\alpha = (L, L)/(F, F)\}$), and a sequential play under perfect information otherwise (with the order of moves announced by the players). Denote Π_i^S, $i = o, c$ as firm i's profit when it is engaged in a simultaneous game, where S stands for *simultaneous*. The resulting production quantity is denoted as q_i^S. Also, denote Π_i^L (Π_i^F), $i = o, c$ as firm i's profit when it is the Stackelberg leader (follower). Let q_i^L (q_i^F) represent the corresponding production quantity.

The subgame perfect equilibrium of this extended game leads to a quantity decision timing sequence, and the resulting payoffs of each player are listed in Table 1. Comparing the equilibrium payoffs in the simultaneous and Stackelberg settings allows derivation of the conditions under which the OEM and competitive CM would prefer Stackelberg leadership. For simplicity, in the following analysis, the term CM means the competitive CM.

Table 1: Quantity and Leadership Decisions

CM \ OEM	Leader	Follower
Leader	Π_o^S, Π_c^S	Π_o^F, Π_c^L
Follower	Π_o^L, Π_c^F	Π_o^S, Π_c^S

4 Exogenous Wholesale Price and Outsourcing Decisions

This section begins with the case of exogenous wholesale price and outsourcing decision parameters, and investigates the quantity leadership preferences of the OEM and the competitive CM.

4.1 Equilibrium of three basic games

The closed-form expressions for the equilibrium outcomes under the three basic games are summarized in the following proposition.

Proposition 1. *In the simultaneous game, if* $m > \frac{2}{2-b}w$*, then the equilibrium production quantities and profits are:*

(1) $q_o^S = \frac{(2-b)m-2w}{4-b}$, $q_c^S = \frac{m+w}{4-b}$;

(2) $\Pi_o^S = \frac{[(2-b)m-2w]^2}{(4-b)^2}$, $\Pi_c^S = \frac{(m+w)^2}{(4-b)^2} + \frac{[(2-b)m-2w]\theta w}{4-b}$.

In the OEM-as-leader game, if $m > \frac{2}{2-b}w$*, then the equilibrium production quantities and profits are:*

(1) $q_o^L = \frac{(2-b)m-2w}{2(2-b)}$, $q_c^F = \frac{(2-b)m+2w}{4(2-b)}$;

(2) $\Pi_o^L = \frac{[(2-b)m-2w]^2}{8(2-b)}$, $\Pi_c^F = \frac{[(2-b)m+2w]^2}{16(2-b)^2} + \frac{[(2-b)m-2w]\theta w}{2(2-b)}$.

In the CM-as-leader game, if $m > \frac{4-b^2\theta-b}{4-3b}w$*, then the equilibrium production quantities and profits are:*

(1) $q_o^F = \frac{(4-3b)m-(4-b^2\theta-b)w}{4(2-b)}$, $q_c^L = \frac{m+(1-b\theta)w}{2(2-b)}$;

(2) $\Pi_o^F = \frac{[(4-3b)m-(4-b^2\theta-b)w]^2}{16(2-b)^2}$, $\Pi_c^L = \frac{[m+(1+b\theta)w][m+(1-b\theta)w]}{8(2-b)} + \frac{[(4-3b)m-(4-b^2\theta-b)w]\theta w}{4(2-b)}$.

Therefore, if the market size is too small relative to the wholesale price, such that $m \leq \frac{2}{2-b}w$, then both the simultaneous and OEM-as-leader games are reduced to a monopoly setting in which the CM alone produces its monopolistic quantity and the OEM is expelled from the market. A similar situation results if $m \leq \frac{4-b^2\theta-b}{4-3b}w$ in the CM-as-leader game. The explanation lies in the difference between the profit margins of the OEM and CM. Their objective functions, (2) and (3), indicate that the OEM must pay the CM a wholesale price w that is larger than the latter's production cost; that is, the OEM has to bear a larger cost than the CM. Condition $m \leq \frac{2}{2-b}w$ in the simultaneous and OEM-as-leader games suggests a wholesale price so high that the market price does not even cover the OEM's cost (i.e., the wholesale price paid to the CM). Condition $m \leq \frac{4-b^2\theta-b}{4-3b}w$ in the CM-as-leader game has a similar implication.

Proposition 1 provides several conclusions about the wholesale price's impact on the equilibrium outcome. In both the simultaneous and sequential games, a higher transfer wholesale price to the CM always results in a smaller production quantity and a smaller profit for the OEM. However, the wholesale price's impact on the CM depends on the production proportion outsourced to it. A high wholesale price can hurt the CM, as it reduces the order quantity from the OEM.

Proposition 1 also provides interesting conclusions concerning the impact of θ. In both the simultaneous and OEM-as-leader games, the equilibrium production quantities of the OEM and CM are independent of θ. The best response function of the CM, $q_c(q_o) = \frac{m-q_o}{2}$, is clearly independent of θ, because the market price of the CM's own-brand product is affected by the OEM's production quantity decision, not by its manufacturing outsourcing decision. Anticipating such independence, the OEM's decision is also independent of θ. However, in the CM-as-leader game, θ does affect the CM's production quantity decision because it affects the tradeoff between the CM's two revenue streams: that generated from contracted manufacturing and that generated from self-manufacturing. Counterintuitively, the OEM's profit is increasing in θ. To maximize its own profit, the OEM should thus outsource all of its production ($\theta = 1$) to the CM. One explanation is that the CM's profits come from two sources: contract manufacture and sales in the consumer market. When the CM is the quantity leader, the OEM can reduce the CM's incentive to produce its own-brand products, and thus face less competition in the consumer market, by outsourcing more product manufacturing to it.

4.2 Equilibrium of the extended timing game

Drawing on Proposition 1, the conditions under which moving first and being the Stackelberg leader is beneficial for the OEM/CM are now derived by comparing their sequential payoffs with their simultaneous payoffs.

The equilibrium outcome of the extended endogenous timing game depends on certain conditions. If $m < \min\left\{\frac{2}{2-b}, \frac{4-b^2\theta-b}{4-3b}\right\} w$, then the OEM is always expelled from the market and the CM is always the monopolist. If $\frac{4-b^2\theta-b}{4-3b} w < m < \frac{2}{2-b} w$, then the OEM is expelled from the market in the simultaneous and OEM-as-leader games. If $\frac{2}{2-b} w < m < \frac{4-b^2\theta-b}{4-3b} w$, then it is expelled in the CM-as-leader game. These three reduced cases are omitted here.

When $m > \max\left\{\frac{4-b^2\theta-b}{4-3b}, \frac{2}{2-b}\right\} w$, both the OEM and CM exist in the market in all three basic games. To characterize the equilibrium, define

$$w_{AL} = \frac{16 - 10b + b^2}{8\theta(2-b)(4-b) - 16 + 6b}m, \text{ and } w_{AF} = \frac{1}{(4-b)\theta - 1}m.$$

Proposition 2. *Assume that* $m > \max\left\{\frac{4-b^2\theta-b}{4-3b}, \frac{2}{2-b}\right\} w$ *or* $w \leq \min\left\{\frac{2-b}{2}, \frac{4-3b}{4-b^2\theta-b}\right\} m$. *Comparing the three basic games shows that, at the quantity timing decision stage, the extended timing game can have the following possible outcomes.*

(1) L is a dominant strategy if $\theta \in \left[0, \frac{1}{2-b}\right)$ *or* $\theta \in \left[\frac{1}{2-b}, 1\right]$, *but* $w < w_{AL}$.

(2) If $\theta \in \left[\frac{1}{2-b}, 1\right]$, *then (L, F) is the unique pure NE for* $w \in [w_{AL}, w_{AF})$, *and (L, F) and (F, L) are the two NE for* $w \in \left[w_{AF}, \frac{2-b}{2}m\right]$. w_{AL} *and* w_{AF} *are decreasing in* θ, *and* $w_{AL} \leq w_{AF}$ *for* $\theta \in \left[\frac{1}{2-b}, 1\right]$.

(3) F cannot be the dominant strategy because $\Pi_o^L \geq \Pi_o^S$ *and* $\Pi_c^L \geq \Pi_c^S$.

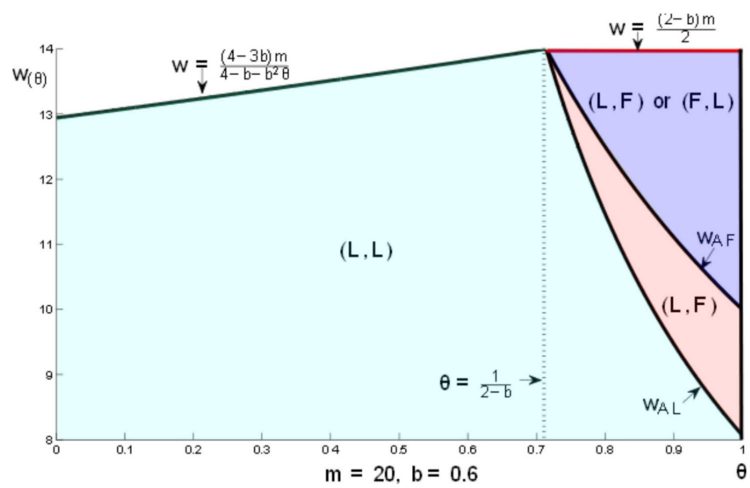

Figure 1: Impact of Wholesale Price on Quantity Timing Equilibrium

The results in Proposition 2 show that the outsourcing relationship between the OEM and CM, the relative size of the wholesale price and the outsourcing proportion all affect both parties' quantity leadership preference in the consumer market. Figure 1 illustrates the impact of the wholesale price on the Stackelberg leadership preference of each. When θ, the proportion outsourced to the CM, is low ($< \frac{1}{2-b}$), regardless of how high the wholesale price is, the CM will be aggressive in the consumer market and choose Stackelberg leadership. Even when θ is high, it will still choose Stackelberg leadership if the OEM offers a low wholesale price ($< w_{AL}$). Only when θ is large ($> \frac{1}{2-b}$) and the wholesale price is moderate ($w \in (w_{AL}, w_{AF})$) will the CM definitely choose the follower position. Interestingly, when the outsourcing percentage θ is large ($> \frac{1}{2-b}$) and the wholesale price is high ($> w_{AF}$), (F, L) can also be the NE in the quantity timing game, and the OEM faces the possibility of losing its Stackelberg leadership. This occurs because when w is very high, the OEM's profit margin is too small. In such a scenario, the OEM's payoff as the follower is higher than that in the simultaneous game. Knowing this to be the case, the CM is motivated to take the leadership position. In addition, w_{AL} is decreasing in θ, which implies that when the OEM outsources a large proportion of its product manufacturing to the CM, the latter is willing to play the quantity followership role even if the wholesale price offered is not high.

Let $k = \frac{(2-b)m}{w}$, where $k > 2$. Define

$$\theta_{AL} = \frac{k(8-b) + 16 - 6b}{8(2-b)(4-b)}, \text{ and } \theta_{AF} = \frac{k + (2-b)}{(2-b)(4-b)}.$$

Fixing k, the following corollary is obtained on the equilibrium strategy for different ranges of θ.

Corollary 1. *Assume that* $\frac{1}{2-b} \leq \theta \leq 1$.

(1) L is a dominant strategy if $\theta \in \left[\frac{1}{2-b}, \theta_{AL}\right)$.

(2) (L, F) is a NE if $\theta \in [\theta_{AL}, \theta_{AF})$.

(3) (L, F) and (F, L) are two NE if $\theta \in [\theta_{AF}, 1]$.

(4) $\theta_{AF} > \theta_{AL}$; θ_{AL}, θ_{AF} *and* $\theta_{AF} - \theta_{AL}$ *are all increasing in b.*

Figure 2 illustrates how the OEM's outsourcing decision and the substitutability of the CM's product over that of the OEM affect the quantity timing equilibrium outcome. For any given substitution rate b, when the amount outsourced to the CM is relatively high, $\theta \in (\theta_{AL}, \theta_{AF})$, and the CM will surely assume the Stackelberg followership position. Otherwise, it is motivated to take the leadership position. The figure also shows that when substitution rate b increases, both θ_{AL} and θ_{AF} are increasing. It thus becomes more difficult to retain the CM as the follower, as the degree of homogeneity between the two parties' products increases, rendering it easier for the simultaneous game to appear.

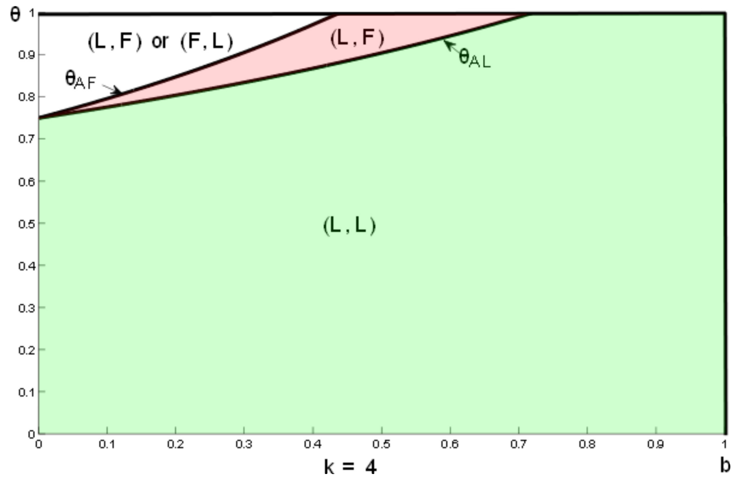

Figure 2: Impact of Outsourcing on Quantity Timing Equilibrium

4.3 Impact of CM product substitutability

Note that a larger b implies greater homogeneity between the OEM and CM's products. As the competitive CM enhances such abilities as learning, design, production and quality control, the degree of its product's substitutability b increases and may even reach 1. The following proposition summarizes the impact of b on the outcomes of the three basic games.

Proposition 3. *For the three basic games,*

(1) q_o^S, q_o^L *and* q_o^F *are decreasing in* b, *and* q_c^S, q_c^F *and* q_c^L *are increasing in* b.

(2) Π_o^S, Π_o^L *and* Π_o^F *are decreasing in* b.

(3) Π_c^S, Π_c^F *and* Π_c^L *are increasing in* b *if* $\theta \in [0, \frac{1}{2-b}]$. *If* $\theta \in (\frac{1}{2-b}, 1]$, *then*

- *(i)* Π_c^S *is decreasing in* b *for* $m \in [\frac{2}{2-b}w, ((4-b)\theta - 1)w]$; *otherwise,* Π_c^S *is increasing in* b;
- *(ii)* Π_c^F *is decreasing in* b *for* $m \in [\frac{2}{2-b}w, \frac{4(2-b)\theta-2}{2-b}w]$; *otherwise,* Π_c^F *is increasing in* b; *and*
- *(iii)* Π_c^L *is decreasing in* b *for* $m \in [\frac{4-b^2\theta-b}{4-3b}w, ((4-b)\theta-1)w]$; *otherwise,* Π_c^L *is increasing in* b.

Proposition 3 shows that in the three basic games, the OEM's equilibrium production quantities are higher if the CM's product has a lower degree of substitutability, whereas the situation is reversed for the CM. In other words, if the OEM's/CM's products are favored over those of the CM/OEM, then the OEM/CM will produce more. Moreover, the OEM always obtains a higher profit when the CM has a lower degree of product substitutability. Hence, it is beneficial for OEMs to make large investments in R&D and product quality improvement. An interesting finding is that if the market is not very large, but the proportion outsourced to the competitive CM is high, then the CM is also better off with a lower substitution rate b. Hence, the OEM always prefers a less-substitutable CM product, a preference sometimes shared by the competitive CM.

Moreover, when $b = 0$, that is, the OEM and competitive CM's products are not substitutes for each other, the two parties are indifferent to which basic game they play (according to Proposition 1). When $b = 1$, however, both parties prefer leadership, and the simultaneous game is played (see Corollary 1).

