An Emigration Genetic Algorithm and Its Application to Multiobjective Optimal Designs of Electromagnetic Devices

Yuhuai Wang, Shiyou Yang, Guangzheng Ni, S. L. Ho, and Z. J. Liu

***Abstract*—The emigration genetic algorithm, which is a genetic-based algorithm, is proposed to obtain the Pareto optimal solution of vector optimal designs of electromagnetic devices. The proposed algorithm differs from the traditional ones in its design of an emigration operator as well as the inclusion of some useful approaches such as the fitness sharing, clustering, and elitism strategy. Detailed numerical results on three different multiobjective design problems are reported to demonstrate the effectiveness and advantages of the proposed algorithm for solving practical engineering multiobjective optimal design problems.**

***Index Terms*—Emigration operator, genetic algorithm (GA), numerical method, vector optimization.**

I. Introduction

MOST practical design problems involve several incommensurable and sometimes conflicting objectives. As it is well known, for a multiobjective solver, the following two issues must be addressed carefully: 1) means to accomplish the fitness assignment and selection in order to guide the search toward the Pareto-optimal set and 2) means to maintain a diversified population in order to prevent premature convergence and to smoothen the sampled Pareto front. In this sense, evolutionary algorithms have been proven to be one of the most efficient multiobjective or vector optimal problem (MOP) solvers and, thus, they have attracted a lot of attentions from different engineering branches [1], [2], [4]–[8]. Although significant efforts have been made in the study of multiobjective genetic algorithms (GAs), the performance of the existing algorithms relating to the two aforementioned issues is still unsatisfactory. A Pareto emigration GA is proposed in this paper. To preserve the diversity and to smoothen the Pareto front, an emigration operator is introduced. The approaches such as fitness sharing, clustering, and the elitism strategy are also improved and used. To validate and to show the advantages of the proposed algorithm, three numerical examples are presented.

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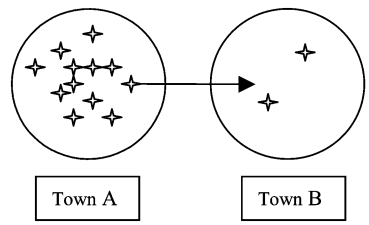

Fig. 1. Schematic diagram of the emigration operation.

II. A Pareto Emigration Genetic Algorithm

A. Emigration Operator

In a real society in which a town is densely populated with many residents, life becomes highly competitive and the resources would become increasingly scarce. Hence, the less competitive ones are forced to immigrate to the less developed and, hence, less competitive, towns as illustrated in Fig. 1. For an optimization problem, the effect of this emigration operation is to maintain some diversities in the entire population. Based on this analogy between our society and a vector optimal problem, an emigration operator is introduced and explained in the following steps.

Step 1) Find out the maximum distance among every two solutions in the population, and define a town radius which is proportional to this maximum distance. The number of solutions which will emigrate is proportional to the population size.

Step 2) Determine the neighborhood size of every individual by comparing the distances of it and its neighborhood solutions with a predefined town radius. The neighborhood size of an individual is proportional to the number of the neighborhood solutions whose distances to the specified individual are less than the predefined town radius.

Step 3) Identify, respectively, the solutions with the maximum and minimum neighborhood sizes, and then replace the maximum one by a newly generated individual. The new individual is generated from the individual of the minimum neighborhood size by adding a small perturbation to it. Two different approaches are used in adding the perturbations.

a) Use the solution with the minimum neighborhood size only as the parent to generate new solutions. To do so, one first selects the last three bits of the chromosome (here, one

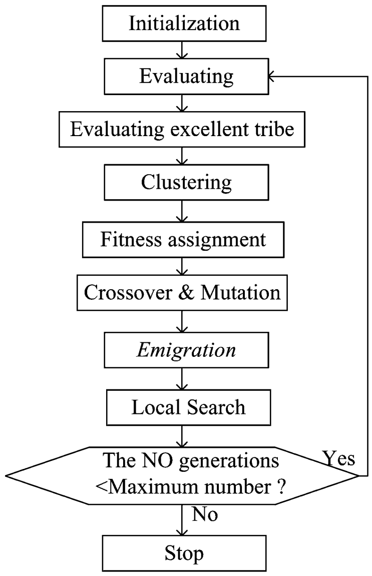

Fig. 2. Flowchart of the proposed algorithm.