5 Outsourcing with Endogenized Wholesale Price Determined by the CM

In contrast to the previous section, in which the wholesale price w and outsourcing decision θ were exogenously given, this section considers a price-only contract in which the CM decides w and the OEM makes the optimal decision about θ. A similar assumption can be found in other operations management and marketing research studies, including those of Lariviere and Porteus (2001) and Cui et al. (2008). It is also consistent with current industry practice. Such CMs as Asustek, Quanta and Foxconn, for example, usually offer price quotes to their OEMs, including Apple, Dell and Sony. The OEMs then decide whether and what kind of contract to sign. For simplicity, here, the non-competitive CMs charge a wholesale price p_0, and $p_0 \geq w$; otherwise, the OEM would have no incentives to source from the competitive CM. The profit functions of the OEM and competitive CM are, respectively,

$$\Pi_o = (m - q_o - bq_c)q_o - \theta w q_o - (1-\theta)p_0 q_o, \quad (4)$$

$$\Pi_c = (m - q_c - q_o)q_c + \theta w q_o. \quad (5)$$

It is possible that the competitive CM decides the wholesale price w first, followed by the OEM's outsourcing decision θ (named *Decision order 1*). Alternatively, the OEM decides the outsourcing proportion first, followed by the CM's decision on w (named *Decision order 2*). The main results are the same regardless of the sequence. Hence, the decision order 1 results alone are listed here, with the decision order 2 results relegated to online Appendix B. In the following, superscript $*$ denotes the optimal results when w and θ are endogenous.

5.1 Simultaneous game

The game sequence in the simultaneous game is defined as follows and illustrated in Figure 3. The CM first decides the wholesale price w, and the OEM then makes its outsourcing decision θ. Finally, the CM and OEM simultaneously decide their production quantities. Solving the game backwards obtains the following proposition.

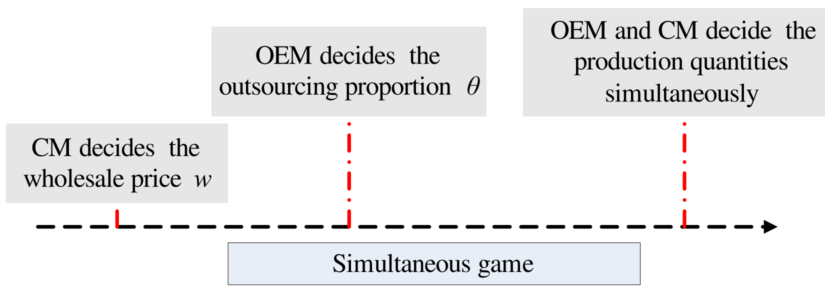

Figure 3: Game Sequence for the Simultaneous Game

Proposition 4. *For the simultaneous game,*

(1) $w^{S*} = \min\left\{p_0, w^S\right\}$, *where* $w^S = \frac{10-6b+b^2}{14-4b}m$; $\theta^* = 1$.

(2) If $p_0 > w^S$, *then* $\Pi_o^{S*} = \frac{(1-b)^2}{(7-2b)^2}m^2$, $\Pi_c^{S*} = \frac{8-4b+b^2}{4(7-2b)}m^2$; *otherwise,* $\Pi_o^{S*} = \frac{[(2-b)m-2p_0]^2}{(4-b)^2}$, $\Pi_c^{S*} = \frac{(m+p_0)^2}{(4-b)^2} + \frac{[(2-b)m-2p_0]p_0}{4-b}$.

As Proposition 4 shows, the OEM will prefer to source its entire production from the competitive CM as long as $p_0 \geq w^{S*}$. Will the competitive CM then have the incentive to produce nothing for the OEM, thus expelling it from the market? Note that a monopolist CM needs to charge a wholesale price of $w = \frac{2-b}{2}m$, and its monopolist profit (denoted as Π_c^m) is thus $\Pi_c^m = \frac{m^2}{4}$. The following corollary is then obtained.

Corollary 2. $\Pi_c^{S*} \geq \Pi_c^m$ *for* $p_0 \geq \frac{6-b}{14-4b}m$.

The wholesale price offered by non-competitive CMs, p_0, is often the result of a price war among them. Corollary 2 shows that when p_0 is higher than a threshold value, the CM has no incentive to charge a wholesale price sufficiently high to expel the OEM from the market, a rather surprising result. A mixture model, that is, selling its own products in the low-end market and carrying out contract manufacturing for the OEM in the high-end market, allows the CM to enjoy higher profits than does a pure model in which it acts as a monopolist and provides only low-end products in the consumer market. If the wholesale price war leads to $p_0 \leq \frac{6-b}{14-4b}m$, then the competitive CM cannot be a monopolist even if it wants to, as the OEM will source from non-competitive CMs.

5.2 OEM-as-leader game

Recall that "leader" in this paper refers to the quantity leader, not the price leader. In the OEM-as-leader game, the game sequence is as follows. The CM first decides its wholesale price; the OEM then jointly decides its production quantity and the fraction of production to source from the competitive CM; and, finally, the CM decides the production quantity of its own-brand products (see Figure 4). Again solving the game by backward induction leads

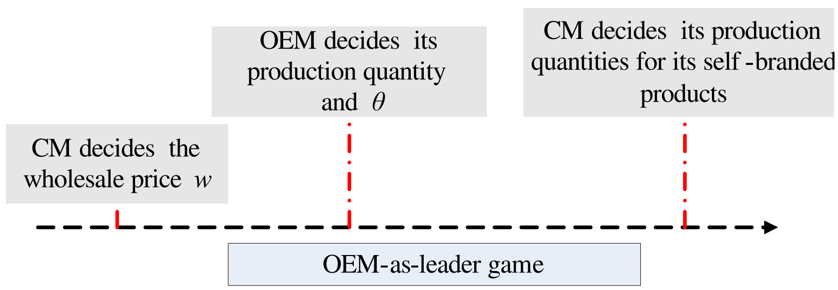

Figure 4: Game Sequence for the OEM-as-leader Game

to Proposition 5.

Proposition 5. *For the OEM-as-leader game,*

(1) $w^{F*} = \min\{p_0, w^F\}$, *where* $w^F = \frac{(2-b)(5-2b)}{14-8b}m$; $\theta^* = 1$.

(2) If $p_0 > w^F$, *then* $\Pi_o^{L*} = \frac{(2-b)(1-b)^2}{2(7-4b)^2}m^2$, $\Pi_c^{F*} = \frac{(2-b)(4-b)}{4(7-4b)}m^2$; *otherwise,* $\Pi_o^{L*} = \frac{[(2-b)m-2p_0]^2}{8(2-b)}$, $\Pi_c^{F*} = \frac{[(2-b)m+2p_0]^2}{16(2-b)^2} + \frac{[(2-b)m-2p_0]p_0}{2(2-b)}$.

Similarly, comparing the competitive CM's optimal profit with its monopolist profit leads to the following corollary.

Corollary 3. $\Pi_c^{F*} \geq \Pi_c^m$ *for* $p_0 \geq \frac{3(2-b)}{14-8b}m$.

Again, the competitive CM is better off keeping the OEM in the consumer market. This analysis is similar to that in the simultaneous game.

5.3 CM-as-leader game

Figure 5 depicts the game sequence for the CM-as-leader game. First, the CM decides its wholesale price and the production quantity of its own branded products. Second, the OEM decides its production quantity and the fraction to outsource to the competitive CM. Proposition 6 is obtained by solving the game backwards.

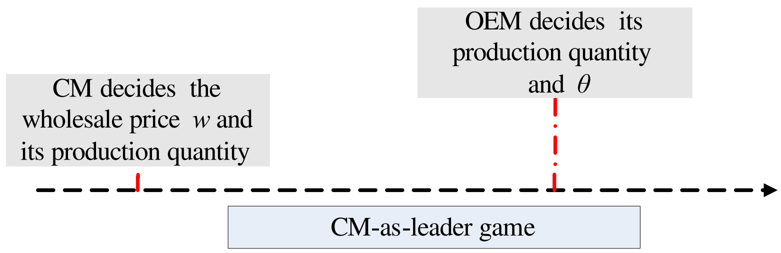

Figure 5: Game Sequence for the CM-as-leader Game

Proposition 6. *For the CM-as-leader game,*

(1) $w^{L*} = min\{p_0, w^L\}$ *where* $w^L = \frac{5-3b}{7-2b-b^2}m$, $\theta^* = 1$.

(2) If $p_0 > w^L$, *then* $\Pi_o^{F*} = \frac{(1-b)^2}{(7-2b-b^2)^2}m^2$, $\Pi_c^{L*} = \frac{(2-b)}{7-2b-b^2}m^2$; *otherwise,* $\Pi_o^{F*} = \frac{[(4-3b)m-(4-b-b^2)p_0]^2}{16(2-b)^2}$, $\Pi_c^{L*} = \frac{[m+(1+b)p_0][m+(1-b)p_0]}{8(2-b)} + \frac{[(4-3b)m-(4-b-b^2)p_0]p_0}{4(2-b)}$.

Again, comparing the competitive CM's optimal profit with its monopolist profit leads to the following corollary.

Corollary 4. $\Pi_c^{L*} \geq \Pi_c^m$ *for* $p_0 \geq \frac{(5-3b)-(1-b)\sqrt{2(2-b)}}{7-2b-b^2}m$.

Here, the CM still prefers to charge a low wholesale price and earn contract manufacturing revenue as long as p_0 is relatively high.

The preceding analyses support the following proposition.

Proposition 7. *For the three basic games,*

(1) The optimal wholesale price for the competitive CM is always the one that keeps both itself and the OEM in the consumer market.

(2) If the competitive CM offers a wholesale price no higher than that of the non-competitive CMs ($w \leq p_0$)*, then the OEM will outsource its entire production to the competitive CM (i.e.,* $\theta^* = 1$*).*

As the proofs of Propositions 4-6 in online Appendix A show, the OEM's profit functions in different scenarios all have a term of θ with the coefficient parameter $p_0 - w$. Consequently, as long as $p_0 > w$, the OEM's profit function is increasing in θ, and hence the optimal decision $\theta^* = 1$. Otherwise, if $p_0 < w$, then the OEM's profit function is decreasing in θ, which leads to the optimum $\theta^* = 0$. For the boundary case in which $p_0 = w$, the OEM is indifferent to the choice of different CMs. However, in this case, the competitive CM always has the incentive to lower its wholesale price slightly to attract contract manufacturing business from the OEM. Therefore, in equilibrium, $\theta^* = 1$ holds.

Considering that it is very common for an OEM to target the high-end market while its CM targets the low-end market, Proposition 7 is very insightful for contract manufacturing practice. Part a implies that the OEM need not worry about being expelled from the consumer market by a competitive CM, as it is in the latter's best interests to keep the OEM in the market and earn greater revenue from selling its own-brand products and engaging in contract manufacturing. One possible explanation is that if the OEM chooses to outsource to non-competitive CMs, then the resulting loss in profits will force the competitive CM to become more aggressive in producing and selling its own branded products, which will harm the OEM in the consumer market. For example, Asustek believes that "the capability to create innovative technology (for its self-branded business) and maintain manufacturing strength is crucial for IT players as they try to outflank their competitors in fast-changing times" (Chung 2004). Part b of the proposition states that the OEM should outsource all of its production to the competitive CM as long as that CM's wholesale price is not higher than the other options available to the OEM. In reality, a competitive CM will normally have accumulated such special expertise as trained workers, advanced production technology and good quality control system. From the viewpoint of transaction cost economics (Williamson 1985), such expertise can be considered to constitute transaction costs for the OEM, which hinder it from switching to non-competitive CMs. For example, Asustek, a competitive CM, snatched back an Apple production order for the 14-inch wide-screen iBook by charging a wholesale price lower than that of the non-competitive CM Quanta (Lin 2005).

5.4 Equilibrium of the extended timing game

The following Lemma facilitates the derivation of the extended timing game equilibrium.

Lemma 1. $w_{AL} \leq w_{AF} \leq w^F \leq w^S \leq w^L$.

Consequently, the competitive CM's optimal wholesale prices in the three basic games have the following relationship: $w^{F*} \leq w^{S*} \leq w^{L*}$. In other words, the competitive CM charges the highest wholesale price in the CM-as-leader game and the lowest in the OEM-as-leader game. Moreover, $w^F \leq w^{F*} \leq w^{S*} \leq w^{L*}$ if $p_0 \geq w^F$, whereas $w^{F*} = w^{S*} = w^{L*} = p_0 \leq w^F$ if $p_0 < w^F$. Recall that in all of the basic games, $\theta^* = 1$. Proposition 2 then leads to the following conclusion.

Proposition 8. *L is a dominant strategy if $p_0 < w_{AL}$; (L, F) is the unique pure NE if $p_0 \in [w_{AL}, w_{AF})$; and (L, F) and (F, L) are the two NE if $p_0 > w_{AF}$.*

Proposition 8 shows that p_0, the wholesale price charged by the non-competitive CMs, has a significant influence on the competitive CM's endogenized wholesale price decision, and thus on the outcome of the quantity timing game; see Figure 6. If p_0 is very low, then the simultaneous game is preferable; if p_0 is moderate, then the OEM-as-leader game is preferable; and if p_0 is large, then the OEM-as-leader and CM-as-leader games are equally preferable.

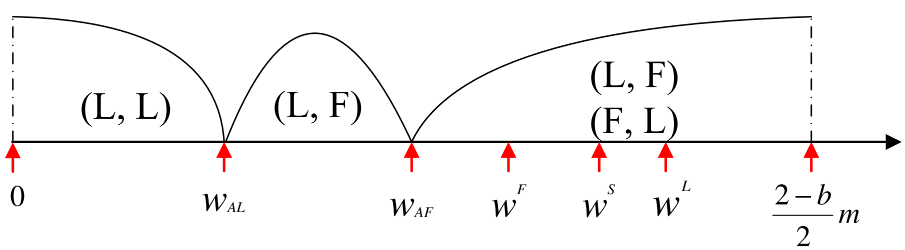

Figure 6: Impact of p_0 on Quantity Timing Equilibrium

6 Outsourcing with Endogenized Wholesale Price via Nash Bargaining

The assumption in §5 was that the CM determines the outsourcing wholesale price. In practice, the wholesale price can also be determined through negotiation. If this is the case, then both the OEM and the CM have bargaining power, which enables them to influence the outcome of the negotiated wholesale price. This section considers such a scenario and discusses both parties' quantity leadership preference.

6.1 Generalized Nash bargaining scheme

The GNB scheme first proposed by Nash (1950) and later extended by Roth (1979) is a common methodology for studying price negotiation. A number of recent papers in the operations management arena use the GNB scheme to investigate the endogenized pricing issue, for example, Nagarajan and Bassok (2008), Nagarajan and Sosic (2008), Feng and Lu (2009) and İşlegen and Plambeck (2009).

In this paper, the GNB scheme is defined to solve the following optimization problem.

$$\max_{w} \quad \Omega = (\Pi_c)^{\alpha}(\Pi_o)^{1-\alpha}$$

$$s.t. \quad 0 \leq w \leq min\left\{p_0, \left(\frac{2-b}{2}m \ or \ \frac{4-3b}{4-b-b^2}m\right)\right\}, \tag{6}$$

$$\Pi_c \geq \Pi_c^r, \tag{7}$$

where Ω is the *Nash product* and Π_i is party i's corresponding profit, $i = o, c$. α ($\alpha \in [0,1]$) and $1-\alpha$ correspond to the bargaining powers of the competitive CM and OEM, respectively. The value $\alpha = 1/2$ refers to the equal bargaining power case, whereas the extreme values $\alpha = 0$ and $\alpha = 1$ reduce the two-player bargaining setting to a one-player setting. Both the OEM and the competitive CM are rational and risk-neutral, and their bargaining powers are exogenously given. Condition (6) is the participation constraint for the OEM. First, w must not be larger than p_0; otherwise, the OEM will have no incentive to source from the competitive CM. Second, assume that $w \leq \left(\frac{2-b}{2}m \ or \ \frac{4-3b}{4-b-b^2}m\right)$ (for the simultaneous and OEM-as-leader games, $w \leq \frac{2-b}{2}m$; for the CM-as-leader game, $w \leq \frac{4-3b}{4-b-b^2}m$) to ensure that the OEM and the competitive CM can coexist in the market. Condition (7) is the participation constraint for the competitive CM to participate in the contract manufacturing business, where Π_c^r is its reserved profit if it does not engage in such business, but competes directly with the OEM in the market.

The GNB-characterized wholesale price in the three basic games is denoted as w^{Nj}, $j = S, F, L$, where the superscript j stands for the CM's quantity leadership position.

6.2 GNB-characterized wholesale price in three basic games

Here, the game sequence remains the same as that in §5 except that in the first stage, instead of the CM deciding the wholesale price w, the competitive CM and OEM cooperate to negotiate the price. The game is solved via backward induction. Moreover, as argued in §5, $\theta^* = 1$ as long as the GNB-characterized wholesale price $w^{Nj} \leq p_0$, $j = S, F, L$. Solving the constrained optimization problem leads to the following proposition on the negotiated wholesale price.

Proposition 9. *For game j, $j = S, F, L$, denote K^j as the optimum maximizing the Nash product without considering the constraints and $\underline{w}^j$ as the wholesale price leading to the binding nature of the CM's participation constraint (7). Then,*

$$
\begin{aligned}
K^S &= \frac{2(10-6b+b^2)+(1-b)(4-b)\alpha-(4-b)\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}}{4(7-2b)}m;\\
\underline{w}^S &= \frac{(10-6b+b^2)m-\sqrt{(10-6b+b^2)^2m^2-4(7-2b)(p_0^2+2p_0m)}}{2(7-2b)};\\
K^F &= \frac{(2-b)(5-2b)+(2-b)(1-b)\alpha-(2-b)\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}}{2(7-4b)}m;\\
\underline{w}^F &= \frac{(2-b)(5-2b)m-\sqrt{(2-b)^2(5-2b)^2m^2-4(7-4b)(p_0^2+(2-b)p_0m)}}{2(7-4b)};\\
K^L &= \frac{(5-3b)(4-b-b^2)+2(1-b)(2-b)\alpha-2(2-b)\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}}{(4-b-b^2)(7-2b-b^2)}m;\\
\underline{w}^L &= \frac{(5-3b)m-\sqrt{(5-3b)^2m^2-(7-2b-b^2)(p_0^2+2p_0m)}}{7-2b-b^2}.
\end{aligned}
$$

For the three basic games, the Nash product $\Omega^j, j = S, F, L$ is unimodal in w, and the GNB-characterized wholesale price is

1. *$w^{Nj} = min(p_0, max(\underline{w}^j, K^j))$, $j = S, F, L$;*
2. *K^j is increasing in α and $K^j = w^j$ $(j = S, F, L)$ if $\alpha = 1$, where w^j is the wholesale price determined by the CM, as discussed in §5.*

In the expression w^{Nj}, $\underline{w}^j$ provides a lower bound on the negotiated wholesale price. In other words, if the negotiated wholesale price is lower than $\underline{w}^j$, then the competitive CM will give up the contract manufacturing business and become purely a competitor to the OEM. In addition, p_0, the wholesale price offered by a non-competitive CM, provides an upper bound for the negotiated wholesale price. Part 2 of Proposition 9 shows that the GNB-characterized wholesale price increases in the competitive CM's bargaining power α, but decreases in that of the OEM. Moreover, it shows that in each basic game j, $j = S, F, L$, the GNB-characterized wholesale price w^{Nj} is less than the corresponding CM-determined wholesale price w^j (they become equal when $\alpha = 1$, provided that p_0 is sufficiently high).

6.3 Equilibrium of the extended timing game

Analogous to the discussion in §5.4, the equilibrium outcome of the quantity timing game depends on the value of the negotiated wholesale prices in the three basic games—w^{NS}, w^{NF} and w^{NL}—and the wholesale price charged by the non-competitive CM, p_0. w^{NS}, w^{NF} and w^{NL} are all increasing in α, which allows examination of the impact of α and p_0 on the equilibrium of the quantity timing game.

6.3.1 $\alpha = 0$

First we consider the special case in which $\alpha = 0$, which is equivalent to the OEM-determines-the wholesale-price setting. As the OEM's profits in the three basic games —$\Pi_o^S(w)$, $\Pi_o^L(w)$ and $\Pi_o^F(w)$—are always *decreasing* in the wholesale price, the OEM will offer the CM a wholesale price that is as low as possible. Proposition 9 suggests that there is a lower bound on the wholesale price for the competitive CM to participate in contract manufacturing business. Here, p_0 is assumed to be higher than that lower bound. Therefore, the OEM will offer the CM the lower bound of the wholesale price, which leads to the following proposition.

Proposition 10. *If the OEM determines the wholesale price, then the competitive CM always prefers leadership.*

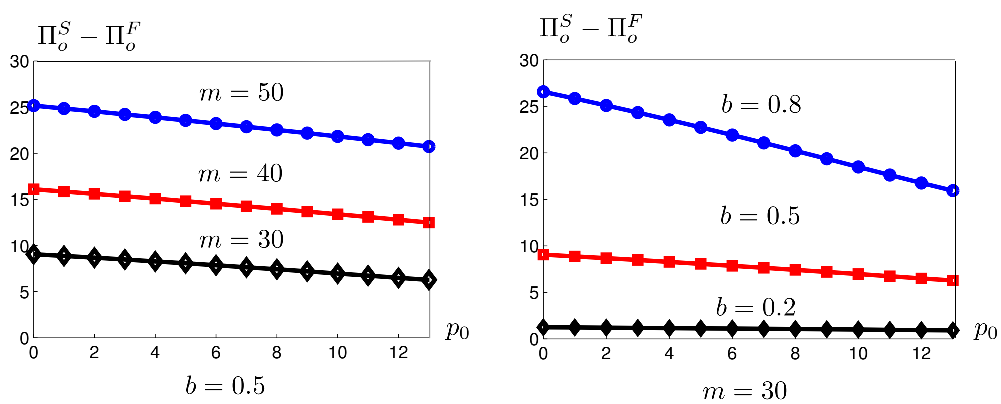

Figure 7: Comparison between Π_o^S and Π_o^F if $\alpha = 0$

Proposition 10 suggests that we need only compare $\Pi_o^F(\underline{w}^L)$ and $\Pi_o^S(\underline{w}^S)$ to determine whether (L, L) or (F, L) is the equilibrium. When $b = 0$, it can be shown that $\Pi_o^F(\underline{w}^L) =$

$\Pi_o^S(\underline{w}^S)$. When $b > 0$, extensive numerical study shows that $\Pi_o^F(\underline{w}^L) < \Pi_o^S(\underline{w}^S)$ always holds; see Figure 7. Thus, the OEM also prefers leadership, and (L, L) is the unique equilibrium for the extended timing game when the OEM determines the wholesale price.

This conclusion appears paradoxical, as it suggests a weak CM actually behaves aggressively in the end-product market. A possible explanation is as follows. Because the OEM sets a very low wholesale price, and the revenue generated from contract manufacturing thus becomes minimal, the competitive CM is forced to become aggressive in the end-product consumer market.

6.3.2 $\alpha > 0$

This subsection considers the general case in which the CM's negotiating power $\alpha > 0$. In extensive numerical study, α is first fixed and p_0 varied, and p_0 is then fixed and α varied; see Table 2 for the list of parameters.

Table 2: Parameters

varying p_0 case	varying α case
$\alpha = 0.2, \quad 1$	$p_0 = 1, \quad 20$
$m = 30, b = 0.5$	$m = 30, b = 0.5$
$p_0 = 0 : 20$ (steplength: 0.5)	$\alpha = 0 : 1$ (steplength: 0.05)

Several patterns are displayed across the various parameter settings, as illustrated in Figures 8 and 9, which depict the payoff differences among the three basic games for each player, thus allowing equilibrium analysis.

Figure 8 shows the impact of p_0 on the quantity leadership preferences of the OEM and CM. When α is small, say $\alpha = 0.2$, (L, L) is the unique equilibrium for the extended timing game. Again, the limited revenue from contract manufacturing forces the competitive CM to become aggressive in the end market. However, as α increases, many different equilibria appear. In particular, when $\alpha = 1$, the equilibrium outcomes are exactly the same as those in Proposition 8; that is, when p_0 is small, (L, L) is the equilibrium; when p_0 is moderate, (L, F) is the equilibrium, as the shaded area of Figure 8 shows; and when p_0 is large, the two equilibria (L, F) and (F, L) coexist.

Figure 9 shows the results for a given p_0. The kinks in the curves are due to the expression of the optimal wholesale price, as shown in Proposition 9: $w^{Nj} = min(p_0, max(\underline{w}^j, K^j))$,

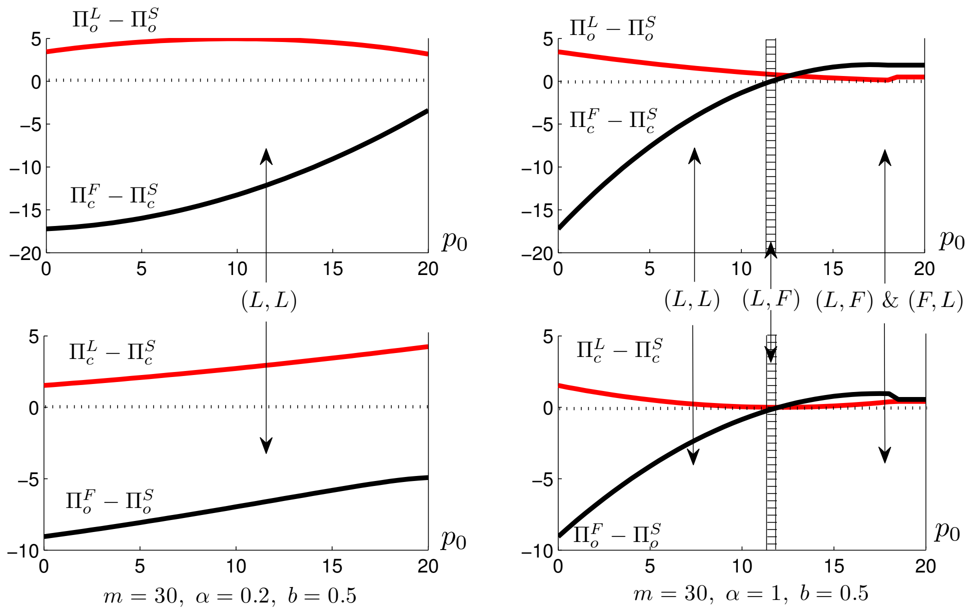

Figure 8: Impact of p_0 on the Equilibrium of the Quantity Timing Game

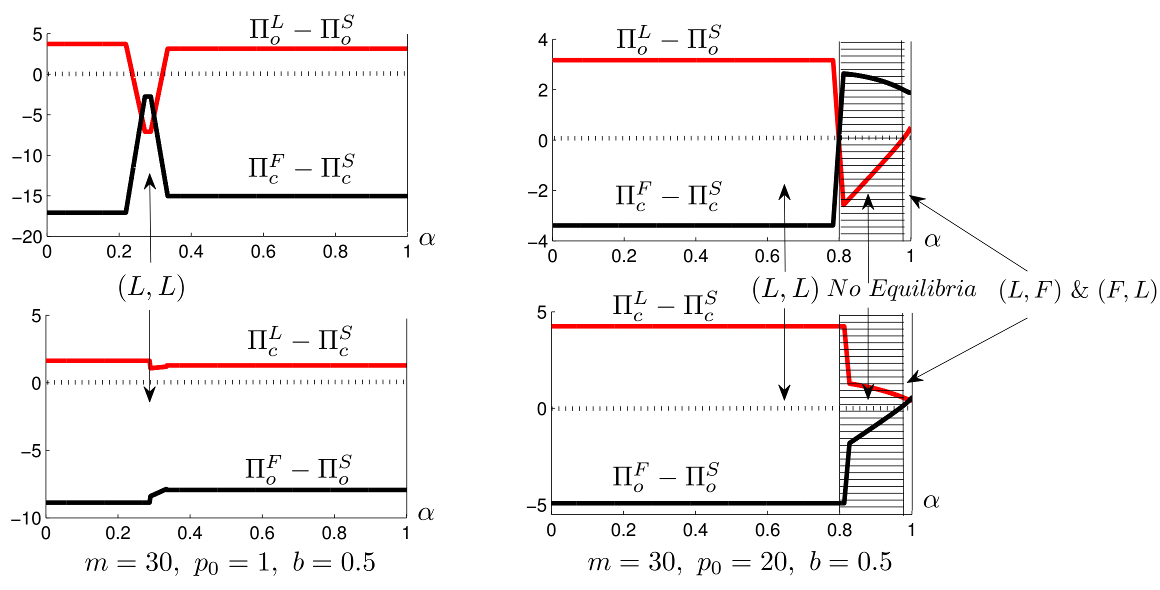

Figure 9: Impact of the CM's bargaining power α on the Equilibrium of the Quantity Timing Game

$j = S, F, L$. Clearly, w^{Nj} is determined by the relative value of three items, p_0, $\underline{w}^j$ and K^j. The first two items are independent of the bargaining power α, whereas the third is increasing in α. When α changes, the optimal wholesale price may take different values from among p_0, $\underline{w}^j$ and K^j, which generates the kinks. When $p_0 = 1$, (L, L) is always the equilibrium no matter how large α is. The CM's revenue again provides an explanation. Although the CM's degree of bargaining power is large here, a low outside option for the OEM, p_0, still allows the OEM to offer a low wholesale price to the CM, which forces the latter to be the market leader. When $p_0 = 20$, the equilibrium outcome depends on α. If α is less than 0.8, then the equilibrium is (L, L). An interesting observation arises when α is larger than 0.8, but less than 0.95 (the shaded area in the figure). In this case, the OEM prefers a simultaneous game, and the competitive CM prefers a sequential-move game, which implies that no pure-strategy equilibrium exists. A closer look at the negotiated wholesale prices in Table 3 explains this phenomenon. The numbers in bold show that the wholesale price in the simultaneous game is the lowest for $0.8 \le p_0 \le 0.95$, which reduces the OEM's incentive to assume market leadership.

Table 3: Impact of Bargaining Power on Negotiated Wholesale Prices

p_0	α	w^{NS}	w^{NF}	w^{NL}
20	0.75	10.26	10.00	10.83
20	**0.80**	**10.26**	**10.40**	**10.83**
20	**0.85**	**11.31**	**11.70**	**11.49**
20	**0.90**	**12.90**	**13.20**	**13.10**
20	**0.95**	**14.89**	**15.06**	**15.10**
20	1.00	18.13	18.00	18.26

The table also shows that when both α and p_0 are large, the two equilibria (L, F) and (F, L) coexist. Complementing the conclusion under $\alpha = 0$ in §6.3.1, the results of numerical studies suggest that the OEM and CM will tend to collaborate by engaging in a sequential-move game when the latter's bargaining power is strong and the former has no favorable outside option (a high p_0).

7 Conclusion

It is quite common today for a CM to be both the upstream partner and downstream competitor of an OEM. The two parties' preferences for quantity leadership and the intensity of competition in the consumer market are intriguing but under-explored issues in the literature. This paper considered asymmetric Cournot competition between an OEM and a competitive CM, and showed that both parties can prefer either Stackelberg leadership or followership depending on the circumstances. Whether the OEM and CM play a simultaneous, OEM-as-leader or CM-as-leader game depends on multiple factors, including the market size, the wholesale price, the outsourcing percentage and the degree to which their products are substitutable. Contrary to the conventional wisdom that the OEM may need to penalize its "competitor", this paper demonstrated that it is actually in the OEM's best interests to treat the competitive CM as a partner. Further, the OEM must be very *cautious* in adopting penalizing practices, such as reducing the amount of production it outsources to this CM or decimating its surplus by offering a very low wholesale price, because doing so can actually spur the competitive CM to develop its own-brand business and intensify the competition in the end market. This paper showed that outsourcing a high proportion of products to a competitive CM at a moderate wholesale price can effectively reduce the CM's incentive to become the Stackelberg leader. If, in contrast, the outsourcing proportion is small or the wholesale price is lower than a threshold value, then both the OEM and the CM will prefer Stackelberg leadership.

The paper further demonstrated that the OEM's profit and production quantities decrease when the competitive CM's products have a higher degree of substitutability, whereas the competitive CM's production quantities increase when this is the case. Interestingly, if the proportion of production the OEM outsources to the competitive CM is high, but the market is small in size, then the CM's profit actually decreases with greater product substitutability. Again, this conclusion implied that as long as the profit generated from contract manufacturing is sufficiently large and the market size is not attractive, the CM has little incentive to develop highly substitutable products.