chooses three bits, but it depends on how long the chromosome is in general). For each selected bit, a random number between 0 and 1 is generated. If it is less that 0.5, one changes the number of the bit (form "0" to "1," or vice versa).

b) Use the solution with minimum neighborhood size and the one with the maximum neighborhood size to generate a new individual. Select the last few bits of the chromosomes (the size depends on the chromosome length) of the solution with the minimum neighborhood size, and then change the selected bits with those of the solution with maximum neighborhood size.

Fig. 2 shows a flowchart to facilitate understanding of the proposed algorithm. From this flowchart, it can be seen that the emigration operator is following the mutation operator immediately to strengthen the robustness of a GA to maintain diversity in the solutions.

B. Fitness Assignment

As similar, and yet different, from a reported Pareto-based fitness assignment strategy [1], the fitness assignment of an individual in the proposed algorithm is conducted in the following steps.

Step 1) Introduce an external population which is called the excellent tribe, and copy the nondominated solutions to it. The fitness value of a solution i in the external excellent tribe is decided by

$$f_i = 1 + (N - n)/N \tag{1}$$

where n is the number of individuals in the population which is dominated by the specific solution in the excellent tribe, N is the size of the population.

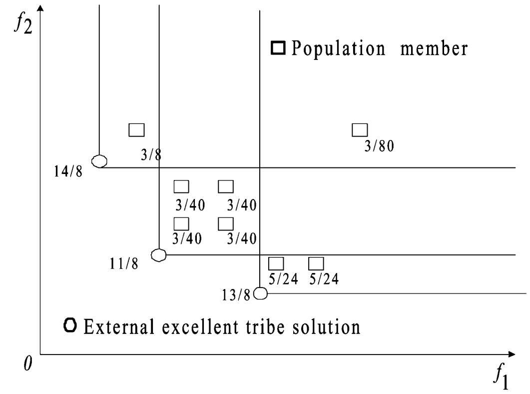

Fig. 3. Fitness assignment for a minimization problem with two objectives.

Step 2) Fitness assignment of the population. The fitness assignment of an individual j in the population is explained as: (1) firstly, one identifies the solution with the smallest fitness value among solutions in the excellent tribe which dominates individual j, and define the smallest fitness value as *smallestfitness*; (2) one then sums up all the *num* of the nondominated solutions in the excellent tribe which dominate the individual j as *totalnum* (the num of a nondominated solution is given by num $= N\times(2-f_i)$); (3) finally, one will calculate the fitness value of the individual j using

$$f_j = \frac{\text{smallestfitness} - 1}{\text{totalnum}}. \tag{2}$$

From Fig. 3 and (2), one can see that a nondominated solution always has fitness values that are larger than those of individuals in the population. Moreover, the solutions in densely populated regions are having smaller fitness values than those in the sparsely populated regions, other things being equal; and the individuals of a population which is dominated by more nondominated solutions have relatively smaller fitness values. Thus, the proposed fitness assignment strategy would have the ability to obtain a uniform and smooth Pareto front.

C. Local Search

Although the GA is very efficient in global searches, it is very inefficient for local searches. Thus, some special local search techniques are designed in the proposed algorithm. Contrary to the commonly used local searches, the proposed approach does not need a search radius to start, and can be summarized as the following.

Step 1) Determine the region where a local search is required. The number of bits in the last chromosome, instead of the traditional local search radius, is used to define the regions.

Step 2) Copy the selected individual to a tentative individual. For each selected bit of the chromosome of the tentative individual, randomly change the state of each bit.

Step 3) Evaluate the object values of the tentative individual. If it is better than that of the original one, replace the

TABLE I
ALGORITHM PARAMETERS USED BY THE PROPOSED AND A TRADITIONAL GENETIC ALGORITHM

Population size:	80
Size of excellent tribe:	20
Crossover probability:	0.8
Mutation probability:	0.001
Max generation:	200
Chromosome length	10+10
Local search size (k):	4

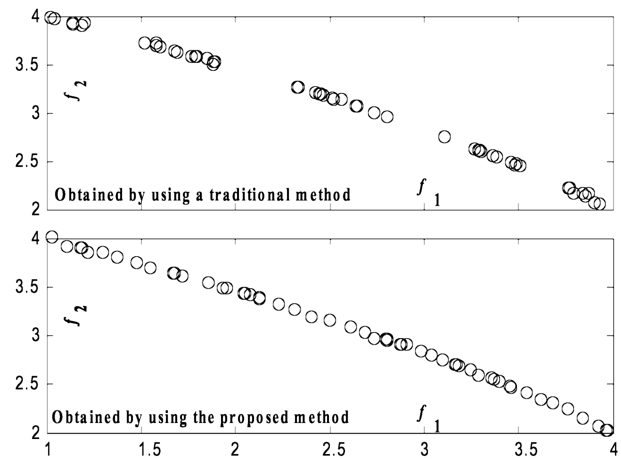

Fig. 4. Searched Pareto front of different methods for the test problem.

original individual with the tentative one and terminate the local search; otherwise, go to step 1 unless the number of iterations exceeds a predefined value K.

III. NUMERICAL RESULTS

A. Mathematical Test Function

A two-decision variable and two-objective minimization problem, as defined in the following, is used to validate the proposed algorithm. Mathematically

$$\min\begin{cases} f_1(x_1,x_2)=2.0\times\sqrt{x_1}\\ f_2(x_1,x_2)=x_1\times(1.0-x_2)+5.0\end{cases}$$
$$\text{s.t.}\begin{cases}1.0\le x_1\le 4.0\\ 1.0\le x_2\le 2.0\end{cases}. \quad (3)$$

The parameters used by the proposed and traditional GAs are given in Table I. The searched Pareto solutions by the two algorithms for this mathematical problem are depicted in Fig. 4. Obviously, although the traditional GA can sample some parts of the Pareto front, the proposed one can give a nearly ideal uniform sampling of the Pareto front. Thus, these primary numerical results have positively validated the feasibility and demonstrated the advantages of the proposed algorithm in solving multiobjective optimal problems.

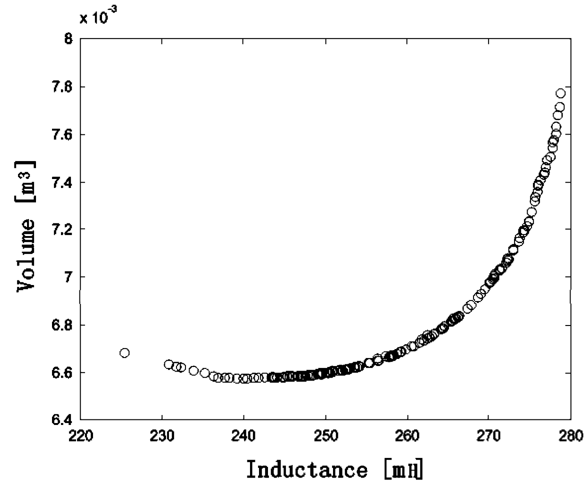

Fig. 5. Searched Pareto front by EPEA for the case study reported.