The paper also investigated the impact of bargaining power on competition between the two parties in a scenario in which the wholesale price is determined through Nash bargaining. The results suggested that the two parties tend to collaborate by engaging in a sequential-move game when the CM's bargaining power is strong and the non-competitive

CM's wholesale price is sufficiently high. Otherwise, the CM becomes aggressive in the end market, and the simultaneous game is likely to be played. In short, a powerful CM is actually more cooperative, a conclusion that may explain the non-market-entry strategies of such large CMs as Foxconn and Flextronics. In the extreme case of the CM deciding the wholesale price as the Stackelberg leader, the findings suggested that the CM will set a low wholesale price to allow itself and the OEM to coexist in the market. Furthermore, the OEM will outsource all of its manufacturing to the competitive CM as long as this CM's wholesale price is lower than the other options available to the OEM.

In summary, this paper's main conclusion is that when a competitive CM is able to obtain a reasonable level of profit from contract manufacturing, it has little incentive to develop self-branded products, mitigating the intensity of the competition between it and the OEM. This paper thus provides another angle from which to view the relationship between OEMs and competitive CMs. Admittedly, outsourcing activities are highly complex in practice. Deciding which functions to outsource and which type of CM to choose involves consideration of multiple factors, such as IP leakage, potential competition from the CM, the product cost structure, tax issues, the lead-time for new product development and inventory liabilities. This paper focuses on the issue of competition and cooperation between an OEM and a CM. It would be interesting in future research to consider a richer model in which some of these other factors are included.

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References

Amir, R., and A. Stepanova. 2006. Second-mover advantage and price leadership in Bertrand duopoly. *Games and Economic Behavior*. **55(1)**, 1-20.

Arruñada, B., and X. Vázquez. 2006. When your contract manufacturer becomes your competitor. *Harvard Business Review*. **84(9)**, 135-144.

Arya, A., B. Mitendorf, and D. Yoon. 2008a. The make-or-buy decision in the presence of a rival: Strategic outsourcing to a common supplier. *Management Science*. **54(10)**, 1747-1758.

Arya, A., B. Mitendorf, and D. Yoon. 2008b. Friction in related-party trade when a rival is also a customer. *Management Science*. **54(11)**, 1850-1860.

Arya, A., B. Mitendorf, and D.E.M. Sappington. 2007. The bright side of supplier encroachment. *Marketing Science*. **26(5)**, 651-659.

Baljko, J. 2006. OEM pressure and changing market demands blur the ODM and EMS boundaries. http://eetimessupplynetwork.com. October 1.

Bernstein, F., and Federgruen A. 2004. Comparative statics, strategic complements and substitutes in oligopolies. *Journal of Mathematical Economic*. **40(6)**, 713-746.

Cachon, G. and P. Harker. 2002. Competition and outsourcing with scale economies. *Management Science*. **48(10)**, 1314-1333.

Chen, K., M. Kaya, and Ö. Özer. 2008. Dual sales channel management with service competition. *Manufacturing & Service Operations Management*. **10(4)**, 654-675.

Chen, Y., S. Shum, and W. Xiao. 2010. Should an OEM retain sourcing when outsourcing to a competing CM? *Production and Operations Management*, Forthcoming.

Chiang, W., D. Chhajed, and J. Hess. 2003. Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management Science*. **49(1)**, 1-20.

Chung, A. 2004. Experts differ on manufacturing. http://www.taipeitimes.com. July 19.

Cui, H., J. Raju, and Z. Zhang. 2008. A price discrimination model of trade promotions. *Marketing Science*. **27(5)**, 779-795.

Dannen, C. 2008. Dell announces low-cost sub-notebook. http://www.fastcompany.com. May 29.

Elmaghraby, W. 2000. Supply contract competition and sourcing policies. *Manufacturing & Service Operations Management*. **2(4)**, 350-371.

Farahat, A.and G. Perakis. 2011. A comparison of Bertrand and Cournot profits in oligopolies with differentiated products. *Operations Research*. **59(2)**, 507-513.

Feng, Q., and L. Lu. 2009. Design outsourcing in a differentiated product market: The role of bargaining and scope economics. Working paper. University of Texas, Austin and

University of North Carolina, Chapel Hill.

Gray, J., B. Tomlin, and A. Roth. 2009. Outsourcing to a powerful contract manufacturer: The effect of learning-by-doing. *Production and Operations Management*. **18(5)**, 487-505.

Gray, J., A. Roth, and B. Tomlin. 2009. The influence of cost and quality priorities on the propensity to outsource production. *Decision Science*. **40(4)**, 697-726.

Hamilton, J., and S. Slutsky. 1990. Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. *Games and Economic Behaviour*. **2(1)**, 29-46.

Himola, O., P. Helo, and M. Holweg. 2005. On the outsourcing dynamics in the electronics sector: the evolving role of the original design manufacturer. Working Paper. University of Cambridge.

Hoetker, G. 2005. How much you know versus how well I know you: Selecting a supplier for a technically innovative component. *Strategic Management Journal*. **26(1)**, 75-96.

Horng, C., and W. Chen. 2007. From contract manufacturing to own brand management: The role of learning and cultural heritage identity. *Management and Organization Review*. **4(1)**, 109-133.

İşlegen, O., and E. L. Plambeck. 2009. Capacity leadership in supply chains with asymmetric demand information and noncontractible capacity. Working paper. Stanford University.

Kaya, M., and Ö. Özer. 2009. Quality risk in outsourcing: Noncontractible product quality and private quality cost information. *Naval Research Logistics*. **56(7)**, 669-685.

Lariviere, M. and E. Porteus. 2001. Selling to the newsvendor:An analysis of price-only contracts. *Manufacturing & Service Operations Management*. **3(4)**, 293-305.

Leiblein, M. and D. Miller. 2003. An empirical examination of transaction-and firm-level influences on the vertical boundaries of the firm. *Strategic Management Journal*. **24(9)**, 839-859.

Laptops. 2007. World's easiest PC' going for $199 - ASUS Eee PC. http://www.nforcershq.com. June 11.

Lim, W. and S. Tan. 2010. Outsourcing supplier as downstream competitors: Biting the hand that feeds. *European Journal of Operational Research*. **203(2)**, 360-369.

Lin, Z. 2005. Asustek snatches Apple iBook orders. http://pro.udnjob.com. June 20.

Liu, J. 2009. Foreign traders shift export-oriented goods to domestic sales. http://en.ce.cn. May 8.

McGovern, G., and J. Quelch. 2005. Outsourcing marketing. *Harvard Business Review.* **83(3)**, 22-26.

Nagarajan, M., and G. Sosic. 2008. Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *European Journal of Operational Research.* **187(3)**, 719-745.

Nagarajan, M., and Y. Bassok. 2008. A bargaining framework in supply chains. *Management Science.***54(8)**, 1482-1496.

Nystedt, D. 2007. Acer passes Lenovo in Q1, next up: Dell. http://pcworld.about.com. April 27.

Nash, 1950. The bargaining problem. *Econometrica.* **18(2)**, 155-162.

Ozkan, B., and S. Wu. 2009a. Market entry analysis when releasing distinctive products in independent markets. Working paper. Lehigh University.

Ozkan, B., and S. Wu. 2009b. Capacity rationing for contract manufacturers serving multiple markets. Working paper. Lehigh University.

Parmigiani, A. 2007. Why do firms make and buy? An investigation of concurrent sourcing. *Strategic Management Journal.* **28(3)**, 285-311.

Roth, A. 1979. *Axiomatic Models in Bargaining.* Springer-Verlag, Germany.

Spiegel, Y. 1993. Horizontal subcontracting. *Rand Journal of Economics.* **24(4)**, 570-590.

Stremersch, S., A. Weiss, B. Dellaert and R. Frambach. 2003. Buying modular systems in technology-intensive markets. *Journal of Marketing Research.* **40(3)**, 335-350.

Smith A. 2008. Dell plans to shake up business model, sell factories, reports say. http://www.dallasnews.com. September 8.

Shilov, A. 2007. Asustek's reformation plans remain vague. http://www.xbitlabs.com. March 30.

Tomlin, B. 2006. On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science.* **52(5)**, 639-657.

Tsay, A. and N. Agrawal. 2004a. Modeling conflict and coordination in multi-channel distribution systems: A Review. *Handbook of Quantitative Supply Chain Analysis: supply* *chain analysis in the ebusiness era*. D. Simchi-Levi, D. Wu and M. Shen, eds. Kluwer Academic Publishers, Boston, MA.

Tsay, A., and N. Agrawal. 2004b. Channel conflict and coordination in the e-commerce age. *Production and Operations Management*. **13(1)**, 93-110.

Ülkü, S., B. Toktay, and E. Yucesan. 2007. Risk ownership in contract manufacturing. *Manufacturing & Service Operations Management*. **9(3)**, 225-241.

van Damme, E., and S. Hurkens. 2004. Endogenous price leadership. *Games and Economic Behavior*. **47(2)**, 404-420.

Vives, X. 2001. *Oligopoly pricing: Old ideas and new tools*. MIT Press.

Vilches, J. 2007. Second generation Asus Eee PC to come April 2008? http://www.techspot.com. September 6.

Wang, L. 2006. BenQ's European failure sparks record third-quarter losses. http://www. taipeitimes.com. October 25.

Wang, A. 2008. The changing outsourcing landscape in China. http:// www.edn.com. January 1.

Wang, T., D. Thomas and N. Rudi. 2009. The effect of competition on the efficient responsive supply chain choice. Working paper. National University of Singapore.

Williamson, O. 1985. *The economic institutions of capitalism: Firms, markets, relational contracting*. Free Press.

Yang, Y. 2006. The Taiwanese notebook computer production network in China: Implication for upgrading of the Chinese electronics industry. Working paper. Manchester University.

Online Appendices

"On the Advantage of Quantity Leadership

When Outsourcing Production to a Competitive Contract Manufacturer"

Y. Wang, B. Niu, P. Guo

Appendix A Proofs

Proof of Proposition 1. For the simultaneous game, by maximizing (2) and (3) the best response functions are**:**

$$q_o(q_c) = \frac{m - bq_c - w}{2};\ q_c(q_o) = \frac{m - q_o}{2}.$$

Solving these two equations yields the equilibrium quantities $q_o^S = \frac{(2-b)m-2w}{4-b}$, $q_c^S = \frac{m+w}{4-b}$. The corresponding equilibrium profits can be obtained by substituting q_o^S and q_o^S into functions (2) and (3):

$$\Pi_o^S = \frac{[(2-b)m-2w]^2}{(4-b)^2};\ \Pi_c^S = \frac{(m+w)^2}{(4-b)^2} + \frac{[(2-b)m-2w]\theta w}{4-b}.$$

For the OEM-as-leader game, substituting $q_c(q_o)$ into the OEM's profit function yields $\Pi_o = (m - q_o - b\frac{m-q_o}{2} - w)q_o$. Maximizing the above objective function yields the optimal production quantity for the OEM: $q_o^L = \frac{(2-b)m-2w}{4-2b}$. Moreover, the corresponding optimal decision for the CM is $q_c^F = \frac{m-q_o^L}{2} = \frac{(2-b)m+2w}{8-4b}$. The procedure for the CM-as-leader game is similar to the foregoing analysis.

Proof of Proposition 2. When $m > max\left\{\frac{4-b^2\theta-b}{4-3b}, \frac{2}{2-b}\right\}w$, all three basic games exist. In the following, we compare the performance of the OEM and the CM under the three basic games derived in Proposition 1. First**,** we show that

$$\begin{aligned}
\Pi_o^L - \Pi_o^S &= \frac{[(2-b)m-2w]^2}{16-8b} - \frac{[(2-b)m-2w]^2}{(4-b)^2} = \frac{[(2-b)m-2w]^2 b^2}{8(4-b)^2(2-b)} > 0,\\
\Pi_c^L - \Pi_c^S &= \frac{[m+(1+b\theta)w][m+(1-b\theta)w]}{8(2-b)} + \frac{[(4-3b)m-(4-b^2\theta-b)w]\theta w}{4(2-b)}\\
&\quad - \frac{(m+w)^2}{(4-b)^2} - \frac{[(2-b)m-2w]\theta w}{4-b}\\
&= \frac{b^2[(b^2\theta^2 - 8b\theta^2 + 16\theta^2 - 8\theta + 2b\theta + 1)w^2 + 2(1-4\theta+b\theta)mw + m^2]}{8(2-b)(4-b)^2}\\
&= \frac{b^2[(b\theta+1-4\theta)w+m]^2}{8(2-b)(4-b)^2} \geq 0.
\end{aligned}$$

Hence, $\Pi_o^L > \Pi_o^S$ and $\Pi_c^L > \Pi_c^S$. Next, we have

$$\begin{aligned}\Pi_c^F - \Pi_c^S &= [\frac{(2-b)m+2w}{4(2-b)} + \frac{w+m}{4-b}][\frac{(2-b)m+2w}{4(2-b)} - \frac{w+m}{4-b}] + \theta w[(2-b)m-2w](\frac{1}{2(2-b)} - \frac{1}{4-b}) \\ &= \frac{[(b^2-10b+16)m+(16-6b)w][b((b-2)m+2w)]}{16(2-b)^2(4-b)^2} + \frac{[(2-b)m-2w]\theta wb}{2(2-b)(4-b)} \\ &= \frac{b[(2-b)m-2w]}{16(2-b)^2(4-b)^2}[8\theta(2-b)(4-b)-16+6b]w-(16-10b+b^2)m].\end{aligned}$$

Then, the sign of $\Pi_c^F - \Pi_c^S$ depends on that of

$$[8\theta(2-b)(4-b)-16+6b]w-(16-10b+b^2)m. \tag{8}$$

Note that if $\theta \leq \frac{1}{2-b}$, $\frac{4-b^2\theta-b}{4-3b}w \geq \frac{2}{2-b}w$, then $m \geq \frac{4-b^2\theta-b}{4-3b}w$; if $\frac{1}{2-b} < \theta \leq 1$, then $\frac{4-b^2\theta-b}{4-3b}w < \frac{2}{2-b}w$ and $m \geq \frac{2}{2-b}w$.

Case 1: $\theta \in [\frac{1}{2-b}, 1]$. If $\theta \in [\frac{1}{2-b}, 1]$, then $8\theta(2-b)(4-b)-16+6b \geq 8(4-b)-16+6b > 0$. Thus, if $w \geq \frac{(16-10b+b^2)m}{8\theta(2-b)(4-b)-16+6b} = w_{AL}$, then equation (8) is positive. Furthermore, we show that

$$\begin{aligned}w_{AL} - w_{AF} &= \frac{(16-10b+b^2)m}{8\theta(2-b)(4-b)-16+6b} - \frac{1}{(4-b)\theta-1}m \\ &= \frac{(-8b\theta+6b^2\theta+4b-b^3-b^2)m}{[8\theta(2-b)(4-b)-16+6b][(4-b)\theta-1]} \\ &= \frac{(4-b)[b-\theta b(2-b)]m}{[8\theta(2-b)(4-b)-16+6b][(4-b)\theta-1]} \leq 0, \\ w_{AF} - \frac{2-b}{2}m &= \frac{(4-b)[1-(2-b)\theta]m}{2[(4-b)\theta-1]} \leq 0\end{aligned}$$

Thus, $w_{AL} \leq w_{AF} \leq \frac{2-b}{2}m$ if $\theta \in [\frac{1}{2-b}, 1]$, and $\Pi_c^F \geq \Pi_c^S$ if $w \in [w_{AL}, \frac{2-b}{2}m]$. Otherwise, $\Pi_c^F < \Pi_c^S$.