B. Case Study

The multiobjective shape optimization of a coreless solenoid with rectangular cross-section $a\times b$ and a mean radius c is selected as a case study [3]. If the electric current is uniformly distributed over the cross section, it can be shown that if the number of turns (N) of the solenoid is given, the inductance $L[\mu H]$ can be approximated from

$$L=\frac{31.49\frac{a^2N^2}{b}}{9+6\frac{a}{b}+10\frac{a}{b}} \quad (4)$$

This multiobjective design problem can then be formally defined in the following two terms: maximize the inductance $L(a,b,c)$ and minimize the volume $V(a,b,c)$ for the given length $k_1=10$ m and $k_2=10^{-6}$ m^2 of the current carrying wire. In order to simplify the analysis, two variables, a and b, are considered. Correspondingly, the computation of L and V are simplified, respectively, to

$$F_1=\frac{31.49\frac{k_1^2}{4\pi^2 b}}{9+6\frac{a}{b}+5\frac{k_1k_2}{\pi ab^2}} \quad (5)$$
$$F_2=\frac{\pi a^2 b}{4}+\frac{k_1^2k_2^2}{4\pi a^2 b}+\frac{k_1k_2}{2}. \quad (6)$$

Now, the problem reads: maximize $F_1(a,b)$ and minimize $F_2(a,b)$ subject to

$$a>\sqrt{\frac{k_1k_2}{4\pi b}}. \quad (7)$$

It should be noted that to use the proposed fitness assignment strategy, one should transfer the maximization problem to a minimization one by defining $F_3=-F_1$. The searched Pareto front using the proposed algorithm is illustrated in Fig. 5. Clearly, the proposed algorithm produces a uniform sampling of the Pareto front for this case study.

C. Application

The feasibility of the proposed algorithm for solving engineering multiobjective design problems is finally tested on a two

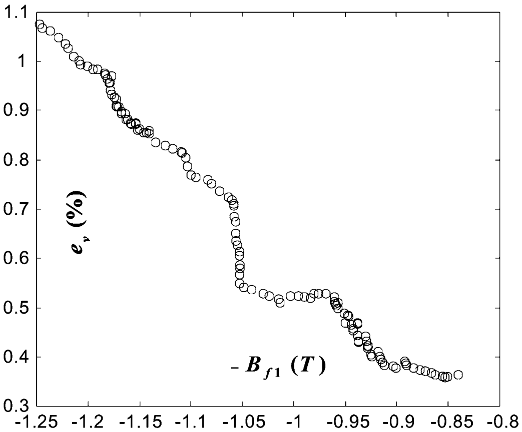

Fig. 6. Computed Pareto front by Emigration Pareto Evolutionary Algorithm (EPEA) for the 400-MW, 44-pole hydrogenerator.

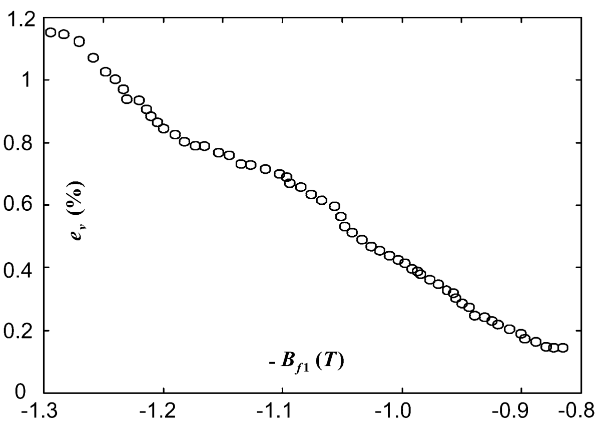

Fig. 7. Calculated Pareto solutions by EPEA for a 200-MW, 22-pole hydrogenerator.

objective, five decision variable multiobjective design problem which is a geometrical optimization problem of the multisectional pole arcs in large hydrogenerators [4], that is

$$\begin{aligned} \max \quad & B_{f1}(X) \\ \min \quad & e_v \end{aligned} \tag{8}$$

$$\begin{aligned} \text{subject to} \quad & \mathrm{SCR} - \mathrm{SCR}_0 \geq 0 \\ & X'_d - X'_{d0} \leq 0 \\ & \mathrm{THF} - \mathrm{THF}_0 \leq 0 \end{aligned} \tag{9}$$

where, B_{f_i} is the amplitude of the ith component of the flux density in the air gap, e_v is the distortion factor of a sinusoidal voltage of the machine on no-load, THF is the telephone harmonic factor, and SCR is the short circuit ratio.