Case 2: $\theta \in [0, \frac{1}{2-b})$. Equation (8) implies that $\Pi_c^F \geq \Pi_c^S$ if $m \leq \frac{[8\theta(2-b)(4-b)-16+6b]w}{16-10b+b^2}$. However,

$$\begin{aligned}&\frac{[8\theta(2-b)(4-b)-16+6b]w}{16-10b+b^2} - \frac{(4-b^2\theta-b)w}{4-3b} \\ &= \frac{w\{\theta(2-b)[(1-b)(128+b^2)+31b^2]-[(1-b)(128+b^2)+31b^2]\}}{(16-10b+b^2)(4-3b)} \\ &= \frac{w[(1-b)(128+b^2)+31b^2][\theta(2-b)-1]}{(16-10b+b^2)(4-3b)} < 0.\end{aligned}$$

Hence, $\Pi_c^F \geq \Pi_c^S$ requires $m \leq \frac{(4-b^2\theta-b)w}{4-3b}$, which cannot hold. Thus, $\Pi_c^F < \Pi_c^S$.

Similarly, we can show that the sign of $\Pi_o^F - \Pi_o^S$ is the same as that of

$$[(4-b)\theta-1]w-m. \tag{9}$$

Case 1: $\theta \in [\frac{1}{2-b}, 1]$. If $\theta \in [\frac{1}{2-b}, 1]$, then $(4-b)\theta - 1 \geq \frac{4-b}{2-b} - 1 > 0$. Then, equation (9) is positive if $w \geq \frac{1}{(4-b)\theta-1}m = w_{AF}$. Therefore, $\Pi_o^F \geq \Pi_o^S$ if $w \in [w_{AF}, \frac{2-b}{2}m]$. Otherwise, $\Pi_o^F < \Pi_o^S$.

Case 2: $\theta \in [0, \frac{1}{2-b})$. Equation (9) shows that $\Pi_o^F \geq \Pi_o^S$ requires $m \leq [(4-b)\theta - 1]w$. Note that here, $m > \frac{[4-(1+\theta)b^2]w}{4-3b}$, but

$$
\begin{aligned}
[(4-b)\theta - 1]w - \frac{(4-b^2\theta-b)w}{4-3b} &= \frac{w[(4\theta - b\theta - 1)(4-3b) - (4-b^2\theta-b)]}{4-3b} \\
&= \frac{4w(2-b)[\theta(2-b)-1]}{4-3b} < 0.
\end{aligned}
$$

Therefore, $\Pi_o^F \geq \Pi_o^S$ cannot hold. Hence, $\Pi_o^F < \Pi_o^S$ if $\theta \in [0, \frac{1}{2-b})$. In summary, we have

$$
\begin{cases}
\Pi_c^F \geq \Pi_c^S, & \text{if} \quad \theta \in [\frac{1}{2-b}, 1] \;\&\; w \in [w_{AL}, \frac{2-b}{2}m], \\
\Pi_c^F < \Pi_c^S, & \text{if} \quad \theta \in [0, \frac{1}{2-b}) \text{ or } \theta \in [\frac{1}{2-b}, 1] \;\&\; w \in [0, w_{AL}).
\end{cases} \tag{10}
$$

$$
\begin{cases}
\Pi_o^F \geq \Pi_o^S, & \text{if} \quad \theta \in [\frac{1}{2-b}, 1] \;\&\; w \in [w_{AF}, \frac{2-b}{2}m], \\
\Pi_o^F < \Pi_o^S, & \text{if} \quad \theta \in [0, \frac{1}{2-b}) \text{ or } \theta \in [\frac{1}{2-b}, 1] \& \; w \in [0, w_{AF}).
\end{cases} \tag{11}
$$

Because $\Pi_o^L \geq \Pi_o^S$ and $\Pi_c^L \geq \Pi_c^S$, based on Table 1, we have (L, F) which is a NE if $\Pi_c^F \geq \Pi_c^S$; (F, L) is a NE if $\Pi_o^F \geq \Pi_o^S$; if $\Pi_c^S \geq \Pi_c^F$, then L is a dominant strategy for the CM; if $\Pi_o^S \geq \Pi_o^F$, then L is a dominant strategy for the OEM; and F is never a dominant strategy for the OEM or the CM. Therefore, we have proved Proposition 2 based on equations (10) and (11).

Proof of Corollary 1. Assume that $\theta \in [\frac{1}{2-b}, 1]$, then, $\max(\frac{4-b^2\theta-b}{4-3b}w, \frac{2}{2-b}w) = \frac{2}{2-b}w$. Let $m = \frac{k}{2-b}w$, $k > 2$. Then, $w = \frac{(2-b)m}{k}$. Equation (10) indicates that $\Pi_c^F \geq \Pi_c^S$ requires that $w \geq w_{AL}$, which is equivalent to $\theta \geq \frac{k(8-b)+16-6b}{8(2-b)(4-b)} = \theta_{AL}$. Because

$$
\theta_{AL} - \frac{1}{2-b} = \frac{k(8-b) + 16 - 6b - 8(4-b)}{8(2-b)(4-b)} = \frac{(k-2)(8-b)}{8(2-b)(4-b)} > 0,
$$

$\Pi_c^F \geq \Pi_c^S$ requires $\theta_{AL} \leq \theta \leq 1$. Similarly, we can show that $\Pi_c^F < \Pi_c^S$ requires $\frac{1}{2-b} \leq \theta < \min(\theta_{AL}, 1)$.

From equation (11), we have $\Pi_o^F \geq \Pi_o^S$ which requires $w \geq w_{AF}$. This condition is equivalent to $\theta \geq \frac{k+(2-b)}{(2-b)(4-b)}$. Denote $\frac{k+(2-b)}{(2-b)(4-b)}$ as θ_{AF}. $k > 2$ implies that $\theta_{AF} > \frac{1}{2-b}$. Thus, $\Pi_o^F \geq \Pi_o^S$ if $\theta \in [\theta_{AF}, 1]$. We can also show that $\Pi_o^F < \Pi_o^S$ if $\frac{1}{2-b} \leq \theta < \min(\theta_{AF}, 1)$. In summary, we have

$$
\begin{cases}
\Pi_c^F \geq \Pi_c^S, & \text{if} \quad \theta \in [\theta_{AL}, 1], \\
\Pi_c^F < \Pi_c^S, & \text{if} \quad \theta \in [\frac{1}{2-b}, \min(\theta_{AL}, 1)).
\end{cases}
\quad \text{and} \quad
\begin{cases}
\Pi_o^F \geq \Pi_o^S, & \text{if} \quad \theta \in [\theta_{AF}, 1], \\
\Pi_o^F < \Pi_o^S, & \text{if} \quad \theta \in [\frac{1}{2-b}, \min(\theta_{AF}, 1)).
\end{cases} \tag{12}
$$

In addition, because

$$\theta_{AL} - \theta_{AF} = \frac{k(8-b)+16-6b}{8(2-b)(4-b)} - \frac{k+(2-b)}{(2-b)(4-b)} = \frac{b(2-k)}{8(2-b)(4-b)} < 0,$$

$\theta_{AL} < \theta_{AF}$. Further, we can show that

$$\begin{aligned}
\frac{\partial \theta_{AL}}{\partial b} &= \frac{(-k-6)(2-b)(4-b) - [k(8-b)+16-6b](-4+b-2+b)}{8(2-b)^2(4-b)^2} \\
&= \frac{-(k+6)(2-b)(4-b) + 2[k(8-b)+16-6b](3-b)}{8(2-b)^2(4-b)^2} \\
&= \frac{(40-16b+b^2)k + 48 - 32b + 6b^2}{8(2-b)^2(4-b)^2} > 0, \\
\frac{\partial \theta_{AF}}{\partial b} &= \frac{-(2-b)(4-b) - (k+2-b)(-4+b-2+b)}{(2-b)^2(4-b)^2} \\
&= \frac{-8+6b-b^2+2(k+2-b)(3-b)}{(2-b)^2(4-b)^2} = \frac{2(3-b)k+4-4b+b^2}{(2-b)^2(4-b)^2} > 0, \\
\frac{\partial(\theta_{AF}-\theta_{AL})}{\partial b} &= \frac{(k-2)(2-b)(4-b) - b(k-2)(-4+b-2+b)}{8(2-b)^2(4-b)^2} = \frac{(k-2)(8-b^2)}{8(2-b)^2(4-b)^2} > 0.
\end{aligned}$$

Similar to Proposition 2, we derive Corollary 1 based on equation (12).

Proof of Proposition 3: We first consider the simultaneous game. Taking the first-order condition (FOC) of the equilibrium production quantities yields

$$\frac{\partial q_o^S}{\partial b} = \frac{-2(m+w)}{(4-b)^2} < 0, \ \frac{\partial q_c^S}{\partial b} = \frac{(m+w)}{(4-b)^2} > 0.$$

Correspondingly, we have

$$\begin{aligned}
\frac{\partial \Pi_o^S}{\partial b} &= \frac{\partial \Pi_o^S}{\partial q_o^S}\frac{\partial q_o^S}{\partial b} = 2q_o^S\frac{\partial q_o^S}{\partial b} < 0, \\
\frac{\partial \Pi_c^S}{\partial b} &= \frac{2(m+w)^2}{(4-b)^3} + \frac{-(4-b)\theta mw + [(2-b)m-2w]\theta w}{(4-b)^2} \\
&= \frac{2[m^2 + (2-(4-b)\theta)mw + (1-(4-b)\theta)w^2]}{(4-b)^3} \\
&= \frac{2(m+w)(m-((4-b)\theta-1)w)}{(4-b)^3}.
\end{aligned}$$

Hence, $\frac{\partial \Pi_c^S}{\partial b} > 0$ if $m > ((4-b)\theta-1)w$. Note that we have assumed $m > \frac{2}{2-b}w$. Comparing $((4-b)\theta-1)w$ with $\frac{2}{2-b}w$ yields $\frac{2}{2-b}w - ((4-b)\theta-1)w = \frac{(4-b)(1-(2-b)\theta)}{2-b}w$. Therefore, Π_c^S is increasing in b if $\theta \in [0, \frac{1}{2-b}]$; if $\theta \in (\frac{1}{2-b}, 1]$, then it is decreasing in b when $m \in [\frac{2}{2-b}w, ((4-b)\theta-1)w]$ and is increasing in b otherwise.

Now we consider the OEM-as-leader game. We similarly take the FOC of the equilibrium production quantities and obtain

$$\frac{\partial q_o^L}{\partial b} = \frac{-w}{(2-b)^2} < 0, \; \frac{\partial q_c^F}{\partial b} = \frac{w}{2(2-b)^2} > 0, \quad \frac{\partial \Pi_o^L}{\partial b} = \frac{-[(2-b)m-2w][(2-b)m+2w]}{8(2-b)^2} < 0.$$

In addition,

$$\begin{aligned}\frac{\partial \Pi_c^F}{\partial b} &= \frac{2[(2-b)m+2w](-m)(2-b)^2 + 2[(2-b)m+2w]^2(2-b)}{16(2-b)^4} + \frac{[(2-b)m-2w-(2-b)m]\theta w}{2(2-b)^2} \\ &= \frac{[(2-b)m-(4(2-b)\theta-2)w]w}{4(2-b)^3}.\end{aligned}$$

Hence, $\frac{\partial \Pi_c^F}{\partial b} > 0$ if $m > \frac{4(2-b)\theta-2}{2-b}w$. Combining our assumption that $m > \frac{2}{2-b}w$, we have $\frac{2}{2-b}w - \frac{4(2-b)\theta-2}{2-b}w = \frac{4[1-(2-b)\theta]}{2-b}$. Similarly, we can show that Π_c^F is increasing in b if $\theta \in [0, \frac{1}{2-b}]$; if $\theta \in (\frac{1}{2-b}, 1]$, then it is decreasing in b when $m \in [\frac{2}{2-b}w, \frac{4(2-b)\theta-2}{2-b}w]$ and is increasing in b otherwise.

Lastly, we consider the CM-as-leader game. The FOCs of the equilibrium production quantities are

$$\frac{\partial q_o^F}{\partial b} = \frac{-2m+(-2+4b\theta-b^2\theta)w}{4(2-b)^2}; \frac{\partial q_c^L}{\partial b} = \frac{m-(2\theta-1)w}{2(2-b)^2}.$$

Note that our assumption in this basic game is that $m > \frac{4-b^2\theta-b}{4-3b}w$; therefore,

$$\begin{aligned}-2m+(-2+4b\theta-b^2\theta)w &< \frac{-2(4-b^2\theta-b)+(4-3b)(-2+4b\theta-b^2\theta)}{(4-3b)}w \\ &= \frac{(2-b)[(8-3b)b\theta-8]}{(4-3b)}w < 0; \\ m-(2\theta-1)w &> \frac{4-b^2\theta-b-(4-3b)(2\theta-1)}{4-3b}w \\ &= \frac{(2-b)(4-(4-b)\theta)}{4-3b}w > 0.\end{aligned}$$

Thus, $\frac{\partial q_o^F}{\partial b} < 0$ and $\frac{\partial q_c^L}{\partial b} > 0$. We can then show that

$$\frac{\partial \Pi_o^F}{\partial b} = \frac{\partial \Pi_o^F}{\partial q_o^F}\frac{\partial q_o^F}{\partial b} = 2q_o^F\frac{\partial q_o^F}{\partial b} < 0.$$

We also have

$$\begin{aligned}\frac{\partial \Pi_c^L}{\partial b} &= \frac{m^2+2(1-2\theta)mw+(1-4\theta+4b\theta^2-b^2\theta^2)}{8(2-b)^2} \\ &= \frac{[m+(1-b\theta)w][m-((4-b)\theta-1)w]}{8(2-b)^2}.\end{aligned}$$

Hence, $\frac{\partial \Pi_c^L}{\partial b} > 0$ if $m > ((4-b)\theta-1)w$. Note that $\frac{4-b^2\theta-b}{4-3b}w-((4-b)\theta-1)w = \frac{4(2-b)[1-(2-b)\theta]}{4-3b}w$. Similarly, we can show that Π_c^L is increasing in b if $\theta \in [0, \frac{1}{2-b}]$; if $\theta \in (\frac{1}{2-b}, 1]$, then it is decreasing in b when $m \in [\frac{4-b^2\theta-b}{4-3b}w, ((4-b)\theta-1)w]$ and is increasing in b otherwise.

Proof of Proposition 4:

First, given the outsourcing decision θ and wholesale price w, and drawing on Proposition 1, we can derive the equilibrium production quantities of the OEM and the CM as

$$q_o^{S*}(\theta, w) = \frac{(2-b)m + 2(p_0 - w)\theta - 2p_0}{4-b}, \quad q_c^{S*}(\theta, w) = \frac{m - (p_0 - w)\theta + p_0}{4-b}.$$

Next, substituting q_o^{S*} and q_c^{S*} into the OEM's profit function (4) yields the OEM's equilibrium profit

$$\Pi_o^S(\theta) = \frac{[(2-b)m + 2(p_0 - w)\theta - 2p_0]^2}{(4-b)^2},$$

which increases in θ. Therefore, the OEM will set $\theta^* = 1$.

In addition, substituting q_o^{S*}, q_c^{S*} and $\theta^* = 1$ into the CM's profit function (5) results in

$$\Pi_c^S(w) = \frac{[m - (p_0 - w)\theta + p_0]^2}{(4-b)^2} + \frac{[(2-b)m + 2(p_0 - w)\theta - 2p_0]w}{4-b},$$

which is concave in w. Taking the first order derivative of $\Pi_c^S(w)$ w.r.t. w yields

$$\frac{\partial \Pi_c^S(\theta, w)}{\partial w} = \frac{[(10 - 6b + b^2)m - (14 - 4b)w}{(4-b)^2}.$$

Setting the above to be equal to 0 generates the optimal solution, $\frac{10-6b+b^2}{14-4b}m$. According to the analysis in §3.1, $m > \frac{2}{2-b}w$ or $w < \frac{2-b}{2}m$ is required for the OEM to remain in the market. It can be verified that $\frac{10-6b+b^2}{14-4b}m - \frac{2-b}{2}m = \frac{-(4-b)(1-b)}{14-4b}m \leq 0$.

As $w \leq p_0$, the optimal pricing decision of the CM is to set

$$w^{S*} = \min\left\{p_0, \frac{10 - 6b + b^2}{14 - 4b}m\right\}.$$

Substituting w^{S*} and θ^* into (4) and (5), we can obtain the optimal profits of the OEM and the competitive CM.

Proof of Corollary 2: If $p_0 > \frac{10-6b+b^2}{14-4b}m$, then $w^{S*} = \frac{10-6b+b^2}{14-4b}m$. The maximum profit that the CM can obtain with the OEM in the market is $\Pi_c^{S*} = \frac{(8-4b+b^2)}{4(7-2b)}m^2$. It can be verified that $\frac{(8-4b+b^2)}{4(7-2b)}m^2 - \frac{m^2}{4} = \frac{(1-b)^2}{4(7-2b)}m^2 \geq 0$.

If $p_0 \le \frac{10-6b+b^2}{14-4b}m$, then $w^{S*} = p_0$. The maximum profit that the CM can obtain with the OEM in the market is $\Pi_c^{S*} = \frac{(m+p_0)^2}{(4-b)^2} + \frac{[(2-b)m-2p_0]p_0}{4-b}$. Comparing Π_c^{S*} and Π_c^m yields

$$\begin{aligned}
\Pi_c^{S*} - \Pi_c^m &= \frac{(m+p_0)^2}{(4-b)^2} + \frac{[(2-b)m-2p_0]p_0}{4-b} - \frac{m^2}{4} \\
&= \frac{4(m+p_0)^2 + 4(4-b)[(2-b)m-2p_0]p_0 - (4-b)^2m^2}{4(4-b)^2} \\
&= \frac{-4(7-2b)p_0^2 + 4(10-6b+b^2)mp_0 - (2-b)(6-b)m^2}{4(4-b)^2} \\
&= \frac{[2(7-2b)p_0 - (6-b)m][(2-b)m-2p_0]}{4(4-b)^2}.
\end{aligned}$$

Hence, $\Pi_c^{S*} \le \Pi_c^m$ if $p_0 \in [0, \frac{6-b}{14-4b}m]$. Otherwise, $\Pi_c^{S*} > \Pi_c^m$. It can be verified that $\frac{6-b}{14-4b}m \le \frac{10-6b+b^2}{14-4b}m$.

Proof of Proposition 5: First, given the wholesale price w and the OEM's production quantity $q_o(\theta, w)$ and outsourcing decision θ, the CM maximizes its profit by choosing an optimal production quantity. It can be shown that the optimal production quantity the CM should produce for its own-brand products is

$$q_c(q_o, \theta, w) = \frac{m - q_o(\theta, w)}{2}.$$

Second, anticipating the CM's optimal production decision, the OEM makes its production and outsourcing decisions to maximize its own profit. Substituting the production decision of the CM into (4) yields

$$\max_{q_0, \theta} \quad \Pi_o^L(w) = \frac{-(2-b)q_o^2 + [(2-b)m + 2(p_0 - w)\theta - 2p_0]q_o}{2}.$$

The above objective function is increasing in θ and concave in w. It can be shown that the optimal decisions of the OEM and the corresponding quantity decision of the CM are

$$\theta^*(w) = 1, \quad q_o^{L*}(w) = \frac{(2-b)m - 2w}{2(2-b)}, \quad q_c^{F*}(w) = \frac{(2-b)m + 2w}{4(2-b)}.$$

Finally, substituting the above production quantities and outsourcing decision into the CM's profit function $\Pi_c^F(w)$ yields

$$\max_{w} \quad \Pi_c^F = \frac{[(2-b)m + 2w]^2}{16(2-b)^2} + \frac{[(2-b)m - 2w]w}{2(2-b)}.$$

Taking the FOC, we have

$$\frac{\partial \Pi_c^F}{\partial w} = \frac{4[(2-b)m + 2w]}{16(2-b)^2} + \frac{[(2-b)m - 2w] - 2w}{2(2-b)} = \frac{(2-b)(5-2b)m - 2(7-4b)w}{4(2-b)^2}.$$

Hence, the optimal wholesale price is

$$w^{F*} = \min\{p_0, \frac{(2-b)(5-2b)}{14-8b}m\}.$$

It can be verified that $w^{F*} < \frac{2-b}{2}m$. Substituting w^{F*} and θ^* into (4) and (5), we can obtain the optimal profits of the OEM and the competitive CM listed in Proposition 5.

Proof of Corollary 3: If $p_0 > \frac{(2-b)(5-2b)}{14-8b}m$, then $w^{F*} = \frac{(2-b)(5-2b)}{14-8b}m$. The corresponding profit becomes $\Pi_c^{F*} = \frac{(2-b)(4-b)m^2}{4(7-4b)}$. It can be verified that $\Pi_c^{F*} - \Pi_c^m = \frac{(2-b)(4-b)m^2}{4(7-4b)} - \frac{m^2}{4} = \frac{(1-b)^2}{4(7-4b)}m^2 \geq 0$.

If $p_0 \leq \frac{(2-b)(5-2b)}{14-8b}m$, then $w^{F*} = p_0$. Thus, we have

$$\begin{aligned}
\Pi_c^{F*} - \Pi_c^m &= \frac{[(2-b)m+2p_0]^2}{16(2-b)^2} + \frac{[(2-b)m-2p_0]p_0}{2(2-b)} - \frac{m^2}{4} \\
&= \frac{[(2-b)m+2p_0]^2 + 8(2-b)[(2-b)m-2p_0]p_0 - 4(2-b)^2m^2}{16(2-b)^2} \\
&= \frac{-4(7-4b)p_0^2 + 4(2-b)(5-2b)mp_0 - 3(2-b)^2m^2}{16(2-b)^2} \\
&= \frac{[2(7-4b)p_0 - 3(2-b)m][(2-b)m-2p_0]}{16(2-b)^2}.
\end{aligned}$$

Note that $\frac{3(2-b)}{14-8b}m - \frac{(2-b)(5-2b)}{14-8b}m = \frac{-2(2-b)(1-b)}{(14-8b)}m \leq 0$. Hence, if $p_0 \in [0, \frac{3(2-b)}{14-8b}m]$, then $\Pi_c^{F*} \leq \Pi_c^m$; otherwise, $\Pi_c^{F*} > \Pi_c^m$.

Proof of Proposition 6: First, given production quantity q_c and wholesale price w, the OEM will choose θ and a production quantity to maximize its profit. As the OEM's profit function (4) is increasing in θ, the OEM will set $\theta^* = 1$. Taking the FOC with respect to (4) yields

$$q_o^*(q_c, w) = \frac{m - bq_c - w}{2}.$$

Next, anticipating the OEM's optimal decisions, the CM makes decisions about its production quantity and wholesale price to maximize its own profit. Substituting $q_o^*(q_c, w)$ and $\theta^* = 1$ into (5) generates the following decision problem for the CM.

$$\max_{q_c, w} \Pi_c^L = \frac{[m-(2-b)q_c+w]q_c}{2} + \frac{(m-bq_c-w)w}{2}.$$

The FOCs are

$$w(q_c) = \frac{m+(1-b)q_c}{2}, \quad q_c(w) = \frac{m+(1-b)w}{2(2-b)}.$$

Solving these two equations yields $w^{L*} = min\{p_0, \frac{5-3b}{7-2b-b^2}m\}$. It can be verified that $w^{L*} < \frac{4-3b}{4-b-b^2}m$. Substituting w^{L*} and θ^* into (4) and (5) yields the optimal profits of the OEM and the competitive CM.

Proof of Corollary 4: If $p_0 > \frac{5-3b}{7-2b-b^2}m$, then $w^{L*} = \frac{5-3b}{7-2b-b^2}m$. The competitive CM's optimal profit is $\Pi_c^{L*} = \frac{(2-b)}{7-2b-b^2}m^2$. It can be verified that $\Pi_c^{L*} - \Pi_c^m = \frac{(1-b)^2}{4(7-2b-b^2)}m^2 \geq 0$.

If $p_0 \leq \frac{5-3b}{7-2b-b^2}m$, then $w^{L*} = p_0$. The corresponding profit is

$$\Pi_c^{L*} = \frac{[m+(1+b)p_0][m+(1-b)p_0]}{8(2-b)} + \frac{[(4-3b)m-(4-b-b^2)p_0]p_0}{4(2-b)}.$$

Taking the FOC on p_0 yields

$$\begin{aligned}\frac{\partial \Pi_c^{L*}}{\partial p_0} &= \frac{(1+b)(m+p_0-bp_0)+(1-b)(m+p_0+bp_0)}{8(2-b)} + \frac{(4-3b)m-(4-b-b^2)p_0-(4-b-b^2)p_0}{4(2-b)} \\ &= \frac{(5-3b)m-(7-2b-b^2)p_0}{4(2-b)} \geq 0.\end{aligned}$$

Hence, Π_c^{L*} is increasing in p_0 and there exists a unique p_0 at which $\Pi_c^{L*} - \Pi_c^m = 0$. Note that

$$\Pi_c^{L*}(p_0=0) - \Pi_c^m = \frac{-3+2b}{8(2-b)}m^2 \leq 0, \text{ and } \Pi_c^{L*}(p_0 = \frac{5-3b}{7-2b-b^2}m) - \Pi_c^m = \frac{(1-b)^2}{4(7-2b-b^2)}m^2 \geq 0.$$

Solving

$$\Pi_c^{L*} - \Pi_c^m = \frac{[m+(1+b)p_0][m+(1-b)p_0]}{8(2-b)} + \frac{[(4-3b)m-(4-b-b^2)p_0]p_0}{4(2-b)} - \frac{m^2}{4} = 0$$

yields the feasible root $\frac{(5-3b)-(1-b)\sqrt{2(2-b)}}{7-2b-b^2}m$. Therefore, if $p_0 \in [0, \frac{(5-3b)-(1-b)\sqrt{2(2-b)}}{7-2b-b^2}m]$, then $\Pi_c^{L*} \leq \Pi_c^m$; otherwise, $\Pi_c^{L*} > \Pi_c^m$.

Proof of Lemma 1: Comparing w_{AF}, w^S, w^F and w^L yields

$$\begin{aligned}
w^S - w^F &= \frac{10-6b+b^2}{14-4b}m - \frac{(2-b)(5-2b)}{14-8b}m \\
&= \frac{(10-6b+b^2)(7-4b)-(2-b)(5-2b)(7-2b)}{2(7-2b)(7-4b)}m \\
&= \frac{b(1-b)}{2(7-2b)(7-4b)}m \geq 0; \\
w^S - w^L &= \frac{10-6b+b^2}{14-4b}m - \frac{5-3b}{7-2b-b^2}m \\
&= \frac{(10-6b+b^2)(7-2b-b^2)-(5-3b)(14-4b)}{2(7-2b)(7-2b-b^2)}m \\
&= \frac{-3b^2+4b^3-b^4}{2(7-2b)(7-2b-b^2)}m. \\
&= \frac{-b^2(3-b)(1-b)}{2(7-2b)(7-2b-b^2)}m \leq 0. \\
w^F - w_{AF} &= \frac{(2-b)(5-2b)}{14-8b}m - \frac{1}{3-b}m \\
&= \frac{(2-b)(5-2b)(3-b)-(14-8b)}{2(7-4b)(3-b)} \\
&= \frac{16-29b+15b^2-2b^3}{2(7-4b)(3-b)} \\
&= \frac{(1-b)(16-13b+2b^2)}{2(7-4b)(3-b)} \geq 0.
\end{aligned}$$

Thus, $w_{AL} \leq w_{AF} \leq w^F \leq w^S \leq w^L$.

Proof of Proposition 9:

Simultaneous game For the simultaneous game, when $\theta^* = 1$, the CM's and the OEM's profit functions are respectively

$$\Pi_c^S(w) = \frac{(m+w)^2}{(4-b)^2} + \frac{[(2-b)m-2w]w}{4-b}, \quad \Pi_o^S(w) = \frac{[(2-b)m-2w]^2}{(4-b)^2}.$$

The corresponding Nash product becomes

$$\begin{aligned}
\underset{w}{\text{Max}} \quad & \Omega^S = (\Pi_c^S(w))^\alpha (\Pi_o^S(w))^{1-\alpha} && (13) \\
s.t. \quad & 0 \leq w \leq min\left\{p_0, \frac{2-b}{2}m\right\}, \\
& \Pi_c^S(w) \geq \Pi_c^{RS}, && (14)
\end{aligned}$$

where Π_c^{RS} is the reserved profit for the CM if not participating in the negotiation and giving up the contract manufacturing business. It can be shown that $\Pi_c^{RS} = (m+p_0)^2/(4-b)^2$.

We solve the above constrained optimization problem as follows.

First, taking the first order derivative of Ω^S with respect to w yields

$$\begin{aligned}\frac{\partial \Omega^S}{\partial w} &= \alpha\left(\Pi_c^S(w)\right)^{\alpha-1}\left(\Pi_o^S(w)\right)^{1-\alpha}\frac{\partial \Pi_c^S(w)}{\partial w} + (1-\alpha)\left(\Pi_c^S(w)\right)^{\alpha}\left(\Pi_o^S(w)\right)^{-\alpha}\frac{\partial \Pi_o^S(w)}{\partial w} \\ &= \underbrace{\left(\Pi_c^S(w)\right)^{\alpha-1}}_{1}\underbrace{\left(\Pi_o^S(w)\right)^{-\alpha}}_{2}\underbrace{\left[\alpha\Pi_o^S(w)\frac{\partial \Pi_c^S(w)}{\partial w} + (1-\alpha)\Pi_c^S(w)\frac{\partial \Pi_o^S(w)}{\partial w}\right]}_{3}.\end{aligned}$$

As the first two terms are positive, the first order condition (FOC) $\frac{\partial \Omega^S}{\partial w} = 0$ is reduced to let the third term be zero, and substituting the CM's and the OEM's profit functions into the third term yields

$$\begin{aligned}(1-\alpha)\Pi_c^S(w)\frac{-\partial \Pi_o^S(w)}{\partial w} &= \alpha\Pi_o^S(w)\frac{\partial \Pi_c^S(w)}{\partial w} \\ (1-\alpha)\Pi_c^S(w)\frac{4[(2-b)m-2w]}{(4-b)^2} &= \alpha\Pi_o^S(w)\frac{(10-6b+b^2)m-2(7-2b)w}{(4-b)^2},\end{aligned}$$

which can be simplified as

$$4(7-2b)w^2 - 2[2(10-6b+b^2)+(1-b)(4-b)\alpha]mw + [(24-22b+8b^2-b^3)\alpha-4]m^2 = 0,$$

a quadratic function of w. Solving it yields two optimal solutions:

$$\begin{aligned}w_1^S &= \frac{2(10-6b+b^2)+(1-b)(4-b)\alpha+(4-b)\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}}{4(7-2b)}m, \\ w_2^S &= \frac{2(10-6b+b^2)+(1-b)(4-b)\alpha-(4-b)\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}}{4(7-2b)}m.\end{aligned}$$

Note that $w_2^S = K^S$.

Next, taking the first order derivative of w_1^S with respect to α yields

$$\begin{aligned}\frac{\partial w_1^S}{\partial \alpha} &= \frac{m}{4(7-2b)}\left[(1-b)(4-b)+\frac{(4-b)[2(1-b)^2\alpha-4(8-4b+b^2)]}{2\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}}\right] \\ &= \frac{(4-b)m}{4(7-2b)\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}} \\ &\quad \times[(1-b)\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}-(16-8b+2b^2-(1-b)^2\alpha)] \\ &< 0,\end{aligned}$$

where the last inequality is due to

$$\begin{aligned}
& \left[(1-b)\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}\right]^2-[16-8b+2b^2-(1-b)^2\alpha]^2 \\
= \ & (1-b)^2[(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)]-(16-8b+2b^2)^2 \\
& -(1-b)^4\alpha^2+2(16-8b+2b^2)(1-b)^2\alpha \\
= \ & -(16-8b+2b^2)^2+4(8-4b+b^2)(1-b)^2 \\
= \ & -4(8-4b+b^2)(7-2b) \\
< \ & 0.
\end{aligned}$$

Hence, w_1^S is decreasing in α. Thus, $w_1^S \geq w_1^S|_{\alpha=1} = \frac{2-b}{2}m$, which is in conflict with our pricing constraint $w < \frac{2-b}{2}m$. Therefore, w_1^S cannot be the optimal solution.