The corresponding geometrical parameters to be optimized are the center positions and radii of the multisectional arcs of the pole shoes. The computed Pareto solutions for a 400-MW, 44-pole hydrogenerator and for a 200-MW, 22-pole hydrogenerator are shown, respectively, in Figs. 6 and 7. Comparing the results of the proposed algorithm with those of a tabu-based one as reported in [9], it can be seen that the Pareto solutions as computed by the two different methods for the 400-MW, 44-pole hydrogenerator are nearly the same. Consequently, the feasibility of the proposed method for finding the Pareto solutions of multiobjective functions is confirmed further by the numerical results of this example.

In summary, from these three numerical examples one can see that the Pareto solutions obtained by the proposed algorithm are more uniform and smoother when compared with those of the traditional ones, which suggests that the proposed algorithm is more promising in solving engineering multiobjective optimal design problems.

IV. Conclusion

The classical GAs have gone a long way to preserve diversity, but the sampled Pareto front obtained by these algorithms is not as smooth as one expects. To address this problem, a multiobjective GA called emigration Pareto evolutionary algorithm, in which a newly designed operator named emigration is included alongside with the classical GAs operators, is proposed in this paper. Moreover, some other improvements to enhance the robustness of the proposed algorithm such as a special fitness assignment strategy to preserve the diversity of the solution, a modified local search approach to improve the local search ability, are also introduced and integrated into the algorithm. The efficiency and advantages of the proposed algorithm have been demonstrated by solving a mathematical test problem and to comparing its performances thus obtained with those of a classical GA. The numerical results of the proposed algorithm on two engineering multiobjective problems are also reported. The computed results are encouraging and suggest that the proposed EPEA could be used to solve more complex practical engineering multiobjective optimization problems.

References

[1] Z. Eckart and T. Lothar, "Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach," *IEEE Trans. Evol. Comput.*, vol. 3, pp. 257–271, Aug. 1999.
[2] H. Ishibuchi and T. Murata, "A multi-objective local search algorithm and its application to flowshop scheduling," *IEEE Trans. Syst., Man, Cybern. C*, vol. 28, pp. 392–403, Aug. 1998.
[3] P. Di Barba, M. Farina, and A. Savini, "An improved technique for enhancing diversity in Pareto evolutionary optimization of electromagnetic devices," *COMPEL*, vol. 20, pp. 482–496, 2001.
[4] R. Y. Tang, S. Yang, Y. Li, and G. Wen, "Combined strategy of improved simulated annealing and genetic algorithm for inverse problems," *IEEE Trans. Magn.*, vol. 32, pp. 1326–1329, May 1996.
[5] Z. Ecakart, T. Lother, and D. Kalyanmoy, "Comparision of multiobjective evolutionary algorithms: Empirical results," *Evol. Comput.*, vol. 8, pp. 173–195, 2000.
[6] A. Carlos and C. Coello, "An updated survey of GA-based multiobjective optimization techniques," *ACM Comput. Surveys*, vol. 3, no. 2, pp. 204–217, 2000.
[7] D. Kalyanmoy, "Solving goal programming problems using multi-objective genetic algorithms," in *Proc. Evolutionary Computation Conf.*, vol. 1, 1999, pp. 77–84.
[8] S. A. Kazarlis, S. E. Papadakis, J. B. Theocharis, and V. Petridis, "Microgenetic algorithms as generalized hill-climbing operators for GA optimization," *IEEE Trans. Evol. Comput.*, vol. 5, pp. 204–217, June 2001.
[9] S. Y. Yang, J. R. Cardoso, S. L. Ho, P. H. Ni, and J. M. Machado, "An improved tabu based vector optimal algorithm for design optimizations of electromagnetic devices," in *Conf. Rec. COMPUMAG'2003*, pp. 180–181.