Similarly, we can show the first order derivatives of w_2^S with respect to α as

$$\begin{aligned}
\frac{\partial w_2^S}{\partial \alpha} &= \frac{m}{4(7-2b)}\left[(1-b)(4-b)-\frac{(4-b)[2(1-b)^2\alpha-4(8-4b+b^2)]}{2\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}}\right] \\
&= \frac{(4-b)m}{4(7-2b)\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}} \\
&\quad \times[(1-b)\sqrt{(1-b)^2\alpha^2+4(8-4b+b^2)(1-\alpha)}+(16-8b+2b^2-(1-b)^2\alpha)] \\
&> 0,
\end{aligned}$$

so w_2^S is increasing in α. Letting $w_2^S=0$ yields $\alpha^S(0)=\frac{4}{(4-b)(6-4b+b^2)}$. Therefore, if $\alpha \geq \alpha^S(0)$, then $0 \leq w_2^S \leq w_2^S|_{\alpha=1} = \frac{10-6b+b^2}{2(7-2b)}m = w^S \leq \frac{2-b}{2}m$. However, is w_2^S the maximizer of Ω^S? To answer this question, we need to check the sign of $\frac{\partial \Omega^S}{\partial w}|_{w_2^S=0}$ and $\frac{\partial \Omega^S}{\partial w}|_{w_2^S=w^S}$. If the former is positive and the latter is negative, then w_2^S maximizes Ω^S and thus $\Omega^S(w)$ is unimodal for $w \in [0, \frac{2-b}{2}m]$.

We can show that

$$\begin{aligned}
\frac{\partial \Omega^S}{\partial w}|_{w_2^S=0} &= \left(\Pi_c^S(0)\right)^{\alpha-1}\left(\Pi_o^S(0)\right)^{-\alpha}\left[\alpha\Pi_o^S(0)\frac{\partial \Pi_c^S(w)}{\partial w}|_{w_2^S=0}+(1-\alpha)\Pi_c^S(0)\frac{\partial \Pi_o^S(w)}{\partial w}|_{w_2^S=0}\right] \\
&= \left(\Pi_c^S(0)\right)^{\alpha-1}\left(\Pi_o^S(0)\right)^{-\alpha}\left[\alpha\frac{(2-b)^2m^2}{(4-b)^2}\frac{(10-6b+b^2)m}{(4-b)^2}+(1-\alpha)\frac{m^2}{(4-b)^2}\frac{-4(2-b)m}{(4-b)^2}\right] \\
&= \left(\Pi_c^S(0)\right)^{\alpha-1}\left(\Pi_o^S(0)\right)^{-\alpha}\frac{(2-b)m^3}{(4-b)^4}\left[(4-b)(6-4b+b^2)\alpha-4\right].
\end{aligned}$$

Note that $w_2^S \geq 0$ requires $\alpha \geq \alpha^S(0) = \frac{4}{(4-b)(6-4b+b^2)}$. Substituting this condition into

$\frac{\partial \Omega^S}{\partial w}|_{w_2^S=0}$, then we have $\frac{\partial \Omega^S}{\partial w}|_{w_2^S=0} \geq 0$. Similarly we have

$$\begin{aligned}
\frac{\partial \Omega^S}{\partial w}|_{w_2^S=w^S} &= \left(\Pi_c^S(w^S)\right)^{\alpha-1}\left(\Pi_o^S(w^S)\right)^{-\alpha}\left[\alpha \Pi_o^S(w^S)\frac{\partial \Pi_c^S(w)}{\partial w}|_{w_2^S=w^S} + (1-\alpha)\Pi_c^S(w^S)\frac{\partial \Pi_o^S(w)}{\partial w}|_{w_2^S=w^S}\right] \\
&= \left(\Pi_c^S(w^S)\right)^{\alpha-1}\left(\Pi_o^S(w^S)\right)^{-\alpha}\left[(1-\alpha)\Pi_c^S(w^S)\frac{\partial \Pi_o^S(w)}{\partial w}|_{w_2^S=w^S}\right] < 0.
\end{aligned}$$

Thus we prove that w_2^S is the optimal solution that maximizes Ω^S, and Ω^S is unimodal for $w \in [0, \frac{2-b}{2}m]$.

Next, let

$$\Pi_c^S(w) = \frac{(m+w)^2}{(4-b)^2} + \frac{[(2-b)m-2w]w}{4-b} = \Pi_c^{RS} = \frac{(m+p_0)^2}{(4-b)^2},$$

then it can be shown that the unique solution is

$$w = \frac{(10-6b+b^2)m - \sqrt{(10-6b+b^2)^2m^2 - 4(7-2b)(p_0^2+2p_0m)}}{2(7-2b)} = \underline{w}^S,$$

which is smaller than w^S. As $0 \leq p_0 < \frac{2-b}{2}m$ (otherwise the OEM cannot source from the non-competitive CM as it will get negative profit), we can show that

$$\begin{aligned}
& (10-6b+b^2)^2m^2 - 4(7-2b)(p_0^2+2p_0m) \\
> \; & (10-6b+b^2)^2m^2 - 4(7-2b)\left[\frac{(2-b)^2}{4}m^2 + (2-b)m^2\right] \\
= \; & [(10-6b+b^2)^2 - (2-b)(6-b)(7-2b)]m^2 \\
= \; & (16-40b+33b^2-10b^3+b^4)m^2 \\
= \; & (1-b)^2(4-b)^2m^2 \geq 0.
\end{aligned}$$

Then the participation constraint is reduced to that the negotiated price is no less than $\underline{w}^S$.

Based on the foregoing analysis and recalling that $K^S = w_2^S$, we obtain the GNB-characterized wholesale price as $w^{NS} = min(p_0, max(\underline{w}^S, K^S))$. Thus, the simultaneous game is solved.

OEM-as-leader game

For the OEM-as-leader game, when $\theta^* = 1$ the CM's and the OEM's profit functions are respectively

$$\Pi_c^F(w) = \frac{[(2-b)m+2w]^2}{16(2-b)^2} + \frac{[(2-b)m-2w]w}{2(2-b)}; \quad \Pi_o^L(w) = \frac{[(2-b)m-2w]^2}{8(2-b)}.$$

The corresponding Nash product becomes

$$\begin{aligned} \underset{w}{\text{Max}} \quad & \Omega^F = (\Pi_c^F(w))^{\alpha}(\Pi_o^L(w))^{1-\alpha} \qquad (15) \\ s.t. \quad & 0 \le w \le min\left\{p_0, \frac{2-b}{2}m\right\}, \\ & \Pi_c^F(w) \ge \Pi_c^{RF}, \qquad (16) \end{aligned}$$

where Π_c^{RF} is the CM's reserved profit if not participating in the negotiation. It can be shown that $\Pi_c^{RF} = \frac{[(2-b)m+2p_0]^2}{16(2-b)^2}$.

Then we optimize the above Nash product as follows.

Similar to the proof of simultaneous game, we let the first-order derivative of Nash product Ω^F with respect to w to be zero to derive the extreme-value points. Substituting $\Pi_c^F(w)$ and $\Pi_o^L(w)$ into Ω^F, the FOC can be rewritten as

$$\begin{aligned} (1-\alpha)\Pi_c^F(w)\frac{-\partial\Pi_o^L(w)}{\partial w} &= \alpha\Pi_o^L(w)\frac{\partial\Pi_c^F(w)}{\partial w} \\ (1-\alpha)\Pi_c^F(w)\frac{4[(2-b)m-2w]}{8(2-b)} &= \alpha\Pi_o^L(w)\frac{[(2-b)(5-2b)m-2(7-4b)w]}{4(2-b)^2}. \end{aligned}$$

Rearranging the foregoing equation yields a quadratic function of w,

$$4(7-4b)w^2 - 4[(2-b)(5-2b)+(1-b)(2-b)\alpha]mw + (2-b)^2[2(3-b)\alpha-1]m^2 = 0,$$

which has two roots

$$\begin{aligned} w_1^F &= \frac{(2-b)(5-2b)+(2-b)(1-b)\alpha+(2-b)\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}}{2(7-4b)}m, \\ w_2^F &= \frac{(2-b)(5-2b)+(2-b)(1-b)\alpha-(2-b)\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}}{2(7-4b)}m. \end{aligned}$$

Note that $w_2^F = K^F$.

Next we check whether w_1^F and w_2^F are the optimal solution. Taking the first-order derivative of w_1^F with respective to α yields

$$\begin{aligned} \frac{\partial w_1^F}{\partial \alpha} &= \frac{(2-b)m}{2(7-4b)}\left[(1-b)+\frac{2(1-b)^2\alpha-4(2-b)(4-b)}{2\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}}\right] \\ &= \frac{(2-b)m}{2(7-4b)} \times \frac{(1-b)\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}-[16-12b+2b^2-(1-b)^2\alpha]}{\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}} \\ &< 0, \end{aligned}$$

where the last inequality is because of

$$\begin{aligned}
& \left[(1-b)\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}\right]^2-[16-12b+2b^2-(1-b)^2\alpha]^2 \\
= & (1-b)^2[(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)]-(16-12b+2b^2)^2 \\
& -(1-b)^4\alpha^2+2(16-12b+2b^2)(1-b)^2\alpha \\
= & -4(2-b)(4-b)(7-4b)<0.
\end{aligned}$$

Hence, w_1^F is decreasing in α. Thus, $w_1^F \geq w_1^F|_{\alpha=1} = \frac{2-b}{2}m$, which contradicts our assumption that $w < \frac{2-b}{2}m$. Thus, w_1^F cannot be the optimal solution.

Taking the first-order derivative of w_2^F with respect to α yields

$$\begin{aligned}
\frac{\partial w_2^F}{\partial \alpha} &= \frac{(2-b)m}{2(7-4b)}\left[(1-b)-\frac{2(1-b)^2\alpha-4(2-b)(4-b)}{2\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}}\right] \\
&= \frac{(2-b)m}{2(7-4b)}\times\frac{(1-b)\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}+16-12b+2b^2-(1-b)^2\alpha}{\sqrt{(1-b)^2\alpha^2+4(2-b)(4-b)(1-\alpha)}} \\
&> 0.
\end{aligned}$$

Hence, w_2^F increases in α. Thus, $0 \leq w_2^F \leq w_2^F|_{\alpha=1} = \frac{(2-b)(5-2b)}{2(7-4b)}m = w^F \leq \frac{2-b}{2}m$. Letting $w_2^F(\alpha)=0$ yields $\alpha^F(0)=\frac{1}{2(3-b)}$. We then check the sign of $\frac{\partial \Omega^F}{\partial w}|_{w_2^F=0}$ and $\frac{\partial \Omega^F}{\partial w}|_{w_2^F=w^F}$. If the former is positive and the latter is negative, then w_2^F maximizes Ω^F and the constrained Ω^F is unimodal.

$$\begin{aligned}
\frac{\partial \Omega^F}{\partial w}|_{w_2^F=0} &= \left(\Pi_c^F(0)\right)^{\alpha-1}\left(\Pi_o^L(0)\right)^{-\alpha}\left[\alpha\Pi_o^L(0)\frac{\partial \Pi_c^F(w)}{\partial w}|_{w_2^F=0}+(1-\alpha)\Pi_c^F(0)\frac{\partial \Pi_o^L(w)}{\partial w}|_{w_2^F=0}\right] \\
&= \left(\Pi_c^F(0)\right)^{\alpha-1}\left(\Pi_o^L(0)\right)^{-\alpha}\left[\alpha\frac{(2-b)m^2}{8}\frac{(2-b)(5-2b)m}{4(2-b)^2}+(1-\alpha)\frac{m^2}{16}\frac{-4(2-b)m}{8(2-b)}\right] \\
&= \left(\Pi_c^F(0)\right)^{\alpha-1}\left(\Pi_o^L(0)\right)^{-\alpha}\frac{m^3}{32}\left[2(3-b)\alpha-1\right].
\end{aligned}$$

As $w_2^F \geq 0$ requires $\alpha \geq \alpha^F(0) = \frac{1}{2(3-b)}$, substituting this condition into $\frac{\partial \Omega^F}{\partial w}|_{w_2^F=0}$ leads to $\frac{\partial \Omega^F}{\partial w}|_{w_2^F=0} \geq 0$.

Similarly, we can show that

$$\begin{aligned}
\frac{\partial \Omega^F}{\partial w}|_{w_2^F=w^F} &= \left(\Pi_c^F(w^F)\right)^{\alpha-1}\left(\Pi_o^L(w^F)\right)^{-\alpha}\left[\alpha\Pi_o^L(w^F)\frac{\partial \Pi_c^F(w)}{\partial w}|_{w_2^F=w^F}+(1-\alpha)\Pi_c^F(w^F)\frac{\partial \Pi_o^L(w)}{\partial w}|_{w_2^F=w^F}\right] \\
&= \left(\Pi_c^F(w^F)\right)^{\alpha-1}\left(\Pi_o^L(w^F)\right)^{-\alpha}\left[(1-\alpha)\Pi_c^F(w^F)\frac{\partial \Pi_o^L(w)}{\partial w}|_{w_2^F=w^F}\right]<0.
\end{aligned}$$

Therefore, w_2^F is the optimal solution that maximizes Ω^F, and the constrained Ω^F is thus unimodal.

Next, letting $\Pi_c^F(w) = \Pi_c^{RF}$ and solving it yields

$$\underline{w}^F = \frac{(2-b)(5-2b)m - \sqrt{(2-b)^2(5-2b)^2m^2 - 4(7-4b)[p_0^2 + (2-b)mp_0]}}{2(7-4b)},$$

which is smaller than w^F. As $0 \leq p_0 < \frac{2-b}{2}m$ (otherwise the OEM cannot source from the non-competitive CM as it will get negative profit), we can show that

$$\begin{aligned}
& (2-b)^2(5-2b)^2m^2 - 4(7-4b)[p_0^2 + (2-b)mp_0] \\
> \;& (2-b)^2(5-2b)^2m^2 - 4(7-4b)\left[\frac{(2-b)^2}{4}m^2 + \frac{(2-b)^2}{2}m^2\right] \\
= \;& [(2-b)^2(5-2b)^2 - 3(7-4b)(2-b)^2]m^2 \\
= \;& (2-b)^2[(5-2b)^2 - 3(7-4b)]m^2 \\
= \;& (1-b)^2(2-b)^2m^2 \geq 0,
\end{aligned}$$

thus $\underline{w}^F$ does exist and the participation constraint is reduced to the negotiated wholesale price is no less than $\underline{w}^F$.

Based on the foregoing analysis and recalling that $K^F = w_2^F$, we obtain the GNB-characterized wholesale price as $w^{NF} = min(p_0, max(\underline{w}^F, K^F))$. Thus, the OEM-as-leader game is solved.

CM-as-leader game

For the CM-as-leader game, when $\theta^* = 1$, the CM's and the OEM's profit functions are respectively

$$\Pi_c^L(w) = \frac{[(m + (1+b)w)][m + (1-b)w]}{8(2-b)} + \frac{[(4-3b)m - (4-b-b^2)w]w}{4(2-b)};$$

$$\Pi_o^F(w) = \frac{[(4-3b)m - (4-b-b^2)w]^2}{16(2-b)^2}.$$

The corresponding Nash product becomes

$$\max_w \quad \Omega^L = (\Pi_c^L(w))^\alpha(\Pi_o^F(w))^{1-\alpha} \tag{17}$$

$$s.t. \quad 0 \leq w \leq min\left\{p_0, \frac{4-3b}{4-b-b^2}m\right\},$$

$$\Pi_c^L(w) \geq \Pi_c^{RL}, \tag{18}$$

where Π_c^{RL} is the CM's reserved profit if not participating in the negotiation. It can be shown that $\Pi_c^{RL} = \frac{(m+p_0)^2}{8(2-b)}$.

We solve the game as follows.

Similar to the proof of simultaneous game, we let the first-order derivative of Nash product Ω^L with respect to w to be zero to derive the extreme-value points. Substituting $\Pi_c^L(w)$ and $\Pi_o^F(w)$ into Ω^L, the FOC can be rewritten as

$$\begin{aligned}(1-\alpha)\Pi_c^L(w)\frac{-\partial\Pi_o^F(w)}{\partial w} &= \alpha\Pi_o^F(w)\frac{\partial\Pi_c^L(w)}{\partial w}\\(1-\alpha)\Pi_c^L(w)\frac{(4-b-b^2)[(4-3b)m-(4-b-b^2)w]}{8(2-b)^2} &= \alpha\Pi_o^F(w)\frac{[(5-3b)m-(7-2b-b^2)w]}{4(2-b)}.\end{aligned}$$

Rearranging the foregoing equation yields a quadratic function of w as follows:

$$(4-b-b^2)(7-2b-b^2)w^2-2[(5-3b)(4-b-b^2)+2(1-b)(2-b)\alpha]mw+[4(6-7b+2b^2)\alpha-(4-b-b^2)]m^2=0,$$

which has two roots,

$$w_1^L=\frac{(5-3b)(4-b-b^2)+2(1-b)(2-b)\alpha+2(2-b)\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}}{(4-b-b^2)(7-2b-b^2)}m,$$

$$w_2^L=\frac{(5-3b)(4-b-b^2)+2(1-b)(2-b)\alpha-2(2-b)\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}}{(4-b-b^2)(7-2b-b^2)}m.$$

Note that $w_2^L=K^L$.

Taking the first-order derivative of w_1^L with respect to α yields

$$\begin{aligned}\frac{\partial w_1^L}{\partial\alpha} &= \frac{2(2-b)m}{(4-b-b^2)(7-2b-b^2)}\left[(1-b)+\frac{2(1-b)^2\alpha-2(16-8b-7b^2+2b^3+b^4)}{2\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}}\right]\\&= \frac{2(2-b)m}{(4-b-b^2)(7-2b-b^2)}\\&\quad\times\frac{(1-b)\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}-[16-8b-7b^2+2b^3+b^4-(1-b)^2\alpha]}{\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}}\\&< 0,\end{aligned}$$

where the last inequality is due to

$$\begin{aligned}&\left[(1-b)\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}\right]^2-[16-8b-7b^2+2b^3+b^4-(1-b)^2\alpha]^2\\=\;&(1-b)^2[(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)]-(16-8b-7b^2+2b^3+b^4)^2\\&-(1-b)^4\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-b)^2\alpha\\=\;&-(16-8b-7b^2+2b^3+b^4)(14-4b-9b^2+2b^3+b^4)<0.\end{aligned}$$

Hence, w_1^L is decreasing in α. Thus, $w_1^L\geq w_1^L|_{\alpha=1}=\frac{4-3b}{4-b-b^2}m$, which violates our assumption that $w<\frac{4-3b}{4-b-b^2}m$. Therefore, w_1^L cannot be the optimal solution.

Next, taking the first-order derivative of w_2^L with respect to α yields

$$\begin{aligned}\frac{\partial w_2^L}{\partial \alpha} &= \frac{2(2-b)m}{(4-b-b^2)(7-2b-b^2)}\left[(1-b)-\frac{2(1-b)^2\alpha-2(16-8b-7b^2+2b^3+b^4)}{2\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}}\right]\\ &= \frac{2(2-b)m}{(4-b-b^2)(7-2b-b^2)}\times\\ &\quad \frac{(1-b)\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}+[16-8b-7b^2+2b^3+b^4-(1-b)^2\alpha]}{\sqrt{(1-b)^2\alpha^2+2(16-8b-7b^2+2b^3+b^4)(1-\alpha)}}\\ &> 0.\end{aligned}$$

Hence, w_2^L is increasing in α. Thus, $0 \leq w_2^L \leq w_2^L|_{\alpha=1} = \frac{5-3b}{7-2b-b^2}m = w^L \leq \frac{4-3b}{4-b-b^2}m$. Letting $w_2^L(\alpha)=0$ yields $\alpha = \frac{4-b-b^2}{4(2-b)(3-2b)} \equiv \alpha^L(0)$. Then we check the sign of $\frac{\partial \Omega^L}{\partial w}|_{w_2^L=0}$ and $\frac{\partial \Omega^L}{\partial w}|_{w_2^L=w^L}$. If the former is positive and the latter is negative, then w_2^L maximizes Ω^L and the constrained Ω^L is unimodal. We can show that

$$\begin{aligned}\frac{\partial \Omega^L}{\partial w}|_{w_2^L=0} &= \left(\Pi_c^L(0)\right)^{\alpha-1}\left(\Pi_o^F(0)\right)^{-\alpha}\left[\alpha\Pi_o^F(0)\frac{\partial \Pi_c^L(w)}{\partial w}|_{w_2^L=0}+(1-\alpha)\Pi_c^L(0)\frac{\partial \Pi_o^F(w)}{\partial w}|_{w_2^L=0}\right]\\ &= \left(\Pi_c^L(0)\right)^{\alpha-1}\left(\Pi_o^F(0)\right)^{-\alpha}\left[\alpha\frac{(4-3b)^2m^2}{16(2-b)^2}\frac{(5-3b)m}{4(2-b)}+(1-\alpha)\frac{m^2}{8(2-b)}\frac{-2(4-b-b^2)(4-3b)m}{16(2-b)^2}\right]\\ &= \left(\Pi_c^L(0)\right)^{\alpha-1}\left(\Pi_o^F(0)\right)^{-\alpha}\frac{(4-3b)m^3}{64(2-b)^3}\left[4(2-b)(3-2b)\alpha-(4-b-b^2)\right].\end{aligned}$$

As $w_2^L \geq 0$ requires $\alpha \geq \frac{4-b-b^2}{4(2-b)(3-2b)}$, substituting this requirement into $\frac{\partial \Omega^L}{\partial w}|_{w_2^L=0}$ leads to $\frac{\partial \Omega^L}{\partial w}|_{w_2^L=0} \geq 0$.

Similarly, we have

$$\begin{aligned}\frac{\partial \Omega^L}{\partial w}|_{w_2^L=w^L} &= \left(\Pi_c^L(w^L)\right)^{\alpha-1}\left(\Pi_o^F(w^L)\right)^{-\alpha}\left[\alpha\Pi_o^F(w^L)\frac{\partial \Pi_c^L(w)}{\partial w}|_{w_2^L=w^L}+(1-\alpha)\Pi_c^L(w^L)\frac{\partial \Pi_o^F(w)}{\partial w}|_{w_2^L=w^L}\right]\\ &= \left(\Pi_c^L(w^L)\right)^{\alpha-1}\left(\Pi_o^F(w^L)\right)^{-\alpha}\left[(1-\alpha)\Pi_c^L(w^L)\frac{\partial \Pi_o^F(w)}{\partial w}|_{w_2^L=w^L}\right] < 0.\end{aligned}$$

Therefore, w_2^L is the optimal solution that maximizes Ω^L, and the constrained Ω^L is unimodal.

Next, solving $\Pi_c^L(w) = \Pi_c^{RL}$ yields $\underline{w}^L = \frac{(5-3b)m-\sqrt{(5-3b)^2m^2-(7-2b-b^2)[p_0^2+2p_0m]}}{7-2b-b^2}$.

As $0 \leq p_0 < \frac{4-3b}{4-b}m$ (otherwise the OEM cannot source from the non-competitive CM as it will get negative profit), we can show that

$$\begin{aligned}&(5-3b)^2m^2-(7-2b-b^2)[p_0^2+2p_0m]\\ >\ &(5-3b)^2m^2-(7-2b-b^2)\left[\frac{(4-3b)^2}{(4-b)^2}m^2+\frac{2(4-3b)}{4-b}m^2\right]\\ =\ &[(4-b)^2(5-3b)^2-(4-3b)(12-5b)(7-2b-b^2)]\frac{m^2}{(4-b)^2}\\ =\ &4(4-8b+6b^2)(2-b)^2\frac{m^2}{(4-b)^2} \geq 0,\end{aligned}$$

where the last inequality is due to the fact that $4 - 8b + 6b^2$ is always positive for $b \in [0, 1]$. Therefore, $\underline{w}^L$ does exist and the participation constraint is reduced to the negotiated wholesale price is no less than $\underline{w}^L$.

Based on the foregoing analysis and recalling that $K^L = w_2^L$, we obtain the GNB-characterized wholesale price as $w^{NL} = min(p_0, max(\underline{w}^L, K^L))$. The CM-as-leader game is solved.

Proof of Proposition 10

If the OEM determines the wholesale prices, the price lower bounds will be reached under three basic games. The competitive CM will receive the reserved profits respectively: Π_c^{RS}, Π_c^{RF} and Π_c^{RL}. We have the following relationship among them.

$$\begin{aligned}
\Pi_c^{RL} - \Pi_c^{RS} &= \frac{(m+p_0)^2}{8(2-b)} - \frac{(m+p_0)^2}{(4-b)^2} \\
&= \frac{b^2(m+p_0)^2}{8(2-b)(4-b)^2} \\
&\geq 0 \\
\Pi_c^{RS} - \Pi_c^{RF} &= \frac{(m+p_0)^2}{(4-b)^2} - \frac{[(2-b)m+2p_0]^2}{16(2-b)^2} \\
&= \left[\frac{(2-b)m+2p_0}{4(2-b)} + \frac{m+p_0}{4-b}\right]\left[\frac{(2-b)m+2p_0}{4(2-b)} - \frac{m+p_0}{4-b}\right] \\
&= \frac{b[(2-b)(8-b)m+(16-6b)p_0][(2-b)m-2p_0]}{16(2-b)^2(4-b)^2} \\
&\geq 0.
\end{aligned}$$

The last inequality is due to our assumption that $p_0 \leq \frac{2-b}{2}m$. Therefore, we have $\Pi_c^{RL} \geq \Pi_c^{RS} \geq \Pi_c^{RF}$ and the competitive CM always prefers the price leadership.

Appendix B

Here, we consider decision order 2 in which the OEM decides the outsourcing proportion θ first and then the competitive CM decides its wholesale price w.

Simultaneous game

The game sequence in the simultaneous game is as follows. First, the OEM makes the outsourcing decision θ, and then the competitive CM announces the wholesale price w. Next, the OEM and the competitive CM decide their production quantities simultaneously. We solve the game backwards.

First, we can derive the equilibrium production quantities as

$$q_o^{S*}(\theta, w) = \frac{(2-b)m + 2(p_0 - w)\theta - 2p_0}{4-b}, \quad q_c^{S*}(\theta, w) = \frac{m - (p_0 - w)\theta + p_0}{4-b}.$$

Next, by substituting $q_o^{S*}(\theta, w)$ and $q_c^{S*}(\theta, w)$ into (5), the competitive CM's profit is

$$\Pi_c^S(w) = \frac{[m - (p_0 - w)\theta + p_0]^2}{(4-b)^2} + \frac{[(2-b)m + 2(p_0 - w)\theta - 2p_0]w}{4-b}.$$

Setting the FOC to zero generates the optimal solution

$$w^S = \frac{(8 - 6b + b^2 + 2\theta)m - 2(1-\theta)(4 - b - \theta)p_0}{2\theta(8 - 2b - \theta)}.$$

Therefore, $w^{S*} = \min\left\{p_0, w^S\right\}$. It can be verified that if $p_0 \geq \frac{8-6b+b^2+2\theta}{2(4-b+3\theta-b\theta)}m$, then $w^{S*} = w^S$; otherwise, $w^{S*} = p_0$. When $w^{S*} = p_0$,

$$\Pi_o^S(\theta) = \frac{[(2-b)m + 2(p_0 - w^{S*})\theta - 2p_0]^2}{(4-b)^2},$$

and Π_o^S is constant in θ if $w^{S*} = p_0$. Then, if $p_0 \in [\frac{8-6b+b^2+2\theta}{2(4-b+3\theta-b\theta)}m, \frac{2-b}{2}m]$, substituting $w^{S*} = w^S$ into Π_o^S and rearranging it yields

$$\begin{aligned}
\Pi_o^S &= \left[\frac{(2-b-\theta)m - 2(1-\theta)p_0}{8 - 2b - \theta}\right]^2 \\
\frac{\partial \Pi_o^S}{\partial \theta} &= \frac{2[(2-b-\theta)m - 2(1-\theta)p_0]}{8-2b-\theta}\frac{(-6+b)m + (14-4b)p_0}{(8-2b-\theta)^2}.
\end{aligned}$$

Because

$$\begin{aligned}
\frac{2[(2-b-\theta)m - 2(1-\theta)p_0]}{8-2b-\theta} &\geq \frac{2[(2-b-\theta)m - (1-\theta)(2-b)m]}{8-2b-\theta} = \frac{2\theta(1-b)}{8-2b-\theta}m \geq 0; \\
\frac{(-6+b)m + (14-4b)p_0}{(8-2b-\theta)^2} &\geq \frac{(-6+b)(4-b+3\theta-b\theta) + (7-2b)(8-6b+b^2+2\theta)}{(4-b+3\theta-b\theta)(8-2b-\theta)^2}m \\
&= \frac{(1-b)(4-b)((8-2b-\theta)}{(4-b+3\theta-b\theta)(8-2b-\theta)^2}m \\
&= \frac{(1-b)(4-b)}{(4-b+3\theta-b\theta)(8-2b-\theta)} \geq 0,
\end{aligned}$$

$\frac{\partial \Pi_o^S}{\partial \theta} \geq 0$. Thus, the OEM will set $\theta^* = 1$.

OEM-as-leader game

Under this game, the OEM first decides its production quantity and the outsourcing proportion. Then, the CM decides its wholesale price and production quantity. We also solve this problem backwards. First, we can show that Π_c^F in (5) is increasing in w, and thus the CM will set $w^{F*} = p_0$. The production quantity is $q_c(q_o) = \frac{m - q_o(\theta)}{2}$. Substituting w^{F*} and $q_c(q_o)$ into the OEM's profit function yields

$$\Pi_o^L = \frac{-(2-b)q_o^2 + [(2-b)m - 2p_0]q_o}{2},$$

which is constant in θ and reaches the maximum when $q_0 = \frac{(2-b)m - 2p_0}{4-2b}$. We assume that the OEM will outsource all of its production orders to the competitive CM if $w^{F*} = p_0$[4]; thus, $\theta^* = 1$.

CM-as-leader game

The game sequence here is as follows. First, the OEM makes the outsourcing decision θ. Then, the competitive CM decides the wholesale price w and its production quantity q_c. Next, the OEM decides its production quantity q_o.

We can show that the best response function of the OEM's production decision is

$$q_o^*(q_c, w) = \frac{m - bq_c - \theta w - (1-\theta)p_0}{2}.$$

Then, we can derive the competitive CM's profit function as

$$\Pi_c^L = \frac{[m - (2-b)q_c + \theta w + (1-\theta)p_0]q_c}{2} + \frac{(m - bq_c - \theta w - (1-\theta)p_0)\theta w}{2}.$$

The competitive CM will decide w and q_c simultaneously. Their best response functions are

$$q_c(w) = \frac{m + (1-\theta)p_0 + (1-b)\theta w}{2(2-b)}, \text{ and } w(q_c) = \frac{m + (1-b)q_c - (1-\theta)p_0}{2\theta}.$$

Solving these two equations yields

$$w^L = \frac{(5-3b)m - (3-b)(1-\theta)p_0}{(7-2b-b^2)\theta}, \; q_c^L = \frac{(3-b)m + (1-\theta)(1+b)p_0}{7-2b-b^2}.$$

Therefore, if $p_0 < \frac{5-3b}{3-b+(4-b-b^2)\theta}m$, then we have $w^{L*} = p_0$; otherwise, $w^{L*} = w^L$.

When $w^{L*} = p_0$, the OEM's profit function is

$$\Pi_o^F = (q_o^F)^2 = \left[\frac{(7-5b)m - (7-b)p_0 + b(1+b)\theta p_0}{7-2b-b^2}\right]^2,$$

which is increasing in θ, and so $\theta^* = 1$. When $w^{L*} = w^L$, the OEM's profit function is

$$\Pi_o^F = \left[\frac{2(1-b)m - 4p_0 + 4\theta p_0}{7-2b-b^2}\right]^2,$$

[4] This assumption is reasonable as, in practice, a competitive CM can normally offer value-added services in addition to product manufacturing. Thus, it can win the OEM's orders when non-competitive CMs do not have a price advantage.

which is also increasing in θ, and thus $\theta^* = 1$.

As the results for the three basic games under decision order 2 are similar to those under decision order 1, the equilibrium quantity timing decisions are also the same as those in Proposition 8.